Competitive Equilibrium Fraud in Markets for Credence-Goods

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Abstract

This is a study of the nature and prevalence of persistent fraud in a competitive market for credence-quality goods. We model the market as a stochastic game of incomplete information in which the players are customers and suppliers and analyze their equilibrium behavior. Customers characteristics, idiosyncratic search cost and discount rate, are private information. Customers do not possess the expertise necessary to assess the service they need either ex ante or ex post. We show that there exists no fraud-free equilibrium in the markets for credence-quality goods and that fraud is a prevalent and persistent equilibrium phenomenon.

Keywords: Competitive equilibrium fraud; Credence-quality goods markets; Search with learning;

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1 Introduction

There are markets in which customers seeking to purchase services that involve specialized knowledge might be defrauded by suppliers who prescribe unnecessary services. Examples include, medical tests and treatments, auto repairs, equipment maintenance, and taxi cab service. In these markets the service suppliers make diagnostic determinations of the service required and offer to provide it, and the customers must decide whether to purchase the prescribed service or to seek, at a cost, a second service prescription. Typically in these situations, the customer can judge, ex post, whether or not the service provided was *sufficient* to solve the problem, but is unable to assess whether the prescribed service was also *necessary.*

Darby and Karni (1973) were the first to identify the fundamental ingredients of the problem underlying the provision of what they dubbed *credence-quality goods.* First, information asymmetry between the customer who lacks the expertise required to assess the service needed and service provider who possess the required expertise and, second, the cost saving of the joint provision of diagnosis and services.[1] They proceeded to discuss and analyze the economic implications of transactions involving this type of asymmetric information. Specifically, Darby and Karni argued that in competitive market equilibrium for credence-quality goods there is persistent tendency of suppliers to over-prescribe services (that is, to prescribe services that are sufficient but are not necessary to solve the problem at hand).

Obviously the nature and extent of fraudulent practices depend on the specific

[1] This bundling of information and service is crucial. See Wolinsky (1993) for an analysis of the implication of separation of diagnosis and service.

characteristics of the credence-good market. For example, the demand for auto-repair at a given service station depends on the waiting time (that is, the length of the queue of customers waiting to be served) which is not an issue when it comes to taxi cab service. It will also depend on the information the customer may acquire before choosing the service provider and the cost of searching for a second opinion. For instance, in some cases medical diagnosis may require an invasive procedure, making the cost of obtaining a second opinion prohibitively high. It seems obvious, therefore, that modeling of credence-goods markets, while incorporating the fundamental ingredients of the problem – information asymmetry and the bundling of diagnosis and service – must be based on specifics of the market under consideration. In this paper we focus on markets for the provision of services, such as mechanical services, in which the capacity limitations of the service suppliers may result in waiting for service. We underscore this point to avoid the impression that this is a general model of credence-good markets. We believe, however, that the game-theoretic approach invoked here is not specific to the analysis of the model we study in this paper, rather it is natural framework for the analysis of credence-good markets in general.

Since the publication of Darby and Karni (1973), numerous studies confirm the prevalence of fraudulent behavior in the markets for credence-quality goods.[2] For medical services, especially physicians' services, over treatment, a phenomenon known in medical literature as supplier induced demand, is widely documented (see McGuire [2000], Currie, et. al [2011], Dranove [1988]). Domenighetti (1993) found that in Swiss canton of Ticino on average the population has one third more oper-

[2] Dulleck and Kerschbamer (2006) includes a survey of the literature and provides numerous references.

ations than medical doctors and their relatives, suggesting that greater information symmetry tends to reduce overprescription of surgical procedures. The same type of conclusion was reached by Balafoutas et. al. (2013). They report the results of a natural field experiment on taxi rides in Athens, Greece, designed to measure different types of fraud and to examine the influence of passengers' presumed information on the extent of fraud. Their findings indicate that passengers with inferior information about optimal routes are taken on significantly longer detours. Iizuka (2007) finds physicians drugs prescriptions are influenced by markup. Schneider (2012) reports the results of a field experiment designed to assess the accuracy of service provision in the auto repair market. He finds evidence for over prescription of services as well as under prescription. Beck et. al (2014) reports that in experimental setting, car mechanics are significantly more prone to supplying unnecessary services than student subjects.

