Expertise and Reputations in Markets for Credence Services

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6th August 2025

Abstract

The waste of resources spent on unnecessary services–such as sham car maintainance and repairs, dental and medical treatments–is prevalent and well documented. This paper studies the role of providers' expertise and reputations in credence service markets and their effect on the incidence of fraudulent prescription of unnecessery services and treatments.

Keywords: Credence service markets, reputation, expert's competence, dynamic stochastic games of incomplete information

1 Introduction

Credence services-such as car repairs and maintenance, medical and dental treatments, plumbing repairs, and taxi services to mention just a few examples–are char-

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acterized by information asymmetry between providers who have the expertise required to assess the need for service, and customers, who lack the ability to assess the necessity of the prescribed service *both before and after* the service is rendered. When, in addition, cost considerations lead to the bundling of the diagnosis and the provision of service, combined with the providers' incentives to prescribe unnecessary services, this asymmetry leads to fraudulent prescriptions and provision of unnecessary services.

In the wake of Darby and Karni (1973), who introduced the concept of credence goods, numerous studies documenting and analyzing fraudulent prescriptions of unnecessary services and treatments in the markets for credence goods and services have been published.[1] Most of the literature dealing with credence goods and services presumes that the experts (i.e., service providers) can always correctly identify the problem and the necessary treatment. This presumption, while a convenient simplification, disregards the fact that, in reality, there are degrees of expertise that determine the providers' ability to correctly identify the problem and the necessary treatment. The inability of an expert to always diagnose a problem correctly leads to two types of errors: overestimating and underestimating the severity of a problem. Their expertise is the providers' private information; their records of recommendations and subsequent outcomes, are public and determine providers' reputations.[2] Reputation concerns, in turn, affect providers' behavior.

[1] Dulleck and Kerschbamer (2006) and Balafoutas and Kerschbamer (2020) provide extensive surveys of these studies and references.

[2] The assumption that the record is public information is invoked to simplify the exposition. It is intended to capture information spread by channels, such as word of mouth or reviews platforms, that affect providers' reputations.

The main objective of this paper is the study of the roles of providers' expertise and reputation concerns in shaping their prescription strategies, customers' demand for service, and the perpetration of fraud. Toward this end, I consider a market for credence service populated by customers and two service providers. When a customer detects symptoms (e.g., equipment malfunction, physical malaise) she chooses a provider to diagnose the problem. The provider prescribes remedial treatment that, if accepted, provides the prescribed service or treatment. The presumption is that underlying the symptoms is a true state, which is the cause of the problem. The provider obtains a signal indicating the likelihoods of the underlying states. The quality of the provider's signal depends on his expertise. In particular, the greater the provider's expertise, the more informative is his signal.

Providers' reputations reflect customers' perceptions of their expertise and the adequacy of their prescriptions. These perceptions evolve as evidence accumulates about the providers' prescriptions and the outcomes of the services they provide. Earlier studies of reputations in credence goods markets, discussed in the concluding section, all presume that experts can observe the true state without fail. These studies examined different market structures, and invoke different measures and definitions of reputation. They identify fraudulent behavior with intentional undertreatment and overtreatment.

This paper introduces a novel and more realistic take on all these issues, and examines the effect of reputation on the equilibrium prices charged by the competing providers.

Credence good and service markets is a term that encompasses a variety of institutions with specific essential features. Consequently, the analysis of these markets requires market-specific models that capture these idiosyncratic characteristics.[3]

To highlight the roles of expertise and reputations, I model the market of credence services as a dynamic stochastic game of incomplete information consisting of a sequence of stage games. Each stage game consists of two parts. In the first part, dubbed the *pricing game*, providers set their prices. In the second part, dubbed *prescription subgame*, the sequential interaction of a customer and the two providers takes place. This interaction is modeled as a game of incomplete information and analyzed using the concept of perfect Bayesian equilibrium. The pricing game and the successor prescriptions subgame are linked by the mutual effects of the strategies employed in these games. The stage games are linked by the evolution of the providers' reputations.

The next section describes the credence service market and provides a formal definition of reputation in this market. Section 3 describes the providers' pricing game that precedes the prescription subgame. Section 4 depicts the prescriptions subgame and establishes the existence of perfect Bayesian equilibrium (PBE) of the stage game. Section 5 discusses the economic implications of expertise, reputations and liability in shaping the equilibrium in the credence service market and the ensuing fraudulent behavior. This section also includes comments on the literature. The Appendix presents the proofs of the main results.

[3] See Chiu and Karni (2021) and Karni (2024) for examples.

2 The Credence Service Market, Expertise, and Reputation

2.1 Overview

Two providers, A and B, operate service facilities. They are characterized by their levels of expertise, which are their private information. Providers' expertise determine their likelihoods of correctly identifying the true state. At the outset of each stage game, providers' reputations are based on the records of their past performance (i.e., their recommendations and subsequent outcomes). Before the arrival of customers, providers post their hourly services prices.

Upon experiencing problems the customer must decide whether to seek or forgo remedial service (e.g., to service or to retire malfunctioning equipment). If she decides to seek remedial service, she chooses a provider. The provider diagnoses the problem and recommends treatment. In choosing her first provider the customer is guided by the posted service prices and the providers' reputations. After receiving the first provider's prescription, the customer must choose between accepting it (i.e., purchasing the recommended service), seeking a second opinion, or forgoing the service altogether. Seeking a second opinion involves personal cost, $\theta \in \Theta := \left[0, \bar{\theta}\right]$, which is the customer's private information. The presence of these "search, or switching, costs" makes the bundling of diagnosis and provision of service cost-effective. To simplify the exposition, I assume that returning to the first provider for service is prohibitively costly, so the customer must choose between accepting the second provider's prescription or foregoing the service altogether.

Customers are assumed to assign a value, $v \in V := [\underline{v}, \bar{v}]$, to fixing the problem. This value is the customer's private information. If the cost of the prescribed service exceeds that value, the customer forgoes the service. In addition, in the event that the recommended service fails to address the problem, the customer bears an idiosyncratic cost (e.g., inconvenience and loss of time), $b \in \mathbb{B} := \left[0, \bar{b}\right]$. A cost–which is borne by the customer even if the provider rectifies his mistake by fixing the problem at no additional cost–is the customer's private information.

The customer has no way of knowing, in advance, whether the prescribed service is necessary. If she accepts the recommended service and the problem is solved, she concludes that the recommended service was sufficient. She cannot, however, be sure that it was necessary (i.e., whether the service provided was the minimal required to solve the problem).

To simplify the exposition, I assume that there are two states: a minor problem, denoted by ω_L, and a major problem, denoted by ω_H, one of which is the true state. The prior probabilities of the states are $1-\mu$ and μ, respectively, and are assumed to be common knowledge. I assume that the major problem requires more service time to be fixed. With slight abuse of notation, let ω_L and ω_H denote the number of service hours required to fix the minor and major problems, respectively (i.e., $\omega_L < \omega_H$). Let $\Omega := \{\omega_L, \omega_H\}$ denote the state space.

Problems that require service occur randomly. Consequently, the arrival of customers on the market is an exogenous stochastic process. The customers' arrival at a provider's station, however, is a random process whose rate also depends on the prices and reputations of the two providers. In particular, as we shall see, other things equal, the lower the provider's price and the higher his reputation, the higher the arrival rate at his service station. I assume that providers are not constrained by capacity and are able to accommodate all customers in need of service without delay.[4]

The information asymmetry in this market is two-sided. Customers' private information is depicted by the parameters (θ, v, b). Service providers' private information is their expertise. A provider's expertise, e_j, $j \in \{A, B\}$, reflects his inherent talent and/or acquired skill and is his private information. More formally, when a customer shows up reporting symptoms, the provider conducts a diagnostic test and receives a noisy signal in the set $S = \{s_L, s_H\}$ that informs him about the true state (i.e., the cause of the symptoms). The quality of the signal determines the provider's posterior distribution on the states in the information set, Ω. By definition, the greater the provider's expertise, the higher the quality of the signal he receives. Formally, $e_j := \Pr_j (s_k \mid \omega_k)$, $k \in \{L, H\}$. I assume that providers are capable of identifying the true state more often than not. Hence, e_A and e_B taking values in $[1/2, 1]$.[5]

I further assume that from the customers' viewpoint, the providers' expertise are independent and identically distributed random variables. Let η denote the prior, full support and differentiable, cumulative distribution function of e_j, $j \in \{A, B\}$, and assume that it is common knowledge.

Ex post, after the inspection, there is additional information asymmetry; The

[4]Chiu and Karni (2012) and Karni (2024) adress the possible effect of waiting time.

[5]There is no essential loss of generality by including $1/2$ in the range of the expertise. The analysis that follows will not change if the range is $[q, 1]$ for some $q > 1/2$.

provider obtains a signal that informs him about the likelihoods of the states. The higher the level of expertise, the greater the correlation between the signal and the true state.

Let p_j, $j \in \{A, B\}$, denote the price per hour of service of provider j, and let c denote the (constant) marginal hourly operating cost of the service station. Then provider j's hourly profit is $p_j - c$ when the station is occupied and $-c$ when it is idle. Consequently, providers have incentives to sell more service time, (i.e., to sell ω_H as often as possible).

