Investment Spending, Equilibrium Indeterminacy, and the Interactions of Monetary and Fiscal Policy

Thomas A. Lubik*

Department of Economics†
The Johns Hopkins University

January 2003

Abstract

This paper investigates determinacy of equilibrium in a canonical New Keynesian model under different monetary and fiscal policy rules. It is shown that a simple monetary rule that responds aggressively to inflation is a necessary condition for equilibrium determinacy, when fiscal policy is accommodating. If there is a high degree of structural distortions in the economy, then the interesting possibility arises that both aggressive monetary and fiscal policies are required to guarantee existence. When investment adjustment costs are introduced, the monetary and fiscal policy dichotomy is in principle maintained. The determinacy region is, however, highly dependent on the degree of distortion in the economy. The more prices are sticky, and the less competitive firms are, the economy is likely to exhibit indeterminacy even if monetary policy is active.

Keywords: Indeterminacy, Investment, Monetary and Fiscal Policy

JEL Classification: E22, E52, E63,

*The author wishes to thank Larry Ball, Chuck Carlstrom and Massimiliano Marzo for useful comments. He is also grateful to the faculty of the Department of Economics at the University of Bologna and at the Johns Hopkins Bologna Center for their hospitality during his visit. Additional thanks go to seminar participants at Johns Hopkins, Brescia, Bologna, the Bank of England, and the Money, Macro, and Finance Research Group Conference in Belfast. Naturally, the author is solely responsible for the contents of this paper.

†Correspondence address: Mergenthaler Hall, 3400 N. Charles Street, Baltimore, MD 21218, USA. Tel.: +1 410 516 5564, Fax: +1 410 516 7600. Email: thomas.lubik@jhu.edu.

1 Introduction

Recent research in monetary economics has focused on the analysis of monetary policy rules in optimizing models with nominal rigidities. Although the behavior of private agents in these models is based on forward-looking investment and consumption equations, the government sector is typically described by ad-hoc policy rules[1]. Since there is convincing empirical evidence that monetary authorities do follow simple policy rules, the partial ad-hoc nature of this modeling approach seems a small price to pay for empirical relevance. Recent research[2], however, has drawn attention to the fact that exogenous, rule-based behavior may lead to indeterminacy and sunspot equilibria, which, by their very nature, are welfare-reducing. The task, therefore, seems to identify ad-hoc monetary policy rules that 'do no harm'.

However, the general impression that emerges from this literature is that the question of equilibrium indeterminacy is heavily model-dependent. In order to be able to give meaningful policy advice, the emphasis should be on developing dynamic stochastic general equilibrium models that are empirically relevant. In this paper, I present a canonical monetary business cycle model variants of which have been subjected to empirical testing[3].

The model in this paper differs from the related literature in the following aspects. First, I model nominal price stickiness by introducing quadratic price adjustment costs. It is well known that this model produces qualitatively similar aggregate dynamics as the pricing mechanisms based on Calvo (1983). Furthermore, this is less restrictive than assuming that prices are preset for a fixed time period as in Carlstrom and Fuerst (2000a). The modeling framework in this paper is similar to Kim (2000) who performs a structural estimation and finds that the model's fit is comparable to an unrestricted VAR. There's a case to be made that in studying issues of determinacy preference should be given to models that are empirically useful.

Secondly, I introduce investment and capital-based production. At business cycle frequencies, fluctuations in investment contribute substantially to GDP volatility. Furthermore, the real return on capital in an optimizing forward-looking model is determined by future supply and demand conditions. In a model with government debt, the real interest

[1] A notable exception is Erceg, Henderson, and Levin (2000).

[2] Among others, Benhabib, Schmitt-Grohé and Uribe (2001), Carlstrom and Fuerst (2000a, b), Dupor (2001, 2002).

[3] Kim (2000) estimates a similar model and finds that its forecasting performance is comparable to a small-scale VAR. Lubik and Schorfheide (2002) extend the approach to the open economy.

rate is thus related via an arbitrage condition to the consumption-smoothing opportunities available to private agents.[4] Abstracting from investment in a monetary business cycle model would therefore miss an important component of the monetary transmission mechanism.[5]

Finally, while earlier literature restricted the discussion of potential equilibrium indeterminacy to simple Taylor rules this paper extends the literature in an important direction by explicitly introducing a fiscal policy rule. Leeper (1991) demonstrates that existence and uniqueness of equilibrium depend jointly on the behavior of monetary and fiscal authorities. Previous contributions implicitly assume that fiscal policy is extremely passive in that tax rates are instantaneously adjusted to rebate any inflation tax revenue from the government's nominal liabilities. By considering a wider behavioral range of the fiscal authority, I show that Leeper's (1991) conclusion of determinacy under active and passive monetary and fiscal rules has to be qualified for empirically plausible parameter values. Specifically, researchers have to take into account the structural characteristics of the economy at hand. A low degree of monopolistic competition and substantial price rigidity render an economy's production very elastic, which opens up the possibility of self-fulfilling equilibria. Similarly, adjustment costs in investment work in the opposite direction. Monetary and fiscal policies can therefore ensure determinacy in combinations that run counter to the results by Leeper.

The modeling framework in this paper, although highly stylized, is sufficiently rich so that analytical solutions are not obtainable. I therefore characterize regions of determinacy in the parameter space using numerical simulation. I also provide a qualitative and quantitative assessment of the model dynamics for determinate and indeterminate equilibria. The theoretical model is presented in the next section. Section 3 discusses determinacy issues with reference to the monetary policy only. I study a model with instantaneous investment adjustment first before introducing adjustment costs. In Section 4 I investigate the interaction of monetary and fiscal policy behavior. Section 5 concludes and discusses avenues for further research.

[4] This point is particularly emphasized by Dupor (2001)

[5] McCallum and Nelson (1997) offer a contrasting view.

2 A Canonical New Keynesian Model with Investment

2.1 Household

The economy is populated by a representative household that derives utility from consumption c and real balances $\frac{M}{P}$, and disutility from working, where n denotes the labor supply:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \chi \log \frac{M_t}{P_t} - n_t \right]. \tag{1}$$

P is the economy-wide nominal price level which the household takes as given. The household owns the capital stock k and makes all investment decisions i subject to the constraint:

$$k_{t+1} = i_t + (1 - \delta) k_t, \tag{2}$$

where δ is the rate of depreciation. The agent supplies labor services and rents out his capital to the firms period by period for which he receives real factor payments w and r, respectively. The household has access to nominal government bonds B that pay interest R. Furthermore, it receives aggregate residual profits Π from the firms and has to pay lump-sum taxes τ. Consequently, the household maximizes (1) subject to (2) and its budget constraint:

$$c_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + i_t + \tau_t = w_t n_t + r_t k_t + \frac{M_{t-1}}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + \Pi_t. \tag{3}$$

The usual transversality condition on asset accumulation applies which rules out Ponzi-schemes. Initial conditions are given by k_0, B_0.

A detailed derivation of the first order conditions can be found in the Appendix. The household's behavior can then be described by:

$$\begin{aligned}
c_{t+1} &= \beta \frac{R_t}{\pi_{t+1}} c_t, && (4)\\
c_{t+1} &= \beta \left[r_{t+1} + (1 - \delta) \right] c_t, && (5)\\
\frac{M_t}{P_t} &= \chi \frac{R_t}{R_t - 1} c_t, && (6)\\
w_t &= c_t. && (7)
\end{aligned}$$

Equation (4) is the usual Euler-equation. Consumption growth is thus an increasing function of the real interest rate, which is defined as the nominal interest rate adjusted for (expected) inflation.

Note that (4) and (5) imply an arbitrage condition between bonds and capital: $\frac{R_t}{\pi_{t+1}} = r_{t+1} + (1 - \delta)$. Investment is chosen such that the return on an additional unit of capital, as measured by its future cost r_{t+1} and adjusted for depreciation, is equal to the expected real return from holding nominal debt, i.e. the real interest rate. Monetary policy, via control of the nominal interest rate, exerts a direct influence on capital accumulation. Although capital is not very volatile over the business cycle, investment is. The arbitrage condition shows how investment, and thus future output, are crucially linked to policy decisions. Contrary to the view embraced in McCallum and Nelson (1997), modeling investment decisions should be an integral part in the analysis of monetary policy. Finally, equation (6) depicts a standard, interest-sensitive money-demand schedule. Since labor enters the utility function linearly, labor supply is simply determined by (7).

2.2 Firms

The production sector is described by a continuum of monopolistically competitive firms each of which faces a downward-sloping demand curve for its differentiated product:

$$p_t(j) = \left(\frac{y_t(j)}{y_t} \right)^{-1/\nu} P_t. \tag{8}$$

Such a demand function can be derived in the usual way from Dixit-Stiglitz preferences, where $p_t(j)$ is the profit-maximising price consistent with production level $y_t(j)$. The parameter ν is the elasticity of substitution between two differentiated goods. With $\nu \to \infty$ the demand function becomes perfectly elastic and the differentiated goods become substitutes. The aggregate price level and aggregate demand/output y_t are beyond the control of the individual firm.

Nominal price stickiness is introduced via the assumption of quadratic adjustment costs. When a firm wants to change its price beyond the general trend in prices, given by the economy-wide inflation rate π, it incurs 'menu costs' in the form of lost output. The parameter $\varphi \geq 0$ governs the degree of stickiness in the economy. Assuming Cobb-Douglas production $y_t(j) = k_t^{1-\alpha}(j) n_t^{\alpha}(j)$ the firm j chooses factor inputs $n_t(j), k_t(j)$ to maximize:

$$\Omega_t(j) = \sum_{t=0}^{\infty} \rho_t \Pi_t(j) = \sum_{t=0}^{\infty} \rho_t \left[\frac{p_t(j)}{P_t} y_t(j) - w_t n_t(j) - r_t k_t(j) - \frac{\varphi}{2} \left(\frac{p_t(j)}{p_{t-1}(j)} - \pi \right)^2 y_t \right] \tag{9}$$

subject to (8), the production function, and the constraint that output is demand-determined. ρ is the (potentially) time-dependent discount factor that firms use to evaluate future profit streams. Under the assumption of perfect insurance markets $\rho_t = \beta \frac{c_t}{c_{t+1}}$. In other words, since the household is the recipient of the firms' residual payments it 'directs' firms to make decisions based on the households intertemporal rate of substitution.

