PROGRESSIVE TAXATION AND THE EQUAL SACRIFICE PRINCIPLE

H.P. YOUNG*

School of Public Affairs, University of Maryland, College Park, MD 20742, USA

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A classical problem in public finance is: when does equal sacrifice imply progressive taxation? Suppose that a tax schedule imposes equal sacrifice on all taxpayers in loss of utility, and that this property is preserved under re-indexing of the schedule. Then the utility function must exhibit constant relative risk aversion, i.e. it must be logarithmic or a power function. If equal sacrifice relative to such a utility function holds for all taxable incomes, and if tax rates do not exceed 100 percent, then the degree of relative risk aversion must be at least unity, and the tax must be nonregressive.

1. Introduction

'Equality of taxation ... as a maxim of politics, means equality of sacrifice. It means apportioning the contribution of each person towards the expenses of government so that he shall feel neither more nor less inconvenience from his share of the payment than every other person experiences from his. This standard, like other standards of perfection, cannot be completely realized; but the first object in every practical discussion should be to know what perfection is.' [J.S. Mill (1848, Book V, Chapter II)]. The equal sacrifice principle has often been invoked to justify specific types of tax schedules. In particular, some have claimed that it justifies progressive, or at least nonregressive, taxation. Since a dollar of tax falls more lightly on a rich man than on a poor one, it seems right that the rich should pay at a higher rate than the poor if all are to sacrifice equally. But in fact, equal sacrifice only implies that the rich should pay more in tax. As Schumpeter remarks, 'This error can be found, as a witness to our loose habits of thinking, in the writings of quite reputable economists, though it should be obvious that, given the intention to take away equal "amounts" of satisfaction, nothing follows from the "law" of decreasing marginal utility of income except that higher incomes should pay higher absolute sums than

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lower incomes...' [Schumpeter (1954, p. 1070, n.3)]. The fallacy was also pointed out by Cohen Stuart (1889) and Edgeworth (1897). Samuelson seems to have been the first to give an explicit condition for equal absolute sacrifice to yield strictly progressive taxation; namely the elasticity of marginal utility \( xu''(x)/u'(x) \) should be less than \(-1\) [Samuelson (1947, p. 227)]. Musgrave (1959) and Richter (1983) discuss other aspects of equal sacrifice in taxation.

Whether equality of sacrifice entails progressive taxation depends to some extent on how the term 'equal sacrifice' is interpreted. Equal absolute sacrifice means that everyone foregoes the same amount of utility in paying taxes. Equal proportional sacrifice means that everyone foregoes the same percentage of utility in paying taxes. (Equal proportional sacrifice makes the most sense if the utility of income is positive.) The question as to whether equal absolute or equal proportional sacrifice is the more appropriate standard will not be taken up here. The former criterion says that people tend to evaluate inequality in terms of differences, the latter that they think of inequality in terms of ratios. We know of no empirical evidence that settles this question one way or the other. From a purely formal point of view there is no loss of generality in restricting attention to equal absolute sacrifice, because equal proportional sacrifice relative to the positive function \( u(x) \) amounts to equal absolute sacrifice relative to \( \ln u(x) \).

Under either criterion of equal sacrifice, strict progressivity over the whole range of taxable incomes may not result even from quite reasonable utility functions. For example, consider the concave function \( u(x) = ax + b \) where \( a > 0 \). Equal absolute sacrifice means that \( a\sqrt{x} - a\sqrt{x-t} = c \geq 0 \) is a constant for all \( x > 0 \). For \( x \geq c^2/a^2 \) the solution is \( t = (c/a)(2\sqrt{x} - c/a) \), which is strictly regressive. For \( 0 < x < c^2/a^2 \) there is no real-valued solution. Suppose instead that equal proportional sacrifice is the criterion. In this case it is natural to take \( b = 0 \). Then \( \sqrt{x} / \sqrt{x-t} \) is a constant for all \( x > 0 \), which means that \( t \) is merely proportional to \( x \) (the flat tax). If \( b > 0 \), a strictly regressive tax results (except for small incomes, where again the tax is undefined). This underscores the point that equal proportional sacrifice is not preserved under translations of the utility function.

