1 Data Construction

The Survey of Professional Forecasters (SPF) data are available from the Philadelphia Federal Reserve website. I downloaded their Excel file SPFmicrodata.xlsx. Owing to my poor Matlab skills, I had to replace all of the #N/A observations in that Excel file with blanks. I renamed this file SPFmicrodata_relabelMissing.xlsx. It is on my webpage. Also on my webpage is an Excel file dailyGSWdata.xlsx. As the name suggests, it contains daily yields constructed by Federal Reserve Board staff using the technique of Gurkaynak, Sack, and Wright (2007). As the name does not suggest, it also contains daily three-month Treasury bill yields. These are from the Fed’s H15 release, available on the FRED database. Moreover, this Excel file has daily value-weighted aggregate stock returns.

The Matlab code construct SPFLevelMeasures.m on my website uses the SPF data to construct consensus forecasts of levels of real GDP and levels of the GDP price index. These are stored in the Excel file gdp_price_levelSurveyFebruary2021.xls, also on my website. The Matlab code constructSPFGrowthRateInnovsAndRelatedData.m uses these data, along with the data in dailyGSWdata.xlsx, to construct mid-quarter observations of macroeconomic news, excess stock returns, and changes in bond yields.

Mid-quarter ex ante real one-year yields are defined as mid-quarter nominal one-year yields less expected annual inflation. Mid-quarter expected annual inflation for quarter \( t \) is the consensus forecast of the log price level at quarter \( t + 4 \) divided by the consensus forecast of the log price level at quarter \( t \). There are five observations in the database with missing consensus forecast of the four-quarter-ahead price level. These missing observations are replaced with the consensus forecast of the three-quarter-ahead log price level plus the consensus forecast of the change in the log price level from \( t + 2 \) to \( t + 3 \). In other words, for these missing five observations, I assume a constant expected inflation rate from quarter \( t + 2 \) to quarter \( t + 4 \). All of these steps are in the Matlab code on my website.
2 Estimation

The Matlab code I use to construct all of the tables and figures is on my website. Standard errors are constructed by treating everything as a GMM estimation. For example, Table 2 tests whether the covariances are stable across the two periods. The moments are the quarterly products of residuals less $\rho_E$ in the early period and less $\rho_E + \delta$ in the later period. GMM is used to test the hypothesis that $\delta = 0$.

3 Inflation Variance Ratios in Other Papers

Here I describe how I calculate inflation variance ratios for estimated models in three papers. Inflation variance ratios are defined in Duffee (2018).

3.1 Song (2017)

In Song (2017), Dongho calculates inflation variance ratios for each of the three estimated regimes. They are reported in the paper’s E-7. Dongho graciously provided me with the numerators and denominators for these ratios, in the accompanying Excel spreadsheet Table E-7(Duffee).xlsx. The spreadsheet reports variances of monthly innovations in yields and expected inflation measured in decimal points per month. In the spreadsheet I convert the units to quarterly innovations in yields and expected inflation measured in percent per year. In Table IA1 I report only the values for a five-year bond. The spreadsheet also has values for one-year through four-year bonds.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Std Dev of Inflation News (b.p.)</th>
<th>Std Dev of Yield Innovations (b.p.)</th>
<th>Variance Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countercyclical/Active Fed</td>
<td>79</td>
<td>71</td>
<td>1.22</td>
</tr>
<tr>
<td>Countercyclical/Passive Fed</td>
<td>104</td>
<td>99</td>
<td>1.12</td>
</tr>
<tr>
<td>Procyclical/Active Fed</td>
<td>44</td>
<td>54</td>
<td>0.65</td>
</tr>
</tbody>
</table>
3.2 David and Veronesi (2013)

In David and Veronesi (2013) there are $n = 6$ states of the economy, unobserved by agents. Agents have subjective probabilities $\pi_t$ of the time-$t$ state, where $\pi_t$ is a length-$n$ vector that sums to one. The Ito process for the subjective probability vector is in their equation (6), written here:

$$d\pi_t = \Lambda'\pi_t dt + \Sigma\pi_t d\tilde{W}_t.$$  (1)

The $n \times n$ generator matrix $\Lambda$ contains the instantaneous probability $\lambda_{ij}$ of transitioning from state $i$ to state $j$.

