Sacrifice Ratios with Long-Lived Effects

April 20, 2001

Lawrence Huiyan Zhang
Department of Economics
The Johns Hopkins University

Abstract

This paper contains a theoretical and empirical study of sacrifice ratios with long-lived effects, including possible strong persistence effects, or even hysteresis effects. The empirical analysis is based on G-7 quarterly output data as well as unemployment data from 1960 to 1999. In this paper, I develop some new methods to measure sacrifice ratios with long-lived effects. I reach four conclusions: **First,** sacrifice ratios with long-lived effects are larger than sacrifice ratios that do not account for long-lived effects. **Second,** from a theoretical model and simulation, the "standard method" of measuring sacrifice ratios by Ball (1994) has a larger downward bias for countries with larger long-lived effects. **Third,** both random and fixed effect models show that there is a negative relationship between sacrifice ratios and initial inflations, which can provide one explanation of the large magnitude of sacrifice ratios with long-lived effects in the 1990s, compared with other periods. **Fourth,** there is no significant negative relationship between sacrifice ratios with long-lived effects and nominal wage rigidities.

Key Words: Sacrifice Ratio, Long-Lived Effect, Persistence, Hysteresis, and Disinflation Monetary Policy

JEL: E52, E58

I am grateful for comments from Laurence Ball, Christopher Carroll, Carl Christ, James Cover, Fred Gale, Michael Hoke, Louis Maccini, Adrian Pagan, and seminar participants at the Southern Economic Association 2000 Annual Meeting and 2001 Midwest Macroeconomics Conference. Specially, I would like to thank Laurence Ball for his encouragement and innumerable discussions during the preparation of this paper. The author also thanks the John D. and Catherine T. MacArthur Foundation and Social Science Research Council, Program in Applied Economics for financial support. All errors are mine. Contact information: Department of Economics, The Johns Hopkins University, Baltimore, MD 21218; E-mail: hzhang@jhu.edu; Phone: 703-566-8670; Fax: 410-516-7600.

I. Introduction

The sacrifice ratio is the cost of reducing inflation, the loss of output or unemployment that must be sustained by the economy in order to achieve a reduction in inflation. It is defined as a ratio of the percentage loss of real output or unemployment to the change of trend inflation. Economists have paid a lot of attention to estimating sacrifice ratios since this ratio plays a key role in setting monetary policy.

Sacrifice ratios are commonly estimated from the Phillips Curve relationship between output and inflation in a long time series, as in Okun (1978) and Gordon and King (1982). In recent years, many authors have used episode-specific methods that concentrate on individual disinflation episodes. The most delicate issue for this method is the measurement of potential output. Ball's (1994) approach assumes that output is at its potential level at the start of a disinflation episode and returns to its potential level four quarters after the end of an episode. He assumes that trend output grows log-linearly following the initial deviation until it returns to its potential trend value. Many economists have followed Ball's "Standard Method," including Jordan (1997), Partow and Yuravlivker (1998), Junguito (1998), Bernanke, Laubach, Mishkin, and Posen (1999), and Boschen and Weise (1999).

In this paper, I investigate the robustness of sacrifice ratio estimates by questioning assumptions about the path taken by potential output. In particular, theoretical and empirical evidence suggest that persistence effects are stronger than assumed by Ball (1994). Thus, the assumption that output returns to its potential level four quarters after the end of an episode may understate the cost of disinflation episode.

A number of authors have offered theoretical and empirical evidence suggesting the existence of long-lived effects. Blinder (1987) finds a long lag for the Volcker disinflation in the U.S. during the 1980s. Romer and Romer (1989 and 1994) also found that monetary shocks have a high persistence effect. Even Ball (1997) himself realized that there should be a stronger persistence effect than is assumed in the standard method. Moreover, evidence for long-lived effects has also been found in recent VAR analysis; Christiano, Eichenbaum, and Evans (1996) assess the impact of a monetary shock on the U.S. economy and find contractionary monetary policy shocks are associated with persistence declines in real GDP and employment. Dolado and Lopez Salido (1996), using a structural VAR model to examine the output and unemployment data of Spain, also find that disinflation policy in an economy suffering from high persistence can become very costly.

Some economists go even further and assume a hysteresis effect, which can be regarded as the limiting case of a strong persistence effect. A hysteresis effect occurs when a disinflationary monetary policy affects potential output. This idea was pioneered by Blanchard and Summers (1986), who use the term "hysteresis" to describe the permanent effects of disinflation. Romer (1989) and Romer and Romer (1989 and 1994) argue that demand shifts may reduce output permanently. That is, contractionary monetary policy can reduce trend output as well as cause temporary deviations from the trend. It is likely that a larger recession leads to a larger permanent loss. Ball (1997 and 1999) also finds that disinflationary monetary policy has a substantial effect on the NAIRU and that the hysteresis effect can explain the increase in the NAIRU in some European countries.

In the presence of persistence effects, therefore, sacrifice ratios that do not take into account the long-lived effects will understate the true cost of disinflationary monetary policy. In this paper, I develop a new method to measure the sacrifice ratio that takes into account possible strong persistence, or even hysteresis effects. I modify Ball's (1994) method, relaxing the assumption that output returns to its potential level four quarters after the trough. Instead, I use the Hodrick Prescott Filter to project potential output. Using this method, I calculate sacrifice ratios from unemployment and output data for G-7 countries. There are four main conclusions in this paper.

1. *Larger sacrifice ratios*

As we expected, sacrifice ratios with long-lived effects are larger than sacrifice ratios that do not account for long-lived effects. The average sacrifice ratio for all G-7 countries is about 2.5 by my new method, compared to 1.4 by the standard method.

2. *Relative ranking of G-7 sacrifice ratios*

Allowing for long-lived effects also permits more accurate comparison of sacrifice ratios across countries with varying degrees of persistence. While previous estimates have suggested a much larger sacrifice ratio for the U.S. compared with European countries, persistence effects are stronger in European countries than in the U.S. (Ball 1999). I find that the U.S. sacrifice ratio is comparable to ratios of average European countries when persistence effects are taken into account by my new method.

3. *Initial inflation and sacrifice ratio*

For a long-time, economists have puzzled over sacrifice ratios and initial inflation. Theoretically, a higher initial inflation is associated with a lower sacrifice ratio through many channels; one possible channel is that a higher inflation rate may reduce nominal rigidities because it encourages more frequent contract renegotiations. However, there is no conclusive empirical evidence for this relationship. In this paper, using the sacrifice ratios estimated by my new method, I find strong empirical evidence to support the negative relationship from both random and fixed effect models. This can explain the fact that sacrifice ratios with long-lived effects in the 1990s are larger than in other episodes.

4. *Nominal wage rigidities and sacrifice ratio*

Previous research has shown that one of the reasons why sacrifice ratios for the U.S. are much higher than for G-7 European countries is that nominal wage rigidity in the U.S. is much higher than in other countries. In this paper, the adjusted sacrifice ratio for the U.S. is in the middle among G-7 countries. Thus, there is no significant negative relationship between the sacrifice ratio and nominal wage rigidity.

Previously, most empirical studies have concentrated on output data. Given the difficulties and problems with output data, I also make some robust analyses from unemployment data. Nevertheless, most of my results from unemployment data are consistent with my results from output data.

The rest of this paper contains five sections. Section 2 examines a simple model to show that standard method produces a larger downward bias for countries with a higher degree of long-lived effects. The results from the standard method may be misleading. Section 3 calculates the sacrifice ratios from output data using the new method. Section 4 calculates the sacrifice ratios from unemployment data. Section 5 examines what determines sacrifice ratios. Section 6 is the conclusion.

II. A Simple Model

In this section, I use a simple model and some numerical examples to show that the standard method for calculating sacrifice ratios by Ball (1994) generates a downward bias and the downward bias is greater with stronger persistence effect. First, I will examine a basic model with only persistence effects and then I will extend my conclusions to the case that allows for both persistence and hysteresis effects.

1. Basic Model

The main equation is the following Lucas supply curve with backward expectation.

$$y_t - y_t^* = a(p_t - p_{t-1}) + b(y_{t-1} - y_{t-1}^*) + e_t,\ a \succ 0,\ 0 \le b \prec 1 \tag{1}$$

In equation (1), y_t^* is potential output at period t. y_t is actual real output,[1] e_t is noise, and b represents the degree of persistence. The larger b is, the stronger the persistence effect and the longer the time required for actual output to return to its potential level following a disinflation episode. If b is equal to zero, there is no persistence effect. As b approaches 1, the degree of persistence increases. To insure an equilibrium solution, I assume that $0 \le b \prec 1$.

The term $y_t - y_t^*$ is the deviation from potential output resulting from the disinflation policy at period t. If there is no disinflation ($p_t - p_{t-1} = 0$), output will stay at its potential level if the last period's output was at its potential level; when disinflation occurs ($p_t - p_{t-1} \prec 0$), there is a short-term output loss; however, output returns to its potential level in the long run.

2. What is Wrong with the Standard Method?

2.1. Standard method to calculate sacrifice ratio

[1] In this paper, all outputs (e.g., trend and actual output) refer to the natural log of real output.

The sacrifice ratio is defined as a ratio of the percentage loss of real output or unemployment to the change of trend inflation. There are many methods to calculate the sacrifice ratio. The standard or most commonly used method (Ball 1994) defines potential output for a disinflation episode based on the following three assumptions:

- Output is at its natural level at the start of a disinflation episode or the inflation peak.
- Output returns to its potential level four quarters after the end of an episode, that is, four quarters after an inflation trough.
- Potential output grows log-linearly between the two points when actual and potential outputs are equal.

Using these three assumptions, Ball (1994) pins down the potential output at the starting and ending point for an episode then calculates the output loss through the episode. The sacrifice ratio (SR) for a disinflation episode (starts at period S and ends at period E) is the ratio of output loss to the change in trend inflation, $SR = \left[\sum_{t=S}^{E+4}\left(y_t - y_t^*\right)\right] / \left(\pi_t - \pi_{t-1}\right)$. The assumption that output returns to its potential level after four quarters ($b^4 \approx 0$) may underestimate the sacrifice ratio if there are persistence effects.

2.2. First problem: underestimating the true sacrifice ratio

Figure 1 illustrates how the standard method may understate the output loss. In Figure 1, a disinflation episode starts at period S and ends period E. The standard method assumes that output is at its potential level at period S (point O) and returns to its potential level at period E+4, four quarters after the trough (point F). Potential output is assumed to grow log-linearly between points O and F, thus it is a straight line. Output loss by the standard method is area A.

However, if monetary policy has strong persistence effects, output may not return to its potential until period E+4+T. In this case, the true short-term loss is the larger area A+B. Area B is the extra output loss due to persistent effects. The standard method has downward bias if there are strong persistence effects. Moreover, this downward bias is not equal for all disinflation episodes. The bias in the standard method varies with the strength of the persistence effect, the stronger the persistence effect, the greater the downward bias.

The cost of reducing inflation includes a short-term and a long-term loss. The short-term loss is defined as the output loss from the inflation peak to four quarters after the inflation trough. The long-term loss includes output loss after four quarters beyond the trough, the area A+B+C in Figure 1. Since there is more disagreement on potential output in the long run,[2] for a short cut, most economists only think about short-term output loss; see Ball (1994), Jordan (1997), Bernanke, Laubach, Mishkin, and Posen (1999), and Boschen and Weise (1999). In my empirical studies, I follow previous studies and concentrate on the short-term loss.

[2] There are two kinds of problems in the long run: one is that there is more uncertainty about potential output. When will the potential output return to potential level? The other is that in the short-run, we can argue monetary contractions are the essential reason for a recession, but in the long run, it is difficult to say this.