If the true state is ω_H and the service provider prescribes and provides ω_L, the problem persists. In this case, it is assumed that the provider must fix the problem at no additional charge to the customer. This aspect of the transaction is dubbed *liability*.[6] Recommending insufficient service thus entails a loss to the provider, it is also costly to the customer.

2.2 Expertise

The main objective of this paper–the analysis of the effects of providers' expertise and reputations on the level of fraud perpetuated in credence service markets–requires a clear concept of what is meant by expertise and reputation.

[6] In some markets (e.g., medical and dental services), misdiagnosis may result in malpractice payouts.

Given $e_j \in [1/2, 1]$, the joint probability distribution on $S \times \Omega$ is given by:

$$\left| \begin{array}{cccc} \text{Prior} & 1-\mu & \mu & \\ \text{States} & \omega_L & \omega_H & \text{Marginal} \\ s_L & e_j(1-\mu) & (1-e_j)\mu & \Pr(s_L \mid e_j) \\ s_H & (1-e_j)(1-\mu) & e_j\mu & \Pr(s_H \mid e_j) \end{array} \right|.$$

Hence, the higher the provider's expertise, the greater is the probability that the signal indicates the true state. Formally, by application of Bayes' rule, provider $j's$ posterior beliefs about the likelihoods of the two states are:

$$\mu_j(\omega_H \mid s_H) = \frac{e_j\mu}{e_j\mu + (1-e_j)(1-\mu)}, \ \mu_j(\omega_L \mid s_H) = \frac{(1-e_j)(1-\mu)}{e_j\mu + (1-e_j)(1-\mu)} \quad (1)$$

and

$$\mu_j(\omega_H \mid s_L) = \frac{(1-e_j)\mu}{(1-e_j)\mu + e_j(1-\mu)}, \ \mu_j(\omega_L \mid s_L) = \frac{e_j(1-\mu)}{(1-e_j)\mu + e_j(1-\mu)}. \quad (2)$$

2.3 Reputation

The conditions preceding the stage game include the reputations of the two providers, which are the perceived probabilities of diagnosing the problems correctly and prescribing the necessary service. To formalize this assertion, I denote by $\omega_k(j)$, $k \in \{L, H\}$, the prescribed level of service of provider $j \in \{A, B\}$.

Define $\Phi := \{\varphi : [1/2, 1] \to \Delta(\Omega)^S\}$ as the set of providers' prescription strategies. In words, a *prescription strategy* is a map, φ, from the levels of expertise to the set of randomized prescription decision contingent on the signal received. Specifically, for each $s \in S$, $\varphi(e) = (\alpha(s_L, e)(\omega_H), \alpha(s_H, e)(\omega_H))$, where $\alpha(s, e)(\omega_H) \in [0, 1]$ and $\alpha(s, e)(\omega_L) = 1 - \alpha(s, e)(\omega_H)$ denote the probabilities of prescribing ω_H and ω_L, respectively, upon receiving the signal s. Thus, the probability that provider j prescribes *truthfully*, (i.e., prescribes ω_k when he receives the signal s_k, $k \in \{L, H\}$), is $\alpha(s_k, e_j)(\omega_k)$, $k \in \{L, H\}$).

The probability that provider j prescribes *correctly* (i.e., prescribes the true state) is $e_j \alpha(s_k, e_j)(\omega_k) + (1 - e_j)\alpha(s_{k'}, e_j)(\omega_k)$, $k, k' \in \{L, H\}$, $k \neq k'$. Because his expertise is the provider's private information, the customer's belief that the provider j prescribes correctly is

$$\rho_j(\omega_k(j) \mid \omega_k, \bar{x}) := \int_{1/2}^{1} [e\alpha(s_k, e)(\omega_k) + (1 - e)\alpha(s_{k'}, e)(\omega_k)]\, d\eta_j(e \mid \bar{x}), \ \ k, k' \in \{L, H\}, k \neq k', \tag{3}$$

where $\bar{x}$ denotes the record of prescriptions and subsequent outcomes at the outset of the stage game, and $\eta_j(\cdot \mid \bar{x})$ denotes the customers' posterior beliefs regarding provider j's expertise. Implicit in $\rho_j(\omega_k \mid \omega_{k'}, \bar{x})$, $k, k' \in \{L, H\}$ are the customer's beliefs about the expertise and the prescription strategy of provider j.

A provider's reputation is based on the perceived probabilities that he prescribes each state correctly. Formally,

Definition 1: Provider j's, $j \in \{A, B\}$, reputation is:

$$\mathsf{r}_j(\bar{x}) = (\rho_j(\omega_H \mid \omega_H, \bar{x}), \rho_j(\omega_L \mid \omega_L, \bar{x})).$$

As made clear by equation (3), a provider's reputation amalgamates his expertise and his prescription strategy.

I assume that for every given e, providers are more likely to prescribe the state that in their judgment is more likely to obtain given the signal they received. Formally, $\alpha_j(s_k, e)(\omega_k) > \alpha_j(s_{k'}, e)(\omega_k)$, for $k, k' \in \{L, H\}$ and $j \in \{A, B\}$.[7]

Let j_1 and j_2 denote the first and second providers, respectively. Listed below are four possible prescription-outcome scenarios that are informative and affect the providers' reputations. Let X be the set of these scenarios, and denote by $\bar{x}^n = (x_1, ..., x_n) \in X^n$ the record of scenarios after the n^{th} stage game. Denote by $x_{n+1} \in X$ the $n+1$ scenario.

Scenario 1. The first provider, j_1, prescribes service that is *revealed to be unnecessary.* The first provider prescribes ω_H; the customer seeks a second opinion; and the second provider prescribes ω_L, which happens to be the true state. In this case, the customers revised their beliefs about the provider's expertise downward. Formally, $\eta_{j_1}(e \mid \bar{x}^{n+1})$ is first-order stochastically dominated by $\eta_{j_1}(e \mid \bar{x}^n)$, where x_{n+1} denote Scenario 1. Consequently, $\rho_{j_1}(\omega_H(j_1) \mid \omega_H, \bar{x}^n) > \rho_{j_1}(\omega_H(j_1) \mid \omega_H, \bar{x}^{n+1})$, and the reputation of the first provider diminishes.

By contrast the second provider, j_2, prescribes the service that is necessary and sufficient. Customers therefore revise their beliefs regarding the expertise of the provider upward. Formally, $\eta_{j_2}(e \mid \bar{x}^{n+1})$ first-order stochastically dominates $\eta_{j_2}(e \mid \bar{x}^n)$, and $\rho_{j_2}(\omega_L(j_2) \mid \omega_L, \bar{x}^n) < \rho_{j_2}(\omega_L(j_2) \mid \omega_L, \bar{x}^{n+1})$. The reputation of the second provider is enhanced.

[7] In fact, in equilibrium no provider prescribes ω_L if the signal he recieves in ω_H. Thus, $\alpha_j(s_H, e)(\omega_H) = 1$, $j \in \{A, B\}$. The assumption also implies that $\alpha_j(s_L, e)(\omega_H) < 1/2 < \alpha_j(s_L, e)(\omega_L)$.

Scenario 2. The first provider, j_1, prescribes ω_H that is *revealed to be necessary* (and sufficient). He prescribes ω_H, which happens to be the true state. The customer seeks a second opinion, the second provider prescribes ω_L, which turns out to be insufficient. In this case, the customers revised their beliefs about the expertise of the first provider upward. Formally, $\eta_{j_1}\left(e \mid \bar{x}^{n+1}\right)$ is first-order stochastically dominates $\eta_{j_1}\left(e \mid \bar{x}^{n}\right)$, where x_{n+1} denote Scenario 2. Consequently, $\rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}^n\right) < \rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}^{n+1}\right)$.

By contrast the second provider, j_2, prescribes service that is *insufficient.* As a result, customers revise their beliefs about his expertise downward. Formally, $\eta_{j_2}\left(e \mid \bar{x}^{n+1}\right)$ is first-order stochastically dominated by $\eta_{j_2}\left(e \mid \bar{x}^{n}\right)$, and $\rho_{j_2}\left(\omega_H\left(j_2\right) \mid \omega_H, \bar{x}^n\right) > \rho_{j_2}\left(\omega_H\left(j_2\right) \mid \omega_H, \bar{x}^{n+1}\right)$. The reputation of the first provider is enhanced while that of the second provider diminishes.

Scenario 3. The first provider, j_1, prescribes ω_L, which is *revealed to be sufficient.* The first provider prescribes ω_L, which happens to be the true state. The customer accepts the prescription, which turns out to be sufficient. In this case, the customers revise their beliefs about the expertise of the first provider upward. Formally, $\eta_{j_1}\left(e \mid \bar{x}^{n+1}\right)$ first-order stochastically dominates $\eta_{j_1}\left(e \mid \bar{x}^{n}\right)$, where x_{n+1} denote Scenario 3. Consequently, $\rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}^n\right) < \rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}^{n+1}\right)$.

The second provider was not consulted, so beliefs regarding his expertise remain unchanged. The reputation of the first provider is enhanced while that of the second provider is unchanged.