As shown in their equation (16) and discussed in their Appendix A, nominal bond prices are analytic functions of maturity and the subjective probabilities of the states. I use their reported parameters to calculate the analytic function for a five-year bond. In anticipation of a discussion below, I include their (16) here, in slightly revised form. Denoting the nominal state-price density by $N_t$, the bond price is

$$B(\pi_t, \tau; \theta) = E_t\left(\frac{N_{t+\tau}}{N_t}\right) = \pi'_t B(\tau; \theta),$$  (2)

where $B(\tau)$ is a length-$n$ vector that depends on the maturity $\tau$ and the model parameters $\theta$. Denoting by $r_i$ the instantaneous nominal interest rate (negative of the drift of the relative state-price density) conditional on investors knowing the true state is $i$, the vector $B$ is given by

$$B(\tau; \theta) = e^{(\Lambda - \text{diag}(r_i))\tau}1_{n \times 1}.$$  (3)

Matrix exponentiation is used in (3), as described in the paper’s equation (A5).

I discretize the Ito process (1) to generate a long time series of simulated probability vectors, one quarter apart. I then combine the analytic function for the five-year nominal bond price with these simulated probabilities to produce a long time series of five-year bond yields observed one quarter apart.

In line with the empirical measure of innovations in long-term bond yields, I define the one-quarter innovation as the first-difference in this time series. The unconditional variance of the innovation is the variance of the simulated innovations. I also calculate variances conditional on the subjective probability at $t$ for state $j$ exceeds 0.5. For example, consider all dates $t$ for which $\pi_{1,t} > 0.5$. Using only these dates, calculate the variance of the subsequent change in the five-year bond yield.

The paper’s notation for the price level at $t$ is $Q_t$. I define the time-$t$ expected log inflation rate, measured at an annual frequency, from $t + \tau_1$ to $t + \tau_2$, $t \leq \tau_1 < \tau_2$ as

$$E_t(\text{infl}_{t+\tau_1,t+\tau_2}) \equiv \frac{1}{\tau_2 - \tau_1} (\log E_t(Q_{t+\tau_2}) - \log E_t(Q_{t+\tau-1})).$$  (4)
The special case of the time-$t$ expectation of the gross inflation rate from $t$ to $\tau + \tau$ is

$$E_t(infl_{t,t+\tau}) \equiv \frac{1}{\tau} \log \left( \frac{E_t(Q_{t+\tau})}{Q_t} \right).$$

(5)

Note that (4) can be calculated with two applications of (5):

$$E_t(infl_{t+\tau_1,t+\tau_2}) \equiv \frac{1}{\tau_2 - \tau_1} \left( \log E_t \left( \frac{Q_{t+\tau_2}}{Q_t} \right) - \log E_t \left( \frac{Q_{t+\tau_1}}{Q_t} \right) \right).$$

(6)

Equation (6) is solved using the same math used to solve (2) and (3). Replace the relative drift of the nominal state-price density used in (3) with the relative drift of the price level. Equation (4) in DV denotes this relative drift by $\beta_t$. The result is

$$E_t \left( \frac{Q_{t+\tau}}{Q_t} \right) = \pi_t' Z(\tau; \theta),$$

(7)

$$Z(\tau; \theta) = e^{(\Lambda + \text{diag}(\beta_t))\tau} 1_{n \times 1}.$$  

(8)

An estimate of the $n$-vector $\beta$ is in the paper.

I calculate one-quarter inflation innovations for the simulated data. At one observation I calculate the forecast of inflation from $t + (1/4)$ to $t + (5/4)$. At the next quarterly observation, which is at $t + (1/4)$, I calculate the new expectation of inflation from $t + (1/4)$ to $t + (5 1/4)$. The difference between them is the inflation innovation. Unconditional and conditional variances are calculated following the description for the five-year bond yield.

The Matlab code DV_calculateInflationVarnfo.m, available on my website, produces the results. It calculates the Table IA2 below.