While equal sacrifice is an appealing principle, it seems difficult to apply because of the apparent arbitrariness in specifying \( u(x) \). To be sure, certain general characteristics can be stated with reasonable confidence. It seems certain that \( u(x) \) is continuous and nondecreasing. It is probably strictly increasing. Most would argue that it is concave. A good case can be made, based on studies of behavior under risk, that the degree of absolute risk aversion \(-u''(x)/u'(x)\) should be a decreasing function of \( x \) [Pratt (1964), Arrow (1974)]. Otherwise, the amount that a person invests in risky assets (e.g. securities) would be expected to decrease with increasing wealth, a conclusion that is at variance with observation. Arrow (1974) argues further
that the presumption is in favor of utility functions with the property that the degree of relative risk aversion \(-xu''(x)/u'(x)\) is nondecreasing. Beyond these properties there seems to be little reason for choosing one utility function over another except mathematical convenience. A fairly standard approach is to simply assume that relative risk aversion is constant and absolute risk aversion is decreasing, which implies that \(u(x)\) is of the form \(a \ln x + b\) or \((a/p)x^p + b\) for some \(a > 0\) and \(p < 1\).

In this paper we shall show how the constant relative risk aversion utility functions can be axiomatized from first principles. Indeed, we shall show that they follow from the single principle that perceived equality (or inequality) of sacrifice depends only on relative income levels. What matters is how much people give up relative to their initial positions. This 'relativity principle' is consistent with two types of empirical data. First, it is consistent with the practice of re-indexing tax schedules. If we assume that tax schedules embody some generalized perception of equal sacrifice, then the practice of re-indexing says that equality of sacrifice is preserved under changes of scale in income. Second, relativity is consistent with a wide range of evidence from experimental psychology. Experiments on sensory perception tend to confirm the proposition that subjectively perceived differences in the intensity of a stimulus are preserved under changes of scale in its intensity. This property essentially characterizes the logarithmic and power functions.

The result has interesting consequences for the design of tax schedules. If equal sacrifice is to hold over the whole range of taxable incomes, if this property is preserved under re-indexing, and if tax rates do not exceed 100 percent, then the tax schedule must either be flat, or strictly progressive and of form \(t = x - [x^p + \lambda^p]^{1/p}\) where \(p\) is negative, \(1 - p\) is the degree of relative risk aversion, and \(\lambda\) is a parameter whose value depends on the total amount of tax to be raised. The latter tax schedules are essentially constant elasticity of substitution (CES) functions [Arrow, Chenery, Minhas, Solow (1961)], and do not appear to have been studied before in the context of taxation except in the particular case \(p = -1\), which corresponds to a method proposed by G. Cassel in 1901.

2. The relativity principle in perceptions of sacrifice

Let \(x > 0\) be some index of ability to pay, such as income or wealth. We shall always assume that \(x\) is net of minimum subsistence requirements [as J.S. Mill (1848), Edgeworth (1919) and others have advocated], since otherwise it does not represent ability to pay in anything but the short run. For brevity we shall simply refer to \(x\) as 'taxable income'. Let \(t\) represent a possible tax on \(x\) and let \(y = x - t\) be 'after-tax' income. Both taxable income and tax are assumed to be measured in constant ('real') monetary units.
For ‘sacrifice’ to make sense we must have \( t \geq 0 \); otherwise we are speaking of a subsidy. On the other hand, taxing according to ability to pay requires that no one be taxed beyond his means, hence \( t \leq x \). A pair \((x, t)\) is admissible if \( x > 0 \) and \( 0 \leq t \leq x \).

The notion of ‘comparative sacrifice’ in taxation can be defined quite generally as follows. Assume that for every two admissible pairs \((x, t)\) and \((x', t')\), a comparison can be made as to which constitutes the greater sacrifice. Write \((x, t) \gtrless (x', t')\) if \((x, t)\) represents at least as great a sacrifice as does \((x', t')\). The relation \((x, t) \sim (x', t')\) denotes equal sacrifice.

The relativity principle states that the comparison of sacrifice at any two income levels \( x \) and \( x' \) depends on three terms: the rate of taxation at \( x \), the rate of taxation at \( x' \), and the relative size of \( x \) as compared with \( x' \). That is, whether \((x, t)\) involves more or less sacrifice than \((x', t')\) depends on the relative income levels and tax rates at these levels, but not on the absolute meaning of the units in which income (or wealth) are measured.

An equivalent and somewhat more transparent way of stating the matter is that if all incomes and taxes are scaled up or down in the same proportion, then the sacrifice relation is left unchanged. If for all admissible pairs \((x, t)\) and \((x', t')\), and for all \( \theta > 0 \),

\[
(x, t) \gtrless (x', t') \iff (\theta x, \theta t) \gtrless (\theta x', \theta t'),
\]

then the sacrifice relation \( \gtrless \) is said to be scale-invariant.