2.3. Second problem: misleading ranking of cross-country sacrifice ratios

Since downward bias is not equal for all disinflation episodes, problems arise when comparison of sacrifice ratios are made for countries with different persistence effects. Figure 2 shows the possible mistake caused by the standard method. If there is stronger persistence effect in country B than in country A, by the standard method, country A ends up with a larger sacrifice ratio (area OMN) than country B (area OMF). However, after taking into account stronger persistence effects in country B, the true sacrifice ratio in country A (area OMNG) is smaller than that for country B (area OMFG).[3]

Why does the standard method produce misleading ranking of cross-country sacrifice ratios? The answer is straightforward; strong persistence causes actual output to stay low. The standard method's assumption that actual output returns to its potential level four quarters after the trough results in a lower potential output than it should be and thus a downward bias is generated. The true sacrifice ratio could be the reversal.

Studies have shown that persistence effects are stronger in G-7 European countries than in the U.S. as in Ball (1999). Allowing for this difference will result in a better cross-country comparison of sacrifice ratios. Thus, it is possible that the sacrifice ratio of the U.S. calculated by the standard method is larger than those of most G-7 European countries. But, in fact, the true sacrifice ratio for the U.S. is smaller than those of G-7 European counties.

2.4. Numerical examples

Some numerical examples show the problems of the standard method. There are many possible assumptions for a disinflation path. But it will be complicated and inefficient to include many periods. To simplify the problem while still capturing the long-lived effect (related to parameters a and b in equation 1), I make the following three assumptions:

1. Potential levels of trend output are zero at period 0 and period 1.
2. Initial levels of inflation are two percent at period –1 and period 0.
3. Inflation declines to one percent in period 1 and zero percent in period 2.

These three assumptions capture the essential features of the problem by using the simplest assumptions about the disinflation path. Based on these three assumptions, Table 1A shows the true output loss and the sacrifice ratio calculated by the standard method. In Table 1A, true potential output is always equal to 0, y_t^* is the potential output assumed by the standard method at period t, and y_t is actual output.

From equation (1), I calculate the true output loss for each period in the disinflation episode. For example, there is no disinflation at periods –1 and 0, actual output is zero and the true output loss is zero for these two periods. For period 1, there is a 1% decline in inflation, from 2% to 1%.

[3] I proved the following proposition mathematically (not shown in the paper). There exists a range of parameter b that makes the following reversal true: $SR_A^S \succ SR_B^S$ and $SR_A^T \prec SR_B^T$, SR_A^S and SR_B^S are sacrifice ratios for country A and country B calculated by the standard method; SR_A^T and SR_B^T are true sacrifice ratios for country A and country B.

Since potential output at period 0 and 1 remains at zero and actual output at period 1 is –a, the output loss at period 1 is -a.

Similarly, I calculate true output loss for period 2 and other periods.[4] The sum of true output loss from the inflation peak (period 0) to four quarters after the trough (period 2) is the total true output loss, L^T, shown in Table 1A:

$$L^T = -a\left(2 + 2b + 2b^2 + 2b^3 + b^4\right) \tag{2}$$

The larger b is, the stronger the persistence effect, and the larger the true output loss and sacrifice ratio.[5] Appendix A shows the calculation of the output loss by the standard method, L^S:

$$L^S = -a\left[\left(2 + 2b + 2b^2 + 2b^3 + b^4\right) - \frac{5}{2}(1+b)b^4\right] \tag{3}$$

Comparing equations (2) and (3), the difference between true output loss and output loss calculated by the standard method is $\frac{5}{2}a(1+b)b^4$. As Ball (1994) points out that under assumption 1 ($b^4 \approx 0$),[6] the standard method is a good approximation for the true output loss, but b may be large enough to make the difference important.

Table 1C shows some numerical results for different degrees of persistence. Columns four through six in Table 1C are three cases in which b equals 0.5, 0.85, and 0.95 respectively. A larger value of b implies a stronger persistence effect. Case one is the moderate persistence case (b=0.5) while case two and three are the cases of larger persistence.[7] Comparing these three cases, the standard method underestimates true sacrifice ratios. A stronger persistence effect implies a larger downward bias in the calculation by the standard method. In addition, when case two is compared to cases three, the true sacrifice ratio for case three is larger than that for case two. The standard method, however, produces the opposite result: case two has a higher ratio than case three.

3. New Method

The most delicate issue for calculating the sacrifice ratio is the measurement of trend output since small differences in fitted trends can make large differences for sacrifice ratios. The new

[4] For period 2, there is 1% decline of inflation, from 1% to 0. Similarly, the trend output for period 1 and 2 is still zero. The output loss at period 2 includes two parts; one is from 1% decline of inflation in period 2, thus, the decline of output is –a; the other is due to the persistence effect of the output loss from period 1 (-a); it continues to cause loss, but by a smaller amount (-ab). Total output loss in the second period is –a(1+b).

[5] The sacrifice ratio is total percentage true output loss divided by total decline of inflation (2%), so in this case, the sacrifice ratio is $\frac{1}{2}a + \frac{1}{2}a(1+b)\sum_{i=0}^{4} b^i$. Note that this sacrifice ratio only captures the short-term loss, up to four quarters after the trough. If we are interested in the long-term loss, in this case, when t approaches infinity, the output loss is $\frac{2a}{1-b}$.

[6] Basically, assumption 1 shows that there is no strong persistent effect.

[7] There is no hysteresis effect, thus c=0.

method takes into account the possible persistence effect and does not make any assumption about the degree of persistence; output could return to its potential at any time after the trough.[8]

3.1. How to predict potential output

Following Ball (1994), I assume output is at its natural level at the start of a disinflation episode and make a reasonable prediction about potential output based only on its past growth path. I use the Hodrick-Prescott filter (HP filter) to predict potential output. First, I calculate the HP filter of log real GDP. Second, I calculate the growth rate of the HP filter. Third, I assume that potential output will grow at the rate estimated by the HP filter at the beginning of the episode and use this growth rate to forecast the potential output for calculation of the sacrifice ratio.[9] Since the HP filter is affected by recessions, I used two alternative smoothing parameters of 1600 and 16000.[10]

3.2. Numerical simulation

Since it is difficult to estimate the potential output from the actual data, I use numerical examples to compare the sacrifice ratios calculated by the new method with those by the standard method. In Table 1C, I maintain the same assumptions about the disinflation path I assumed before, and calculate sacrifice ratios with the new method. As a result, sacrifice ratios calculated by the new method are much closer to true sacrifice ratios than those given by the standard method.

Figure 3 shows the simulation results for case two, the third column at Table 1C. I assume that the disinflation starts at period 0 (point O) and continues until period 2 (point A). Actual output stays at 0 until period 0, after which disinflation causes it to fall below potential for the following seven periods. Finally, actual output returns to its trend at period 7 (point C). Thus, the serrated-shape line at Figure 3 is the actual output path and the dot line is its HP filter with smoothing parameter 1600. Since the standard method assumes that output returns to its trend at four quarters after the trough, point B is the trend output assumed by the standard method at period 6.

Line OC shows the true trend output without disinflation (stays at 0); line OB shows the trend output predicted by the standard method while line OD shows the trend output predicted by the new method. It is easy to see that the output loss calculated by the standard method is the area OAB while the output loss by the new method is the area OABD, which is much closer to the true output loss, the area OABC.

[8] The difference between these two methods can also be seen from their implications about short-term loss and long-term loss. The standard method assumes actual output returns to its potential level by four quarters after the trough. Thus, short-term loss and long-term loss are the same; there is no further output loss beyond four quarters after the trough. The new method assumes there is a possible strong persistence effect, thus, short-term loss and long-term loss are not the same. In this paper, I concentrate on the short-term loss or the sacrifice ratio.

[9] For example, if a disinflation episode starts at period S and ends at E and the growth rate of the HP filter trend output at the start of the episode is g_S, then I would assume the growth rate of the potential output from period S to E+4 would be constant and equal to g_S.

[10] The most commonly used smoothing parameter for quarterly data is 1600. An HP filter with parameter 16000 is smoother than with parameter 1600. Some economists criticize using the HP filter method to forecast trend output; so, this is why I use two different smoothing parameters. I will discuss the disadvantages of the HP filter method later on.

The downward bias problem with the standard method can also be seen from the numerical examples. In the third column of Table 1C, the output loss calculated by the new method is 7.769, which is close to the true output loss, 7.861. By the standard method, output loss is only 4.481. Therefore, the new method is a better one; it measures the true output loss more accurately than the standard method. The standard method is a good approximation only if there is no persistence effect.

3.3. Potential bias and robustness

As can be seen from the above numerical examples, there is a potential downward bias for the new method since the HP filter is also affected by recessions. The true potential output should be zero all the time. However, empirically, the new method uses 0, -0.44 percent, -0.88 percent, -1.32 percent, -1.76 percent, -2.20 percent, and –2.63 percent, respectively for periods 0 to 6. Thus, the new method generates some negative bias.[11] However, the bias is much less than that of the standard method. While these results suggest that the smoothing parameter 16000 may be more appropriate than a parameter of 1600, I will maintain both parameters for my empirical studies in the next section.[12]

There are many other ways to predict potential output; one way is to use univariate time series models to predict the trend, such as the quadratic and ARIMA model. I tried these methods and obtained quite similar results. Therefore, the downward bias problems caused by the standard method do not depend on these specific ways to predict potential output. In fact, by using some of the most common methods to predict potential output, such as the HP filter, quadratic, and ARIMA model, this downward bias problem always exists for the standard method. Thus, in this following numerical as well as empirical analysis, I keep Ball's (1994) assumption that output is at its potential at the beginning of the episode and make a reasonable projection about potential output by HP filter.

4. Hysteresis Effect

One extension of the above idea is to include a possible hysteresis effect, which can be regarded as an extreme case of a strong persistence effect. While costs of disinflation are usually regarded

[11] This kind of potential bias can be limited by using an only "one-sided" HP filter. That is to say, for each episode, an HP filter is used only up to the beginning point of the episode, not though the whole sample. Thus, the HP filter will not be affected by the following recession.

[12] Two other problems are related to the new method: one is the potential upward bias. Friedman (1994) argues that output is above its trend at the beginning of an episode, thus, the sacrifice ratio may be overestimated. This is a common problem for the standard method. In this paper, I only relax one assumption and keep all other assumptions in the standard method, leaving those for future research. The other problem is that the new method is more sensitive to noise in the data than the standard method. If an incorrect beginning point is chosen due to noise in the data, trend output calculated by the new method will make a parallel shift up or down, but trend output by the standard method will only make a partial shift. In fact, the data is noisy. For example, the sample range of the U.S. disinflation episode in the 1990s is different for different authors since they use different data sets: the episode in Boschen and Weise (1999) is 89:4-93:4; in Bernanke, Laubach, Mishkin, and Posen (1999), the episode is 90:1-94:4. Italy's big negative sacrifice ratio in the 1970s is a result of this kind of problem. The noise problem can be partially solved by averaging the sacrifice ratios. I calculate the sacrifice ratios by trying different episodes, beginning from one quarter earlier to four quarters earlier than the start point I should use. I then take the average of these four sacrifice ratios. For example, for Italy's negative episode in the 1970s, by taking the average of the sacrifice ratios, I get a reasonable positive sacrifice ratio.

as being transitory, another school of thought (Blanchard and Summers, 1986) argues that disinflation can result in "hysteresis," a permanent reduction in unemployment and output.

Several other studies also support the possibility of hysteresis effects, such as Romer (1989) and Romer and Romer (1989 and 1994). Ball (1997 and 1999) also argues that disinflation monetary policy has a substantial effect on the NAIRU, and the hysteresis effect can explain the increase in the NAIRU in some European countries. So disinflation will decrease the short-term unemployment rate as well as the NAIRU.