Scenario 4. The first provider, j_1, prescribes ω_L that is *revealed to be insufficient.* The first provider prescribes ω_L, when ω_H is the true state. The customer accepts the prescription, which turns out to be insufficient. In this case, customers revise their beliefs about the expertise of the first provider downward. Formally, $\eta_{j_1}(e \mid \bar{x}^{n+1})$ is first-order stochastically dominated by $\eta_{j_1}(e \mid \bar{x}^{n})$, where x_{n+1} denote Scenario 4. Consequently, $\rho_{j_1}(\omega_H(j_1) \mid \omega_H, \bar{x}^n) > \rho_{j_1}(\omega_H(j_1) \mid \omega_H, \bar{x}^{n+1})$.

The second provider was not consulted, so beliefs regarding his expertise remain unchanged. The reputation of the first provider diminishes while that of the second provider is unchanged.

If the first provider prescribes ω_H and the customer either is accepts it, or rejects it the second provider prescribes ω_H, the problem is solved regardless of whether the true state is ω_L or ω_H. Thus, no information is revealed regarding the necessity of the service. In this cases, the reputations of both providers remain unchanged.[8]

A provider's reputation varies as a function of the change in customers' beliefs about the correctness of their prescriptions. Formally, for $j \in \{A, B\}$,

$$\mathsf{r}_j\left(\bar{x}^{n+1}\right)-\mathsf{r}_j\left(\bar{x}^{n}\right)=\left(\left[\rho_j\left(\omega_H \mid \omega_H, \bar{x}^{n+1}\right)-\rho_j\left(\omega_H \mid \omega_H, \bar{x}^{n}\right)\right],\left[\rho_j\left(\omega_L \mid \omega_L, \bar{x}^{n+1}\right)-\rho_j\left(\omega_L \mid \omega_L, \bar{x}^{n}\right)\right]\right). \tag{4}$$

[8]These are the only scenarios on the equilibrium paths of the stage games. Specifically, if a customer plans on rejecting the prescription ω_L of provider j, she will not choose j for her first provider.

3 The Providers' Pricing Game

3.1 Overview

At the outset nature assigns customers and providers their types. These types–$(\theta, v, b) \in \Theta \times V \times \mathbb{B}$ in the case of customers and $e_j \in [1/2, 1]$, $j \in \{A, B\}$, in the case of providers–are maintained throughout the sequence of stage games that model the market, and their prior distributions are common knowledge. The players' strategies are mappings from their set of types to contingent plans of action in the pricing games and in the prescriptions subgames that follow.

At the outset of each stage game, the reputations of the two providers, having been established by the historical record of prescriptions and subsequent outcomes, are inherited from the preceding stage game. In the pricing game, providers post their hourly service prices. I assume that the more reputable provider (the leader) posts his price first. The less reputable provider (the follower) observes the leader's price before posting his own.

In the second part of the stage game, the outset of the prescription game, nature assigns a random customer a state in Ω. The customer observes the prices of the two providers before deciding whether or not to seek service. If she decides to seek service, the customer must choose which provider to visit first. The first provider diagnoses the problem and prescribes service. Having obtained the first provider's prescription, the customer must choose among accepting it, rejecting it and of seeking a second opinion, and forgoing fixing the problem. If the customer decides to seek a second opinion, she obtains a second prescription, which she must either accept or decline. Declining the second prescription amounts to forgoing fixing the problem.

The link among the stage games is provided by the stochastic evolution of providers' reputations. Henceforth, without loss of generality, assume that $e_A > e_B$. I also assume that in the stage game under consideration, the reputation of provider A exceeds that of provider B, (i.e., $\mathsf{r}_A(\bar{x}) > \mathsf{r}_B(\bar{x})$).[9]

3.2 Demands for credence services

The demand for service is determined by a population of customers characterized by their idiosyncratic parameters, θ, v, and b. I assume that these parameters are stochastically independent random variables and that their cumulative distribution functions Λ, Υ, and F, respectively, have full support, are differentiable, and are commonly known.

Based on the prices posted by the two providers, and their reputations, a customer in need of service chooses a course of action that identifies the provider to visit first, and, contingent on the first provider's prescription, indicate whether to accept it or seek a second opinion. The customer anticipates the expected cost of each course of action and pursues the least costly one.

To analyze the demands for service of the two providers and the effects of reputation, I introduce the following additional notations and definitions. Let $\Psi_j(p_j \mid b, \bar{x})$ denote the anticipated expected cost of service of a customer whose inconvenience cost is $b \in \mathbb{B}$ if she chooses $j \in \{A, B\}$ and anticipates accepting the first provider's

[9]The symmetric case would require that the levels of expertise are the same. Given our assumption on their distributions, this is an event of measure zero.

prescription. By definition:

$$\Psi_j \left(p_j \mid b, \bar{x}\right) = [(\rho_j \left(\omega_H \left(j\right) \mid \omega_H, \bar{x}\right) + \rho_j \left(\omega_H \left(j\right) \mid \omega_{_L}, \bar{x}\right))\mu\omega_H$$
$$+ \left(\rho_j \left(\omega_L \left(j\right) \mid \omega_L, \bar{x}\right) \omega_L + \rho_j \left(\omega_L \left(j\right) \mid \omega_H, \bar{x}\right)\right) \left(\omega_L + b\right)\right) \left(1 - \mu\right)]p_j.$$

3.2.1 The follower's demand

Since A is the more reputable provider, he is the leader, posting his price first. Given p_A, the demand for service of provider B is inversely related to his own price.

Consider the situation in which $\eta_A \left(\cdot \mid \bar{x}\right)$ first-order dominates $\eta_B \left(\cdot \mid \bar{x}\right)$, and suppose that $p_A \leq p_B$. But, if $\eta_A \left(\cdot \mid \bar{x}\right)$ first-order dominates $\eta_B \left(\cdot \mid \bar{x}\right)$ then A is more likely than B to prescribes the true state. Thus, $\rho_A \left(\omega_H \mid \omega_H, \bar{x}\right) > \rho_B \left(\omega_H \mid \omega_H, \bar{x}\right)$. Consequently, by accepting A's prescription ω_H, the customer is less likely to purchase unnecessary services. Moreover, $\rho_A \left(\omega_L \mid \omega_{_L}, \bar{x}\right) > \rho_B \left(\omega_L \mid \omega_{_L}, \bar{x}\right)$. Thus, by accepting A's prescription, $\omega_{_L}$, the customer is less likely to be provided with insufficient services. By assumption, purchasing insufficient services is more costly to the customer than having purchased the necessary service (i.e., $\omega_{_L} + b > \omega_H$). Thus, if $p_A \leq p_B$, it holds that, for all customers, $\Psi_A \left(p_A \mid b, \bar{x}\right) < \Psi_B \left(p_B \mid b, \bar{x}\right)$. Hence, all customers who seek remedial service, (i.e., all customers for whom $v > \Psi_A \left(p_A \mid b, \bar{x}\right)$) choose A for their first provider. Consequently, if provider B is to induce some customers to choose him as their first provider, p_B must be sufficiently lower then p_A so that $\Psi_B \left(p_B \mid b, \bar{x}\right) < \Psi_A \left(p_A \mid b, \bar{x}\right)$ for some $b \in \mathbb{B}$.

Customers that are prescribed ω_H by A are now in possession of new information that allows them to revise their anticipated expected cost of provider B. Let $\Psi_B \left(p_B \mid b, \bar{x}, \omega_H \left(A\right)\right)$ denote the updated anticipated cost of seeking a second opinion from provider B. Customers use the information to update their belief about the likelihoods of the states. The posterior distribution on the states, in turn, allows customers to revise their assessments of the likelihoods of the signals received by B and, given B's recommendation strategy, the corresponding expected cost of seeking a second opinion from B. Formally, by Bayes' rule, the customer's posterior beliefs about the states are:

$$\mu\left(\omega_H \mid \bar{x}, \omega_H(A)\right) = \frac{\rho_A\left(\omega_H \mid \omega_H, \bar{x}\right)\mu}{\rho_A\left(\omega_H \mid \omega_H, \bar{x}\right)\mu + \rho_A\left(\omega_H \mid \omega_L, \bar{x}\right)(1-\mu)} \tag{5}$$

and $\mu\left(\omega_L \mid \bar{x}, \omega_H(A)\right) = 1 - \mu\left(\omega_H \mid \bar{x}, \omega_H(A)\right).$

Thus, if the customer anticipates accepting provider B's prescription, her expected cost conditional on $\omega_H(A)$ is

$$\begin{aligned}\Psi_B\left(p_B \mid b, \bar{x}, \omega_H(A)\right) = \{&(\rho_B\left(\omega_H \mid \omega_H, \bar{x}\right) + \rho_B\left(\omega_H \mid \omega_L, \bar{x}\right))\mu\left(\omega_H \mid \bar{x}, \omega_H(A)\right)\omega_H \\ &+ [\rho_B\left(\omega_L \mid \omega_L, \bar{x}\right)\omega_L + \rho_B\left(\omega_L \mid \omega_H, \bar{x}\right))(\omega_L + b)]\left(1 - \mu\left(\omega_H \mid \bar{x}, \omega_H(A)\right)\right)\}p_B.\end{aligned} \tag{6}$$

Customers for whom $p_A\omega_H - \Psi_B\left(p_B \mid \theta, v, b, \bar{x}, \omega_H(A)\right) \leq \theta$ accept provider $A's$ prescription. All other customers seek a second opinion.