The work of Darby and Karni, while calling attention to a neglected aspect of economic interactions that results in market failure, lacks the formal structure necessary to derive more subtle implications of the concept they introduced.[3] In this work we take a step towards a more formal analysis of competitive markets for credence-quality services. Specifically, taking a game-theoretic approach we analyze the equilibrium behavior in a market in which two suppliers operating service stations are engaged in Bertrand competition. The suppliers are assumed to be ex ante

[3]Theirs was the first paper after the seminal work of Akelof (1970) to discuss market failure due to asymmetric information. Unlike the asymmetry concerning hidden characteristics giving rise to adverse selection problem pointed out by Akerlof, the information asymmetry that concerns Darby and Karni has to do with the ability to assess the service needed which, in conjunction with the provision of service, gives rise to fraud.

identical in every respect and the only asymmetry between them is the lengths of their queues, which arises endogenously. The suppliers post prices per service hour which the competitive force equalizes.[4] The critical aspect of the model is the information asymmetry regarding the service that is requires to address the problem at hand. The suppliers are supposed to possess the expertise necessary to assess the required service while the customers do not. To make it easier to understand the distinctive aspects of this work, we relegate the discussion of earlier attempts at modeling credence good markets to the concluding section.

Customers heterogeneity is the consequence of idiosyncratic costs of obtaining a second prescription and of waiting for service. We assume that these costs are the customers' private information and that the customers discover the lengths of the suppliers queues (that is, the waiting time) only when they visit their service outlet.

We study the market in stationary equilibrium in which normal profits discourage entry or exit. In other words, the idle time at the service stations is short enough so that no supplier loses money but is sufficiently long so as to discourage new entries or installing more service capacity. We show that there exists no fraud-free competitive equilibrium in this market, that the level of fraud committed by the two suppliers depends on the lengths of their queues, and that the short-queue supplier is more likely to overprescribe service that the long-queue supplier.

In the next section we describe the credence good market. The equilibrium analysis appears in section 3. Some economic implications of our analysis are discussed in section 4. Section 5 includes a discussion of related theoretical work and some

[4] The presumption is that the prices are posted and observed at no cost by all customers.

concluding remarks. To allow for uninterrupted reading we collected the proofs of the main results in section 6.

2 The Credence Good Market

2.1 Overview

Consider a market for credence-quality service populated by customers and two suppliers, A and B. The information asymmetry in this market is two sided. The customers' private information consists of their idiosyncratic search cost and discount rate. The suppliers possess expertise that the customers do not have, which allows them to observe the actual state of disrepair and assess the service required to fix the problem. Let $\widetilde{\omega}$ denote the discrete random variable representing the true state of disrepair expressed as the necessary and sufficient number of service hours required to address the problem. We normalize $\widetilde{\omega}$ to take values in $\Omega := \{\omega_1, ..., \omega_n\}$, where $0 < \omega_1 < .... < \omega_n < 1$.[5] Denote the distribution of $\widetilde{\omega}$ by $\mu \in \Delta(\Omega)$, where $\Delta(\Omega)$ denotes the simplex in $\mathbb{R}^n$. We assume that μ is exogenous and commonly known.

Assume that, like the states of disrepair, the prescribed service, denoted by q, is specified in discrete quantities and, to simplify the exposition, we suppose that the prescribed service levels correspond to the states.[6] Moreover, we assume that the prescribed service must fix the problem (e.g., malfunction) or the customer refuses

[5] As will become clear later, the assumption of discrete state space has implications for the customers perception of the difference between the suppliers strategies.