4.1. Model of hysteresis

In this section, I still assume the Lucas supply curve, the equation (1), as before and add the hysteresis effect. Rather than assuming that y_t^* is exogenous, I make it endogenous by assuming equation (4).

$$y_t - y_t^* = a(\pi_t - \pi_{t-1}) + b(y_{t-1} - y_{t-1}^*) + \varepsilon_t,\ a \succ 0,\ 0 \le b \prec 1 \tag{1}$$

$$y_t^* = \alpha + c y_{t-1}^* + (1-c) y_{t-1},\ 0 \le c \le 1 \tag{4}$$

y_t^* depends on last period potential output as well as last period actual output. I assume that potential output is a weighted average of last period's potential and actual output. Under this formulation, an output loss in one period affects potential output in the next period. The change in potential output is proportionate to the deviation in output of earlier periods.

α is the constant growth rate for potential output. If α is assumed to be zero, then the potential output will remain constant. The term $(1-c)$ represents the degree of hysteresis. If c=1, there is no hysteresis effect. If $0 \le c \le 1$, there is a hysteresis effect and the sacrifice ratio increases with the degree of hysteresis. If c=0, potential output is only affected by the previous period's actual output. In this case, the sacrifice ratio approaches its maximum.

Equation (4) is not a new idea. To analyze the hysteresis in unemployment, some economists also use the channel through which current employment is affected by past employment, such as Blanchard (1997). With the hysteresis effect, the economy is described by both equation (1) and equation (4) while equation (4) is the hysteresis effect of a disinflation monetary policy.

4.2. Comparing the standard method and the new method

In the presence of hysteresis effect, the standard method generates an even poorer approximation of the true sacrifice ratio. However, the new method can still provide a better estimation since it does not make any assumption about the degree of persistence or hysteresis.

Second and, maybe more importantly, the hysteresis effect magnifies the following problem caused by the standard method: the incorrect ranking of sacrifice ratios across G-7 countries. I showed earlier how reversals in cross-country rankings could occur with only persistence effect.

Such reversal could happen much more easily when you include both the hysteresis and persistence effects.[13]

I maintain the same three assumptions about the disinflation path and show more numerical examples to include the hysteresis effect. Similar to Table 1A, Table 1B shows the true output loss under both persistence and hysteresis effect. Table 1C shows three more examples, which are the fifth column to the seventh column (cases four through six). For example, case four is the case with small hysteresis (c=0.9) and moderate persistence (b=0.5) effect while case five and six are the cases with larger hysteresis effect (c=0.75 and c=0.6). For all these three cases, the sacrifice ratios calculated by the new method are much closer to true sacrifice ratios. Thus, the new method is a better approximation than the standard method.

Appendix B shows the calculation of the true output loss with long-lived (persistence and hysteresis) effects, L^T :

$$L^T = L^S - 1.5a(1+b)b^4 - 7(1-c)\left(1+b^2+b^3+\frac{1}{2}b^4\right) \tag{5}$$

As can be seen from equation (5), L^T includes two components: one is the sacrifice ratio without the long-lived effects (calculated by standard method), L^S ; the other is the following extra long-lived effect.[14]

$$7(1-c)a\left(1+b+b^2+b^3+\frac{1}{2}b^4\right) - 1.5a(1+b)b^4 \tag{6}$$

With assumption 2 ($1-c \approx 0$),[15] the first term in equation (6) is zero. In addition, under assumption 1 ($b^4 \approx 0$), the second term in equation (6) is zero too. Therefore, output loss according to the standard method is a good approximation of the true output loss only in the case where there is no hysteresis ($c \approx 1$) and no strong persistence effect ($b^4 \approx 0$).

Additionally, as can be seen from equation (5), the true output loss depends on the degree of hysteresis. If there is no hysteresis effect ($c \approx 1$), equation (5) reduces to (2). The sacrifice ratio becomes larger with the degree of hysteresis and persistence effect. The sacrifice ratio also increases with the length of disinflation episode.

III. Sacrifice Ratios from Output Data

[13] Concerning the standard method, Ball (1994) also argues that it is likely that long-lived effects of output loss in different disinflation episodes may move together, thus, we need not worry about long-lived effects. But, in fact, the degree of persistence or hysteresis seems to differ across countries. Ball (1999) argues that the degree of hysteresis seems higher in European countries than in the U.S. Roberts and Morin (1999) also argue that hysteresis evidence from U.S. unemployment data is weak. Thus, the hysteresis effect affects G-7 countries unequally.

[14] The new method itself does not require assumption 1 or assumption 2. These two assumptions are used to show that the true sacrifice ratio can be divided into two components: the part without long-lived effect and the part with long-lived effect.

[15] Assumption 2 requires that c is big enough to ensure that there is almost no hysteresis effect.

In this section, I will calculate sacrifice ratios from output data with the new method as well as with the standard method based on the results from section 2. Then I will compare these two methods to see if the standard method and the new method generate different results and conclusions.

1. Data and Choosing an Episode

In this paper, I estimate sacrifice ratios for G-7 countries. Other countries are excluded due to a lack of reliable quarterly output data. The output and inflation data range from the first quarter of 1960 to the fourth quarter of 1999. Most of the quarterly output and inflation data are from *International Financial Statistics (IFS)*, June 1999.[16]

I maintain Ball's (1994) assumption about selecting episodes. Specifically, the first step is to identify a disinflation episode in which trend inflation falls substantially. Trend inflation is defined in quarter t as the nine-quarter moving average of actual inflation from $t-4$ to $t+4$. Trend inflation reflects a smooth version of actual inflation. A disinflation episode is defined as the following: 1). An inflation peak is a quarter in which trend inflation is higher than it was in both the previous four quarters and the following four quarters; 2). An inflation trough is a quarter in which trend inflation is lower than it was in both the previous four quarters and the following four quarters; 3). A disinflation episode is any period that starts at an inflation peak and ends at a trough with an annual rate at least two points lower than the peak. Thus, an individual disinflation episode must show an inflation difference of at least two points between the beginning and ending points. This is to ensure that an episode does not end with a small increase in inflation.[17]

I take the new method and the standard method I used before for numerical examples and apply them to some empirical analysis. Table 2 shows several disinflation episodes for each country. For example, there are four disinflation episodes in the U.S., the disinflation starts in 1969, 1974, 1980, and 1989, respectively. My disinflation episodes[18] are consistent with the lists of monetary contractions developed by Romer and Romer (1989 and 1994), Ball (1994), and Boschen and Weise (1999). In Table 2, sacrifice ratios from HP filters with smoothing parameters 16000 and 1600 are labeled New SR1 and New SR2, respectively. Sacrifice ratios calculated by the standard method are labeled Ball SR.

2. Results:

From Table 2,[19] the following three conclusions can be drawn:

---

[16] See Data Appendix for more details.

[17] There is one exception in the sample: the France disinflation episode, 89:2-93:3, is included in the sample because I try to compare sacrifice ratios in the 1990s across G-7 countries, and the decline in inflation is 1.72, not too far from 2. Boschen and Weise (1999) use the November 1995 IFS CD-ROM and get a 1.9 decline in trend inflation for the episode: France 90:1-93:4; they also include this episode in their samples.

[18] I checked historical records for each of the disinflation episodes in my quarterly data set. In every episode, there is a significant tightening of monetary policy near the end of the beginning of disinflation. International demand contractions are essentially the only sources of two-point decline in trend inflation.

[19] There are two unusually large negative sacrifice ratios in Table 2. The reasons are different. I have discussed the one in Italy above. For the one in the U.K. in the late 1980s, there were big supply shocks, so the productivity grew

- *Conclusion 1: Sacrifice ratios with long-lived effects are larger than those without long-lived effects*

From Table 2, it is clear that empirical results from output data support the long-lived effect. For most episodes (21 out of 30), sacrifice ratios calculated by the new methods are larger than those calculated by the standard method. For all G-7 countries, the sample average of New SR1, New SR2, and Ball SR are 2.67, 2.40, and 1.40 respectively. For the U.S., the sample average of New SR1, New SR2, and Ball SR are 2.13, 3.62, and 2.64 respectively.

Figure 4A and 4B graphically show the New SR1 and Ball SR for disinflation episodes in the 1990s for the U.S. and Italy. The disinflationary monetary policy affects both trend output and actual output. For example, in Figure 4A, area A is the output loss calculated by the standard method for the 1990s in the U.S.; area B is the extra output loss due to the long-lived effects. A+B is the total output loss calculated by the new method. So, with long-lived effects, sacrifice ratios increase. Figure 4B shows similar results for Italy's disinflation episode in the 1990s.

This result questions the standard method because of its downward bias. The standard method uses actual data to calculate trend output. Basically, it uses a lower trend output than it should be. The new method uses a reasonable trend output, which is not affected by a disinflation monetary policy. For example, the new method uses an average growth rate of trend output for G-7 countries of 2.49 percent for the 1990s, while the standard method uses an average of 1.24 percent. For the U.S., the average growth rate of the trend output in the 1990s used by the new method is 2.46, while it is 1.86 with the standard method. For the U.K., the average growth rate of trend output in the 1990s used by the new method is 2.20, while it is 0.84 with the standard method. Thus, the new method can give us a more reasonable measurement.

- *Conclusion 2: Larger sacrifice ratios in the 1990s*

As can be seen from Table 2, the sacrifice ratios are higher for the 1990s than for other periods; the average sacrifice ratio in G-7 countries during the 1990s, 1980s, 1970s, and 1960s is 5.88, 1.26, 1.84, and 2.03 respectively.[20] For the U.S., the sacrifice ratios for the 1990s, 1980s, 1970s, and 1960s are 7.72, 2.79, 0.50, and 3.46 respectively.

Sacrifice ratio depends on the gap between potential output and actual output; therefore, there are two possible reasons for this result: either the recovery of the economy had slowed down or simply that higher average growth rates for the trend output is assumed by the new method. In

very fast. There are several methods to eliminate these two large negative sacrifice ratios. For example, if I add the following new rule when choosing a episode, then I can combine two short episodes in the U.K. in the 1980s into a longer episode, and the sacrifice ratio for this longer episode is positive. "*If a trough and its next peak are within 4 quarters and also the difference between the trend inflation at the trough and its next peak is less then two points, then regard it as the same episode.*" This new rule is to make sure that the choosing of an episode is not affected by noise in the data. In fact, compared to methods in Ball (1994), with this additional rule, the only change is that the two episodes in the U.K. in the 1980s become one longer episode.

[20] There are two big negative sacrifice ratios in Table 2, one is in the U.K. during the 1980s; the other is in Italy in the 1970s. These big negative numbers affect the average for 1980s and 1970s in some degree.

fact, the true reason was the slow-down of the recovery in the 1990s. As can be seen from the last two columns of Table 2, the new method even uses a higher growth rate of the trend output for the U.S. in the 1980s (annual 2.56 percent) than for the 1990s (2.47 percent). But the recovery rate in the 1990s (1.86 percent) was lower when compared to the 1980s (2.25 percent). For most G-7 countries, the new method assumes that trend output grows lower in 1990s than 1980s,[21] but the actual output was so much lower that I obtain much higher sacrifice ratios in the 1990s for these countries. So, the true reason is the slow recovery in the 1990s. The standard method did not pick up the problem because it calculates the sacrifice ratio from actual data. Thus, the slow-down of recovery will pull down its sacrifice ratios.

There are many ways to explain the larger sacrifice ratio in the 1990s. One possible explanation is the lower initial inflation for disinflation episodes in the 1990s than in the 1980s, which are 5.13 percent and 11.95 percent respectively for the U.S. My results support the idea that a lower initial inflation is related to a larger sacrifice ratio; this idea will be discussed formally later on.