Let $z_j\left(\omega_k; \bar{x}\right)$ denote the probability that provider j prescribes ω_k given record $\bar{x}$. Formally,

$$z_j\left(\omega_k; \bar{x}\right) = \left(\rho_j\left(\omega_k \mid s_k, \bar{x}\right)\Pr\left(s_k \mid e_j\right) + \rho_j\left(\omega_k \mid s_{k'}, \bar{x}\right)\right)\Pr\left(s_{k'} \mid e_j\right). \tag{7}$$

For customers who anticipate seeking a second opinion, the expected cost of choosing

A for their first provider is:

$$\Psi_A(p_A \mid \theta, b, \bar{x}) = z_A(\omega_H, \bar{x})(\Psi_B(p_B \mid b, \bar{x}, \omega_H(A)) + \theta) + z_A(\omega_L, \bar{x})\,\omega_L p_A + \rho_A(\omega_L \mid \omega_H, \bar{x})\,\mu(\omega_H \mid \bar{x}, \omega_H \tag{8}$$

Thus, the probability that a customer who chooses A as her first provider seeks a second opinion from B is

$$D_B(p_B, p_A \mid \bar{x}) := z_A(\omega_H; \bar{x}) \int_{\mathbb{B}} \Lambda\{\theta \in \Theta \mid \theta < p_A\omega_H - \Psi_B(p_B \mid b, \bar{x}, \omega_H(A))\} dF(b). \tag{9}$$

I refer to this demand as the *intensive margin demand*.

In addition, if $\Psi_B(p_B \mid b, \bar{x}_B) < \Psi_A(p_A \mid \theta, v, b, \bar{x}_A)$, then B attracts customers from the *extensive margin,* (i.e., customers whose valuation, v, and trouble cost, b, are such that $v \in [\Psi_B(p_B \mid b, \bar{x}_B), \Psi_A(p_A \mid b, \bar{x}_A)]$). These customers will choose B as their first provider.

Let

$$K_B(p_A, p_B \mid \bar{x}) := D_B(p_B, p_A \mid \bar{x}) + [\Upsilon(\Psi_B(p_B \mid \theta, v, b, \bar{x})) - \Upsilon(\Psi_A(p_A \mid \theta, v, b, \bar{x}))]. \tag{10}$$

$K_B(p_A, p_B \mid \bar{x})$ is the probability that a new customer arriving on the market will, sooner or later, show up at B's facility. The expected hours of service demanded of provider B is

$$Q_B\left(p_A, p_B \mid \bar{x}\right) = K_B\left(p_A, p_B \mid \bar{x}\right)\left(z_B\left(\omega_L; \bar{x}\right)\omega_L + z_B\left(\omega_H; \bar{x}\right)\left(1 - \Upsilon(\omega_H p_B)\right)\omega_H\right), \tag{11}$$

where $1 - \Upsilon(\omega_H, p_B)$ is the probability that the customer accepts provider B's prescription, ω_H.

3.2.2 The leader's demand

The demand for provider A's services consist of the customers who choose A to be their first provider and accept A's prescription. The probability that a new customer arriving on the market will choose A as her first provider is $1 - \Upsilon\left(\Psi_A\left(p_A \mid b, \bar{x}\right)\right)$. Customers who choose B as their first provider are customers whose values v are smaller than $\Psi_A\left(p_A \mid b, \bar{x}\right)$. These customers do not seek a second opinion from A. Hence, the expected hours of service demanded of provider A are

$$Q_A\left(p_A, p_B \mid \bar{x}\right) = \left(1 - \Upsilon\left(\Psi_A\left(p_A \mid b, \bar{x}_A\right)\right)\right) z_A\left(\omega_L; \bar{x}\right)\omega_L + z_A\left(\omega_H; \bar{x}_A\right)\left(1 - D_B\left(p_B, p_A \mid \bar{x}\right)\right)\omega_H. \tag{12}$$

It then follows, from the properties of the distribution Λ, Υ, and F, that the demands for both providers are continuous functions and are monotonic decreasing in the provider's own price and monotonic increasing in the rival's price (i.e., $\partial Q_j\left(p_j, p_{j'} \mid \bar{x}\right)/\partial p_j < 0$ and $\partial Q_j\left(p_j, p_{j'} \mid \bar{x}\right)/\partial p_{j'} > 0$, $j \in \{A, B\}$).

3.3 Equilibrium of the providers' pricing game

At the outset of each stage game, the more reputable provider, A, posts his price followed by provider B. Given p_A, provider B posts a price that maximizes his profit, $\pi_B(p_A, p_B \mid \bar{x}) := (p_B - c) Q_B(p_A, p_B \mid \bar{x})$. Denote the optimal price of B as a function of the price of A by $p_B^*(p_A)$. Anticipating the reaction of B, provider A chooses his price so as to maximize

$$\pi_A(p_A, p_B^*(p_A) \mid \bar{x}) := (p_A - c) Q_A(p_A, p_B^*(p_A) \mid \bar{x}).$$

Let $p_A^* \in \arg\max \pi_A(p_A, p_B^*(p_A) \mid \bar{x})$.

The equilibrium in this market is a pair of prices that are the providers' best responses against each other in the pricing game. Moreover, because the demands for services are derived from the providers' prescriptions and the customers' acceptance strategies in the prescriptions subgame, the pricing strategies are best responses against these strategies as well. As such, they are an integral part of the equilibrium of the stage game.

Definition 2: An *equilibrium of the providers' pricing game* is a pair of prices (p_A^*, p_B^*) such that $p_j^* \geq c$, $j \geq \{A, B\}$, $p_B^* \in \arg\max\ \pi_B(p_A^*, p_B \mid \bar{x})$, and $p_A^* \in \arg\max \pi_A(p_A, p_B^*(p_A) \mid \bar{x})$.

With this definition in mind I, state the first result:

Theorem 1. *There exists an equilibrium of the providers' pricing game* (p_A^*, p_B^*) *in which* A *is the first provider and* $p_j^* > c$, $j \in \{A, B\}$.

The proof is in the Appendix.

The effect of reputation on the equilibrium prices is summarized in the following proposition.

Proposition. *An increase in the reputation of either provider increases his equilibrium price and lower the price of his rival.*

Proof. By definition, the customer perceives that the probability of being prescribed the necessary service is grater the more reputable is the provider. Consequently, the anticipated expected cost to the customer decreases as the reputation of the provider increases. Formally, $\mathsf{r}_A(\bar{x}) > \mathsf{r}_A(\bar{x}')$ implies that $\Psi_A(p_A^* \mid b, \bar{x}') < \Psi_A(p_A^* \mid b, \bar{x})$. Ceteris paribus, this increases the demand for A by increasing his retention rate and attracting additional customers who chose B as their first provider. Consequently, the demand and profits of B decrease. To restore equilibrium, the demands must be readjusted, by an increase in the price A charges and a decrease in the price that B charges. ■

Given the reputations $\mathsf{r}_A(\bar{x})$ and $\mathsf{r}_B(\bar{x})$, the equilibrium price difference, $p_A^* - p_B^*$, represents the *reputation advantage*. The corresponding profit difference, $\pi_A(p_A^*, p_B^* \mid \bar{x}) - \pi_B(p_A^*, p_B^* \mid \bar{x})$, is the *reputation rent*. The proposition implies that the reputation advantage of A increases in his reputation and decreases in that of B.

4 The Prescriptions Subgame

4.1 Preliminaries

At the start of a subscription subgame, Γ_E, nature assigns a customer a state in Ω according to commonly known probabilities. If the customer decides to seek treatment, she chooses her first provider, setting in motion a game in which the players, in the order of their moves, are the first provider; the customer; and, if called upon, the second provider; and the customer.

The first provider diagnoses the problem, updates his probabilities of the two states, and chooses a level of service to prescribe. Upon obtaining the first provider's prescription, the customer updates her beliefs about the likelihoods of the prescriptions of the second provider and chooses between accepting the first provider's prescription, thereby terminating the game, and seeking a second opinion. If the customer decides to seek a second opinion, the second provider diagnoses the problem, updates his probabilities of the two states, and chooses a level of service to prescribe. After receiving the second prescription, the customer must either accept it or forgo the service altogether. In either case the customer's decision terminates the stage game.[10]

4.2 Customers' strategies, beliefs, and payoffs

I assume that customers are myopic, in the sense that, upon noticing a problem, they consider solely the next interaction with providers.[11] Given providers' reputations and prices of the pricing game, the customer's first decision is between forgoing and seeking service. A customer chooses the former alternative if and only

[10] Note that customers from the extensive-margin demand of B never seek a second opinion. Thus, all customers who show up at A's service facility are customers who choose A as first provider. Consequently, provider A knows that he is the customer's first provider. By contrast, if the equilibrium prices are such that the extensive-margin demand of the less reputable provider is positive, then provider B does not know whether he is the customer's first or second provider.

[11] For a model of a credence good market that includes client loyalty and its effect on the perpetration of fraud, see Karni (2024).

if $v \leq \min\{\Psi_A(p_A \mid \theta, v, b, \bar{x}_A), \Psi_B(p_B \mid \theta, v, b, \bar{x}_B)\}$. If she decides to seek service, she chooses A to be her first provider if $\Psi_A(p_A \mid \theta, v, b, \bar{x}_A) \leq \Psi_B(p_B \mid \theta, v, b, \bar{x}_B)$. Otherwise she visits provider B first.