Although firms are heterogeneous ex ante, I impose the usual solution concept for this kind of production structure: I assume that firms behave in an identical way so that the individual firms can be aggregated into a single representative, monopolistically competitive firm. The firm's first-order conditions are (after imposing homogeneity):[6]

$$r_t = (1-\alpha)\frac{y_t}{k_t}\left(1-\frac{1}{\varepsilon_t}\right), \tag{10}$$

$$w_t = \alpha\frac{y_t}{n_t}\left(1-\frac{1}{\varepsilon_t}\right), \tag{11}$$

$$\varepsilon_t = \nu\left[1-\varphi\left(\pi_t-\pi\right)+\beta\frac{c_t}{c_{t+1}}\varphi\left(\pi_{t+1}-\pi\right)\pi_{t+1}^2\frac{y_{t+1}}{y_t}\right]^{-1}. \tag{12}$$

ε_t can be interpreted as the output demand elasticity augmented by the cost of price adjustment. Note that the mark-up of price over marginal cost is $\mu_t = \left(1-\frac{1}{\varepsilon_t}\right)^{-1}$. For instance, we can rewrite (10) as:

$$r_t = \frac{mpk_t}{\mu_t}. \tag{13}$$

In steady state, the mark-up is $\frac{\nu}{\nu-1}$, so that ν indexes the degree of monopolistic distortion in the economy. In the perfectly competitive case, with $\nu \rightarrow \infty$, the mark-up is unity. Furthermore, when prices are perfectly flexible ($\varphi = 0$), the mark-up is constant. With sticky prices, both real and nominal shocks affect real variables via movements in the endogenous mark-up. An increase in ε_t reduces the mark-up and is expansionary due to the reduction of the monopolistic distortion. *Ceteris paribus* this implies an increase in the real return to capital r_t. Similarly, a jump in r_t signals an expansion due to a rise in the marginal productivity of capital mpk_t which due to the one-period rigidity of capital is generated by an increase in effort.

Note that in a model with quadratic adjustment costs prices of individual goods are not completely rigid. Instead, required price changes are optimally spread out over a firm's entire time horizon. This stands in contrast to Calvo's staggered price setting where only a fraction of firms is allowed to change their prices every period. At every time horizon there is thus a non-negligible group of firms that have never adjusted, while in the quadratic adjustment cost model *all* firms adapt their pricing policy immediately.

2.3 Policy Rules

I consider monetary policy rules of the form:

$$R_t = \psi\left(\pi_t\right) = R\left(\frac{\pi_t}{\pi}\right)^{\psi}, \tag{14}$$

[6] See the Appendix for details on the derivation.

where R and π are the steady state levels of the interest rate and inflation. In this simple Taylor-rule the central bank adjust the nominal interest rate in response to changes in the inflation rate only (net of its target level, the steady state inflation rate). A measure of the aggressiveness of this policy is given by ψ which under a log-linear rule corresponds to the response coefficient ψ. The rule is called active when $\psi > 1$, and passive for $0 \leq \psi \leq 1$.[7] This case has been analyzed recently by Dupor (2001) and Carlstrom and Fuerst (2000a,b).

The terminology employed here differs slightly from the earlier literature. An active rule is typically defined as one that leads to an increase in the real rate in response to an increase in inflation. That is, in terms of deviations, $\widetilde{rr}_t = \widetilde{R}_t - \widetilde{\pi}_t = (\psi - 1)\,\widetilde{\pi}_t$. Consequently, an active rule corresponds to $\psi > 1$. Note, however, that in the present model, the equation providing the link between nominal and real interest rates is given by the arbitrage condition between nominal bonds and capital. After log-linearizing: $\widetilde{R}_t - \widetilde{\pi}_{t+1} = \beta r \widetilde{r}_{t+1}$. Interest rate policy thus influences inflation dynamics and the *expected* return to capital. The *current* return on capital (the real rate), on the other hand, depends on the effective marginal productivity of capital (see (10)). There is therefore only an indirect link between these interest rates.

The model is closed by specifying the behavior of the fiscal authority. It is assumed that the government levies a lump-sum tax (or subidy) τ to finance any shortfall in government revenues (or to rebate any surplus):

$$\tau_t = \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - R_{t-1}B_{t-1}}{P_t}. \tag{15}$$

In Leeper's (1991) terminology, the fiscal authority follows a 'passive' rule by endogenously adjusting the primary surplus (proxied by τ) to changes in the government's outstanding liabilities (composed of the monetary base and nominal debt). The monetary authority, on the other hand, is free to pursue an active or a passive policy, whereby the specific classification of a rule is delineated by the coefficent on the inflation rate in (14). If the nominal rate is raised by more than the increase in inflation, so that the real interest rate increases, the policy rule is labelled 'active', and passive otherwise. In Section 4 I will relax the assumption of a passive fiscal rule.

[7] Note that $\psi_1 = 0$ describes an interest rate peg, while $\psi_1 \to \infty$ is the case of a pure inflation peg.

2.4 Local Dynamics

The first order conditions (2) - (7), (10) - (12), and the policy rules (14) - (15) describe a system of non-linear difference equations. I proceed to solving this system by log-linearizing each equation around the unique steady state which is given in the Appendix. Denoting $\widetilde{x}_t \equiv \log x_t - \log x$ the dynamics of the economy in a neighborhood of the steady state are given by:

$$\Gamma_0 z_t = \Gamma_1 z_{t-1}. \tag{16}$$

Γ_0, Γ_1 are coefficient matrices the entries of which are typically non-linear functions of deep structural parameters, such as intra- and intertemporal substitution elasticities, as well as the policy parameter ψ.

The goal of this paper is to derive parameter restrictions on ψ such that the model's equilibrium is unique. For this purpose, I first reduce the system to one in four variables only, where $z_t = \left[\widetilde{\pi}_t, \widetilde{c}_t, \widetilde{r}_t, \widetilde{k}_t\right]$ and Γ_0, Γ_1 are 4×4 matrices the elements of which are given in the Appendix. The model economy is complex enough that no analytical results are obtainable beyond this step. I therefore resort to a numerical solution. If Γ_0 is non-singular we can write $z_t = \Gamma_0^{-1}\Gamma_1 z_{t-1}$. The dynamic behavior of the system is then governed by the eigenvalues of the reduced form coefficient matrix $\Gamma_0^{-1}\Gamma_1$.

It is well known[8] that existence and uniqueness of a perfect foresight equilibrium depend on the number of explosive eigenvalues of the matrix $\Gamma_0^{-1}\Gamma_1$, that is, eigenvalues that are bigger than unity in absolute terms. A general condition for determinacy is that the number of 'stable' eigenvalues has to be equal to the number of 'predetermined' variables.[9] Since capital is the only predetermined variable in the model - the other variables are 'jump' variables -, determinacy requires that three roots lie outside the unit circle. Since the roots are, in turn, functions of the policy parameter, determinacy therefore hinges upon the relative size of ψ, given a specific structural parameterization. The solution procedure is then to numerically compute the eigenvalues of $\Gamma_0^{-1}\Gamma_1$ and check for the number of eigenvalues that are less than three (indeterminacy), exactly equal to three (uniqueness), or more than three (non-existence). Then, I search over the parameter space for the respective regions.

[8] See, for instance, Blanchard and Kahn (1980) or Sims (2000).

[9] An alternative definition is that the number of roots outside the unit circle equal the number of forward-looking equations.

3 Monetary Policy Rules and Determinacy

3.1 Equilibrium Determinacy in the Canonical Model

In general, there is no analytic solution for the system (16). Instead I provide a quantitative assessment of monetary policy rules under a specific parameterization. I set the structural parameters of the model to commonly used values in the business cycle literature. The time period can be thought of as a quarter.

α	β	δ	ν
0.6	0.98	0.025	1.5

Note that $\nu = 1.5$ implies a high degree of monopolistic distortion with a steady state mark-up of 3. Although there are widely diverging estimates in the empirical literature, this value is probably unrealistically high. I will also present results for $\nu = 10$ which is close to the perfectly competitive case.

I characterize regions of the parameter space for which the equilibrium is determinate by computing the number of explosive eigenvalues for combinations of the monetary policy parameter ψ and the price adjustment parameter φ. In the reduced-form four-variable system with $z_t = \left[\widetilde{\pi}_t, \widetilde{c}_t, \widetilde{r}_t, \widetilde{k}_t\right]$ there is one predetermined variable, capital k, while the inflation rate, consumption and the real return on capital are jump variables. A determinate equilibrium therefore requires three explosive eigenvalues in the system. With only two eigenvalues outside the unit circle the equilibrium exhibits one degree of indeterminacy. The results of the simulation are depicted in Figure 1.

The simulation establishes that a necessary condition for a unique equilibrium is that the monetary policy parameter $\psi > 1$. Following the terminology established above this case corresponds to an active policy. Correspondingly, for $\psi < 1$, the economy exhibits at least one degree of local indeterminacy (i.e.the coincidence of two explosive roots and three jump variables).

What is surprising, however, is that the degree of price stickiness, as measured by the menu cost parameter φ, plays a crucial role in determining determinacy. For high enough values - in the benchmark simulation, $\varphi > 8.52$ - the economy can exhibit one degree of indeterminacy even when monetary policy is active. With increases in the inflation coefficient ψ, the boundary between the determinacy regions increases monotonically.[10] In empirical

[10] This patterns persists over a wider range of the parameter space. I performed simulations for extremely large values of the policy parameter, e.g. $\psi = 100$, which effectively amounts to a policy of pegging the inflation rate. In this case the equilibrium remains determinate for all reasonable values of φ.