- *Conclusion 3: The sacrifice ratio for the U.S. is in the middle of the sacrifice ratios for G-7 Countries.*

Previous research suggests that the sacrifice ratio for the U.S is much higher than those of G-7 European countries. Bruno and Sachs (1985) calculate a Nominal Wage Rigidities index and conclude that the U.S. is much less flexible than G-7 European countries. Therefore, disinflation monetary policy has a bigger effect in the U.S. than in other countries. Many other studies also support this idea, such as Ball (1994), Bernanke, Laubach, Mishkin, and Posen (1999), and Boschen and Weise (1999).

I also calculate the average sacrifice ratios for each country. The results in Table 4 suggest that the U.S. is not an outlier, but in the middle of the G-7 European countries. With the standard method, as Table 4 also shows, the U.S. has the highest average sacrifice ratio, much higher than most G-7 European countries. However, using the new method, the results are quite different. Germany, the U.S., France, and Canada have similar sacrifice ratios. Since there are two unusual large negative sacrifice ratios in Italy and the U.K., their sacrifice ratios are below the average.

The standard method has a larger downward bias for G-7 European countries than it does for the U.S. because many authors believe that there are weaker long-lived effects (persistence and hysteresis effects) in the U.S. than in G-7 European countries. With the standard method, the sacrifice ratios for the U.S. are much larger than the sacrifice ratios for G-7 European countries. After taking into account long-lived effects, the downward bias can be corrected so that the U.S. is no longer an outlier.

IV. Sacrifice Ratios from Unemployment Data

[21] For five out of the G-7 countries (excluding France and Germany), I use lower growth rates for trend output during the 1990s, but I get much higher sacrifice ratios in the 1990s for these countries. For France and Germany, although I use slightly higher growth rates of trend output for the 1990s than for the 1980s, I get much bigger sacrifice ratios in the 1990s than in the 1980s. So, in general, the true reason was the slower recovery in the 1990s.

Anderson and Waster (1999) argue that sacrifice ratios measured in terms of unemployment are different from those in terms of output data. Friedman (1994) also argues that the U.S. productivity growth improved in the early years of the 1980s, when compared with the 1970s, so the cost of disinflation in terms of output in the 1980s was less than the cost in terms of unemployment.

Therefore, I believe that comparing unemployment data and output data will raise a lot of interesting questions. But, unfortunately, few people measure sacrifice ratios using unemployment data. In this section, I want to see if the previous results still hold for unemployment data.

1. Data and Sacrifice Ratio

The data is quarterly unemployment data from the 1st quarter of 1965 to the 4th quarter of 1999. It is the Seasonally Adjusted Civilian Standardised Rates of Unemployment from OECD *Quarterly Labor Force Statistics*.[22] I make a parallel analysis for unemployment data by making the following two assumptions about NAIRU:

- Unemployment rate is at its natural level (or NAIRU) at the start of a disinflation episode or the inflation peak.
- NAIRU is constant through each disinflation episode.

I regard the constant-NAIRU assumption as a benchmark. The difference between the actual unemployment rate and NAIRU is regarded as unemployment loss due to a disinflation policy. The result is labeled "New SRU" in Table 3,[23] which lists the same disinflation episodes as Table 2, but with sacrifice ratios calculated from the unemployment data.

I also calculate sacrifice ratios from unemployment data using the standard method. Specifically, I make the following three assumptions: 1). Unemployment is NAIRU at the start of a disinflation episode; 2). Unemployment returns to NAIRU at four quarters after the end of a disinflation episode; 3). NAIRU grows log-linearly between the two points when unemployment and NAIRU are equal. The difference between the NAIRU and actual unemployment rate is interpreted as the unemployment loss. Results are labeled "Ball SRU" in Table 3. I also list NAIRU assumed by the new method and the annual growth rate assumed by the standard method.

2. Results and Interpretation

The sacrifice ratios estimated with unemployment data still support my first conclusion: on average, sacrifice ratios with long-lived effects are larger than sacrifice ratios without taking into account long-lived effects. As can be seen from Table 3,[24] sacrifice ratios with long-lived effect

[22] See Data Appendix for more details.
[23] I also try to calculate the HP filter for unemployment data through the sample, 1965Q1 to 1999Q4. Similar to the output case, I calculate the slope of the HP filter and use the slope as the growth rate of NAIRU to forecast the NAIRU. But the results are not promising and difficult to interpret.
[24] These are two negative sacrifice ratios from unemployment data in Table 2. Thus, it may not be a good idea to apply the standard method to the unemployment data.

(New SRU) are larger than sacrifice ratios by the standard method (Ball SRU) for most episodes. The average sacrifice ratio from unemployment data calculated by the new method is 1.61, while it is 0.53 by the standard method. For the U.S., the average sacrifice ratio from unemployment data is 1.90 by the new method, while it is 1.36 by the standard method.

The unemployment data results do not support the second conclusion obtained from output data estimates. In some European countries, the reversal conclusion is reached; the average sacrifice ratio[25] for European countries from unemployment data in 1980s is 2.79, which is higher than the average sacrifice ratio for the 1990s, 2.09. The reason is that there are stronger institutional factors or hysteresis effects in the 1980s in these European countries. In the 1990s, some European countries began to reform and deregulate their labor markers. As can be seen from Tables 2 and 3, two European countries, Germany and U.K., both experienced this kind of reversal. My results are consistent with Anderson and Wascher (1999).

My third conclusion is strongly supported by the unemployment data. The U.S. sacrifice ratio is in the middle of the G-7 ratios, not an outlier. With the standard method, the U.S. has the highest sacrifice ratio among G-7 countries. But by the new method, the sacrifice ratio for the U.S. is lower than those of Germany, France, and Canada, but higher than ratios for the U.K., Italy, and Japan.

3. Okun's Law

Next, with the results from unemployment data, I can check the robustness of sacrifice ratios calculated from output data. These two measurements should be the same over time if such aspects of macroeconomic activities as productivity growth, labor force participation, population growth, and the like were constant. I will make the following four kinds of comparison. The results are also in Table 4.

First, I check the sample average. Okun's Law tells us there is a ratio between the output losses and unemployment losses, which is around 2 for the U.S. The G-7 average sacrifice ratio from unemployment data is 1.61 while the average sacrifice ratio from output data is 2.67. Thus, the average output-unemployment ratio is 1.66, which may be lower than we expected. One explanation is that there may be a stronger long-lived effect in unemployment data than in output data.

Second, I check the average sacrifice ratio and Okun's Law ratio for each country. The results are in B of Table 4. For example, the Okun's Law ratio for the U.S. is 1.9. Figure 5 is a scatter plot of these two measurements. There is a positive correlation between sacrifice ratios from output data and from unemployment data. The correlation coefficient between these two measurements is 0.56.

Third, for an individual episode, the sacrifice ratio from unemployment data and output data are also positively correlated. The correlation coefficient is 0.48.

[25] The unemployment data for the 1980s and 1990s are only available for Germany and the United Kingdom.

Fourth, as can be seen from B in Table 4, the Okun's Law ratio in the 1990s is much higher than in the 1980s. This is consistent with the "reversal" mentioned by Anderson and Waster (1999).

V. What Determines the Sacrifice Ratio?

In this section, with new results calculated by the new method, I examine what determines the sacrifice ratios with long-lived effects.

1. Initial Inflation and Speed of Inflation

1.1. Sacrifice ratio and initial inflation

For a long-time, economists have puzzled over sacrifice ratios and initial inflation. Theoretically, a higher initial inflation is associated with a lower sacrifice ratio through many channels. Ball, Mankiw, and Romer (1988) show that trend inflation influences the output-inflation trade-off in New Keynesian models. A higher inflation rate going into a disinflation may reduce nominal rigidities because it encourages more frequent contract renegotiations. The application is that the sacrifice ratio during disinflation is decreasing in the initial level of trend inflation.

However, Ball (1994) shows a weak negative effect of initial inflations on the sacrifice ratios. Boschen and Weise (1999) even find a positive sign in a multiple regression, although it is not significant. Anderson and Wascher (1999) also argue that there is no conclusive evidence for the relationship between initial inflation and the sacrifice ratio.

I can now test this relationship again with my new estimated sacrifice ratios. Figure 6 shows the relationship between sacrifice ratios calculated by the new method and initial inflation. For example, US90 represents the disinflation episode for the U.S. in the 1990s.[26] There seems to be a negative relationship and that relationship seems non-linear; when initial inflation is high, it is less costly to decease inflation than when initial inflation is low. A test of this prediction about their non-liner negative correlation can be accomplished by making a simple regression between sacrifice ratio and the log of initial inflation,[27] including the country fixed effect and time fixed effect. The time dummy is the decade when the disinflation occurred.

Table 5 reports the panel regression results. Although the Hausman Tests suggest the random effects are appropriate, I report random effects, country fixed effect, and time fixed effect to compare the new method and the standard method. As can be seen from the first four rows in Table 5, by using the new method, all three panel regressions show that there is significant log-linear negative relationship between the sacrifice ratio and initial inflation. This result also holds even after controlling the length. To interpret the result, for example, if initial inflation decreases from 5 percent to 4 percent, the sacrifice ratio increases by 0.59 percent, while if initial inflation decreases from 20 percent to 19 percent, the sacrifice ratio increases by 0.14 percent. On the other hand, the results from the standard method show weak negative relationship between the sacrifice ratio and initial inflation. None of these regressions are significant, although their signs are still negative.

[26] UK80a and UK80b represent the two episodes for the United Kingdom in the 1980s, see Table 2 for the whole lists.
[27] I also tried the quadratic form and linear form. The log form produced the best regression results.

1.2. Sacrifice ratio and speed of disinflation

The optimal speed of disinflation—gradualism or "cold turkey"— is one of the key issues for a Central Bank. Sargent (1983) argues that a quicker change is more likely to be perceived as a regime-shift, generating credibility, and thus producing a shift in expectations. Ball (1994) shows that the sacrifice ratio is lower if the speed of disinflation is quicker. In this section, I investigate whether this still holds for the adjusted sacrifice ratios with long-lived effects. I test this idea and also the separate effects of inflation change and episode length.

Following Ball (1994), speed of disinflation is the change in trend inflation per quarter: the total change from peak to trough divided by the length of the disinflation episode. The results about sacrifice ratios and speed of disinflation are also in Table 5. Row 5 reports a simple regression of sacrifice ratio on the speed of disinflation. The speed has a significant negative coefficient: the faster the speed of disinflation, the smaller the cost. For example, the sacrifice ratio is about 1.82 if the speed is 0.5 while the sacrifice ratio increases to 3.77 if the speed is only 0.25.[28]

In row 6, after I add initial inflation, neither initial inflation nor speed of disinflation is significant, which suggests there may be some collinearity between them; the correlation coefficient is 0.71. Initial inflation is likely to influence the speed of price and wage adjustment. A high initial inflation may also enhance the effectiveness of the Central Bank's anti-inflationary policy. It is not clear which is more important for reducing the cost of disinflation: the speed of disinflation or the initial inflation.

1.3. Robust Analysis

Next, I perform a robust analysis to determine whether the above results still hold in alternative cases.[29] I use New SR2 in Table 2, the sacrifice ratios calculated from an HP filter with smoothing parameter 1600. As can be seen from Table 5, the results are even stronger in support of the negative relationship between the sacrifice ratio and initial inflation.

---

[28] I also checked the separate effect of the change in inflation and the length of the disinflation episode. The results are quite similar to Ball (1994); the coefficient for the change of inflation is significantly negative, and the length coefficient is significantly positive. Thus, the greater speed reduces the sacrifice ratio regardless of whether it reduces from a larger inflation change over a given period or from a faster completion of a given change.