Customers' strategies. Let a and r denote the customer's decisions to accept or reject a provider's prescription, respectively, and let 1 and 0 denote the customer's decision to seek or forgo service altogether, respectively. Customers' strategies are mappings from the set of types to contingent plans specifying their responses to the providers prescriptions. Formally, the customer strategy consists of three mappings: $\sigma_0 : \Theta \times V \times \mathbb{B} \to \{0, 1\}$, depicts the customer's choice to seek or forgo seeking service. If the customer decides to seek services, her strategy in the subgame played with her first provider is the mapping $\sigma_1 : \Theta \times V \times \mathbb{B} \to \{\Delta(a, r, 0)\}^{\Omega}$, which depicts the customer's plan of action following her visit to the first provider. Specifically, $\sigma_1(\omega_k \mid \theta, v, b)(\ell)$ denotes the probability of choosing $\ell \in \{a, r, 0\}$ if the prescription is ω_k, $k \in \{L, H\}$.[12] If after receiving the first provider's prescription the customer decides to seek a second opinion, her strategy is the mapping $\sigma_2 : \Theta \times V \times \mathbb{B} \times \Omega \to \{\Delta(a, r)\}^{\Omega}$, which depicts the plan of action contingent on the second provider's prescription.[13] Specifically, $\sigma_2(\omega_k \mid \theta, v, b)(\ell)$ denotes the probability of choosing $\ell \in \{a, r\}$ if the second prescription is ω_k, $k \in \{L, H\}$. Because $\sigma_0(\theta, v, b) = 0$ implies that the customer does not look for service, she is not in the market. Henceforth, I consider only the strategies of customers who seek service (i.e., customers for whom $\sigma_0(\theta, v, b) = 1$). I denote by $\Sigma := \{\sigma_1, \sigma_2)\}$ the set of the customer's strategies in

[12] Thus, $\sigma_1(\omega_k \mid \theta, v, b)(\ell) \geq 0$, for all $\ell \in \{a, r, 0\}$, and $\Sigma_{\ell \in \{a,r,0\}} \sigma_1(\omega_k \mid \theta, v, b)(\ell) = 1$

[13] Recall that, should the customer decide to seek a second opinion, she must accept the second provider's prescription or reject it and, thereby, forgo fixing the problem.

the prescription subgame.

The customers' system of beliefs. The customer's prior beliefs regarding the likelihoods of the states ω_H and ω_L are μ and $(1-\mu)$, respectively. Upon receiving the first provider's prescription, $\omega_k(j_1)$, the customer updates her beliefs about the states as follow:

$$\mu\left(\omega_H \mid \bar{x}, \omega_k(j_1)\right) = \frac{\rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}\right)\mu}{\rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}\right)\mu + \rho_{j_1}\left(\omega_H\left(j_1\right) \mid \omega_L, \bar{x}\right)(1-\mu)}, \tag{13}$$

$j_1 \in \{A, B\}$, and

$$\mu\left(\omega_L \mid \bar{x}, \omega_H\left(j_1\right)\right) = 1 - \mu\left(\omega_H \mid \bar{x}, \omega_H\left(j_1\right)\right). \tag{14}$$

Let $\eta\left(\omega_H\right) = \left(\mu, \mu\left(\omega_H \mid \bar{x}, \omega_k\left(j_1\right)\right)\right)$, $j_1 \in \{A, B\}$, denote the customer's system of beliefs following her visit to the first provider. Clearly, the system of beliefs depends on her choice of the first provider and the provider's reputation.

The customer's payoffs. Given prices p_A and p_B, set by the providers in the pricing game, and the first provider's prescription, the customer's expected payoffs are as follows: If the first provider prescribes ω_L and the customer accepts the prescription, her payoff is $v - \omega_L p_{j_1} - \mu\left(\omega_H \mid \bar{x}, \omega_L(j_1)\right)b$. If the first provider prescribes ω_H and the customer accepts the prescription, her payoff is $v - \omega_H p_{j_1} > 0$.

The customer never rejects the first provider's prescription ω_L.[14] She rejects the

[14] If the customer rejects the first provider's prescription ω_L, it must hold that either the prior expected payoff of the second provider exceeds that of the first, in which case she would have chosen to visit the second provider first or the payoff of both providers is negative, in which case she forgoes seeking service.

first provider's prescription ω_H if

$$v - \omega_H p_{j_1} < \max\{v - \theta - \Psi_{j_2}\left(p_{j_2} \mid \theta, v, b, \bar{x}, \omega_H\left(j_1\right)\right), 0\}, \tag{15}$$

where $\Psi_{j2}\left(p_{j2} \mid \theta, v, b, \bar{x}, \omega_L\left(j_1\right)\right)$ is calculated invoking the arguments that produced $\Psi_B\left(p_B^* \mid \theta, v, b, \bar{x}, \omega_L\left(A\right)\right)$ in equation (6).

If the customer rejects the first provider's prescription and seeks a second opinion, her payoffs are as follows: If the second provider prescribes ω_L, the customer accepts it, and her payoff is $v - \omega_L p_{j_2} - \mu\left(\omega_H \mid \bar{x}, \omega_H\left(j_1\right), \omega_L(j_2)\right)b > 0$, where

$$\begin{aligned} &\mu\left(\omega_H \mid \bar{x}, \omega_H\left(j_1\right), \omega_L(j_2)\right) \\ &= \frac{\rho_{j_2}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}\right)\mu\left(\omega_H \mid \bar{x}, \omega_H(j_1)\right)}{\rho_{j_2}\left(\omega_H\left(j_1\right) \mid \omega_H, \bar{x}\right)\mu\left(\omega_H \mid \bar{x}, \omega_H(j_1)\right) + \rho_{j_2}\left(\omega_H\left(j_1\right) \mid \omega_L, \bar{x}\right)\mu\left(\omega_L \mid \bar{x}, \omega_L(j_1)\right)}. \end{aligned} \tag{16}$$

If the second provider prescribes ω_H and the customer accepts her payoff is $v - \omega_H p_{j_2} > 0$, otherwise she forgoes fixing the problem and her payoff is 0.

4.3 The providers' strategies, beliefs and payoffs

The providers strategies were discussed in section 3.2. The providers' prior beliefs about the likelihoods of the states ω_H and ω_L are μ and $1 - \mu$, respectively. Their posterior beliefs are given by equations (1) and (2).

Providers plan their moves in the prescription subgame taking into account the fact that the demand for their service depends on their reputations. Looking ahead, they are concerned with the reputations effect of their current decisions.

To formalize the providers' payoffs, let $W_j(\mathsf{r}_A(\bar{x}^n), \mathsf{r}_B(\bar{x}^n))$ denote provider $j's$ expected present value of the future profits generated by the equilibrium strategies of the two players given their current reputations, $\mathsf{r}_A(\bar{x}^n)$ and $\mathsf{r}_B(\bar{x}^n)$ at the outset of the $n+1$ stage game. I denote by C the cost associated with having to compensate the customer for the misprescription.[15]

If A prescribes ω_L, it is accepted.[16] Then, under Scenario 3 (i.e., $\omega_L(A)$ is sufficient), his payoff is

$$P_A(\omega_L(A) \mid \omega_L) = (p_A - c)\,\omega_L + W_A\left(\mathsf{r}_A\left(\bar{x}^{n+1}\right), \mathsf{r}_B\left(\bar{x}^{n+1}\right)\right) - W_A(\mathsf{r}_A(\bar{x}^n), \mathsf{r}_B(\bar{x}^n)), \tag{17}$$

where x_{n+1} is depicted by Scenario 3. Then $\mathsf{r}_A(\bar{x}^{n+1}) > \mathsf{r}_A(\bar{x}^n)$ and $\mathsf{r}_B(\bar{x}^{n+1}) = \mathsf{r}_B(\bar{x}^n)$. Thus, $W_A(\mathsf{r}_A(\bar{x}^{n+1}), \mathsf{r}_B(\bar{x}^{n+1})) - W_A(\mathsf{r}_A(\bar{x}^n), \mathsf{r}_B(\bar{x}^n)) > 0$.

Under Scenario 4 (i.e., $\omega_L(A)$ is insufficient), the provider's payoff is

$$P_A(\omega_L(A) \mid \omega_H) = (p_A - c)\,\omega_L - \mu_A(\omega_H(A) \mid s)\left[C + (W_A\left(\mathsf{r}_A\left(\bar{x}^{n+1}\right), \mathsf{r}_B\left(\bar{x}^{n+1}\right)\right) - W_A(\mathsf{r}_A(\bar{x}^n), \mathsf{r}_B(\bar{x}^n))\right] \tag{18}$$

where x_{n+1} is depicted by Scenario 4. Then $\mathsf{r}_A(\bar{x}^{n+1}) < \mathsf{r}_A(\bar{x}^n)$ and $\mathsf{r}_B(\bar{x}^{n+1}) = \mathsf{r}_B(\bar{x}^n)$. Thus, $W_A(\mathsf{r}_A(\bar{x}^{n+1}), \mathsf{r}_B(\bar{x}^{n+1})) - W_A(\mathsf{r}_A(\bar{x}^n), \mathsf{r}_B(\bar{x}^n)) < 0$.

Thus, given $s \in S$, provider $A's$ expected payoff is

$$P_A(\omega_L(A) \mid s) := P_A(\omega_L(A) \mid \omega_L)\,\mu_A(\omega_L \mid s) + P_A(\omega_L(A) \mid \omega_H)\,\mu_A(\omega_H \mid s). \tag{19}$$

[15] This cost may include malpractice payouts.

