

# ***Fair Representation***

*Meeting the Ideal of One Man, One Vote*

SECOND EDITION

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## *Preface*

This book concerns a problem of national policy that is both political and mathematical: how to divide the seats in a legislature fairly according to the populations of federal states or party votes. It is a problem universal to representative systems of government and deals with the very substance of political power. Innocent as it may seem at first glance, it involves technical difficulties that have baffled politicians and mathematicians for more than two centuries. The root of the problem is that a perfectly fair division is impossible to achieve owing to the indivisibility of seats. Inevitably, some states will be over-represented while others will be under-represented. The attempt to find a formula that comes closest to the ideal of one person, one vote forms a fascinating historical and scientific tale.

The issue of what formula comes closest to the ideal surfaces after virtually every census. Not surprisingly, these challenges are mounted by states who are threatened by a loss of seats. In 1981, for example, the Indiana delegation led an effort in the House to switch the formula in order to stave off the loss of one seat, but the effort died in committee. Again in 1992, Montana challenged the constitutionality of the current formula after the 1990 census threatened to reduce its representation from two seats to one. Although this suit was ultimately dismissed, the Supreme Court specifically rejected the government's contention that apportionment is a purely political question that lies outside the purview of the Court [United States Department of Commerce et al. v. Montana et al. 503 U.S. 442 (1992)]. Thus both the legislative and judicial routes remain open to future efforts at reform.

The aim of our book is to establish a solid logical foundation for choosing among the available methods of apportioning power in representative systems. It is an example of mathematical reasoning applied to a problem of public policy. The style of analysis is similar to the axiomatic approach used in mathematics, where the object is to discover the logical consequences of certain general principles. The validity of the approach depends on identifying the right principles as revealed through history, political debate, and common sense.

No one method suits in all respects. For the skeptic, there is even an impossibility theorem to show that the problem cannot be "solved." Nevertheless, seats must be apportioned and are. As Daniel Webster

pointed out 170 years ago, the challenge is not to find a perfectly fair solution—which is impossible—but to come as near perfection as may be. The choice of principles to follow and compromises to accept is, of course, ultimately a question of political legitimacy and should be made by a nation’s legislators. Our intent is to clarify the consequences of these choices.

The appeal of the problem lies in its unique combination of history, politics, and mathematics. Although its mathematics are surprisingly challenging, the problem requires no more than simple arithmetic to understand. It is easily accessible to the social scientist, the politician, the student, and the merely curious citizen. To back up the arguments advanced in the text, rather sophisticated mathematical arguments are ultimately required. These are treated separately in an appendix, but even there little more is presumed than some elementary algebra and probability theory—plus a taste for combinatorial reasoning. Appendix A expands and makes explicit the technical points and definitions of the text and contains numerous exercises presented as propositions to be verified. Scientifically inclined readers may wish to read the appendixes in conjunction with the text. The text itself stands on its own.

The first edition of this book was written before the results of the 1980 census were definitively established, hence it only included an analysis of apportionments through 1970. This second edition is essentially unchanged except that Appendix B has been expanded to include results for 1980, 1990 and 2000, and the statistical analysis in the text covers all twenty-two censuses from 1791 to 2000. To avoid unnecessary alterations, however, we continue to use the 1970 census to illustrate various particular points that could equally have been made using more recent census data.

Although apportionment is a problem common to all systems of representative government, we have chosen to study it primarily through the experience in the United States, where it has the longest and richest history. The difficulty of the problem was already clear to the members of the Constitutional Convention in 1787:

The greater the difficulty we find in fixing a proper rule of Representation, the more unwilling ought we to be to throw the task from ourselves on the Gen.<sup>l</sup> Legisl<sup>r</sup>. . . . A Revision from time to time according to some permanent and precise standard [is] essential to ye fair representation.\*

\* George Mason, delegate from Virginia, as quoted by James Madison (Max Farrand, ed., *The Records of the Federal Convention of 1787* [New Haven: Yale University Press, 1911], 1:179).

## CHAPTER 1

# *Apportionment*

*The matter of competition is often Indivisible. An office, or a Mistress, can't be Apportion'd out like a Common.*

JEREMY COLLIER, *Essays Upon Several Moral Subjects*

The ideal of representative democracy—one-man, one-vote—is simple, but to meet it is not. The nations that practice it have tried many models, but in essence they all rely either on one of two systems—a federal system or a proportional representation system—or on a combination of both. In the federal system, the unit of representation is regional: a state or province receives seats according to its population. In the proportional representation system, the unit is political: a party wins seats according to its vote. The United States has a federal system, Israel a proportional representation system, and Germany and Switzerland have combinations that are directly bound both to territory and to party.

Everywhere the stated or unstated understanding is one-man, one-vote. No man should have a greater voice than another: a state should receive a number of representatives in proportion to its population or a party in proportion to its total vote. It seems that to say “proportional” is enough to solve the *problem of apportionment*—to make a precise allocation of seats to states or to parties—but it is not. The difficulty is what to do about the fractions. This has vexed both mathematicians and politicians for hundreds of years.

Some nations resolve the problem in smoke-filled rooms. Some have established nonpartisan commissions to do the job. Others have used mathematical formulas. But rotten boroughs, legal challenges, and good old-fashioned political fights have continuously hounded the problem, for it touches the lifeblood of the politician: his job.