[6] In view of the common practice of informing the customers what are the parts that need to be fixed or replaced before the actual work begins, this assumption is realistic.

payment. Formally, if the state is ω_i then $q \in \{\omega_i, ..., \omega_n\}$.[7] The two suppliers are identical in every respect except the lengths of their queues, expressed in terms of service hours committed to serving customers waiting to be served. We assume that the suppliers observe each other's queue and that customers only discover the length of a supplier's queue upon visiting the supplier.[8] Let $Q^A(t)$ and $Q^B(t)$ denote the lengths of the suppliers queues at time t and suppose that the market is such that the lengths of the queues are bounded by $\bar{Q}$.[9] Formally, $(Q^A(t), Q^B(t)) \in I := \{(Q^A(t), Q^B(t)) \in \mathbb{R}^2_+ \mid Q^A(t) + Q^B(t) \leq \bar{Q}\}$, for all t. Then, each interaction sets up a stage game $\Gamma\left(\omega, Q^A(t), Q^B(t)\right)$ parametrized by a state depicted by the triplet $\left(\omega, Q^A(t), Q^B(t)\right) \in \Omega \times I$. Let υ denote the joint probability distribution on I to be determined in equilibrium.

We assume that the suppliers post their hourly service prices so they are observed by the customers before they choose which supplier to visit first, and that these prices do not vary with the lengths of the queues. This assumption seems justified in view of the fact that prices tend to be sticky and do not respond to transitory changes in the lengths of the queues. Since the suppliers are identical in every respect except of the lengths of their queues, and those are only discovered by the customers upon visiting the suppliers, if the suppliers post different prices it is natural to suppose that the

[7] This assumption is dubbed liability in the literature (see Dulleck and Kreschbamer [2006], Fong an Liu [2016] and Fong et al. [2017]).

[8] The assumption that the suppliers observe each other's queue expresses the presumption that survival in competitive markets requires the players to keep tab of their rivals positions and actions. Relaxing this assumption would require a modification of the suppliers strategies described below, and will complicate the analysis without yielding new insights.

[9] This assumption corresponds to the empirical fact that market sizes are finite. From the analytical point of view this assumption implies the compactness of the domain of the joint distribution of the lengths of the queues.

customer visits the low price supplier first and if the prices are the same the customer chooses the first supplier to visit at random with equal probabilities. Moreover, since being the customer's first call is advantageous, the symmetry between the suppliers requires that, in symmetric equilibria,[10] their prices be the same.[11]

Assume that the installed service capacities of the two suppliers are the same. Let $c > 0$ denote the hourly cost of operating a service outlet regardless of whether the service station is occupied. The profit generated by servicing customers for a fraction, x, of an hour is: $\pi(x) = xp - c$, where p denotes the price per hour of service. We study *stationary symmetric equilibria* in which the suppliers earn normal profit, so that there is no incentive for new suppliers to enter the market or for a current supplier to exit the market or change the level of installed capacity. Formally, the stability of the market requires that the price be such that $\pi(\bar{x}) = \bar{x}p - c = 0$, where $\bar{x}$ denotes the average fraction of occupancy of the service station in equilibrium. To simplify the notation, without loss of generality, we assume throughout that $p = 1$ (that is, the installed capacity is such that $c = \bar{x}$).

We model the credence service market as a stochastic game of incomplete information, denoted Γ. We assume that new customers arrival times follow an underlying stochastic process. A customer's arrival on the market at time t in a state of disrepair ω when the suppliers queues are $Q^A(t)$ and $Q^B(t)$ initiates a dynamic *stage game*, $\Gamma\left(\omega, Q^A(t), Q^B(t)\right)$, depicting the interaction among the customer and the two suppliers. When a new customer shows up at a service station, the supplier

[10] We confine our analysis to symmetric equilibria. The analysis of possible non-symmetric equilibria is beyond the scope of the this paper.

[11] A uniform price may also be imposed by a regulator (e.g., pricing of taxi services). This does not change the analysis that follows.

observes the state of disrepair ω and, consequently, the state $\left(\omega, Q^A(t), Q^B(t)\right)$. The suppliers do not observe the customer's type. Customers know their types but not the state ω, and they discover the length of a supplier's queue upon visiting a service station and receiving a diagnosis. In other words, a customer may discover the lengths of the suppliers queues sequentially, during the process of searching for service. Insofar as the customers are concerned, what matters are the lengths of the queues and not the identity of the suppliers. This assumption rules out suppliers' identity or reputation as a possible factor.[12]