[16] Rejection of ω_L is off the equilibrium path.

I assume that the cost C, and the loss of future profits due to loss of reputation, are sufficiently large as to make $P_A(\omega_L(A) \mid \omega_L) > 0$ and $P_A(\omega_L(A) \mid \omega_H) < 0$. Thus, by equation (19), A would never prescribe ω_L if the signal received is s_H.

If A prescribes ω_H and it is accepted, his payoff is $P_A(\omega_H(A) \mid s, a) = (p_A - c)\,\omega_H > 0$. If it is rejected, his expected payoff depends on the second provider's prescription. In particular, if the second provider prescribes ω_H, the first provider's payoff is zero. By contrast, if the second provider prescribes ω_L and this prescription turns out to be sufficient (i.e., Scenario 1 holds) then the first provider suffers loss of reputation. Formally, the first provider's payoff is

$$P_A(\omega_H(A) \mid \omega_L) = \rho_B(\omega_L(B) \mid \omega_L, \bar{x})\,[W_A\left(\mathsf{r}_A\left(\bar{x}^{n+1}\right), \mathsf{r}_B\left(\bar{x}^{n+1}\right)\right) - W_A(\mathsf{r}_A(\bar{x}), \mathsf{r}_B(\bar{x})) < 0. \tag{20}$$

Provider B's payoffs are as follows: If B prescribes $\omega_L(B)$ it is accepted. If ω_L is the true state then B's payoff is

$$P_B(\omega_L(B) \mid \omega_L) = (p_B - c)\,\omega_L > 0.$$

If the true state is ω_H, B's prescription is revealed to be insufficient (i.e., x_{n+1} corresponds to Scenario 2). In this case, B's payoff is:

$$P_B(\omega_L(B) \mid \omega_H) = (p_B - c)\,\omega_L - \mu_B(\omega_H(B) \mid s)\,[C - (W_B\left(\mathsf{r}_A\left(\bar{x}^{n+1},\right), \mathsf{r}_B\left(\bar{x}^{n+1}\right)\right) - W_B(\mathsf{r}_A(\bar{x}), \mathsf{r}_B(\bar{x})))] \tag{21}$$

If B prescribes ω_H and it is accepted, then, regardless of the true state, his payoff is $P_B(\omega_H(B)) = (p_B - c)\,\omega_H > 0$. If B's prescription is rejected, his payoff is zero.

4.4 Stage-game equilibrium

The concept of equilibrium used in the analysis of the stage game, Γ_E, is perfect Bayesian equilibrium (PBE). This equilibrium consists of a profile of strategies $\vartheta^* = (p_A^*, p_B^*, \sigma^*, \varphi_A^*, \varphi_B^*)$ and system of beliefs η^* specifying a probability $\eta^*(y) \in [0,1]$ for each decision node y in Γ_E such that $\Sigma_{y \in I} \eta^*(y) = 1$, for all informations sets I, such that (a) the strategy profile ϑ^* is sequentially rational given the system of beliefs η^* (i.e., given the system of beliefs η, the player who moves at any information set maximizes his expected utility given the strategies of the other players) and (b) the system of beliefs η^* is derived from the strategy profile ϑ^* using Bayes' rule whenever possible.

Theorem 2: *Given the providers' reputations* $\mathsf{r}_A(\bar{x}), \mathsf{r}_B(\bar{x})$ *there exist PBE of the stage game* Γ_E.

The proof is by application of Kakutani's (1941) fixed point theorem and is given in the appendix.

5 Discussion

5.1 Fraudulent behavior: Reputation and liability effects

Fraud is said to be committed when a provider prescribes ω_H upon receiving the signal s_L, as the provider, knowingly prescribes a service that he knows to be unnecessary more likely than not. Fraudulent behavior is motivated by the extra profit, $(\omega_H - \omega_L)(p_j - c)$, it may generate. Contributing to the perpetration of fraud are higher prices of service and liability cost, C.[17]

The effects of reputation on the inclination to commit fraud are ambiguous. Consider the first provider. The desire to avoid the loss of reputation due to erroneously prescribing insufficient service increases the first provider's inclination to prescribe ω_H when he receives the signal s_L. If, however, a provider prescribes ω_H when his diagnosis indicates that the state is ω_L, he runs the risk of losing reputation if the customer seeks a second opinion and the second provider prescribes the true state ω_L. This potential loss of reputation deters the provider from prescribing unnecessary service. The relative impacts of the opposing pulls depend on the first provider's estimates of the respective likelihoods that his prescription is discovered to be insufficient or unnecessary. In equilibrium, the probabilities of these events are determined, by the strategies of the customer and the second provider. The higher these probabilities, the more likely the first provider is to avoid prescribing unnecessary treatment.

The second (less reputable) provider's incentive to commit fraud is mitigated by the effect it has on (a) the inclination of customers to seek a second opinion (i.e., the smaller is the intensive-margin demand) and (b) by increasing his anticipated cost of service it reduces the second provider's extensive-margin demand. In other words, the second provider's concerns about his reputation makes his equilibrium strategy more likely to prescribe ω_H when the signal he receives is s_L, the less likely is a customer (who was prescribed ω_H by the first provider), to seek a second prescription, and

[17] The provider is liable if he prescribes ω_L when the true state is ω_H. If the liability cost, which may include malpractice payouts is higher, the provider is inclined to prescribe ω_H even if the signal he receives is s_L.

more likely is the customer who chooses him the be his first provider decide to forgo service altogether. In addition, by prescribing ω_H, the second provider loses the income from customers who would have accepted the prescription ω_L but forego the higher level of service.

5.2 Reputation dynamics

The reputations of providers evolve stochastically, with the change between successive stage games depending on the equilibrium strategies of providers and customers. Specifically, if $e_A > e_B$, then A is less likely to misdiagnose the problem. Thus, given the signals received, if providers prescribe honestly then A is less likely than B to misdiagnose the problem and suffer loss of reputation.

Repeated observations make the updated reputations converge to the providers' true levels of expertise (i.e., $\lim_{n\to} \int_{[1/2,1]} e_k d\eta \left(e_k \mid \bar{x}^n\right) = e_k, k \in \{A, B\}$). Consequently, if providers prescribes honestly, the "reputation gap" tends to drift toward the expertise gap, $e_A - e_B$. If, as a result, a provider's equilibrium price increase sufficiently, then, risking loosing reputation, the provider will adopt more aggressive fraudulent strategies. In the long run, the reputations and the providers' prices, will converge to a stationary stochastic equilibrium in which the marginal expected profit from prescribing unnecessary service is equal to the expected marginal cost of loss of reputation. If the expertise levels (and the prices) are high enough, the credence service market equilibrium supports a level of fraud. The welfare effect is the waste of resources devoted to the provision of unnecessary services.

5.3 Related literature

The literature on the role and economic implications of reputations is multifaceted. Hörner (2002) studies the economic effects of reputations in competitive markets for experience goods. Unlike the approach taken in this paper, according to which providers' reputations are based on their records of past performance, Hörner identifies firms' reputations with their customer base. In addition, the main thrust of this paper is on the role of expertise (i.e., a hidden characteristic) and the effect it has on the perpetration of fraud in credence service markets. In contrast, Hörner's main focus is the role of the firms' efforts (i.e., hidden actions) in delivering high-quality products and the evolutions of their reputations.

Closer to the topic of this paper are the studies by Dulleck, Kerschbamer and Sutter, (2011); Mimra, Rasch, and Waibel (2016); and Fong and Ting (2018) on reputations in credence goods markets.

Dulleck et. al (2011) investigate, experimentaly, the impact of reputation on experts' commitment of fraud, with and without liability. In their experiment customers and providers interact over finite number of periods, during which providers may build reputations with customers based on the latter's personal experiences. They find that reputation has little effect on mitigating the welfare loss, due to the inherited asymmetry of information that characterizes credence good markets.

The experimental study by Mimra, Rasch, and Waibel (2016) examine the effects of the pricing and information regimes (competitive versus fixed prices and private versus public information) on providers' performance, reputation, and the perpetration of fraud. Both their setup and objectives are different from those of this paper. In particular, because liability is absent form their design, fraud takes the form of undertreatment rather than overtreatment and of charging for services not performed. Their findings suggest that price competition undermines the incentive to build reputation by offering correct treatment and, consequently, induces higher levels of fraud than a regime of fixed (regulated) prices. Unlike in this paper, the expert in the study observes the state perfectly.

Invoking a repeated game framework, Fong and Ting (2018) study the interactions between a monopoly expert, who diagnoses the true state correctly, and short-lived customers. The main thrust of their analysis is the impact of liability on the provider's incentive to build, and maintain, a reputation for quality service. Fraudulent behavior involves undertreatment of a serious problem. In this context, under liability, customers are fully compensated for undertreatment. Consequently, they do not care about the reputation of the expert, which undermines the provider's incentives to build and maintain his reputation. Their work is quite different from this paper, starting with the market structure (i.e., monopoly versus duopoly); the lack of concern about the level of expertise; and the definition of fraudulent behavior.