[29] Ball (1994) argues there is a potential bias in this estimate. The problem is that the change in inflation is both the denominator of the sacrifice ratio and the numerator of the independent variable, the speed. For a given aggregate demand contraction, a favorable supply shock, that accelerates the change in inflation, will both reduce the estimated sacrifice ratio and increase the speed, creating a negative relationship between these two. Following Ball (1994), I try two approaches. First, I take into account big supply shocks in the 1970s and the 1980s. I split the time period before and after unusually large supply shocks, for example, before and after 1972. Since the demand in the second period is not as stable as in the first period, the negative bias from the supply shock will be bigger. The results from these two sub-samples are similar to the results from the full sample. My previous basic results still hold. Thus, the negative relationship is not mainly driven by the supply shock. As a second approach, I estimate this effect with initial inflation as an instrument variable for the speed, since initial inflation is highly related to speed and it is plausibly uncorrelated with the errors from supply shock. The results are in row 7 of Table 5. The coefficient has no significant change. There is no significant evidence of negative bias from supply shocks in my basic results.

2. Nominal Wage Rigidity

2.1. Nominal Wage Rigidities Index

Previous research suggests that one of the reasons why sacrifice ratios for the U.S. are much higher than for G-7 European countries is that nominal wage rigidity in the U.S. is much higher than in other countries. Ball (1994) finds that greater flexibility reduces the sacrifice ratio. In this section, I check the role of nominal wage rigidity in explaining sacrifice ratios with long-lived effects across countries.

There are two indices of nominal wage rigidities. One, Bruno and Sachs (1985) constructs a nominal wage responsiveness index, which includes the frequency of adjustment across sectors, the degree of indexation, and synchronization. Gordon (1982) also uses the difference in wage-setting institutions to explain the cross-country variation in the cost of disinflation. The other, Grubb, Jackman, and Layard (1983) calculated an index of nominal wage rigidities from a time-series regression of wages on prices and unemployment. These two indices are highly correlated (correlation coefficient is -0.74).

2.2. Basic Results

The regression results are in Table 6. From both pooled effect and between effect, there is no significant relationship between sacrifice ratios calculated by the new method and nominal wage rigidity, although the signs are negative. After controlling the initial inflation and length, it is still not significant. Thus, greater flexibility does not play an important role in reducing the cost of disinflation. Using an alternative new method does not change the results either.

However, similar to Ball (1994), there is a negative correlation between sacrifice ratios calculated by the standard method and nominal wage rigidities. The coefficient on the NWR is significantly negative, implying that greater flexibility reduces the sacrifice ratio calculated by the standard method. For example, on average, the fitted value of the sacrifice ratio by the standard method is 2.2 for the minimum NWR of one but only 0.9 for the maximum NWR of five (the U.S. has a rating of 1 and the United Kingdom has a rating of five).

One possible explanation is that by the standard method, the U.S. is an outlier; its sacrifice ratio is much higher than that of most G-7 European countries. The U.S. has a much less flexible labor market than G-7 European countries. From both NWR indices, the U.S. has the least flexibility among G-7 countries. These two facts make the relationship between the sacrifice ratio by the standard method and NWR significant. In other words, larger downward bias for European countries by the standard method makes the relationship significant. However, by the new method, the sacrifice ratio in the U.S. is in the middle among G-7 countries. In this case, there is less significant correlation between nominal wage rigidities and the sacrifice ratio calculated by the new methods. Furthermore, New Keynesians such as Mankiw (1990) and Ball (1994) argue that monetary non-neutrality derives mainly from rigidities in output price, not in the wage. Thus, to this point, perhaps the wage setting is not important in explaining the sacrifice ratio difference across countries.

VI. Conclusions

This paper offers a correction to previous calculations of sacrifice ratio and this correction allows for the possibility that disinflation monetary policy may have long-lived effects on output. There are four main results in this paper.

- Sacrifice ratios with long-lived effects are larger than sacrifice ratios that do not take into account the long-lived effect.
- The sacrifice ratio for the U.S. falls in the middle of the sacrifice ratios for G-7 countries when long-lived effects are allowed for.
- There is a strong negative relationship between sacrifice ratios and initial inflation. This can explain the fact that the sacrifice ratios in the 1990s are larger than in other periods.
- There is no significant negative relationship between sacrifice ratios with long-lived effects and nominal wage rigidities.

I use a simple model to show that the standard method in Ball (1994) is a good approximation of the true sacrifice ratio only in the case without long-lived effects. The new method used in this paper is robust in either case; it gives the same estimate as the standard method in the case without long-lived effects, but gives a much more accurate estimate in the case with long-lived (strong persistence or even hysteresis) effect.

My results not only solve the initial inflation and sacrifice ratio puzzles, but also shed light on two long-standing problems regarding inflation-output tradeoffs. The "Non-OECD Zero Sacrifice Ratio Puzzle" refers to the argument that some non-OECD countries, such as Colombia and Israel, can reduce inflation at little or even zero cost. My results suggest that the absence of short-run costs of anti-inflationary policy in non-OECD countries may be explained by the high initial inflation in those countries, which can reduce the cost of a disinflationary episode.

The “Credibility-Sacrifice Ratio Puzzle” refers to the surprising positive correlation between central bank independence and the sacrifice ratio found by previous studies such as Fisher (1994), Debelle (1996), Posen (1998), and Ball and Croushore (1998). Many previous studies used Ball’s (1994) constructed measures of the sacrifice ratio in different countries, which has a downward bias if there is a long-lived effect. Therefore, a useful extension is to see if the new method can solve the credibility and sacrifice ratio puzzle after correcting the calculation of sacrifice ratio.

In this paper, I only study the short-term effect of disinflation monetary policy. The long-term effect becomes complicated because of uncertainty about trend output and the disinflation path. One approximation for the long-term effect is the difference between trend output and actual output at four quarters after the trough. This measures the temporary decrease in output (or increase in unemployment) during the recession at four quarters after the trough. Intuitively, it measures how far away the actual output is from its initial trend. This difference is assumed to be zero by the standard method in Ball (1994). Allowing for the long-lived effect, the difference assumed by the new method is not zero, and it becomes larger if the long-lived effects become stronger.

References

Anderson, Palle, and Williams Wascher, 1999, "Sacrifice Ratios and the Conduct of Monetary Policy in Conditions of Low Inflation," *Bank of International Settlements Working Paper No. 82, November.*

Ball, Laurence, 1994, "What Determines the Sacrifice Ratio?" in N. G. Mankiw (eds.), *Monetary Policy*, University of Chicago, 155-193.

Ball, Laurence, 1997, "Disinflation and NAIRU," in Christina Romer and David Romer (eds.) *Reducing Inflation: Motivation and Strategy*, University of Chicago Press, 167-185.

Ball, Laurence, 1999, "Aggregate Demand and Long-Run Unemployment," *Brookings Papers on Economic Activity 2,* 189-251.

Ball, Laurence and Dean Croushore, 1998, "Credibility and Disinflation," manuscript.

Ball, Laurence and N. Gregory Mankiw, and David Romer, 1988, "The New Keynesian Economics and the Output-Inflation Trade-Off," *Brookings Papers on Economic Activities, 1,* 1-65.

Bernanke, Ben, Thomas Laubach, Frederic S. Mishkin, and Adam S. Posen, 1999, "*Inflation Targeting, Lessons from International Experience*," Princeton University Press, Princeton, New Jersey.

Blanchard, Oliver, 1997, Comment, in Christina Romer and David Romer (eds.), *Reducing Inflation: Motivation and Strategy*, University of Chicago Press, 1997, 185-192.

Blanchard, Oliver and Lawrence H. Summer, 1986, "Hysteresis and the European Unemployment Problem," *NBER Macroeconomics Annua1 l.* edited by Stanley Fisher, 15-78.

Blinder, Alan S., 1987, *Hard Head, Soft Heart: Tough-Minded Economics for a Just Society.* Reading, Massachusetts: Addison-Wesley.

Boschen, John and Charles Weise, 1999, "The Ex Ante Credibility of Disinflation Policy and the Cost of Reducing Inflation," manuscript.

Bruno, Michael and Jeffrey Sachs, 1985, *Economics of Worldwide Staginflation*, Cambridge, Mass., Harvard University Press.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans, 1996, "The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds," *Review of Economics and Statistics, 78*, 16-34.

Debelle, Guy, 1996, "The Ends of Three Small Inflation, Australia, Zealand, and Canada," *Canadian Public Policy,* 56-78.

Dolado, Juan and David Lopez Salido, 1996, “Hysteresis and Economic Fluctuation,” *Center for Economic Policy Research, Discussion Paper 1334.*

Fisher, Stanley, 1994, “Modern Central Banking,” in The *Future of Central Banking* by F. Capie Et al., Cambridge University Press, 262-304.

Friedman, Benjamin M, 1994, Comment on “What Determines the Sacrifice Ratio?” in N. G. Mankiw (eds.), *Monetary Policy,* University of Chicago, 182-188.

Gordon, Robert J., 1982, “Why Stopping Inflation May Be Costly: Evidence from Fourteen Historical Episodes”, *In Inflation, Causes and Effects,* ed. Roberts E. Hall, Chicago, University of Chicago Press.

Gordon, Robert J. and Stephen King, 1982, “The Output Cost of Disinflation in Traditional and Vector Autoregressive Models,” *Brookings Papers on Economic Activity 1,*205-242.

Grubb, Dennis, Richard Lackman, and Richard Laryard, 1983, “Wage Rigidities and Unemployment in OECD Countries,” *European Economic Review 21,*11-39.

Jordan, Thomas J. 1997, “Disinflation Costs, Accelerating Inflation Gains, and Central Bank Independence,” *Welt. Archiv* 133, 1-21.

Junguito, Roberto, 1998, “The Costs of and Benefits of Disinflation: Implication for Columbia,” forthcoming in volume published in Central Bank of Columbia.

Mankiw, N. Gregory, 1990, “A Quick Refresher Course in Macroeconomics,” *Journal of Economic Literature, 28,*1645-1660.

Okun, Arthur M. 1978, “Efficient Disinflation Policies,” *American Economic Review, Vol. 68,* 348-352.

Partow, Zeinab and David Yuravlivker, 1998, “Modern Inflation and Costs of Disinflation: Empirical Evidence,” forthcoming in volume published in Central Bank of Columbia.

Posen Adams, 1998, “Central Bank Independence and Disinflation Credibility: a Missing Link,” *Oxford Economic Papers* 50, 335-359.

Roberts, M. John, and Norman Morin, 1999, “Is Hysteresis Important for U.S. Unemployment,” *Board of Governors of Federal Reserve, Finance and Economics Discussion Papers,* 1999-56.

Romer, Christina and David Romer, 1989, “Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz,” *NBER Macroeconomics Annual,* 121-70.

Romer, Christina and David Romer, 1994, "What Ends the Recession?" *NBER Macroeconomics Annual*, 13-57.

Romer, David, 1989, Comment. *Brookings Papers on Economic Activity 2: 117-125.*

Sargent, Thomas, 1983, Stopping Moderate Inflations: The Methods of Poincare and Thatcher. In Dornbush and Simonsen (eds.), *Inflation, Debt, and Indexation*, Cambridge, Mass., MIT Press.

Data Appendix

In this paper, I only concentrate on G-7 countries because reliable quarterly output and unemployment data are available for them.

1. Output[1] and Inflation[2]

The output and inflation data cover the 1st quarter of 1960 to the 4th quarter of 1999. Most of the quarterly output and inflation data is from ***International Financial Statistics*** (IFS), June 1999. Japan's quarterly real GDP data is obviously wrong. In the data set, there is about 10% decline in real output from the 4th quarter of 1978 to the 1st quarter of 1979. Thus, I use the OECD output data for Japan. The output and inflation data for other countries are from IFS.