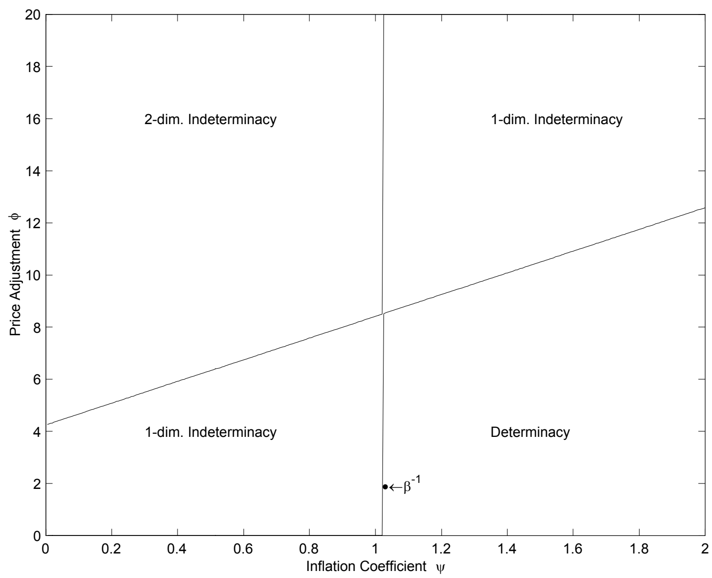

Figure 1: Regions of Determinacy

studies of monetary policy procedures ψ is typically found to lie in the neighborhood of 1.5, for which the boundary value is $\varphi = 10.50$. Interestingly, some estimates of actual costs of price adjustment locate φ near 10 (for instance, Kim, 2000), which is just inside the determinacy region. There is thus the real possibility that typical monetary policy behavior in a generally accepted sticky-price model may actually lead to aggregate instability in the sense of local equilibrium indeterminacy.

What is the intuition behind this result? In an economy with costs of price adjustment nominal shocks can have expansionary effects. In the present model, the transmission mechanism for nominal shocks is essentially the elasticity of output demand ε_t (see equation (12)), which in turn depends on current and future inflation rates. Price stickiness, measured by the size of the parameter φ, thus amplifies the response of the economy to shocks. As pointed out by Benhabib and Farmer (1999), this output amplification-effect, inherent in the New Keynesian framework, shares some similarities with models of increasing returns-to-scale and externalities.

The present model economy is characterized by two types of distortions: monopolistic competition and sticky prices. The larger the deviation from Pareto-optimal perfect competition, the more beneficial is an expansionary shock. In other words, the output response of a monetary relaxation will be relatively larger in a more distorted economy. The degree of this kind of distortion is measured by the steady state mark-up $\frac{\nu}{\nu-1}$. $\nu \to 1$ approxi-

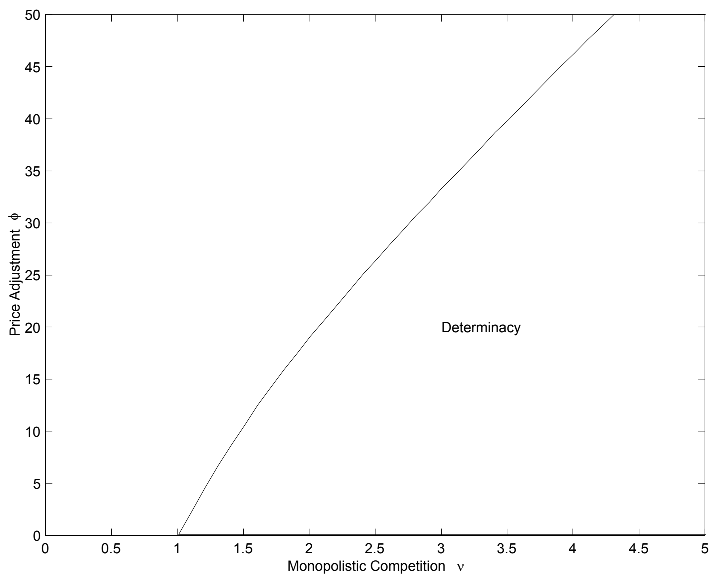

Figure 2: Equilibrium Determinacy for $\psi = 1.5$

mates the case of a monopoly. However, without some form of nominal rigidity, monetary policy shocks have no real effects. By introducing costs of price adjustment, the mark-up $\mu_t = \left(1 - \frac{1}{\varepsilon_t}\right)^{-1}$, with ε_t given by (12), becomes time-varying. The higher the degree of stickiness, as measured by φ, the larger are the output effects of monetary shocks. As a consequence, the distorted economy is more sensitive to shocks the lower the degree of competition and the more rigid goods prices are.

This presumption is borne out by a numerical simulation. Figure 2 depicts the relationship between ν and φ for a given inflation coefficient $\psi = 1.5$. It can be seen that for a low degree of competition even small menu costs lead to indeterminacy. For empirically realistic values of ν, however, prices have to be considerably rigid.[11]

A reading of related work, such as Carlstrom and Fuerst (2000b) and Dupor (2001), reveals markedly different determinacy conditions. The former authors argue that with the type of Taylor-rule used in the present paper determinacy virtually never obtains, while a simple backward-looking rule, such as $R_t = \psi\left(\pi_{t-1}\right)$, produces determinacy for $\psi > 1$. Dupor, on the other hand, finds that only strictly passive monetary policies imply uniqueness. The discrepancies in these results can mainly be explained by different modeling assumptions. Dupor uses a continuous time model with a money-in-the-utility func-

[11] A sensitivity analysis shows that for inflation coefficients above 2.5 monetary policy is sufficiently active so that the indeterminacy problem associated with the distortions disappears.

tion specification and Calvo (1983)-pricing to introduce stickiness. Carlstrom and Fuerst (2000b), however, point out that Dupor's timing conventions, in particular a continuous-time, current-looking rule translates into a discrete backward-looking rule. Furthermore, they argue that continuous-time Calvo-pricing corresponds to a discrete-time pricing equation that implies price stickiness only in future periods.

Similarly, Carlstrom and Fuerst's use of cash-in-advance timing, that is, the amount of cash necessary for making purchases is money balances available at the beginning of the period, essentially rules out determinacy with current-looking rules. Their model implies the arbitrage condition (see Carlstrom and Fuerst, 2000b, p.13):

$$\frac{R_{t+1}}{\pi_{t+1}} = r_{t+1} + (1-\delta),$$

which by using a backward-looking rule leads to the same relationship between inflation dynamics and r_{t+1} as derived in the present paper with reference to a current-looking rule.

I summarize the results from this section in the following conclusion.

Conclusion 1 *In a monopolistically competitive economy with nominal price stickiness and instantaneous investment adjustment a necessary and sufficient condition for equilibrium determinacy is a strictly active monetary policy for which $\psi > 1$, if real and nominal distortions are small enough.*

3.2 Impulse Response Analysis

Having established conditions on equilibrium determinacy I now present impulse response functions to gain further insight into the model dynamics. I assume that the economy is subject to stochastic technology shocks and a monetary policy shock[12]. The stochastic model is solved using the methods described in Sims (2000). I discuss the resulting model dynamics both in the case of a unique equilibrium as well as under indeterminacy. In the latter case it is possible to construct sunspot equilibria.

Consider first the effects of a 1% productivity shock when policy is active ($\psi = 1.5$) and the degree of price stickiness is $\varphi = 10$. The impulse response functions are shown in Figure 3. A purely temporary increase in total facor productivity raises output proportionally more since labor supply increases due to a higher real wage. The additional labor demand

[12] The technology shock is introduced as a multiplicative level shock in the production function. It is assumed to be AR(1). The monetary policy shock enters the (linearized) policy function additively. That is, $y_t = A_t k_t^{1-\alpha} n_t^{\alpha}$ with $\log A_t = \rho \log A_{t-1} + \varepsilon_t^A$; and $\widetilde{R}_t = \psi \widetilde{\pi}_t + \varepsilon_t^R$ and $\varepsilon_t^A, \varepsilon_t^R$ *i.i.d*

comes about because firms decide to lower prices. Since prices are sticky, however, mark-ups become time-varying. The positive technology shock thus increases the elasticity of output demand ε. This allows firms to lower their optimal mark-up below its steady state level, which amplifies the expansion.

Since the existing capital stock cannot change for one period, the marginal productivity of capital rises according to (10). The rise in the current real rate leads to a jump in present consumption (equation (5)). From the arbitrage condition: $\frac{R_t}{\pi_{t+1}} = r_{t+1} + (1-\delta)$, this implies that the expected inflation rate has to decline to increase the current return on holding bonds.[13] The sudden drop in inflation of 3 percentage points prompts the monetary authority to lower the nominal rate by more than 400 basis points. Since policy is active, the expected real rate consequently declines. From the linearized Euler-equation (5): $\widetilde{c}_{t+1} - \widetilde{c}_t = \beta r \widetilde{r}_{t+1}$, this implies that consumption growth has to be negative in all periods after the initial shock which is a requirement for determinacy: In the period when the shock hits the economy jumps onto the saddle path and adjustment dynamics to the steady state are smooth thereafter. In the next period, new capital is put in place, and the marginal productivity of capital falls.

Figure 4 depicts impulse response functions when the technology process is highly persistent ($\rho = 0.9$). The impact effect is qualitatively the same as for the temporary shock, with two important differences, however. The mark-up initially increases, then continually falls towards and below its steady state level. This behavior is due to the optimal pricing behavior by the monopolistically competitive firms. Because of the known persistence of the technology shock and it's long-lasting effect on output, firms continue to cut prices until about 10 periods after the shock. The expectation of future price cuts - and, other things being equal, the loss of revenue - leads firms to increase their mark-up to extract surplus from consumers. The process is reversed when inflation starts to increase again. Secondly, because of the persistent increase in the mark-up the pricing decisions of the firms and the higher production, inflation and the nominal rate remain roughly unchanged for a period before declining.

[13] Bond holders therefore experience an additional windfall arising due to a revaluation effect on nominal assets. Note, however, that this windfall is appropriated in general equilibrium by the government via imposing the lump-sum tax τ. Furthermore, the desire to increase current consumption leads to excess demand for government bonds. Since these bonds are in effect never traded - asset markets are complete internally -, the excess demand is whittled away by a price increase: the expected yield on bonds declines. This again reflects the fact that monetary policy is active: a decline of the interest rate lowers the expected real rate.