Scale invariance is consistent with common-sense experience about how people make comparisons. In evaluating the extent of one person’s sacrifice compared to another’s, what matters is not so much how rich they are relative to some historical standard, or to some theoretically defined poverty level, but how rich they are relative to each other. If A pays a tax of 3,000 on a taxable income of 20,000, and B pays 30,000 on a taxable income of 100,000, then in deciding who sacrifices more it is probably not necessary to know whether these are 1985 or 1986 dollars (or whether they are dollars or pounds). The decision will hinge on a subjective assessment of how much better off A is relative to B, but not on the absolute meaning of the units.

The relativity principle is implicit in the commonly accepted practice of ‘re-indexing’ tax schedules to adjust for secular shifts in income levels. Let \( f(x) \) be an arbitrary tax schedule expressed in real monetary units. Assume further that \( f \) is strictly progressive, i.e. that \( f(x)/x \) is strictly increasing in \( x \). If real incomes rise by some uniform factor \( \theta > 1 \), taxpayers will be shifted into higher tax brackets, hence the real total tax take will increase by a factor larger than \( \theta \). If the tax system is to remain relatively revenue-neutral – in the sense that total taxes remain a fixed percentage of total income – then the tax schedule will have to be changed. A commonly accepted practice is to re-index the schedule by the factor \( \theta \), that is, to define the new
tax schedule \( f^*(x) = \theta f(x/\theta) \) for all \( x > 0 \). If we think of both \( f(x) \) and \( f^*(x) \) as representing equal sacrifice, then \( (x, f(x)) \) and \( (x, f^*(x)) \) are two level curves of some sacrifice relation, where the latter curve represents a uniform outward shift of the former. The effect of re-indexing in this case is to decrease the general level of sacrifice. (See fig. 1, where the tax curve C is obtained from B after re-indexing by the factor \( \theta = 2 \).

A similar but opposite effect occurs when re-indexing occurs implicitly through inflation. Suppose that a tax schedule \( f(x) \) remains in place over a period of years. If there is inflation over the period, but the schedule is expressed in current dollars (a not uncommon occurrence), then in real terms the schedule is being implicitly re-indexed. Of course, the total tax take also increases in real terms. There is no particular reason to think that equality of sacrifice is sacrificed by such an arrangement, however, since everyone remains in the same position relative to each other. They simply sacrifice more. In this case inflation has the effect of contracting the equal sacrifice curve inward by a fixed factor, and increasing the general level of sacrifice.
(In fig. 1 the tax curve A is obtained from B after re-indexing by \( \theta = \frac{1}{3} \); equivalently, A is the result in real terms if B remains in place in nominal terms and there is 100% inflation.) Thus re-indexing lends support to the idea that equal sacrifice, if it is valid as an empirical principle of taxation, is perceived in relative terms.

Further support for relativity as a general cognitive principle comes from the domain of experimental psychology. Experimental results for a wide variety of sensory stimuli (loudness, brightness, weight, salinity, etc.) tend to confirm the hypothesis that perceived differences in intensities depend only on the relative physical intensities of the things being observed. For example, let \( x \) denote the intensity of light measured in photons. Let \( u(x) \) be a function (sometimes called a ‘sensation function’) that measures the subjectively perceived intensity of the stimulus. Suppose that an experimental subject is asked to estimate the perceived difference in intensity between two lights having physical intensity levels \( x \) and \( y \). Suppose that the subject is then asked to compare this difference with the perceived difference in intensity between two lights having intensities \( x' \) and \( y' \); in other words to estimate which of the pairs \( (x, y) \) and \( (x', y') \) represents the larger perceived differential in intensity. It seems reasonable to suppose that, if one differential is perceived to be larger than another, then this perception remains true if the scale of intensity changes; for example, if the subject is moved uniformly further away from all of the light sources. This is precisely the scale invariance property. It was verified by Plateau (1872) in a classical experiment on perception of shades of grey. It is also consistent with most empirical estimates of sensation functions, which tend to confirm (depending on the type of experiment) either a logarithmic relation \( u(x) = a \ln x + b \) [Fechner (1860)] or a power function \( u(x) = (a/p)x^p + b \) [Stevens (1975)]. For a survey of this topic see Luce, Bush and Galanter (1963, Vol. I, Chapter V). Sinn (1985) discusses some implications of these experimental results for the theory of risk bearing.