2. Unemployment

The quarterly unemployment data covers the 1st quarter of 1965 to the 4th quarter of 1999. It is the Seasonally Adjusted Civilian Standardised Rates of Unemployment from OECD ***Quarterly Labor Force Statistics*** CD-ROM 1999, Number 2. In the CD-ROM, for the U.K., the unemployment series ends at the 1st quarter of 1982; for Germany, the series ends at the 1st quarter of 1993. From various previous issues of the hard copy, I extend the available quarterly unemployment series to the 2nd quarter of 1977 for both Germany and the U.K. Standardised Rates of Unemployment are based on definitions of the 13th Conference of Labor Statistics (generally referred to as the ILO Guidelines). Unlike national unemployment rates, which are often based on the numbers of persons registered at employment offices, these standardised rates of unemployment allow valid comparisons to be made between countries and over time.

3. Nominal Wage Rigidities Index

There are two indices of nominal wage rigidities: one is from Bruno and Sachs (1985); the other is from Grubb, Jackman, and Layard (1983).

[1] There may be a problem with the output data since output data was updated and was changed significantly in 2000.
[2] The CPI may also have been problematic since CPI data contains the change of interest rates for some countries.

Figure 1. True Sacrifice Ratio and Sacrifice Ratio By the Standard Method

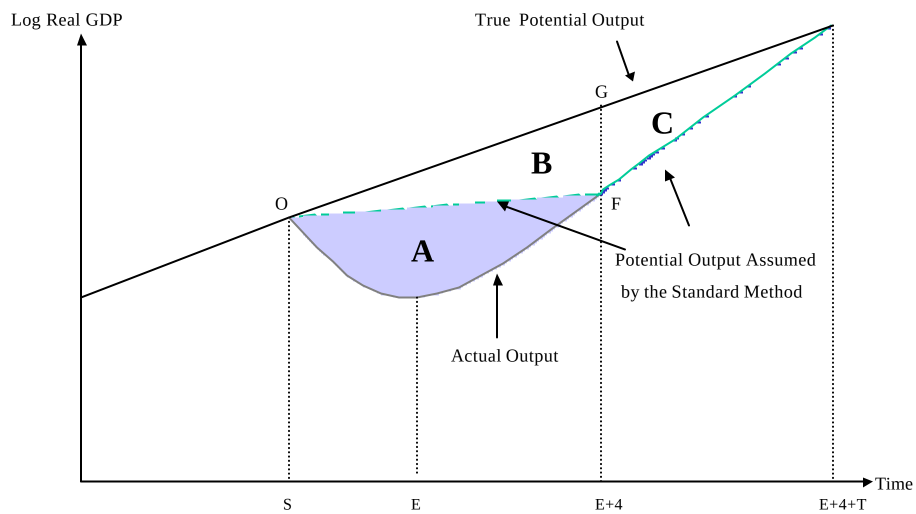


Figure 2. Sacrifice Ratio Reversal by the Standard Method

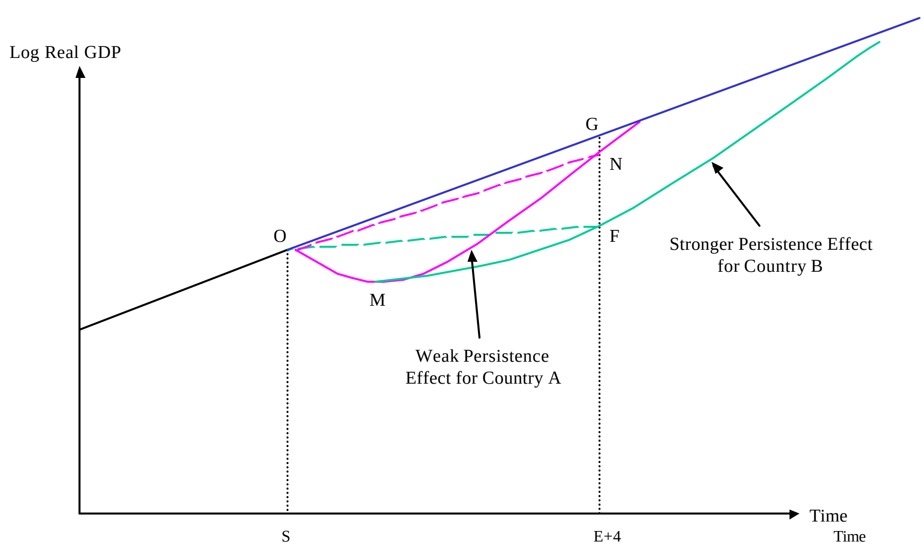


Figure 3. Sacrifice Ratios by the Standard Method and the New Method

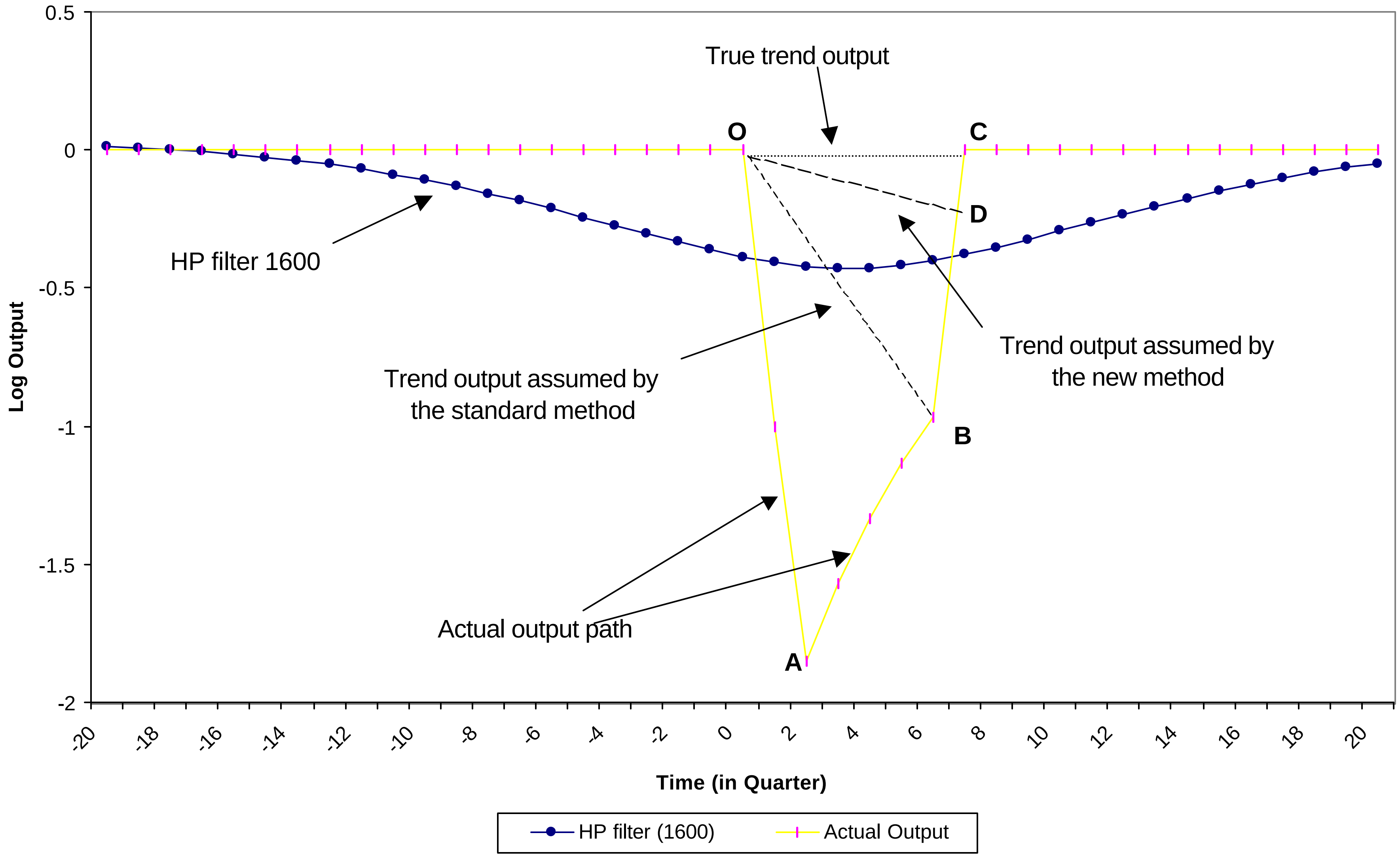


Figure 4A. Disinflation Episode: U.S. 1989:4--1994:3

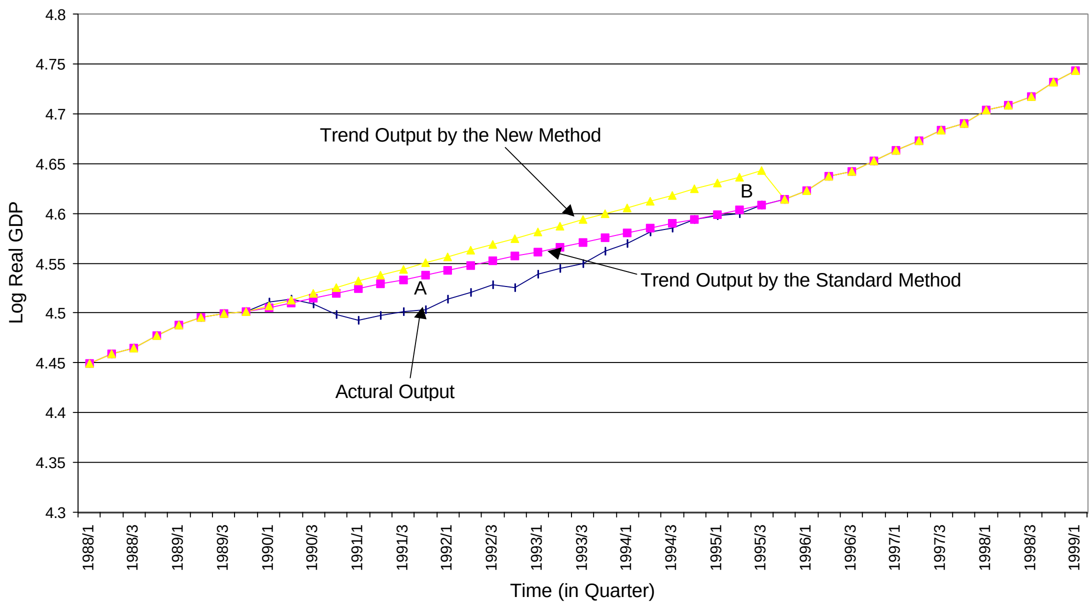
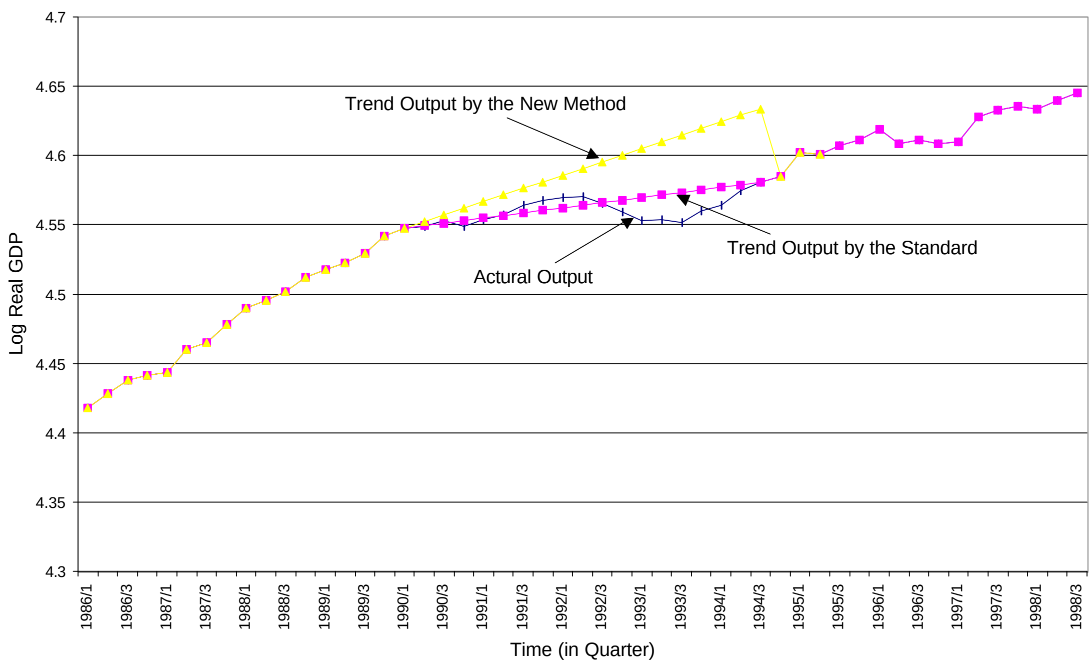