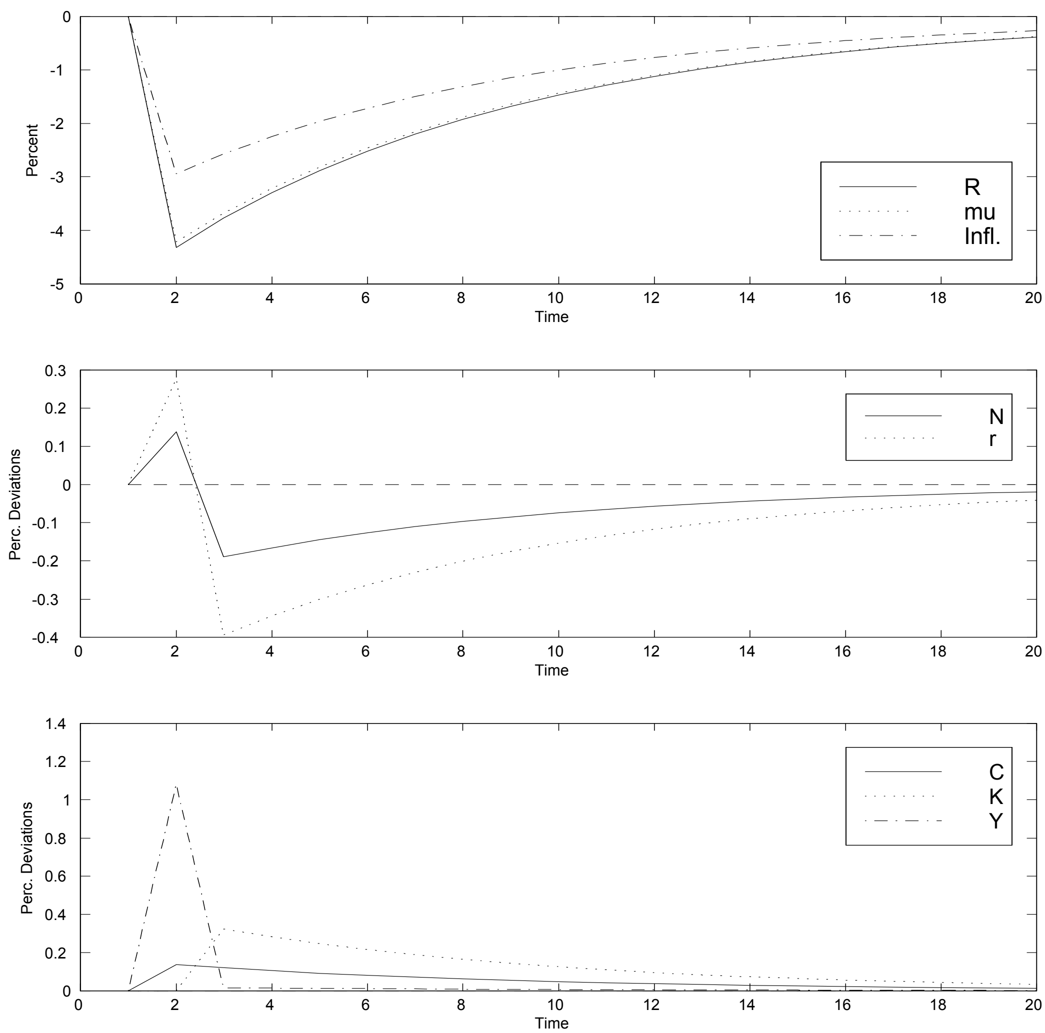

Figure 3: Impulse Response Functions to a Temporary Productivity Shock under Active Monetary Policy

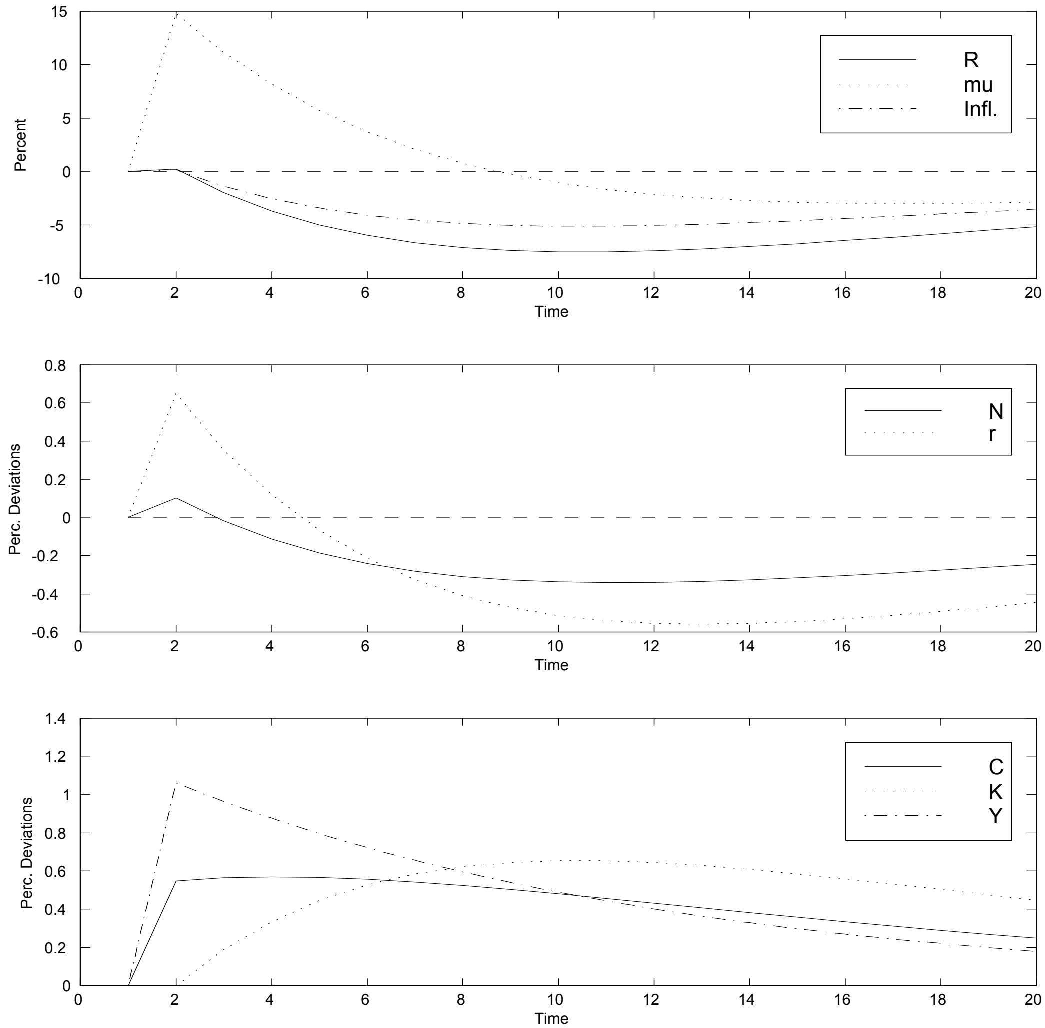

Figure 4: Impulse Response Functions to a Persistent Productivity Shock under Active Monetary Policy

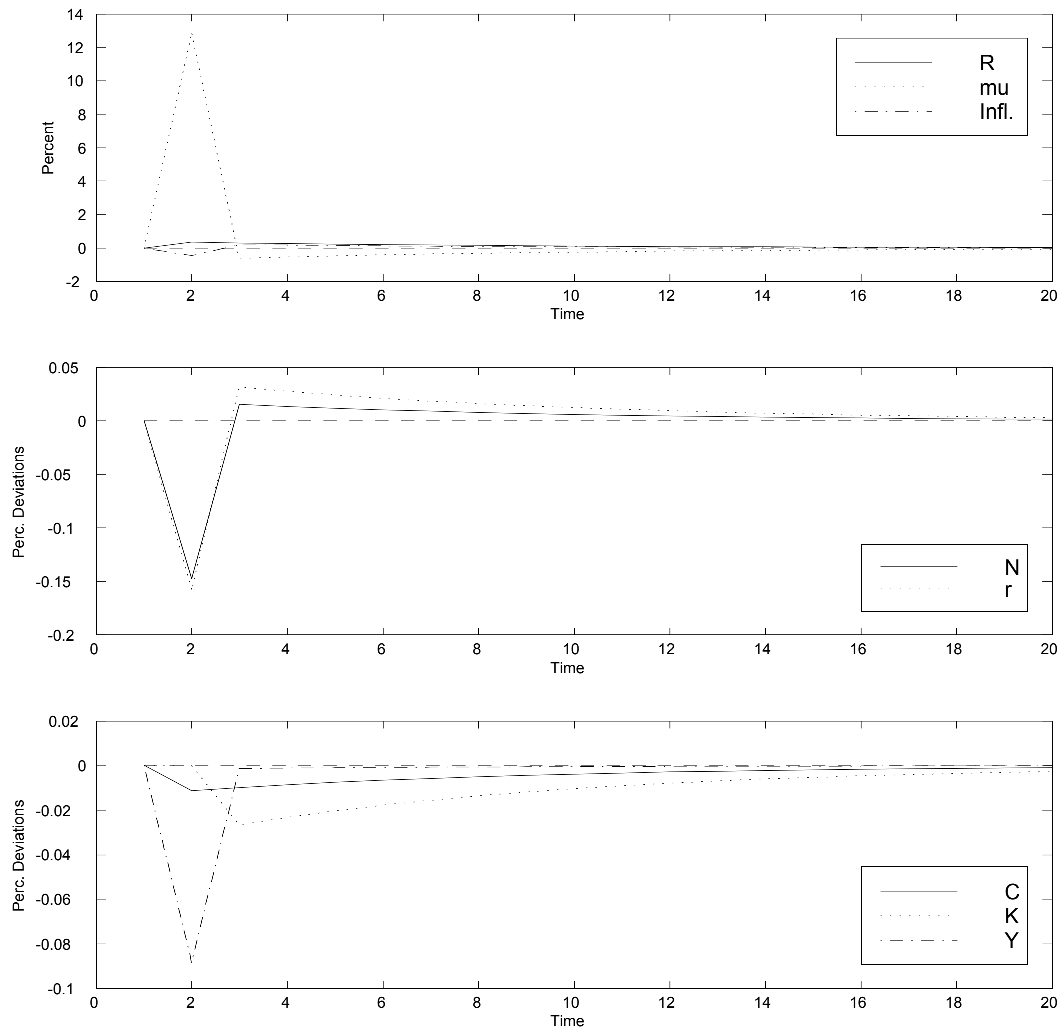

Figure 5: Impulse Response Functions to a Monetary Policy Shock under Active Policy

I now turn to an analysis of the case of random shocks to the monetary policy rule which can be interpreted as implementation errors of open market operations. The associated thought experiment is as follows. Within a period, the monetary authority realizes that conditions on the market for short-term funds[14] are such that the interest rate is out of line with the underlying inflation target. This may be due to a money demand shock. The monetary authority consequently adjusts the interest rate. Since target and instrument variables are both endogenous there is a feedback effect from the interest rate to inflation, and the equilibrium response of the former need not be the same as the initial shock.