3. Utilitarian representations of sacrifice

What we have said up to this point is extremely general. Any tax schedule \( f(x) \) can be thought of as expressing equality of sacrifice, in the sense that \( (x, f(x)) \) is the level curve of some sacrifice relation. The use of re-indexing suggests further that this sacrifice relation is scale-invariant. This assumption is still relatively mild. However, it has quite strong implications if sacrifice is measured relative to some utility function for income.

Let \( u(x) \) represent a utility function that is defined for all taxable incomes \( x > 0 \). We shall generally assume that \( u(x) \) is continuous and nondecreasing. Extend \( u \) by defining \( u(0) = \lim_{x \to 0} u(x) \). For all admissible pairs \( (x, t) \) and


(x', t') define the sacrifice relation \( \succeq \) as follows:

\[
[(x, t) \succeq (x', t') \iff u(x) - u(x - t) \geq u(x') - u(x' - t')].
\]

We shall then say that \((x, t)\) represents at least as much absolute sacrifice as \((x', t')\), relative to \(u(x)\).

**Theorem 1.** Equal absolute sacrifice with respect to a continuous, nondecreasing, nonconstant utility function \(u(x)\) is scale-invariant if and only if \(u(x)\) exhibits constant relative risk aversion; that is, if and only if \(u(x)\) is a positive linear transformation of \(\ln x\) or of \((1/p)x^p\), \(p \neq 0\).

Here we write \((1/p)x^p\) to convey that the coefficient and the exponent must have the same sign.

Theorem 1 will be proved with the aid of a lemma in the theory of functional equations. Suppose that \(\succeq\) represents absolute sacrifice relative to \(u(x)\), which is continuous and nondecreasing. Scale invariance implies that for all \(\theta > 0\) and for all \(0 < y \leq x, 0 < y' \leq x',\)

\[
u(x) - u(y) \geq u(x') - u(y') \iff u(\theta x) - u(\theta y) \geq u(\theta x') - u(\theta y').
\]

In particular, scale invariance implies that

\[
u(x) - u(y) = u(x') - u(y') \iff u(\theta x) - u(\theta y) = u(\theta x') - u(\theta y'). \tag{2}
\]

Any function \(u(x)\) satisfying (2) is said to be homogeneous in differences. Theorem 1 is a consequence of the following.

**Lemma.** If \(u(x)\) is continuous, nondecreasing, and homogeneous in differences for all \(x > 0\), then \(u(x)\) is of form

(i) \( u(x) = b, \)

or

(ii) \( u(x) = a \ln x + b, \quad a > 0, \)

or

(iii) \( u(x) = ax^p + b, \quad ap > 0. \)

**Proof.**\(^1\) If \(u(x)\) is homogeneous in differences, then there is a single-valued

\(^1\)The proof of this result originated in collaboration with Janos Aczel during a pleasant drive together across southern Germany in July, 1985. His contribution and helpful comments on the manuscript are gratefully acknowledged.
function $F$ such that for all $0 < y \leq x$ and all $\theta > 0$,

$$u(\theta x) - u(\theta y) = F(u(x) - u(y), \theta) \geq 0.$$  

Since $u$ is continuous, $F$ is too. Let $D = \{u(x) - u(y): 0 < y \leq x\}$. By continuity and monotonicity, $D$ is an interval of $R_+$ of form $[0, m)$ or $[0, m]$, where $m > 0$ unless $u(x)$ is constant, which is case (i) of the lemma. Suppose then that $u(x)$ is nonconstant, and a fortiori that $m > 0$. Consider first the possibility that $D = [0, m]$ is closed on the right. If $z, z' \geq 0$ and $z + z' \leq m$, then $z + z' = u(x) - u(y)$ for some $x$ and $y$. Since $0 \leq z \leq u(x) - u(y)$, we have

$$u(x) \geq u(x) - z \geq u(y).$$

Thus by the continuity of $u(x)$ there exists some $w$ between $x$ and $y$ such that $u(w) = u(x) - z$; that is, $z = u(x) - u(w)$. Hence also $z' = u(w) - u(y)$. This argument shows that for every fixed $\theta > 0$,

$$F(z + z', \theta) = F(z, \theta) + F(z', \theta) \quad \text{if } z, z' \geq 0 \text{ and } z + z' \leq m.$$  