Figure 4B. Disinflation Episode: Italy 1991:1--1993:3



Data Source: IMF *International Financial Statistics*

Figure 5. Okun's Law, Country Average

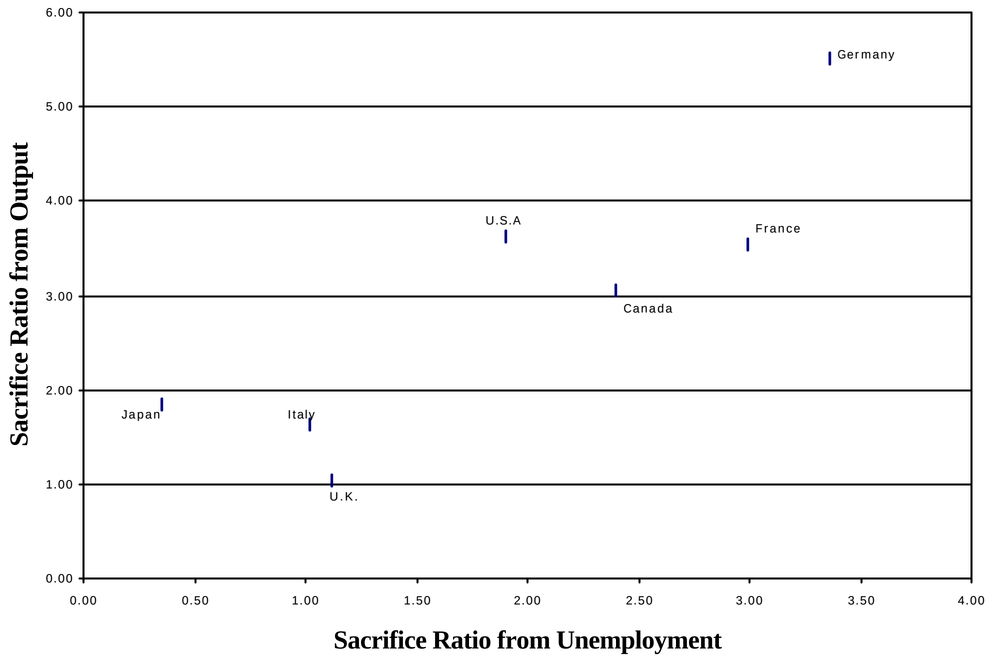
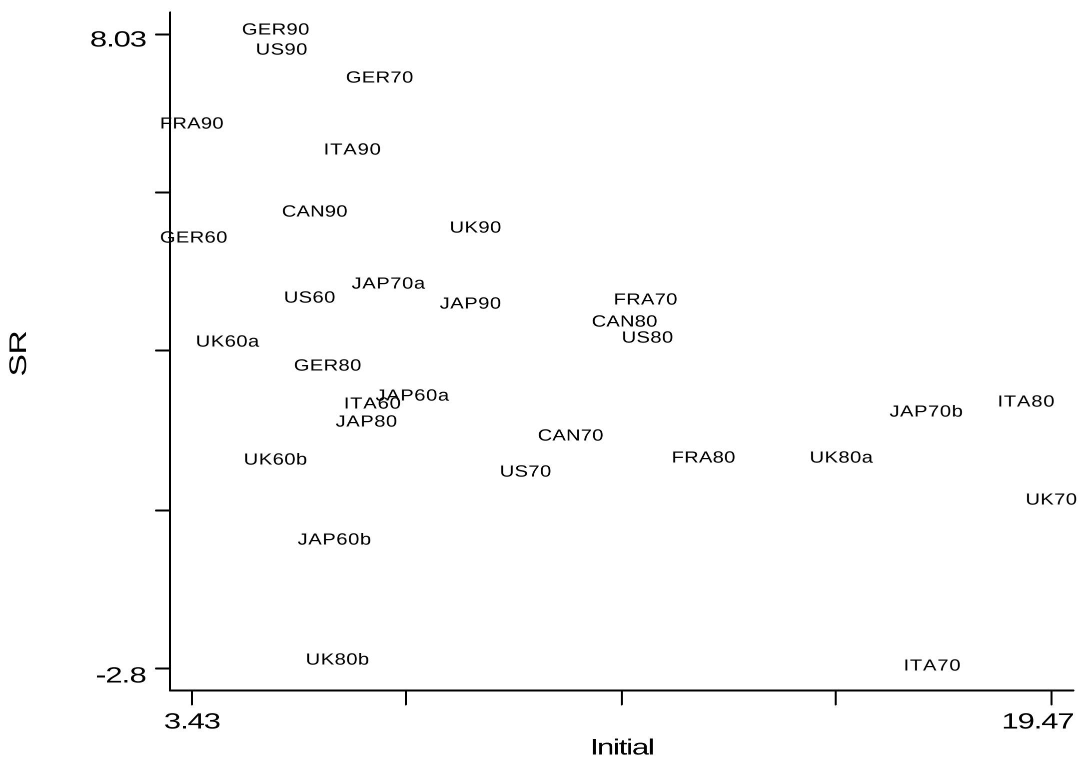


Figure 6. Sacrifice Ratio (SR) and Initial Inflation (Initial)



Data Source: Author's calculation and Bruno and Sachs (1985)

Table 1. Output Loss by the Standard Method and the New Method*

1A. True Output Loss and Output Loss by standard method with Persistence Effect

T	p_t	y_t^*	y_t	L^S	L^T
-1	2	0	0	0	0
0	2	0	0	0	0
1	1	$-\frac{1}{6}a(1+b)b^4$	$-a$	$-a+\frac{1}{6}a(1+b)b^4$	$-a$
2	0	$-\frac{2}{6}a(1+b)b^4$	$-a(1+b)$	$-a(1+b)+\frac{1}{3}a(1+b)b^4$	$-a(1+b)$
3	0	$-\frac{3}{6}a(1+b)b^4$	$-a(1+b)b$	$-a(1+b)b+\frac{1}{2}a(1+b)b^4$	$-a(1+b)b$
4	0	$-\frac{4}{6}a(1+b)b^4$	$-a(1+b)b^2$	$-a(1+b)b^2+\frac{2}{3}a(1+b)b^4$	$-a(1+b)b^2$
5	0	$-\frac{5}{6}a(1+b)b^4$	$-a(1+b)b^3$	$-a(1+b)b^3+\frac{5}{6}a(1+b)b^4$	$-a(1+b)b^3$
6	0	$-a(1+b)b^4$	$-a(1+b)b^4$	0	$-a(1+b)b^4$
Sum				$-a-a(1+b)\sum_{i=0}^{3}b^i+\frac{5}{2}(1+b)b^4$	$-a-a(1+b)\sum_{i=0}^{4}b^i$

1B. True Output Loss under Persistence and Hysteresis Effect

T	p_t	y_t^*	L^T
-1	2	0	0
0	2	0	0
1	1	0	$-a$
2	0	$-(1-c)a$	$-a(1+b)-a(1-c)$
3	0	$-(1-c)a(2+b)$	$-a(1+b)b-a(1-c)(2+b)$
4	0	$-(1-c)a(2+2b+b^2)$	$-a(1+b)b^2-a(1-c)(2+2b+b^2)$
5	0	$-(1-c)a(2+2b+2b^2+b^3)$	$-a(1+b)b^3-a(1-c)(2+2b+2b^2+b^3)$
6	0	$-(1-c)a(2+2b+2b^2+2b^3+b^4)$	$-a(1+b)b^4-a(1-c)(2+2b+2b^2+2b^3+b^4)$
Sum			$-a-a(1+b)\sum_{i=0}^{4}b^i-a(1-c)(9+7b+5b^2+3b^3+b^4)$

1C. Numerical Examples, a=1

	1.C=0 & B=0.5	2.C=0& B=0.85	3.C=0 & B=0.95	4.C=0.9 & B=0.5	5.C=0.75 & B=0.5	6.C=0.6 & B=0.25
L^T	-3.906	-7.861	-9.823	-5.308	-7.414	-7.110
L^S	-3.578	-4.481	-4.263	-3.706	-3.898	-3.369
L^N	-3.874	-7.769	-9.699	-5.269	-7.322	-6.997

* True potential output is zero; L^S , Output loss calculated by the standard method; L^N : Output loss calculated by the new method; L^T , True output loss.

Data Source: Author's calculation.

Table 2. Sacrifice Ratios with Long-Lived Effects: Quarterly GDP Data for G-7 Countries

Episode	Length	Initial inflation	Decline in inflation	New SR 1	New SR 2	Ball SR	g_{sr1}^{new}	g_{sr}^{ball}
U. S.								
69:2—72:1	11	5.63	3.00	3.46	2.30	3.83	3.57	3.72
74:3---76:4	9	9.67	3.91	0.50	0.23	1.21	2.86	3.35
80:1---83:4	15	11.95	8.59	2.79	1.56	1.92	2.56	2.25
89:4---94:3	19	5.13	2.52	7.72	4.41	3.60	2.47	1.86
Japan								
62:1---63:1	4	7.56	2.32	1.80	1.41	1.56	9.44	9.20
65:2---66:3	5	6.09	2.61	-0.65	-0.80	0.59	9.37	10.52
70:2---71:3	5	7.10	1.71	3.72	4.94	1.19	7.74	6.20
74:1---78:3	18	17.14	13.47	1.52	0.20	0.55	5.09	4.26
80:2—87:1	27	6.70	6.69	1.35	1.50	1.84	3.53	3.64
90:2---96:1	23	8.63	8.76	3.36	4.63	0.54	3.27	2.10
Germany								
65:4---67:3	7	3.48	2.29	4.50	3.89	2.23	4.15	2.89
73:1---77:3	18	6.93	4.11	7.22	6.61	2.42	3.12	1.87
81:1---86:3	22	5.97	6.02	2.31	0.20	1.80	1.67	1.53
92:2—95:4	14	5.01	3.63	8.03	8.85	1.22	3.42	1.10
France								
74:3---77:1	10	11.89	2.96	3.43	3.66	1.73	3.43	2.66
81:1---87:1	24	13.00	10.44	0.76	-0.59	0.84	1.83	1.87
89:2---93:3	17	3.43	1.72	6.43	11.87	-0.55	2.09	1.26
Italy								
63:3---68:1	18	6.82	5.66	1.67	0.33	2.74	6.04	6.42
75:2---78:3	13	17.26	4.79	-2.80	-3.32	-0.47	3.46	4.63
80:4---87:2	26	18.99	14.40	1.69	1.11	1.11	2.55	2.26
90:1---93:3	14	6.43	2.41	6.00	6.36	0.79	1.92	0.74
U. K.								
61:2---63:3	9	4.12	2.18	2.72	2.75	2.08	3.28	3.04
65:2---66:3	5	5.02	2.23	0.71	0.66	-0.29	3.04	2.25
74:4---78:2	14	19.47	9.66	0.03	-0.14	1.09	1.88	2.84
80:2---83:3	13	15.57	11.28	0.76	0.02	0.56	1.60	1.36
84:2---86:3	9	6.17	2.99	-2.68	-1.47	0.78	2.46	4.28
89:2—93:4	17	8.74	7	4.66	4.06	1.86	2.20	0.84
Canada								
74:3---77:1	10	10.52	3.18	1.12	1.76	0.35	4.53	4.16
81:2---85:2	16	11.50	7.68	3.08	1.53	2.44	2.94	2.57
90:1---93:2	13	5.75	4.78	4.95	3.34	2.47	2.05	0.81

g_{sr1}^{new} : growth rates used in calculating New SR1; g_{sr}^{ball} : growth rates used in calculating Ball SR.