Figure 5 shows the impulse response functions to a 1% shock to the interest rate. An

[14]For instance, the federal funds market for overnight loans between financial institutions in the US.

interest-rate hike is clearly contractionary; consumption, investment and output contract on impact. The active policy response raises the expected real rate so that current savings increase. The decline in demand prompts firms to lower mark-ups and lower prices. Since capital cannot change for one period, the current real rate falls, firms hire fewer workers, and aggregate production declines. Since consumption (and demand) is expected to increase along the adjustment path firms raise prices and lower their mark-ups in subsequent periods.

As demonstrated above, when monetary policy is passive the equilibrium is indeterminate. In a stochastic economy, we can therefore construct sunspot equilibria. Since the behavior of the endogenous forecast errors is not completely pinned down, the introduction of an extrinsic source of uncertainty, which is unrelated to the fundamental disturbances in the model, can lead to self-fulling, endogenous fluctuations. If the sunspot variable is *i.i.d.* then the resulting equilibrium, if it exists, is a rational expectations equilibrium. For descriptive purposes I assume that the realization of a sunspot affects inflation expectations.[15]

I now consider the case of passive monetary policy ($\psi = 0.5$). In the absence of shocks, the equilibrium of the economy is trivially the steady state. With indeterminacy, however, we can construct equilibrium dynamics outside of the steady state. Suppose that initially the economy is in steady state. If the equilibrium is determinate any expected inflation rate that is not equal to steady state inflation can be ruled out since at some point the first-order conditions would be violated. With indeterminacy, however, the realization of a sunspot variable leads to endogenous fluctuations. Let us assume that agents believe the inflation rate to be 5% instead of zero.[16] The resulting impulse response functions for inflation sunspot dynamics are shown in Figure 6.

The monetary authority accomodates the sunspot increase in inflation by raising the nominal interest rate. Note that since policy is passive inflation increases by more than the interest rate. Because of the sunspot firms believe that prices should be higher than they actually are (i.e. at their steady state levels) which encourages them to lower the mark-up. This increases aggregate demand, and output, consumption, labor supply, and the real interest rate rise. Since the decline in the marginal productivity of capital is not reversed in the period after the shock, current investment is stimulated and output continues to be above its steady state level. Consequently, consumption continues to rise, and the sunspot belief is validated ex-post.

[15] Sunspot equilibria can be constructed for the other endogenous variables in a similar way.

[16] That is, $\pi^S = 1.05 > \pi = 1.00$, which translates (approximately) into $\widetilde{\pi}^S = 0.05$.

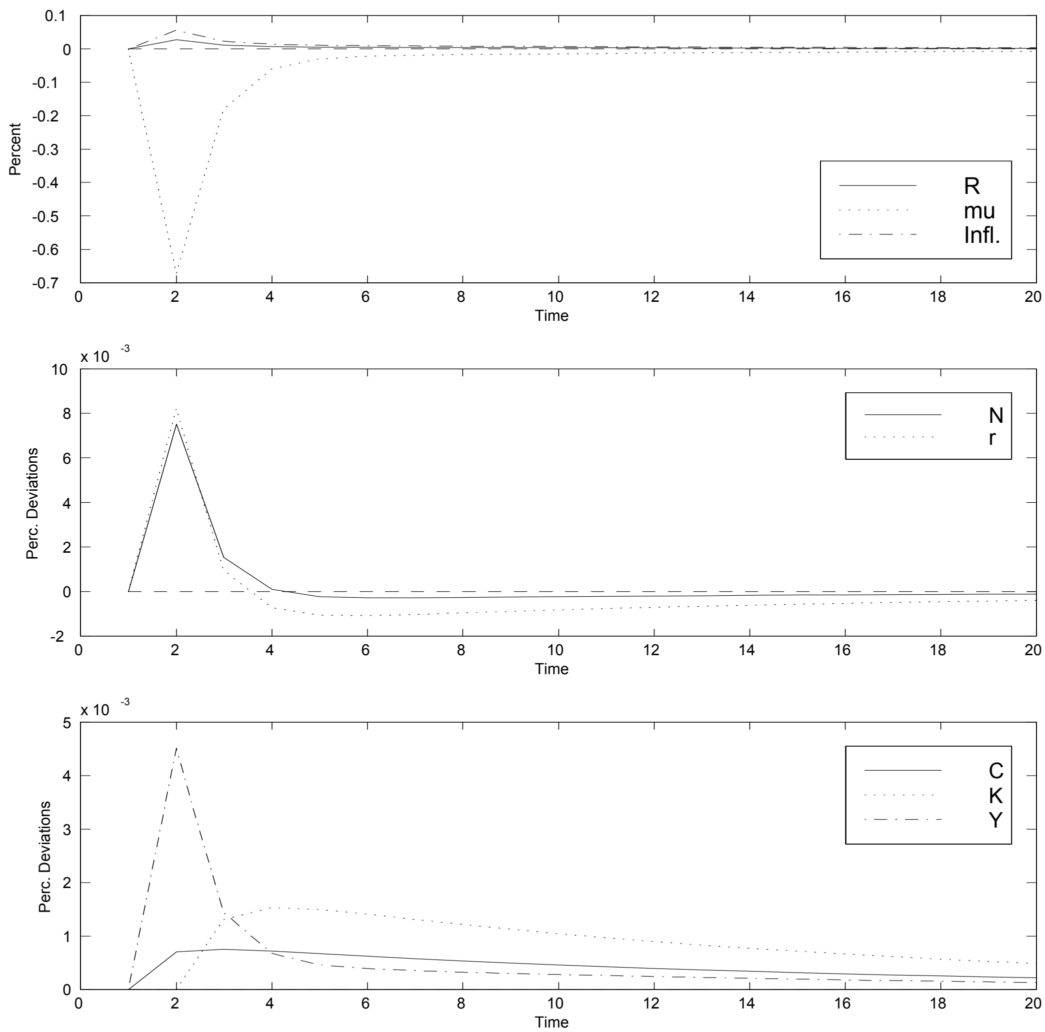

Figure 6: Impulse Response Functions to a 5% Sunspot Shock in Inflation

With passive monetary policy, the real return on nominal debt declines. By force of the arbitrage condition, the expected real interest rate remains above its steady state level. Consumption and expected consumption growth will therefore be positive in the period after the shock, which violates the determinacy condition. Under an active policy, the monetary authority would raise the interest rate by enough so that the expected real rate falls. This in turn would depress investment and production, and firms would find it optimal to lower prices. But this violates the initial assumption of a sunspot belief of a positive inflation rate.[17]

To summarize, in a monetary business cycle model with quadratic price adjustment costs a unique and determinate equilibrium exists if monetary policy follows an active inflation targeting rule, and if the degree of real and nominal distortions is not too large. Although this result is in line with other recent contributions[18], it should be qualified by reference to monetary business cycle models, for instance Kim (2000), that put a quantitatively reasonable value for φ in the neigborhood of 10. Given specific structural characteristics of the technology, this could mean that determinacy is a knife-edge case. In order to determine how robust this result is, I analyse two extensions to the model. First, I introduce adjustment costs in investment in the next section. Secondly, I consider a wider class of fiscal policy functions in Section 4.

3.3 Introducing Investment Adjustment Costs

From a business cycle perspective adjustment costs are important since they reduce the 'excess' volatility of investment. Furthermore, as Dupor (2002) has shown, adding sluggish investment reverses the determinacy properties of an economy with instantaneous adjustment. Assuming that the representative agent owns the capital stock and makes investment decisions, the capital accumulation constraint (2) can be modified for adjustment costs:

$$k_t = \Phi\left(\frac{i_{t-1}}{k_{t-1}}\right) k_{t-1} + (1-\delta)k_{t-1}, \tag{17}$$

The adjustment cost function Φ has the following properties: $\Phi(\delta) = \delta, \Phi(0) = 0, \Phi'(\delta) = 1$, where $\delta = \frac{i}{k}$, the steady state investment-capital ratio. These assumptions imply that the steady state of the model with adjustment costs is identical to that of the previous model.

[17] I also studied the case of indeterminacy when policy is active and price stickiness is high. Unfortunately, this parameter constellation introduces complex eigenvalues which make the interpretation more difficult without adding any new insight. These results are available from the author upon request.

[18] Subject to different modeling conventions employed in alternative models, as was discussed above.