The country that has had the most intense debates over the choice of method to solve the apportionment problem is the United States. The

argument began at the Constitutional Convention in 1787, flared again when the results of the first census were reported in 1791, and was regularly discussed thereafter every ten years.

Apportionment determines the power of the states in Congress and, through the electoral college, directly affects the selection of the president. It determines the number of congressional districts in each state, but it leaves to the states themselves the problem of how to draw the district lines (sometimes called “reapportionment,” sometimes “gerrymandering”). The Constitution makes Congress responsible for fair representation, saying only that it is to be made “according to” the populations of the states. It does not specify any exact rule. The problem of interpreting the Constitution on this point has gone on for nearly two centuries and makes the United States history of apportionment one of the most interesting to study as a case.

Each state is guaranteed at least one representative, but whether a state gets 1 versus 2 or 39 versus 42 is a matter of considerable importance to its influence in national affairs. In 1970, for example, Oregon’s exact share of 435 House seats was 4.500. Should it have gotten 4 or 5 seats? If the number were rounded up to 5 seats, giving its citizens a larger voice than they deserved, another state—say, one with a 4.450 share—might get only 4 seats and therefore less representation than it deserved. One-man, one-vote is in fact a mathematical impossibility.

What is to be done to find a fair solution? This is no small question. The effects of different formulas on Congress’s composition can be dramatic. This may be seen by comparing the first rule used, one advanced by Thomas Jefferson, with another proposed later by John Quincy Adams, and applying them to the 1970 census figures. Jefferson’s method would give New York 41 seats, while Adams’s would give it only 37. Overall the results would differ in 27 states and involve 18 seats.

In virtually every Congress at least one-half of the states have received 7 seats or less, and for those the difference of even one seat is great. These differences depend on which apportionment rule is used. The issue resurfaces with the completion of every census. Important population changes in some states, and so in the balance of power in the House and in the electoral college, have invariably occurred, and will again.

The history of apportionment in the United States teaches two major lessons. The first is the importance of the problem. Political “realists” may belittle the difference of a few seats here or there, yet the effects can be of consequence. One possibility is that the outcome of a presidential election can be changed because of the electoral college, since apportionment determines the number of a state’s electors. Indeed, one has been

changed: the malapportionment of the 1870s was directly responsible for the 1876 election of Rutherford B. Hayes, although his opponent enjoyed 51.6 percent of the vote. Another danger is a significant shift of seats from small rural states to large industrial states—or the reverse—depending upon the method used. The fact is that political realists from Alexander Hamilton in 1792 to Arthur Vandenberg in 1941 have engaged in bitter conflicts over methods of apportionment whose effects were to transfer as few as *one* seat. More fundamentally, of course, the problem is one of political legitimacy: solutions must be acceptable to the nation.

The second major lesson of history is how political acumen—or plain, good common sense—determines whether a method is fair or not. Almost two hundred years of accumulated experience of apportionment are available, spanning nineteen apportionments. This experience provides numbers—the populations and the size of House—on which to test methods. It also provides a collective accumulation of insight into how methods work in practice. Can a method cause a state to lose a seat when the size of the House increases? Can one deprive a state that has grown to benefit a state that has shrunk? Does a method systematically favor the smaller states at the expense of the larger states? Is it fair that New York, with a proportional share by the 1830 census of 38.593, actually received 40 seats? These are questions that have been raised in the face of what some methods actually did and of what one would naturally want them to do.

In short, history and common sense together suggest a set of principles for what methods of apportionment should do and what they should avoid doing. Armed with these principles one can, by logical elimination, deduce which methods are acceptable and which are not. A general theory of apportionment—of what *fair representation means*—emerges from these commonsense principles.

The theory leads to three major conclusions. There are methods in use that may take a seat away from a party whose votes have increased to give it to one whose votes have decreased. There are methods that can take seats away from a state when populations stay the same and more seats are added to the house. To avoid these and other paradoxes, the choice must be restricted to a family of methods called “divisor methods.” These include most of the methods that have ever been used, but not all. This conclusion applies to both federal and proportional representation systems, because the principles upon which it is based are the very heart of the concept of fairness.

How large a deviation from one-man, one-vote can be tolerated? Some countries, such as France and England, use no method of apportionment: solutions are negotiated. If, however, the intent is to eliminate

any systematic advantage to either the small or the large, then only one method, first proposed by Daniel Webster in 1832, will do. In the United States, the case for choosing Webster's method is particularly clear because of the strict standards established by the Supreme Court on one-man-one-vote. In 1973 the Court overturned a state's congressional district plan because the deviations "were not 'unavoidable' and the districts were not as mathematically equal as reasonably possible."<sup>1</sup> This ruling, and other like ones, concerned deviations in representation between electoral districts *within* states. But until now no one seems to have realized that the deviations in representation *between* states caused by the use of the present method of apportionment are worse. This method systematically discriminates against the larger states in a way that is avoidable.

For proportional representation systems, on the other hand, it may sometimes be desirable to sacrifice evenhandedness and deliberately give an advantage to the larger parties so as to encourage the formation of coalitions. To maintain political stability, no advantage in representation should accrue to a party that splinters, and very small parties should be discouraged. If this is the primary desire, then only one method, first proposed by Thomas Jefferson in 1791, will do.