Table IA2. Standard deviations of news about expected inflation and yield innovations

<table>
<thead>
<tr>
<th>Conditioning Information</th>
<th>Std Dev of Inflation News (b.p.)</th>
<th>Std Dev of Yield Innovations (b.p.)</th>
<th>Variance Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconditional</td>
<td>0.28</td>
<td>0.47</td>
<td>0.35</td>
</tr>
<tr>
<td>Regime 1</td>
<td>0.18</td>
<td>0.29</td>
<td>0.38</td>
</tr>
<tr>
<td>Regime 2</td>
<td>0.39</td>
<td>0.66</td>
<td>0.35</td>
</tr>
<tr>
<td>Regime 3</td>
<td>0.26</td>
<td>0.43</td>
<td>0.36</td>
</tr>
<tr>
<td>Regime 4</td>
<td>0.58</td>
<td>1.04</td>
<td>0.31</td>
</tr>
<tr>
<td>Regime 5</td>
<td>0.14</td>
<td>0.22</td>
<td>0.40</td>
</tr>
<tr>
<td>Regime 6</td>
<td>0.27</td>
<td>0.48</td>
<td>0.32</td>
</tr>
</tbody>
</table>
3.3 Campbell, Pflueger, and Viceira (2020)

Like the model of Song (2017), the model in Campbell et al. (2020) uses discrete time. Periods are quarters of a year. I first set up some notation.

Denote the quarter-\( t \) annualized yield on an \( m \)-quarter bond as \( y_t^{(m)} \). As of \( t-1 \), this yield is a random variable with expectation \( E_{t-1}y_t^{(m)} \). At quarter \( t-1 \) there is also an expected average annualized log inflation rate from quarter \( t \) to quarter \( t+m \). This is the mean of the annualized one-quarter log inflation rates realized at \( t+1, \ldots, t+m \).

At quarter \( t \), the bond yield is observed and the inflation expectation is updated. The inflation variance ratio for an \( m \)-quarter bond is the unconditional variance of the inflation news divided by the unconditional variance of the yield innovation.

I calculate the numerator using the model description in the paper, the paper’s appendix and the Matlab code on the JPE website. From the paper’s equation (19), inflation from \( t-1 \) to \( t \) is the sum of a transitory component and a martingale component,

\[
\pi_t = \hat{\pi}_t + \pi_t^*.
\]

The martingale component has dynamics from the paper’s equation (22),

\[
\pi_t^* = \pi_{t-1}^* + v_t^*.
\]

The paper’s length-three state vector is \( \hat{Y}_t \), defined in the paper’s equation (26). The transitory component of inflation is the second element. Define an augmented, length-four state vector

\[
\hat{Y}_t^* \equiv ( \hat{Y}_t' \ \pi_t^* )'.
\]

Write the dynamics of this augmented state vector using the version of the dynamics of \( \hat{Y}_t \) in equation (D.15) of the appendix. The dynamics are

\[
\hat{Y}_t^* = B_*\hat{Y}_{t-1}^* + Q_*\mu_t,
\]

\[
B_* \equiv \begin{pmatrix} B & 0_{3\times1} \\ 0_{1\times3} & 1 \end{pmatrix}, \quad Q_* \equiv \begin{pmatrix} Q \\ e_3' \end{pmatrix} \mu_t.
\]

In (13), \( e_3 \) is a length-three basis vector with one in the third position. By the definition of \( \mu_t \) in Appendix D.1.1, the final element in \( \mu_t \) is \( v_t^* \).

The expectation of inflation at quarter \( t+k \) conditional on \( Y_t^* \) is, using standard VAR mathematics,

\[
E_t(\pi_{t+k}) = c_\pi' E_t(Y_{t+k}^*) = c_\pi' B_*^k Y_t^*,
\]