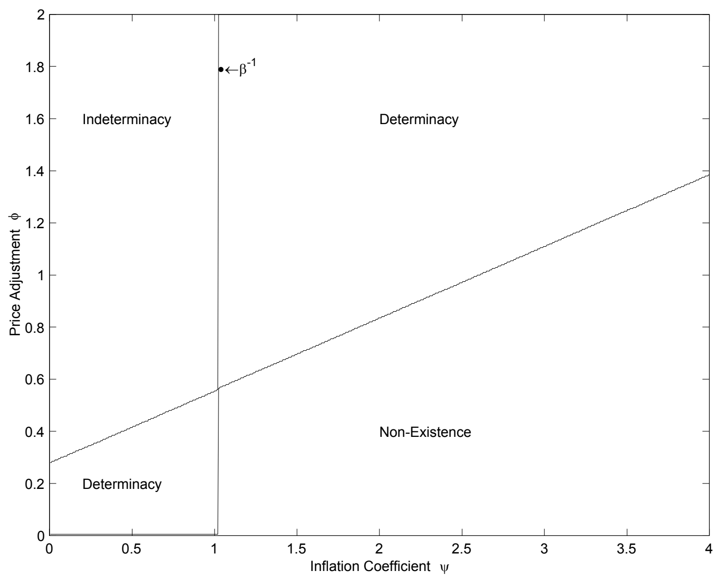

Figure 7: Regions of Determinacy for $\eta = 10$

Introducing adjustment costs modifies the Euler-equation (5) in the following way:

$$c_{t+1} = \beta \Phi' \left(\frac{i_t}{k_t} \right) \left[r_{t+1} + \frac{\Phi\left(\frac{i_{t+1}}{k_{t+1}}\right)}{\Phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)} - \frac{i_{t+1}}{k_{t+1}} + \frac{1-\delta}{\Phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)} \right] c_t. \tag{18}$$

The expected real return to capital is now adjusted for the current output loss due to costs involving the installation of new investment $\Phi'\left(\frac{i_t}{k_t}\right)$, the expected future loss $\Phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)$, and the contribution that newly installed capital makes in reducing adjustment costs. Log-linearizing (18) results in the following equation:

$$\widetilde{c}_{t+1} - \beta r \widetilde{r}_{t+1} + \beta \eta \left(\widetilde{i}_{t+1} - \widetilde{k}_{t+1} \right) = \widetilde{c}_t + \eta \left(\widetilde{i}_t - \widetilde{k}_t \right). \tag{19}$$

Investment dynamics are governed by the parameter $\eta = \frac{\Phi'' \delta}{\Phi'}$, the elasticity of the marginal adjustment cost function. Consumption growth is now no longer a function of the expected real interest rate alone, but it also depends on current and expected investment and capital dynamics. As argued above, determinacy is related to the notion that consumption dynamics are explosive unless the economy is on the stable arm. In the standard model, there is indeterminacy when expected consumption growth (and the expected real rate) is positive along a non-explosive path. With the effective real rate now depending on η the patterns ensuring determinacy may now change.

This suspicion is proven correct by the simulation results that are shown in Figure 7. The region of the parameter space that guarantees a unique equilibrium is divided in two parts.

Active monetary policy leads to determinacy only with sufficiently high price stickiness. For instance, an inflation coefficient of $\psi = 1.5$ requires $\varphi > 0.67$. For smaller degrees of rigidity no equilibrium exists, i.e. the system $z_t = \left[\widetilde{\pi}_t, \widetilde{c}_t, \widetilde{r}_t, \widetilde{k}_t\right]$ contains four explosive eigenvalues. The boundary between these two regions is an increasing and monotonic function mapping ψ into φ. Note, however, that empirically reasonable parameter values are much larger. With investment adjustment costs under an active policy equilibrium determinacy is likely to be the regular case. There is also a small region where a strictly passive monetary policy in combination with very low price stickiness results in a unique equilibrium. A sensitivity analysis[19] reveals that increasing the stickiness parameter φ (or lowering the degree of competition ν), leaves the regions in Figure (7) unchanged. In particular, there are no upper or outer bounds to the determinacy region.

Introducing adjustment costs thus changes the determinacy properties compared to the canonical model from the previous section. The amplification effect, crucial to the indeterminacy results from before, is here superseded by a dampening effect due to adjustment costs in investment. In the four-variable system z_t, an active monetary policy provides the third explosive root required for determinacy. When price stickiness is small, introducing investment adjustment costs adds an additional unstable eigenvalue that converts a previously indeterminate region - associated with passive policy - into a unique equilibrium, while some equilibria under active policy become non-existent.

In the canonical model indeterminacy arises when the economy is sufficiently distorted on account of a high degree of stickiness and a small degree of competition. What changes the situation with adjustment costs is that this dampens the movements of investment. The effective real rate, however, becomes more volatile, which makes an explosive consumption path, and thus determinacy, more likely. In such a setting, an aggressive interest rate policy overshoots its purpose, which result in non-existence. Only when prices become more sticky - and therefore output more volatile -, does an active policy guarantee existence. A similar reasoning applies for a passive monetary policy. Endogenous investment dynamics result in an effective real interest rate so that the consumption path becomes explosive. Monetary policy does not contribute to this, and a passive regime thus ensures determinacy. I summarize the results from this section in the following conclusion:

[19] To conserve space, these results are not reported here, but are available from the author upon request.

Conclusion 2 *In the canonical model with investment adjustment costs active monetary policy guarantees a unique equilibrium if the degree of price stickiness is large enough. Uniqueness also obtains if monetary policy is passive and nominal price rigidity is small enough.*

4 Determinacy and The Interaction of Monetary and Fiscal Policy

In the present model, I assume that the government issues nominal debt that is backed by revenues coming from the primary surplus τ_t and seigniorage $\frac{M_t - M_{t-1}}{P_t}$ (see equation (15)). In many modeling exercises, it is convenient to assume that the government adjusts the primary surplus to keep the budget balanced in every period so that the real value of debt remains constant (see, for instance, Dupor, 2001). This has the advantage that one does not have to keep track of the dynamics of real debt. Although this convention appears innocuous enough, it in fact implies a strong assumption about the government's fiscal policy.

In an important contribution, Leeper (1991) shows that monetary and fiscal policy share joint responsibility in determining a unique equilibrium. If the monetary authority is concerned about price stability and, for instance, follows a simple Taylor-rule monetary growth need not be constant. It follows that revenue from the inflation tax fluctuates. Since seigniorage contributes to the fiscal authority's budget position, this imposes a restriction on the joint movement of the primary surplus and real debt.[20] Monetary and fiscal policy are thus intricately linked via the government budget constraint.

In this section, I consider how taking explicit account of fiscal policy influences the determinacy properties of the canonical model. Following Leeper (1991) I assume a fiscal policy reaction function of the form:

$$\tau_t = \gamma_0 + \gamma_1 \frac{B_{t-1}}{P_{t-1}}. \tag{20}$$

The fiscal authority is said to follow a 'passive' policy when it raises taxes aggressively with changes in real debt, i.e. it automatically adjusts the primary surplus to maintain intertemporal budget balance. Alternatively, fiscal policy is 'active' when taxes are set without giving sufficient consideration to debt dynamics.

[20] Similar reasoning can be applied to derive a link between the behavior of the fiscal authority and monetary policy.

The government has to obey the following budget constraint:

$$\frac{B_t}{P_t} + \frac{M_t}{P_t} + \tau_t = \frac{B_{t-1}}{P_{t-1}}\frac{R_{t-1}}{\pi_t} + \frac{M_{t-1}}{P_{t-1}}\frac{1}{\pi_t}. \tag{21}$$

Note that the presence of nominal assets enables the government to collect seigniorage revenue. Also observe that combining (3) and (21) results in the social resource constraint $c_t + i_t = y_t$. Using (20) to substitute for τ_t in (21), the money demand equation (6), and repeated application of the consumption Euler-equations results in a (reduced-form) government budget constraint that determines the evolution of real debt depending on current and lagged inflation rates only. For given inflation dynamics, the stability properties of this difference equation are determined solely by the coefficients on real debt. The log-linearized budget constraint is:

$$\widetilde{b}_t + (b - m\psi)R\widetilde{\pi}_t = (R - \gamma_1)\widetilde{b}_{t-1} + \left(b - \frac{1+R^2}{R}m\right)\psi\widetilde{\pi}_{t-1}, \tag{22}$$

where $b_t = \frac{B_t}{P_t}$, and b and m are the steady state values of real debt and real balances, respectively.

The stability properties depend on whether $\left|\beta^{-1} - \gamma_1\right| \gtreqless 1$, where $R = \beta^{-1}$. Fiscal policy is passive when $\beta^{-1} - 1 < \gamma_1 < 1 + \beta^{-1}$, and active otherwise. I can therefore append the government budget constraint to the system derived in Section 2.4. This creates a lower triangular system of equations consisting of a block that determines the evolution of the vector $z_t = \left[\widetilde{\pi}_t, \widetilde{c}_t, \widetilde{r}_t, \widetilde{k}_t\right]$ and a single equation block that pins down real debt, given the time path of inflation. Since real debt does not appear in any of the equations that describe z_t, the discussion of local determinacy in this subsystem remains unaffected. With one state variable - the capital stock-, determinacy requires the presence of three roots that lie outside the unit circle, for which a necessary condition is an active monetary policy. Since the dynamics of inflation are determined in the original subsystem, they are exogenous to the equation determining real debt.