This is Cauchy's equation on the restricted domain $\{z, z' \geq 0, z + z' \leq m\}$. Since $F$ is continuous and non-negative, it follows from Aczel (1966, Section 2.1.4, Theorem 3) that for every $\theta > 0$ there is a number $c(\theta)$ such that

$$F(z, \theta) = c(\theta)z \quad \text{for all } z \in [0, m]. \quad (3)$$

Suppose on the other hand that $D = [0, m)$ is open on the right. A similar argument shows that for every $0 < m' < m$, and every $\theta > 0$, there is a number $\gamma(m', \theta)$ such that

$$F(z, \theta) = \gamma(m', \theta)z \quad \text{for all } z \in [0, m'].$$

Since $F$ is single-valued and the intervals $[0, m']$ are nested in $[0, m)$, it follows that $\gamma(m', \theta)$ does not depend on $m'$; hence for some function $c(\theta)$ we can write

$$F(z, \theta) = c(\theta)z \quad \text{for all } z \in [0, m) \text{ and all } \theta > 0.$$  

In either case we therefore have, since $u$ is monotonic, that

$$u(\theta x) - u(\theta y) = c(\theta)[u(x) - u(y)] \quad \text{for all } x, y > 0.$$  

Fix $y^* > 0$, and let $d(\theta) = u(\theta y^*) - c(\theta)u(y^*)$. Then

$$u(\theta x) = c(\theta)u(x) + d(\theta) \quad \text{for all } x, \theta > 0.$$
For all real numbers $r$ define $u^*(r) = u(c^r)$, $c^*(r) = c(e^r)$, and $d^*(r) = d(e^r)$. Making the substitutions $s = \ln \theta$ and $t = \ln x$, the preceding becomes

$$u^*(s + t) = c^*(s)u^*(t) + d^*(s).$$

By Aczel (1966, Section 3.1.3, Theorem 1) this, together with the continuity of $u^*$, implies that

$$u^*(r) = b,$$

or

$$u^*(r) = ar + b, \quad a > 0,$$

or

$$u^*(r) = ae^{pr} + b, \quad a, p \neq 0.$$

Further, since $u^*$ is nondecreasing, $a$ and $p$ must have the same sign. Therefore,

$$u(x) = b,$$

or

$$u(x) = a \ln x + b, \quad a > 0,$$

or

$$u(x) = ax^p + b, \quad ap > 0.$$

If differences in utility are measured relatively rather than absolutely, then the possible forms of $u(x)$ are obtained by exponentiating the functions $a \ln x + b$ and $ax^p + b$. Thus we obtain the following corollary.

**Corollary 1.1.** Equal proportional sacrifice with respect to a positive (continuous, nondecreasing, nonconstant) utility function $u(x)$ is scale-invariant if and only if $u(x)$ is a positive multiple of $x^a$, $a > 0$, or of $e^{ax^p}$ where $ap > 0$.

The class of functions $u(x) = e^{ax^p}$ does not seem to have been studied before in the context of utility theory. Nevertheless, from the standpoint of risk aversion they have quite reasonable properties when $a$ and $p$ are both negative. First, they are bounded both above and below. Second, they are strictly concave except in a neighborhood of the origin. Third, the usual assumptions about risk-averse behavior hold: absolute risk aversion is
decreasing for all sufficiently large $x$, and relative risk aversion is increasing for all $x > 0$.

4. Implications for taxation

Let $t = f(x)$ be a tax schedule that is feasible in the sense that $f(x) \leq x$, and such that all taxpayers sacrifice the same amount of utility relative to $u(x)$. Suppose further that any re-indexing of $f$ also results in equal sacrifice relative to $u(x)$. By Theorem 1, $u(x)$ must be a positive linear transformation of $\ln x$ or of $(1/p)x^p$, where $p \neq 0$. Without loss of generality we may assume that $u(x) = \ln x$ or $u(x) = (1/p)x^p$. Equal absolute sacrifice means that $f(x) \geq 0$ and there is a constant $c \geq 0$ such that

$$\forall x > 0 [u(x) - u(x - f(x)) = c].$$  \hspace{1cm} (4)

Note that if (4) is to hold for some fixed $c > 0$ and all $x > 0$, then it is necessary that $\lim_{x \to 0} u(x) = -\infty$. For suppose that $\lim_{x \to 0} u(x) = L$ were finite. Then $u(x - f(x)) = u(x) - c$ for all $x > 0$. Letting $x \to 0$, and recalling that $0 \leq x - f(x) \leq x$, it follows from the continuity of $u$ that $L = L - c$, which is impossible when $c > 0$.