5
where
\[ c_\pi = \begin{pmatrix} 0 & 1 & 0 & 1 \end{pmatrix}'. \]
The innovation from \( t - 1 \) to \( t \) in expected average inflation from \( t \) to \( t + m \) (again, the average of \( \pi_{t+1} \) through \( \pi_{t+m} \)) is, using (14),
\[ \eta_{t,\pi}^{(m)} = \frac{1}{m} c'_\pi \left( \sum_{k=1}^{m} B_k^* \right) Q_* \mu_t \]
\[ (15) \]
The variance of this innovation is
\[ \text{Var} \left( \eta_{t,\pi}^{(m)} \right) = \frac{1}{m^2} c'_\pi \left( \sum_{k=1}^{m} B_k^* \right) Q_* \text{Cov} \left( \mu_t \right) Q'_* \left( \sum_{k=1}^{m} B_k^* \right) c_\pi. \]
\[ (16) \]
This variance can be calculated for any \( m \). In practice, I calculate it only for \( m = 19 \) quarters because of the computational challenge of calculating unconditional variances of yield innovations. The paper’s Table 3 reports the model-implied annualized standard deviation of excess log one-quarter returns to a five-year nominal bond. I use this to closely approximate the volatility of the innovation in a 19-quarter bond yield.

The log return to the bond is
\[ \text{exr}_t^{(20)} = -(19/4) y_{t-1}^{(19)} + (20/4) y_t^{(20)}. \]
\[ (17) \]
The innovation in the excess return is, recognizing that the return from \( t - 1 \) to \( t \) on a three-month bill is nonstochastic,
\[ \text{exr}_t^{(20)} - E_{t-1} \left( \text{exr}_t^{(20)} \right) = -(19/4) \left( y_t^{(19)} - E_{t-1} y_t^{(19)} \right). \]
\[ (18) \]
Therefore the variance of the one-quarter innovation in the yield on a 19-quarter bond is
\[ \text{Var} \left( y_t^{(19)} - E_{t-1} y_t^{(19)} \right) = (4/19)^2 \text{Var} \left( \text{exr}_t^{(20)} - E_{t-1} \left( \text{exr}_t^{(20)} \right) \right). \]
\[ (19) \]
Table 3 in their paper reports the model-implied standard deviation of the excess return rather than the standard deviation of the innovation in the excess return. One-quarter-ahead predictability in the excess return implies the former exceeds the latter. Thus plugging in the information from their Table 3 into the right side of (19) will produce a variance on the left side slightly larger than the correct model-implied variance of yield innovations. However, since there is little predictability in this excess return (also documented in their Table 3), this wedge is likely negligible.

My Table IA3, below, reports the results from these calculations. It also reports information about inflation variance ratios for a five-year bond (not a 4 3/4 year bond) using
the methodology of Duffee (2018). That paper estimates inflation dynamics using consensus inflation forecasts from the Survey of Professional Forecasters, assuming expected inflation is the sum of a martingale and an AR(1) component. Yield shocks are calculated assuming the five-year yield is a martingale.

The standard deviations of inflation news in Table IA3 are consistent with other information in the paper. Table G.2 in the paper’s appendix reports the standard deviation of excess returns to a five-year real bond. This standard deviation can be used to infer the standard deviation of innovations in the (4 3/4)-year maturity real bond yield. Then using the approximation that the inflation news is orthogonal to real-rate news, and ignoring any volatility owing to news about inflation risk premia, we can write

\[
\text{Var} \left( \eta_{t,\pi}^{(m)} \right)^{1/2} \approx \left( \text{Var} \left( y_t^{(m)} - E_{t-1} \left( y_t^{(m)} \right) \right) - \text{Var} \left( y_{t}^{\text{real,(m)}} - E_{t-1} \left( y_{t}^{\text{real,(m)}} \right) \right) \right)^{1/2}.
\]

For the early sample, the right side is 58 basis points, almost identical to the 59 basis points in Table IA3. For the late sample, the corresponding approximation is 33 basis points, rather than the 37 basis points in Table IA3.

<table>
<thead>
<tr>
<th>Source</th>
<th>Sample Period</th>
<th>Std Dev of Inflation News (b.p.)</th>
<th>Std Dev of Yield Innovations (b.p.)</th>
<th>Variance Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duffee (2018)</td>
<td>1979Q3–2001Q1</td>
<td>24</td>
<td>74</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>2001Q2–2011Q4</td>
<td>8</td>
<td>45</td>
<td>0.03</td>
</tr>
<tr>
<td>Campbell et al. (2020)</td>
<td>1979Q3–2001Q1</td>
<td>59</td>
<td>66</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>2001Q2–2011Q4</td>
<td>37</td>
<td>40</td>
<td>0.85</td>
</tr>
</tbody>
</table>
References