Explicitly considering the government budget constraint now adds another state variable. This, however, does not affect the criterion for determinacy: the system still has to contain three unstable roots. If monetary and fiscal policy are such that equation (22) is a stable difference equation then the previous analysis remains unaltered. In fact, assuming that the government rebates any seigniorage revenue as in (15), or as in Dupor (2001, 2002), renders the five-variable system $\widetilde{z}_t = \left[z_t, \widetilde{b}_t\right]$ deficient in that the GBC reduces to an identity which can be dropped from $\widetilde{z}_t$. Implicitly imposing (15), however, assumes an extremely

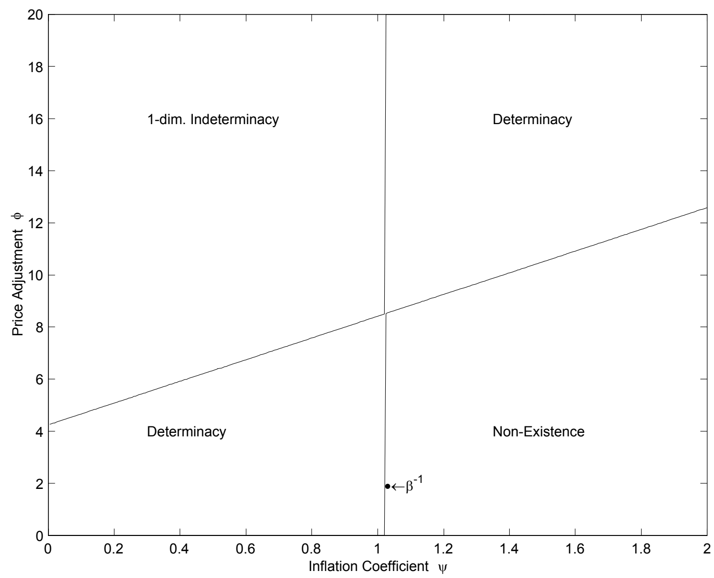

Figure 8: Regions of Determinacy when Fiscal Policy is Active

passive fiscal policy. It requires the government to aggressively raise taxes one-for-one with an increase in the real value of nominal liabilities.

From the analysis in Section 3, we know that active monetary policy assures uniqueness if prices are not too rigid and the degree of competition is large enough. Passive monetary policy, on the other hand, always leads to indeterminacy. If the government budget constraint is a stable difference equation, then adding a fiscal policy function does not change the determinacy properties of the model. Subject to the structural restrictions, the model without capital adjustment costs preserves the dichotomy pointed out by Leeper (1991). Equilibrium determinacy requires active monetary and passive fiscal policy. Note also that passive monetary and fiscal policies lead to at least one-dimensional indeterminacy.

Now suppose that fiscal policy is active, $\left|\beta^{-1} - \gamma_1\right| > 1$, so that the fiscal authority is not concerned about the time path of real debt (see Figure 8). This adds another explosive root to the five-variable system $\widetilde{z}_t = \left[z_t, \widetilde{b}_t\right]$. If the subsystem $\left[\widetilde{\pi}_t, \widetilde{c}_t, \widetilde{r}_t, \widetilde{k}_t\right]$ contains two explosive roots, i.e. it exhibits one degree of indeterminacy, then active fiscal policy renders the whole system determinate. There is a unique equilibrium with passive monetary and active fiscal policy if prices are not too sticky. But there is also a second determinacy region, characterized by higher rigidity, where *both* monetary and fiscal policy are active!

The intuition behind this result can be explained by reference to the argument made in Section 3. Large distortions imply that the economy's output responds very elastically to shocks. A sunspot belief that the inflation rate should be currently above its steady state level is expansionary via a decline in mark-ups unless the monetary authority prevents this by aggressively raising the nominal rate. The expected real rate therefore increases, desired savings rise, and current consumption falls. Expected consumption growth will be positive, which can only be maintained by higher future production. This, in turn, implies that prices will be lower in the future so that the belief cannot become self-validating.

If the output response is potentially very elastic, however, active monetary policy is not enough. The disregard of the fiscal authority for an explosive debt path instead provides the additional condition for uniqueness. If agents belief in rising prices, then this presnts bond-holders with a capital loss on their nominal asset holdings. Consumers would thus try to reduce consumption, which lowers demand. Firms take this as signal to cut prices and raise mark-ups, which runs counter to the initial assumption of a sunspot-driven increase in the inflation rate. When the degree of nominal and real distortions is small enough, then both active monetary and fiscal policies lead to non-existence. Again, this result is reminiscent of Leeper's (1991) assertion. I summarize the results from the previous paragraph in the following conclusion.

Conclusion 3 *In the canonical model without investment adjustment costs, active/passive monetary/fiscal combinations guarantee determinacy if the distortions in the economy are not too large. If price rigidity is high and competitiveness low, then a unique equilibrium only obtains with both active monetary and fiscal policy.*

When capital adjustment costs are introduced, the determinacy conditions change. It was shown in Section 3.3 that active monetary policy implies uniqueness with a menu cost parameter φ that is not too small. Surprisingly, there is also another determinacy region for passive policy with very low rigidity. These conclusions do not change when the fiscal reaction function (20) is introduced, and the fiscal authority is assumed to pursue a passive policy that stabilizes the time path of real debt. The determinacy regions are the same as in Figure 7. The most surprising result is the existence of a unique equilibrium when policies are jointly passive, which runs counter to the argument established in Leeper (1991). The reason behind this has been established before: With investment adjustment costs and small price rigidity, active monetary policy is not needed to guarantee determinacy. Consequently, the path of real debt need not be explosive, which is guaranteed by passive fiscal behavior. Figure 9 depicts determinacy results for active fiscal policy, that is, when an additional

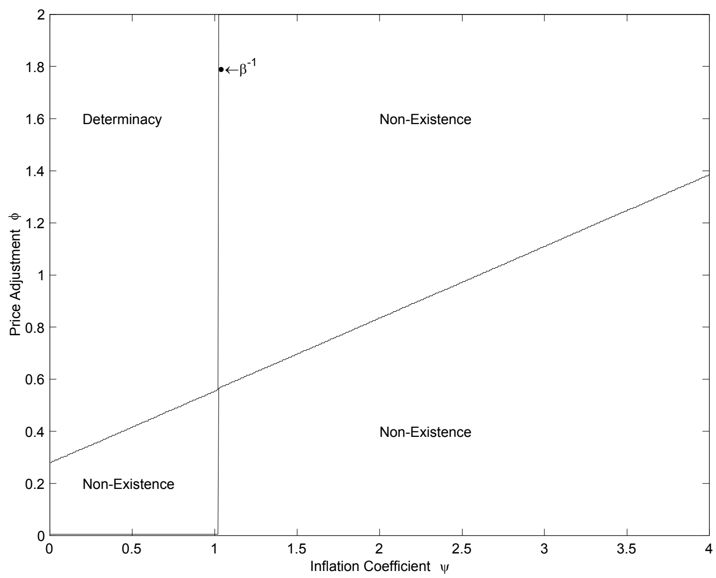

Figure 9: Regions of Determinacy when Fiscal Policy is Active and $\eta = 10$

explosive root is present in the system.

If, on the other hand, fiscal policy is active, the regions of the parameter space that previously guaranteed uniqueness now lead to non-existence. That is, in the model with capital adjustment costs both active monetary and fiscal policies imply non-existence, as does the combination of passive monetary and active fiscal policy when price rigidity is very small. There is now a determinacy region in which monetary policy is passive and fiscal policy active.

Conclusion 4 *In the canonical model with investment adjustment costs, the equilibrium is unique when monetary policy is active (passive) and fiscal policy is passive (active), unless the degree of nominal rigidity is small. In the latter case, determinacy only obtains with active fiscal and passive monetary policies.*

5 Conclusion

This paper contributes to an expanding literature that casts a critical light on the practice of including ad-hoc monetary policy rules in dynamic stochastic general equilibrium. This class of New Keynesian models with nominal rigidities has proven to be very useful in analysing the welfare properties of various policy regimes. Based on substantive empirical evidence that central banks world-wide use various kinds of Taylor-rules, researchers tend to append monetary-policy reaction functions to forward-looking optimizing models. This practice is problematic since the policy rules are typically not derived from optimizing behavior. The feedback from a set of endogenous 'target' variables to an endogenous 'instrument' thus creates the possibility of self-fulfilling expectations and equilibrium indeterminacy.

An immediate extension of this paper is to consider a wider class of target variables which could include deviations of output from some target level, the exchange rate, asset prices, etc. Because of the monopolistically competitive production structure, deviations from the suboptimal steady state output level are welfare-improving and could thus be integrated in a monetary policy framework. Additionally, an extension of the model in this paper to an open economy setting, for instance along the lines of Lubik (2000), seems straightforward. Of particular interest would be to investigate how forward- or backward-looking monetary policy rules change the determinacy properties of this model. Work along these lines by Carlstrom and Fuerst (2000a) shows that all forward-looking rules result in indeterminacy while backward-looking rules produce unique equilibria. This result, however, seems to depend on the timing conventions inherit in the money demand function they use. Lubik and Marzo (2002), in fact, show that for a wide class of monetary policy rules with inflation and output-gap targeting an active monetary policy is a necessary condition for determinacy.

Another interesting extension derives from the observation that a sunspot equilibrium is welfare-reducing. Endogenous fluctuations arise that are not driven by fundamentals, so that agents have to adjust their consumption and leisure decisions, which they would otherwise prefer not to do. A second qualitative observation is that an *i.i.d.* sunspot shock induces a sizeable degree of persistence in real variables as well as in the inflation rate. The work by Farmer (1997) shows that the study of indeterminacy in monetary business cycle models holds promise for matching monetary business cycle facts. A quantitative evaluation of this issue within the present framework is a worthwhile exercise. Given that central banks appear to target only a few variables, indeterminacy could be a defining feature of aggregate economic data. Therefore, it would be interesting to study whether a stochastic optimizing model with sunspot equilibria generated by the presence of policy rules can replicate the stylized business cycle facts.[21]

[21] Schmitt-Grohé (1997) shows that a real business model with endogenous mark-ups or externalities performs as well as the canonical stochastic growth model along this dimension. For further discussion see Benhabib and Farmer (1999).

References

[1] **Benhabib**, Jess and Roger E. A. **Farmer** (1999): "Indeterminacy and Sunspots in Macroeconomics". In: John B. Taylor and Michael Woodford (eds.): *Handbook of Macroeconomics*, Vol. 1A. Elsevier Science B.V.

[2] **Benhabib**, Jess, Stephanie **Schmitt-Grohé**, and Martin Uribe (2001): "Monetary Policy and Multiple Equilibria". *American Economic Review*, **91(1)**, pp. 167-186.

[3] **Blanchard**, Olivier and Charles **Kahn** (1980): "The Solution of Linear Difference Equations under Rational Expectations". *Econometrica*, **48**, pp. 1305-1311.

[4] **Calvo**, Guillermo A. (1983): "Staggered Prices in a Utility-Maximizing Framework". *Journal of Monetary Economics*, **12**, pp. 983-98.