At a state $s(t,\omega) := \left(\omega, Q^A(t), Q^B(t)\right)$ the suppliers and customers make their decisions, after which the game proceeds to the next state as follows. Suppose that the next customer arrives at time t' in a state ω'. If the customer accepts the prescription q_A of supplier A then the new state is

$$s_A(t',\omega') := \left(\omega', \max\{Q^A(t) - \Delta t' + q_A, 0 + q_A\}, \max\{Q^B(t) - \Delta t', 0\}\right),$$

where $\Delta t' := t' - t$, and if she accepts the prescription q_B of supplier B then the new state is

$$s_B(t',\omega') := \left(\omega', \max\{Q^A(t) - \Delta t', 0\}, \max\{Q^B(t) - \Delta t' + q_B, 0 + q_B\}\right).$$

The transition probability from the state $s(t,\omega)$ to the state $s_j(t',\omega')$, denoted $\rho(s(t,\omega), s_j(t',\omega'))$, $j \in \{A,B\}$, is the product of the probability $f(t')$ that the next customer arrives at time t', the probability $\mu(\omega')$ that the state of disrepair is

[12] We revisit the issue of reputation in the discussion section.

ω', the probability that supplier j prescribes q_j in equilibrium, and the probability that the newly arrived customer accepts the prescription q_j. A detailed exposition of these probabilities and the stochastic evolution of the queues are developed in Section 3 below.

2.2 The customers

A customer's type, (θ, β), consists of idiosyncratic search cost, θ, and discount rate, β, both taking values in $[0,1]$. Thus, the set of customers' types is $T = [0,1]^2$. Let $\mathcal{B}(T)$ be the Borel sigma algebra on T and denote by ξ a continuous probability measure on the measurable type space $(T, \mathcal{B}(T))$.

Upon identifying an equipment malfunction indicating potential mechanical failure, the customer engages in sequential search for repair and maintenance service. Diagnosis of the problem and determination of the service needed to solve it or to maintain the equipment requires expert knowledge, which the customer does not have.

The customers' strategies: Since the posted service prices are the same, the customer chooses one of the two service outlets at random with equal probabilities.[13] Upon visiting a service outlet the customer obtains a service prescription, expressed in terms of service-hours, and learns the waiting time for service (that is, the length of the supplier's queue). The customer must then choose between accepting the prescribed service and waiting in the queue, and rejecting it in favor of seeking a

[13] This assumption does not rule out customers loyalty to suppliers or that each customer visits first the supplier whose location is closer provided that the loyalty or proximity are equally devided between the suppliers.

second prescription. If she chooses the latter, the customer visits the second supplier, receives a second prescription and observes the length of the second supplier's queue. The customer must then decide whether to accept the second prescription and waiting to be served or return to the first supplier. We assume that the search is with full and costless recall. Hence, if the customer decides to seek a second prescription and then return to the first supplier, she maintains her place in the queue and is entitled to obtain the service prescribed by the first supplier. Formally, a customer's search strategy is a mapping $\sigma : T \to \Sigma_1 \times \Sigma_2$, where $\Sigma_1 := \{\sigma_1 : \Omega \times \left[0, \bar{Q}\right] \to \{0,1\}\}$, $\Sigma_2 := \{\sigma_2 : \Omega^2 \times I \to \{0,1\}\}$. In other words, the strategy assigns to a customer of type (θ, β) two acts depicted by the functions $\sigma_1^{(\theta,\beta)} : \Omega \times \left[0, \bar{Q}\right] \to \{0,1\}$ and $\sigma_2^{(\theta,\beta)} : \Omega^2 \times I \to \{0,1\}$, where $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 1$ means that the customer accepts the prescription of the first supplier she visits and terminates the search, and $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 0$ means that she seeks a second prescription. Similarly, $\sigma_2^{(\theta,\beta)}(q_1, q_2, Q_1, Q_2) = 1$ means that the customer accepts the second supplier's prescription and $\sigma_2^{(\theta,\beta)}(q_1, q_2, Q_1, Q_2) = 0$ means that she rejects the second supplier's prescription and returns to the first supplier. We denote by Σ the set of customers' strategies.