APPENDIX

Proof of theorem 1 - For each $p_A \geq c$, let $p_B^* = p_B^* (p_A)$ be the solutions to the following equation:

$$Q_B (p_A, p_B \mid \bar{x}_A, \bar{x}_B) + (p_B - c) \frac{\partial Q_B (p_A, p_B \mid \bar{x})}{\partial p_B} = 0. \tag{22}$$

Given B's reaction function, $p_B^* (\cdot)$, define p_A^* by the solution to:

$$Q_A (p_A, p_B^* (p_A) \mid \bar{x}) = (c - p_A) [\frac{\partial Q_A (p_A, p_B^* (p_A) \mid \bar{x})}{\partial p_A} + \frac{\partial Q_A (p_A, p_B^* (p_A) \mid \bar{x})}{\partial p_B} \frac{dp_B^* (p_A)}{dp_A}]. \tag{23}$$

Since $Q_j (p_j, p_{j'} \mid \bar{x}) > 0$ and $\partial Q_j (p_j, p_{j'} \mid \bar{x}) / \partial p_j < 0$, if $p_B \leq c$, then the expressions on the left-hand side of equation (22) are positive. A contradiction. Thus, $p_B^* > c$. Furthermore, $p_A^* > p_B^*$. Hence, $p_A^* > c$. Thus, (p_A^*, p_B^*), where $p_B^* = p_B^* (p_A^*)$ is an equilibrium of the providers' pricing game. ■

Proof of theorem 2 - Since the prices may not exceed $\bar{v}$, the upper bound on the value assigned by the customer to fixing the problem, $p_j \in [0, \bar{v}]$, $j \in \{A, B\}$.

Given $(\theta, v, b) \in \Theta \times V \times \mathbb{B}$ and $(e_A, e_B) \in [1/2, 1]^2$, define a best-response correspondence from $[0, \bar{v}] \times [0, \bar{v}] \times \Sigma \times \Delta (\Omega)^S \times \Delta (\Omega)^S$ to itself

$$\Xi\left(p_A, p_B, \sigma\left(\theta, v, b\right), \varphi_A\left(e_A\right), \varphi_B\left(e_B\right)\right) \rightrightarrows \left(p_A^*, p_B^*, \sigma^*\left(\theta, v, b\right), \varphi_A^*\left(e_A\right), \varphi_B^*\left(e_B\right)\right), \tag{24}$$

such that p_A^* and p_B^* are profit-maximizing prices given $\sigma\left(\theta, v, b\right), \varphi_A\left(e_A\right), \varphi_B\left(e_B\right)$, and $\sigma^*\left(\theta, v, b\right), \varphi_A^*\left(e_A\right), \varphi_B^*\left(e_B\right)$ are the customer's and providers' best-response correspondents to $\left(p_A, p_B, \sigma\left(\theta, v, b\right), \varphi_A\left(e_A\right), \varphi_B\left(e_B\right)\right)$.

To prove that Ξ has a fixed point we need to show that: (a) $[0, \bar{v}] \times [0, \bar{v}] \times \Sigma \times \Delta\left(\Omega\right)^S \times \Delta\left(\Omega\right)^S$ is compact (in the product topology) and convex, (b) given a system beliefs η^*, Ξ is upper hemicontinuous and $\Xi\left(p_A, p_B, \sigma\left(\theta, v, b\right), \varphi_A\left(e_A\right), \varphi_B\left(e_B\right)\right)$ is nonempty and convex for every point in its domain and (c) that η^* is derived from the strategy profile $\vartheta^* = \left(p_A^*, p_B^*, \sigma^*\left(\theta, v, b\right), \varphi_A^*\left(e_A\right), \varphi_B^*\left(e_B\right)\right)$, using Bayes' rule, whenever possible.

Since $[0, \bar{v}]$, Σ and $\Delta\left(\Omega\right)^S$ are compact (in the product topology), so is the product $[0, \bar{v}] \times [0, \bar{v}] \times \Sigma \times \Delta\left(\Omega\right)^S \times \Delta\left(\Omega\right)^S$. Given

$$\left(\sigma\left(\theta, v, b\right), \varphi_A\left(e_A\right), \varphi_B\left(e_B\right)\right), \left(\sigma'\left(\theta, v, b\right), \varphi_A'\left(e_A\right), \varphi_B'\left(e_B\right)\right) \in \Sigma \times \Delta\left(\Omega\right)^S \times \Delta\left(\Omega\right)^S,$$

and $\gamma \in [0, 1]$, $k \in \{L, H\}$, define

$$\left(\gamma\sigma_1 + (1-\gamma)\sigma_1'\right)\left(\omega_k \mid \theta, v, b\right)(\ell) = \gamma\sigma_1\left(\omega_k \mid \theta, v, b\right)(\ell) + (1-\gamma)\sigma_1'\left(\omega_k \mid \theta, v, b\right)(\ell),$$

$\ell \in \{a, r, 0\}$, and

$$(\gamma\sigma_2 + (1-\gamma)\sigma_2')(\omega_k \mid \theta, v, b)(\ell) = \gamma\sigma_2(\omega_k \mid \theta, v, b)(\ell) + (1-\gamma)\sigma_2'(\omega_k \mid \theta, v, b)(\ell),$$

$\ell \in \{a, r\}$.

Recall that $\varphi(e) = (\alpha(s_L, e)(\omega_H), \alpha(s_H, e)(\omega_H))$. Define $\gamma\varphi_j(e_j) + (1-\gamma)\varphi_j'(e_j)$ by

$$\left(\gamma\alpha_j(s_L, e)(\omega_H) + (1-\gamma)\alpha_j'(s_L, e)(\omega_H)\right), \left(\gamma\alpha_j(s_H, e)(\omega_H) + (1-\gamma)\alpha_j'(s_H, e)(\omega_H)\right)$$

$j \in \{A, B\}$ and $k \in \{L, H\}$. Then, $\gamma\varphi_j(e_j) + (1-\gamma)\varphi_j'(e_j) \in \Delta(\Omega)^S$.

$$((\gamma\sigma + (1-\gamma)\sigma')(\theta, v, b), (\gamma\varphi_A + (1-\gamma)\varphi_A')(e_A), (\gamma\varphi_B + (1-\gamma)\varphi_B')(e_B)) \in \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S.$$

Hence, $\Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$ is convex, and so is $[0, \bar{v}] \times [0, \bar{v}] \times \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$.

Define $\xi : [0, \bar{v}] \times \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S \rightrightarrows [0, \bar{v}]$ by $\xi(p_B, \sigma, \varphi_A, \varphi_B) = [0, \bar{v}]$ for all $(p_B, \sigma, \varphi_A, \varphi_B) \in [0, \bar{v}] \times \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$. Then ξ is continuous correspondence. Let $\pi_A : Gr\xi \to \mathbb{R}$ be the profits function of provider A. Define

$$m(p_B, \sigma, \varphi_A, \varphi_B) = \max_{p_A \in \xi(p_B, \sigma, \varphi_A, \varphi_B)} \pi_A(p_A, (p_B, \sigma, \varphi_A, \varphi_B))$$

and the correspondence $\varsigma : [0, \bar{v}] \times \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S \rightrightarrows [0, \bar{v}]$ by

$$\varsigma(p_B, \sigma, \varphi_A, \varphi_B) = \{p_A^* \in \xi(p_B, \sigma, \varphi_A, \varphi_B) \mid \pi_A(p_A^*, (p_B, \sigma, \varphi_A, \varphi_B)) = m(p_B, \sigma, \varphi_A, \varphi_B)\}.$$

Then, by Theorem 1, depicts provider A's best-response strategy in the pricing game. By Berge Maximum Theorem, ς has nonempty compact values and, since $[0, \bar{v}]$ is Housdorff, it is upper hemicontinuous.[18] The same argument applies to p_B.

Consider next the customers best-response correspondence. Regardless of the second provider's strategy, $\sigma_1^* (\omega_L \mid \theta, v, b)(a) = 1$. To see this, suppose by way of negation that this is false. Then the expected payoff of seeking a second opinion must exceed the payoff of accepting the best possible prescription of the first provider. But, in this case the customer could go directly to the second provider and saves herself the search cost. A contradiction.

Consider next, the case in which the customer's first provider prescribes $\omega_H (j_1)$. Define $\bar{\theta} \in \Theta$ by the equation

$$\omega_H p_{j_1} = \Psi_{j_2} (p_{j_2} \mid b, \bar{x}, \omega_H (j_1)) - \bar{\theta}. \tag{25}$$

Then, $\omega_H p_{j_1} < (=, >) \Psi_{j_2} (p_{j_2} \mid b, \bar{x}, \omega_H (j_1)) - \theta$, for all $\theta < (=, >) \bar{\theta}$.[19] Consequently,

$$\sigma_1^* (\omega_H \mid \theta, v, b)(a) = \left| \begin{array}{ll} 1 & \text{if} \quad \theta > \bar{\theta} \\ 0 & \text{if} \quad \theta < \bar{\theta} \\ [0,1] & \text{if} \quad \theta = \bar{\theta} \end{array} \right| . \tag{26}$$

By definition $\Psi_{j2} (p_{j2} \mid b, \bar{x}, \omega_H (j_1))$ is continuous in p_{j2} and in the second provider's strategy φ_{j2} (see (6)). Hence, of $\bar{\theta} (b, \bar{x}, \omega_H (j_1), p_A, p_B)$ is continuous in p_{j1}, p_{j2} and φ_{j2}. Hence, $\sigma_1^* (\omega_H \mid \theta, v, b, \varphi_{j2})$ is nonempty upper hemicontinuous.