[5] **Carlstrom**, Charles T. and Timothy S. **Fuerst** (2000a): "Forward-Looking Versus Backward-Looking Taylor Rules". Mimeo, Federal Reserve Bank of Cleveland.

[6] **Carlstrom**, Charles T. and Timothy S. **Fuerst** (2000b): "The Role of Investment Spending in Sticky Price Models". Mimeo, Federal Reserve Bank of Cleveland.

[7] **Dupor**, Bill (2001): "Investment and Interest Rate Policy". *Journal of Economic Theory*, **98**, pp. 85-113.

[8] **Dupor**, Bill (2002): "Interest Rate Policy and Investment with Adjustment Costs". Mimeo, The Wharton School, University of Pennsylvania.

[9] **Erceg**, Christopher J, Dale W. **Henderson**, and Andrew T. **Levin** (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts". *Journal of Monetary Economics*, **46(2)**, pp. 281-313.

[10] **Farmer**, Roger E. (1997): "Money in a Real Business Cycle Model". J*ournal of Money, Credit, and Banking*, **29**, pp. 568-611.

[11] **Kim**, Jinill (2000): "Constructing and Estimating a Realistic Optimizing Model of Monetary Policy". *Journal of Monetary Economics*, **45(2)**, pp. 329–359.

[12] **Leeper**, Eric M. (1991): "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies". *Journal of Monetary Economics*, **27(1)**, pp. 129-47.

[13] **Lubik**, Thomas A. (2002): "Industrial Structure and the Effectiveness of Monetary Policy in a Small Open Economy". Mimeo, The Johns Hopkins University.

[14] **Lubik**, Thomas A. and Massimiliano **Marzo** (2002): "An Inventory of Monetary Policy Rules in a Simple New Keynesian Macroeconomic Model". Mimeo, The Johns Hopkins University.

[15] **Lubik**, Thomas A. and Frank **Schorfheide** (2002): "Estimating Monetary Policy Rules in Open Economies: A Structural Approach". Mimeo, The Johns Hopkins University.

[16] **McCallum**, Bennett and Edward **Nelson** (1997): "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis". *NBER Working Paper #* **5875**.

[17] **Schmitt-Grohé**, Stephanie (1997): "Comparing Four Models of Aggregate Fluctuations due to Self-Fulfilling Fluctuations". *Journal of Economic Theory*, **72**, pp. 96-147.

[18] **Sims**, Christopher A. (2000): "Solving Linear Rational Expectations Models". Mimeo, Princeton University.

A Appendix

A.1 First Order Conditions and Steady State

The consumer's dynamic optimization problem in Section 2 can be written in a dynamic Lagrangian form, where the current-value Lagrange-multipliers associated with the constraints (3) and (2) are λ and μ, respectively. The necessary and sufficient first-order conditions with respect to c_t, $\frac{M_t}{P_t}$, n_t, i_t,and $\frac{B_t}{P_t}$ are:

$$\begin{aligned} c_t^{-1} &= \lambda_t, && (23) \\ \frac{\lambda_t}{M_t/P_t} &= \lambda_t - \beta\lambda_{t+1}\frac{1}{\pi_{t+1}}, && (24) \\ 1 &= \lambda_t w_t, && (25) \\ \lambda_t &= \beta\mu_{t+1}, && (26) \\ \mu_t &= r_t\lambda_t + \beta\mu_{t+1}(1-\delta), && (27) \\ \lambda_t &= \beta\lambda_{t+1}\frac{1}{\pi_{t+1}}R_t. && (28) \end{aligned}$$

Similarly, the firm's intertemporal problem can be solved by substituting the firm's demand function (8) and the production function $y_t(j) = k_t^{1-\alpha}(j)n_t^{\alpha}(j)$ into (**??**). The resulting intertemporal profit function can then be maximized with respect to $k_t(j)$ and $n_t(j)$. After some simple, but lengthy algebraic steps, we get an individual firm's profit-maximizing choice of labor and capital inputs. Imposing symmetry, i.e. the assumption that all firms behave identically, the necessary and sufficient first-order conditions of a representative monopolistically competitive producer are given by (10), (11), and (12).

From the first-order conditions, we can calculate the steady state values of the endogenous variables:

$$\begin{aligned} r &= \beta^{-1} - (1-\delta), && (29) \\ c &= \alpha\left(\frac{\nu-1}{\nu}\right)^{1/\alpha}\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}}, && (30) \\ k &= \left(\frac{\nu}{\nu-1}\frac{r}{1-\alpha} - \delta\right)^{-1} c, && (31) \\ n &= \left(\frac{\nu}{\nu-1}\frac{r}{1-\alpha}\right)^{1/\alpha} k, && (32) \\ y &= k^{1-\alpha}n^{\alpha}. && (33) \end{aligned}$$

Furthermore, not that $w = c$, and that $\varepsilon = \nu$. In steady state, profits are given by $\Pi = \frac{1}{\nu}y > 0$.

A.2 Linearization and Model Solution

The model is solved by log-linearizing the first-order conditions around the deterministic steady state. Denote $\widetilde{x}_t = \log x_t - \log x$ as the (approximate) percentage deviation of x_t from its steady state level x. The two Euler equations can be approximated as follows:

$$\widetilde{c}_{t+1} = \widetilde{c}_t + \widetilde{R}_t - \widetilde{\pi}_{t+1}, \tag{34}$$

$$\widetilde{c}_{t+1} = \widetilde{c}_t + \widetilde{r}_{t+1}, \tag{35}$$

which implies the arbitrage condition $\widetilde{R}_t - \widetilde{\pi}_{t+1} = \widetilde{r}_{t+1}$. The capital accumulation constraint is:

$$\widetilde{k}_{t+1} = \delta\widetilde{i}_t + (1-\delta)\widetilde{k}_t. \tag{36}$$

Combining the government budget constraint (15), the firm's profit equation (**??**), and the household's constraint delivers the social resource constraint, which is approximated as:

$$\frac{c}{y}\widetilde{c}_t + \frac{i}{y}\widetilde{i}_t = \widetilde{y}_t, \tag{37}$$

where $\widetilde{y}_t = (1-\alpha)\widetilde{k}_t + \alpha\widetilde{n}_t$. Note that money demand (6) depends on the nominal rate and consumption only. Thus, the equation for real balances is not needed for establishing determinacy. The firm's first order conditions (10), (11) yield (using $\widetilde{w}_t = \widetilde{c}_t$):

$$\widetilde{r}_t + \widetilde{k}_t = \widetilde{c}_t + \widetilde{n}_t. \tag{38}$$

The marginal productivity of capital is:

$$\widetilde{r}_t = \left[(1-\alpha)\frac{\nu-1}{\nu} - 1\right]\widetilde{k}_t + \alpha\frac{\nu-1}{\nu}\widetilde{n}_t + \frac{1}{\nu-1}\widetilde{\varepsilon}_t. \tag{39}$$

The intertemporal price setting equation (12) reduces under linearization to:

$$\widetilde{\varepsilon}_t = \varphi\widetilde{\pi}_t - \beta\varphi\widetilde{\pi}_{t+1}. \tag{40}$$

Finally, the policy rule is:

$$\widetilde{R}_t = \psi\widetilde{\pi}_t. \tag{41}$$

The coefficient matrices of the system $\Gamma_0 z_t = \Gamma_1 z_{t-1}$ with $z_t = \left[\widetilde{\pi}_t, \widetilde{c}_t, \widetilde{r}_t, \widetilde{k}_t\right]$ are then given by:

$$\Gamma_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ \beta\varphi/(\nu-1) & 0 & 0 & 0 \\ 0 & 1 & -\beta r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} \beta\psi_1 & 1 & 0 & 0 \\ \varphi/(\nu-1) & \alpha\frac{\nu-1}{\nu} & 1-\alpha\frac{\nu-1}{\nu} & 1/\nu \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{c+\alpha y}{k} & \alpha\frac{y}{k} & \frac{y}{k}+1-\delta \end{bmatrix}.$$

Γ_0 is full rank so that we can write $z_t = \Gamma_0^{-1}\Gamma_1 z_{t-1}$. The size of the eigenvalues of $\Gamma_0^{-1}\Gamma_1$ determines the stability and determinacy properties of the system z_t. Because of the complexity of the system, there are no analytic expressions for the eigenvalues. I therefore study the stability properties numerically by using the structural representation $\Gamma_0 z_t = \Gamma_1 z_{t-1}$.

The solution of the model proceeds as follows. The coefficient matrix can be diagonalized in a Jordan-form which gives us $\Gamma_0^{-1}\Gamma_1 = C\Lambda C^{-1}$, where Λ is a matrix with the eigenvalues along the principal diagonal and C is a matrix containing the associated eigenvectors. Define $\widetilde{z}_t = C^{-1}z_t$. We then have: $\widetilde{z}_t = \Lambda\widetilde{z}_{t-1}$ with $\widetilde{z}_{it} = \lambda_i\widetilde{z}_{it-1}$. A particular solution to this system of difference equations is given by $\widetilde{z}_{it} = \lambda_i^t\widetilde{z}_{i0}$. If $\lambda_i > 1$ then the solution is only bounded, i.e. non-explosive, when $\widetilde{z}_{i0} = 0$. Note that imposing this restriction is equivalent to the requirement that the system moves along the 'stable' arm. Without loss of generality, let $\lambda_1 < ... < \lambda_4$, and $\Lambda_{ii} = \lambda_i, i = 1, ..., 4$. The solution to the system is then given by:

$$z_t = C\widetilde{z}_t = C\begin{bmatrix} \lambda_1^t\widetilde{z}_{10} \\ \lambda_2^t\widetilde{z}_{20} \\ \lambda_3^t\widetilde{z}_{30} \\ \lambda_4^t\widetilde{z}_{40} \end{bmatrix} \tag{42}$$

We can describe a determinate equilibrium as governed by:

$$z_t = C\begin{bmatrix} \lambda_1\widetilde{z}_{1t-1} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{43}$$