The customers' beliefs: Since the customers do not observe the suppliers queues, at the outset the customer's information set is $\Omega \times I$ and her prior beliefs are captured by the commonly known distributions μ on Ω and υ on I. Upon observing the length of the first supplier's queue, Q_1, and obtaining a prescription, $q_1 \in \Omega$, the customer updates her beliefs about the state ω and the waiting time at the second service station. In doing so, the customer applies Bayes' rule.[14] The

[14]This is the sense in which the search involves learning.

updated beliefs regarding the second supplier's queue conditional on the first supplier' prescription, q_1, and queue length, Q_1, is represented by the conditional distribution $m(\omega, Q_2 \mid q_1, Q_1)$ on $\Omega \times \left[0, \bar{Q} - Q_1\right]$.[15]

The customers' payoffs: Accepting a prescribed service q on her first visit from a supplier whose queue length is Q, the utility from fixing the problem to a customer of type (θ, β) is: $u^{\beta}(q, Q; p) = (1 - pq)\, e^{-\beta Q}$. Continuing the search entails a customer-specific additive search cost, $\theta \in [0, 1]$.[16] Thus, the utility of accepting the prescription q' when the queue of the second supplier is Q' is $u^{(\theta,\beta)}(q', Q') = (1 - pq')\, e^{-\beta Q'} - \theta$. Returning to the first supplier after visiting the second supplier, the customer's payoff is $(1 - pq)\, e^{-\beta Q} - \theta$.[17] If the price is such that $1 - pq < 0$ then the customer is better off not fixing the problem. Under our assumptions, $p = 1$ and $\Omega \subset [0, 1]$, implicitly, this presumes that $\omega > 1$ are states of disrepair that are not worth fixing and, consequently, are not included in Ω.[18]

2.3 The suppliers

At every point in time each supplier has a queue representing hours committed to serving customers that have already accepted the supplier's prescriptions. The lengths of the queues are determined by the history of customer arrivals, their service prescriptions, and their acceptance decisions. In other words, the lengths of the queues are determined by the realization of an exogenous stochastic process (that

[15] We examine the updated beliefs in further details below.

[16] Additive search cost is a standard assumption in the literature on optimal stopping rules.

[17] This is the sense in which the recall is costless.

[18] If $p > 1$, then the set of relevant states will exclude the most severe states of disrepair. This generalization would complicate the analysis without adding new insights.

is, the arrival rate and the random state ω) and the endogenous decisions of the suppliers and customers.

The suppliers' strategies: The suppliers' *mixed prescription strategies* are mappings $G : \Omega \times I \to \mathcal{G}$, where $\mathcal{G}$ denotes the set of CDF on Ω.[19] Formally, for each $q_k \in \Omega$ and $(Q^j, Q^{-j}) \in I$, $G(\omega, Q^j, Q^{-j}) := \Sigma_{i=1}^{k} g(\omega, Q^j, Q^{-j})(q_i)\,\delta_{q_i}$, where $(g(\omega, Q^j, Q^{-j})(q_1), ..., g(\omega, Q^j, Q^{-j})(q_n)) \in \Delta(\Omega)$, $j \in \{A, B\}$ and δ_{q_i} denotes the distribution function that assigns the unit probability mass to q_i. For any $x, y \in I$, define $G^j(\omega, Q^j = x, Q^{-j} = y)(\cdot) = G(\omega, x, y)(\cdot)$, $j \in \{A, B\}$. Henceforth, when there is no risk of misunderstanding, we suppress the state and write $g^j(\cdot)$ and $G^j(\cdot)$ instead of $g^j(\omega, Q^j, Q^{-j})(\cdot)$ and $G^j(\omega, Q^j, Q^{-j})(\cdot)$, $j \in \{A, B\}$, and use these notations interchangeably to designate the suppliers' strategies. Since the only asymmetry between the suppliers is the lengths of their queues, the suppliers prescriptions are distinct only as a result of the different images of their queues under the strategies employed and, in the case of mixed strategies, the randomly selected prescription.