Consider now the various utility functions that can arise under scale invariance. If $u(x) = \ln x$ then $f(x) = (1 - e^{-x})x$ and $f$ is the flat tax. If $u(x) = (1/p)x^p$, there are two cases to consider: $p > 0$ and $p < 0$. If $p > 0$, then $u(x)$ is bounded below; hence, as just noted, equal sacrifice cannot hold at all income levels. Indeed the solution $f(x) = x - \lfloor x^p - cp \rfloor^{1/p}$ is only valid when $x \geq \lfloor cp \rfloor^{1/p}$. If $0 < x < \lfloor cp \rfloor^{1/p}$ then either there is no real-valued solution to (4) (which occurs for example when $p = \frac{1}{2}$) or else the solution violates the feasibility constraint $f(x) \leq x$ (which occurs for example when $p = \frac{1}{3}$). Thus equality of sacrifice and taxation subject to ability to pay cannot both hold at all levels of income for a utility function of form $u(x) = (1/p)x^p$ where $p > 0$.

The remaining case is $u(x) = (1/p)x^p$, $p < 0$. Then the solution of (4) is

$$f(x) = x - \lfloor x^p + \lambda p \rfloor^{1/p}, \quad p < 0,$$  \hspace{1cm} (5)

which is valid for all $x > 0$. Here we have made the substitution $\lambda = \lfloor -cp \rfloor^{1/p} \geq 0$ to highlight the connection with CES functions. As may be checked, the tax schedules (5) are strictly progressive whenever $\lambda \neq 0$, that is, whenever a positive amount of tax is collected. The value of $\lambda$ is related to the level of sacrifice (and to the total tax burden), and $p$ is related to the degree of progressivity.

A particularly interesting case occurs when $p = -1$. Then $f(x) = x^2/(x + \lambda)$, which is a type of schedule proposed by Gustav Cassel in 1901. Hence the
schedules defined in (5) can be regarded as a generalization of Cassell's tax that allows any desired degree of progressivity by varying the parameter \( p \). Together with the flat tax they constitute the only family of feasible, equal sacrifice tax schedules that are consistent with re-indexing.

The preceding statement also holds true if equal sacrifice is measured in terms of equal rate of loss in utility. For suppose that \( f(x) \) satisfies

\[
u(x)/u(x - f(x)) = r \quad \text{for all } x > 0 \text{ and some } r \geq 1.\]

where \( u(x) \) is positive (continuous, nondecreasing, and nonconstant). Then

\[
\ln u(x) - \ln u(x - f(x)) = \ln r = c \quad \text{for all } x > 0.
\]

Thus \( f(x) \) equalizes absolute sacrifice relative to \( \ln u(x) \) and we have:

**Theorem 2.** A feasible tax schedule \( f(x) \) and its re-indexings represent equal sacrifice (either absolute or proportional) at all levels of taxable income if and only if \( f(x) \) is the flat tax or \( f(x) = x - [x^p + \lambda^p]^{1/p} \) for some \( \lambda \geq 0 \) and \( p < 0 \).

**Corollary 2.1.** If a feasible tax schedule \( f(x) \) and its re-indexings represent equal sacrifice (either absolute or proportional) at all levels of taxable income, then \( f(x) \) must be nonregressive.

In a separate paper [Young (forthcoming)], we show that the family of schedules in Theorem 2 can be characterized by a set of consistency conditions that do not presuppose any utility function or measure of welfare.

5. **Conclusion**

Empirical studies are beyond the scope of the present paper. It is worth remarking, however, that the preceding results provide a framework for empirically testing the proposition that tax schedules represent equality of sacrifice. We do not mean to suggest that legislators and tax writers explicitly use a utility function in designing tax schedules. Nevertheless, their hands may be guided by a broad sense of what the voting public believes to constitute equal sacrifice. To test this hypothesis, one could examine the rates of taxation that prevail at different levels of income in a particular country. One could then attempt to fit a curve of type (5) to the schedule by estimating values of the parameters \( \lambda \) and \( p \). The value of \( p \) would provide an estimate of the curvature of the utility function, and in particular the degree of relative risk aversion. Such an estimate could then be compared with estimates from other sources, such as studies of behavior under risk.
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