[18] See Aliprantice and Border Theorem 17.31.

[19] By defintion $\bar{\theta} : \Theta \times V \times \mathbb{B} \times \Delta (\Omega)^S \times \Omega \times [0, \bar{v}] \times [0, \bar{v}] \rightarrow \mathbb{R}$ is a function.

Consider next $\sigma_2(\omega \mid \theta, v, b)$. Obviously, if the second provider prescribes ω_L, then $\sigma_2^*(\omega_L \mid \theta, v, b)(a) = 1$. If the second provider prescribes ω_H, then $\sigma_2^*(\omega_H \mid \theta, v, b)(a) = 1$ if $v > \omega_H p_{j_2}$, $\sigma_2^*(\omega_H \mid \theta, v, b)(a) = 0$ if $v < \omega_H p_{j_2}$, and $\sigma_2^*(\omega_H \mid \theta, v, b) = [0, 1]$ if $v = \omega_H p_{j_2}$. Thus, $\sigma_2^*(\omega_H \mid \theta, v, b)$ is continuous in p_{j_2} and is independent of the providers' strategies.

Thus, the customer's best-response strategy, $\sigma^* = (\sigma_1^*(\theta, v, b), \sigma_2^*(\theta, v, b))$ is upper hemicontinuous correspondence with nonempty convex range, and $\sigma_1^*(\omega_k \mid \theta, v, b)$ is nonempty and convex for every $\omega_k \in \Omega$, $k \in \{L, H\}$.[20]

Providers' strategies in the prescription subgame are correspondences $\varphi_j \in \Phi$, $j \in \{A, B\}$, whose domains, $[1/2, 1]$, and ranges, $\Delta(\Omega)^S$, are compact and convex.

Consider the first provider A's best-response correspondence. If the signal received is s_L and A prescribes ω_L then, by the argument above, the customer accepts the prescription with probability 1, and the provider's payoff is: $P_A(\omega_L(A) \mid s_L)$ (see (19). If, instead, he prescribes ω_H, then, by $\sigma_1(\omega_H(A) \mid \theta, v, b, \varphi_{j_2})$, the set of customer types that accept this prescription is $Z := \{(\theta, b) \in \Theta \times \mathbb{B} \mid \omega_H p_A - \Psi_B(p_B \mid \theta, v, b, \varphi_B, \omega_H(A)) \leq \theta\}$. All other customers seek a second opinion and, insofar as A is concerned, are lost. If B prescribes ω_H then A's reputation is unchanged. However, if B prescribes ω_L, this will have reputation consequences for A. Specifically, if the true state turns out to be ω_L, A's reputation diminishes (Scenario 1) ($d\mathsf{r}_A^- = \rho_A(\omega_H \mid \omega_H, \bar{x}^{n+1}) - \rho_A(\omega_H \mid \omega_H, \bar{x}^n) < 0$, x_{n+1} where is Scenario 1) and if it turns out to be ω_H, A's reputation is enhanced (Scenario 2)

[20]Note that under the assumption on the distributions of θ and v, the probability that the customer's best response takes the form of randomized choice is of measure zero. That is, for all practical purposes, the best response correspondence consists of pure strategies.

$(d\mathsf{r}_A^+ = \rho_A(\omega_H \mid \omega_H, \bar{x}^{n+1}) - \rho_A(\omega_H \mid \omega_H, \bar{x}^n) > 0$, x_{n+1} is Scenario 2). Let $W(d\mathsf{r}_A)$ the change of provider A's expected future income due to change of his reputation. Formally,

$$W\left(d\mathsf{r}_A^-\right) = \rho_B(\omega_L \mid \omega_H, \bar{x})\, \mu W(d\mathsf{r}_A^{+-}) + \rho_B(\omega_L \mid \omega_L, \bar{x})\,(1-\mu)\, W(d\mathsf{r}_A^-). \tag{27}$$

By (9) the probability of Z is $(1 - D_B(p_B, p_A \mid \bar{x}))$. Hence, the expected payoff of A is

$$P_A(\omega_H \mid s_L) := (1 - D_B(p_B, p_A \mid \bar{x}))\,(p_A - c)\,\omega_H + D_B(p_B, p_A \mid \bar{x})\, W(d\mathsf{r}_A^-). \tag{28}$$

Thus, A's best response contingent on receiving the signal s_L is:

$$\alpha_A(s_L, e_A)(\omega_H) = \begin{vmatrix} 0 & \text{if} & P_A(\omega_L \mid s_L) < 0 \\ 1 & \text{if} & P_A(\omega_L \mid s_L) > 0 \\ [0,1] & \text{if} & P_A(\omega_L \mid s_L) = 0 \end{vmatrix}. \tag{29}$$

By (6) and (9), $D_B(p_B, p_A \mid \bar{x})$ is continuous in φ_B and $\sigma_1(\theta, v, b, \varphi_B)$.

If the signal received by provider A is s_H and he prescribes ω_H, the expected payoff is:

$$P_A(\omega_H \mid s_H) := (1 - D_B(p_B, p_A \mid \bar{x}))\,(p_A - c)\,\omega_H + D_B(p_B, p_A \mid \bar{x}))\,(1-\mu)\, P_A(\omega_H(A) \mid \omega_L), \tag{30}$$

where $P_A(\omega_H(A) \mid \omega_L) < 0$ is given in (20). If he prescribes ω_L instead then the prescription is accepted and the payoff given (18). By assumption it is negative, and A would never prescribe ω_L if the signal received is s_H (i.e., $\alpha_A(s_H, e_A)(\omega_L) = 0$). Thus, if the signal is s_H the provider A's best response is $\alpha_A(s_H, e_A)(\omega_H) = 1$. Hence, provider A's best-response correspondence φ_A has a closed graph that is bounded. Hence, it is upper hemicontinuous.

If a customer shows up at B's service station, she initiates a substage game in which the players are provider B and the customer. The relevant aspect of the customer's strategy is σ_2. Consider provider B's best-response correspondence φ_B. If B receives the signal s_L and prescribes ω_L then the prescription is accepted and B's payoff is

$$P_B(\omega_L \mid s_L, a) = \omega_L(p_B - c) - \mu(\omega_H \mid s_L)[C - (W_B(\mathsf{r}_A(\bar{x}^{n+1},), \mathsf{r}_B(\bar{x}^{n+1})) - W_B(\mathsf{r}_A(\bar{x}^n), \mathsf{r}_B(\bar{x}^n)))],$$

where x_{n+1} is the Scenario 4. The probability that the customer accepts the second provider prescription ω_L is 1.[21]

If B prescribes ω_H instead, his expected payoff is $P_B(\omega_H \mid s_L) = \Upsilon(\omega_H(p_B - c))\omega_H(p_B^* - c)$. Thus,

$$\alpha_B(s_L, e_B)(\omega_H) = \left| \begin{array}{ll} 0 & \text{if} \quad P_B(\omega_L \mid s_L, a) > \Upsilon(\omega_H(p_B^* - c))\omega_H \\ 1 & \text{if} \quad P_B(\omega_L \mid s_L, a) < \Upsilon(\omega_H(p_B^* - c))\omega_H \\ [0,1] & \text{if} \quad P_B(\omega_L \mid s_L, a) = \Upsilon(\omega_H(p_B^* - c))\omega_H \end{array} \right| .$$

[21] If the customer plans to reject the prescription ω_L then she would refrain from seeking a second opinion.

If B receives the signal s_H, and prescribes ω_L, his prescription is accepted but, by (21) $P_B(\omega_L \mid s_H, a) < 0$. Hence, B never prescribes ω_L when the signal is s_H. Thus, $\alpha_B(s_H, e_B)(\omega_H) = 1$. Consequently, B's best-response correspondence φ_B has a closed graph that is bounded. Hence, it is upper hemicontinuous.

Thus, the best-response correspondences of both providers and the customer are upper hemicontinuous with convex, nonempty, ranges. Hence, by Kakutani's (1941) fixed point theorem, there exist strategy profile $\vartheta^* \in [0, \bar{v}] \times [0, \bar{v}] \times \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$ such that $\Xi(\vartheta^*) = \vartheta^*$.

The customers system of beliefs $\eta_c^*(\omega_H) = (\eta(\cdot \mid \bar{x}), \mu, \mu(\omega_H \mid \bar{x}, \omega_k(A)))$, where $\mu_A^*(\omega_H \mid \omega_k)$ is given by (13) and (14) with $\rho_{j_1}(\omega_H(j_1) \mid \omega_H, \bar{x})$ replaced by $\rho_A(\omega_H(A) \mid \omega_H, \bar{x})$. The providers' systems of beliefs are given by $\eta_j^*(\omega_H) := (\Lambda, V, \mu, \mu_j(\omega_H \mid s_H), \mu_j(\omega_H \mid s_H))$, $j \in \{A, B\}$, where $\mu_j(\omega_H \mid s_H)$ and $\mu_j(\omega_H \mid s_H)$ are given in (1) and (2). The posterior beliefs were obtained by the application of Bayes' rule on the equilibrium paths. Hence, $(\vartheta^*, \eta_c^*, \eta_A^*, \eta_B^*)$ constitutes a PBE. ■

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