Data Source: IMF, *International Financial Statistics* and OECD.

Table 3. Sacrifice Ratios with Long-Lived Effects
Quarterly Unemployment Data for G-7

Episode	NAIRU in New SRU	Annual Growth Rate assumes by Ball SRU	New SRU	Ball SRU
U. S.				
69:2—72:1	3.43	0.40	2.23	1.23
74:3---76:4	5.63	0.32	1.68	1.22
80:1---83:4	6.30	0.21	1.18	0.89
89:4---94:3	5.37	0.06	2.51	2.11
Japan				
62:1---63:1				
65:2---66:3	1.10	0.04	0.16	0.10
70:2---71:3	1.10	0.03	0.25	0.03
74:1---78:3	1.30	0.15	0.27	0.10
80:2—87:1	2.00	0.09	0.67	0.25
90:2---96:1	2.10	0.18	0.40	-0.03
Germany				
65:4---67:3				
73:1---77:3				
81:1---86:3	4.12	0.61	3.83	1.59
92:2—95:4	5.78	0.76	2.89	0.66
France				
74:3---77:1				
81:1---87:1				
89:2---93:3	9.40	0.55	2.99	-1.64
Italy				
63:3---68:1				
75:2---78:3				
80:4---87:2				
90:1---93:3	9.10	0.53	1.02	-1.35
U. K.				
61:2---63:3				
65:2---66:3				
74:4---78:2				
80:2---83:3	5.72	1.26	1.74	0.67
84:2—86:3	11.00	-0.18	0.33	0.68
89:2—93:4	7.30	0.42	1.28	0.41
Canada				
74:3---77:1	5.30	0.88	2.27	0.46
81:2---85:2	7.10	0.50	2.35	1.50
90:1---93:2	7.60	0.71	2.57	1.16

Data Source: IMF, *International Financial Statistics* and OECD.

Table 4. Basic Statistics for Sacrifice Ratio and Okun's Law

A. Country Average of Sacrifice Ratio

Unemployment Data				Output Data			
Ranking	Country	Sacrifice Ratio	Ball Sacrifice Ratio	Ranking	Country	Sacrifice Ratio	Ball Sacrifice Ratio
1	Germany	3.36	1.13	1	Germany	5.52	1.92
2	France*	2.99	-1.64	2	U.S.	3.62	2.64
3	Canada	2.40	1.04	3	France	3.54	0.67
4	U.S.	1.90	1.36	4	Canada	3.05	1.75
5	U.K.	1.12	0.59	5	Japan	1.85	1.05
6	Italy*	1.02	-1.35	6	Italy	1.64**	1.04
7	Japan	0.35	0.09	7	U.K.	1.03**	1.01
All Sample Mean		1.61	0.53	All Sample Mean		2.67	1.40

* There is only one observation for France and Italy.
** There is a big negative sacrifice ratio in Italy and the U.K., so, their average sacrifice ratios are low.

B. Ratio for Okun's Law

Country Average			Period Average	
Ranking	Country	Okun's Ratio	Times	Okun's Ratio
1	Japan	5.29	1990's	3.01
2	U.S.	1.90	1980's	0.75
3	Germany	1.64	1970's	1.65
4	Italy	1.61	1960's	1.70
5	Canada	1.27	All Sample Average	1.66
6	France	1.18		
7	U.K.	0.93		

Data Source: IMF, *International Financial Statistics* and OECD.

Table 5. Sacrifice Ratio and Initial Inflation

Dependent variable is sacrifice ratio[1]

Estimation Methods	Constant,	Log of Initial Inflation,	Speed,	Length,	Adj. R^2
1. New Method[2]					
Random Effect[3]	8.25	-2.68			0.23
	(4.21)	**(-2.92)**			
Random Effect	7.28	-3.17		0.14	0.33
	(3.78)	**(-3.50)**		**(2.00)**	
Country-Fixed Effect	6.95	-2.05			0.16
	(3.26)	**(-2.05)**			
Time-Fixed Effect	7.51	-2.32			0.16
	(3.37)	**(-2.20)**			
Random Effect	5.73		-7.82		0.26
	(5.36)		**(-3.13)**		
Random Effect	7.56	-1.38	-5.16		0.29
	(3.83)	**(-1.10)**	**(-1.48)**		
IV (initial inflation as a IV)	-1.49		-3.98		0.31
	(-1.07)		**(-3.13)**		
2. Standard Method					
Random Effect	2.41	-0.47			0.06
	(2.89)	**(-1.24)**			
Random Effect	2.15	-0.59		0.04	0.10
	(2.53)	**(-1.50)**		**(1.17)**	
Country-Fixed Effect	2.19	-0.38			0.04
	(2.53)	**(-0.93)**			
Time-Fixed Effect	2.28	-0.42			0.02
	(1.87)	**(-0.73)**			
3. Alternative New Method[4]					
Random Effect	9.63	-3.47			0.29
	(4.38)	**(-3.38)**			
Random Effect	8.83	-3.89		0.12	0.34
	(3.96)	**(-3.70)**		**(1.44)**	
Country-Fixed Effect	8.88	-3.11			0.25
	(3.60)	**(-2.69)**			
Time-Fixed Effect	9.84	-3.57			0.30
	(4.27)	**(-3.28)**			

[1] Regression results are written in rows. Please look at it horizontally. Number of observations: 30.
[2] Use New SR1 in Table 2, calculated by using an HP filter with smoothing parameter 16000.
[3] From the Hausman Test, the random effect is better for most cases. Thus, all regression results are from the random effect, except as specially mentioned.
[4] Use New SR2 in Table 2, calculated by using an HP filter with smoothing parameter 1600.

Data Source: Author's calculation.

Table 6. Sacrifice Ratio and Nominal Wage Rigidity (NWR)

Dependent variables are sacrifice ratio

Estimation Methods	Independent Variables				
	Constant,	**NWR,**	**Log of Initial Inflation,**	**Length,**	**Adj. R^2**
1. New Method[1]					
Pooled Effect	4.51	-0.53			0.03
	(3.17)	**(-1.38)**			
Pooled Effect	9.99	-0.51	-2.65		0.24
	(4.46)	**(-1.51)**	**(-2.96)**		
Pooled Effect	8.95	-0.48	-3.13	0.14	0.31
	(4.08)	**(-1.48)**	**(-3.53)**	**(1.96)**	
Between Effect[2]	4.33	-0.44			0.16
	(2.71)	**(-0.96)**			
2. Standard Method					
Pooled Effect	2.61	-0.35			0.15
	(4.98)	**(-2.46)**			
Pooled Effect	3.73	-0.34	-0.54		0.18
	(4.08)	**(-2.49)**	**(-1.48)**		
Pooled Effect	3.50	-0.33	-0.64	0.03	0.18
	(3.72)	**(-2.43)**	**(-1.69)**	**(0.99)**	
Between Effect	2.56	-0.34			0.46
	(4.43)	**(-2.08)**			
1. Alternative New Method[3]					
Pooled Effect	3.24	-0.24			-0.03
	(1.90)	**(-0.52)**			
Pooled Effect	10.38	-0.22	-3.46		0.25
	(3.99)	**(-0.56)**	**(3.33)**		
Pooled Effect	9.50	-0.19	-3.87	0.12	0.27
	(3.60)	**(-0.50)**	**(-3.64)**	**(1.39)**	
Between Effect	3.11	-0.15			0.02
	(1.66)	**(-0.29)**			

[1] Use New SR1 in Table 2, calculated by using an HP filter with smoothing parameter 16000. Number of observations: 30.
[2] NWR index is not a time-varying variable; thus, the between effect is to regress country average sacrifice ratio and NWR.
[3] Use New SR2 in Table 2, calculated by using an HP filter with smoothing parameter 1600.

Data Source: Author's calculation.

Appendix A

Appendix A calculates the sacrifice ratio with persistence effects by the standard method.

From A in Table 1, output at the beginning of disinflation is zero and the output at four quarters after the trough (the sixth quarter) is $-a(1+b)b^4$. By the standard method, the predicted trend outputs grow log-linearly between the two points. Thus, the predicted trend outputs by the standard method at periods 1, 2, 3, 4, 5, and 6 are $-\frac{1}{6}a(1+b)b^4$, $-\frac{2}{6}a(1+b)b^4$, $-\frac{3}{6}a(1+b)b^4$, $-\frac{4}{6}a(1+b)b^4$, $-\frac{5}{6}a(1+b)b^4$, and $-a(1+b)b^4$ respectively. The total output losses are the sum of the difference between the trend output and actual output. Thus, the output losses by the standard method, L^S, are:

$$L^S = -\left[a - \frac{1}{6}a(1+b)b^4\right] - \left[a(1+b) - \frac{2}{6}a(1+b)b^4\right] - \left[a(1+b)b - \frac{3}{6}a(1+b)b^4\right]$$
$$- \left[a(1+b)b^2 - \frac{4}{6}a(1+b)b^4\right] - \left[a(1+b)b^3 - \frac{5}{6}a(1+b)b^4\right] - \left[a(1+b)b^4 - a(1+b)b^4\right] \qquad \text{A1}$$

Or

$$L^S = -a\left[\left(2 + 2b + 2b^2 + 2b^3 + b^4\right) - \frac{5}{2}(1+b)b^4\right] \qquad \text{A2}$$

Appendix B

By equation (1) and (5), I can calculate the true output loss and the output loss using standard method.

1. Output Loss calculated by the standard method

From Table 1B, actual output at the beginning of disinflation is zero; I can calculate the actual output at four quarters after the trough (period 6), which is defined as M,

$$M \equiv y_6 \equiv -a(1+b)b^4 - 2a(1-c)\sum_{i=0}^{3} b^i - a(1-c)b^4 \qquad \text{B1}$$

By the standard method, the predicted trend output at period 1, 2, 3, 4, 5, and 6 are $\frac{1}{6}M$, $\frac{2}{6}M$, $\frac{3}{6}M$, $\frac{4}{6}M$, $\frac{5}{6}M$, and M respectively. The output loss is the sum of the differences between the trend output and actual output. If L^S is defined as total output loss, then

$$L^S = \left(y_1 - \frac{1}{6}M\right) + \left(y_2 - \frac{2}{6}M\right) + \left(y_3 - \frac{3}{6}M\right) + \left(y_4 - \frac{4}{6}M\right) + \left(y_5 - \frac{5}{6}M\right) + (y_6 - M)$$

Plugging y_1 through y_6, and M into the above equation, I get

$$L^S = -a - a(1+b)\sum_{i=0}^{3} b^i + 2.5a(1+b)b^4 - 2a(1-c)(1 - b^2 - 2b^3 - 1.25b^4) \qquad \text{B2}$$

2. True Output Loss

From Table 1B, the true output loss with persistence and hysteresis effects, L^T, is:

$$L^T = -a - a(1+b)\sum_{i=0}^{4} b^i - (1-c)a(9 + 7b + 5b^2 + 3b^3 + b^4) \qquad \text{B3}$$

Equation B3 shows the true output loss; Equation B2 shows the output loss calculated by the standard method. Plugging equation B2 into equation B3, I get:

$$L^T = L^S - 1.5a(1+b)b^4 - 7(1-c)\left(1 + b^2 + b^3 + \frac{1}{2}b^4\right) \qquad \text{B4}$$