The suppliers' payoffs: Consider supplier j's, $j \in \{A, B\}$, problem when a new customer shows up at time t in state ω_i when the queues are Q^A and Q^B, thereby initiating the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$. Denote by $\widetilde{t}$ the random waiting time for the arrival of the next customer taking values in $\mathbb{R}_{++}$. Let F denote the CDF of $\widetilde{t}$ and assume that it is time independent and has full support. Denote by $\alpha_j\left(q_j \mid Q^A, Q^B, \omega_i, \sigma, q_{-j}\right)$ the probability that supplier j's prescription, q_j, is accepted conditional on the rival's prescription q_{-j} and the customer's search strategy, σ. Then, the probability that supplier j's prescription, q_j, is accepted

[19] We are restricting consideration to history-independent, or Markovian, strategies.

conditional on the rival's and customer's strategies, G^{-j} and σ, respectively, is $\alpha_j\left(q_j \mid Q^A, Q^B, \omega_i, \sigma, G^{-j}\right) = \Sigma_{q \in \Omega_{\omega_i}} \alpha_j\left(q_j \mid Q^A, Q^B, \omega_i, \sigma, q\right) g^{-j}(q)$, where $\Omega_{\omega_i} := \{\omega_i, ..., \omega_n\}$. A more detailed discussion of these probabilities appears in section 3.2 below.

Consider the situation at time t. Looking ahead, the suppliers anticipate serving the customers in the queues while waiting for the next customer to show up. Without loss of generality, let $t = 0$ be the present time, $\Delta t' = t'$, for $t' > 0$ and Q^j, $j \in \{A, B\}$, denote the supplier j's queue at $t = 0$. Define the *interim value functions* $v_j : I \times \Omega^2 \times \mathbb{R}_{++} \rightarrow \mathbb{R}$, $j \in \{A, B\}$, as follows: Let

$$\psi\left(t' \mid Q^j\right) = \{ \begin{array}{ll} e^{-rt'} & \text{if} \quad t' < Q^j \\ e^{-rQ^j} & \text{if} \quad t' \geq Q^j \end{array}$$

where $r > 0$ denotes the discount rate. Then given G^{-j} and σ,

$$v^j\left(Q^A, Q^B, \omega_i, q_j, t'\right) \tag{1}$$

$$= \alpha_j\left(q_j \mid Q^A, Q^B, \omega_i, \sigma, G^{-j}\right) \psi\left(t' \mid Q^j + q_j\right) + (1 - \alpha_j\left(q_j \mid Q^A, Q^B, \omega_i, \sigma, G^{-j}\right)) \psi\left(t' \mid Q^j\right). \tag{2}$$

Denote by $V^j : \Omega \times I \times \Omega \rightarrow \mathbb{R}$ supplier j's *continuation value function* (that is, the discounted expected value given the rival and the customer strategies G^{-j} and σ, respectively.

$$V^j\left(\omega_i, Q^j(t'), Q^{-j}(t'), q_j\right) =$$

$$\alpha_j\left(q_j \mid Q^A, Q^B, \omega_i, \sigma, G^{-j}\right) V^j\left(\max\{Q^j - t' + q_j, 0\}, \max\{Q^{-j} - t', 0\}, \omega_i\right) + \tag{3}$$

$\Sigma_{q\in\Omega_{\omega_i}} V^j\left(\max\{Q^j - t', 0\}, \max\{Q^{-j} + q - t', 0\}, \omega_i\right) \alpha_{-j}\left(q \mid Q^A, Q^B, \omega_i, \sigma, q_j\right) g^{-j}(q),$

$j \in \{A, B\}$. Clearly, $V^j\left(\omega, Q^A, Q^B\right)$, $j \in \{A, B\}$ is strictly monotonic increasing function of the supplier's own queue length. Furthermore, considering that, even though they are not obliged to, the suppliers never refuse to serve a customer, suggest that regardless of the length of their queues, supplier j's payoff satisfies $V^j\left(\omega, Q^j + q, Q^{-j}\right) > V^j\left(\omega, Q^j, Q^{-j} + q'\right)$, for every $q' \in \Omega_\omega$, $j \in \{A, B\}$. Finally, anticipating the analysis that follows, we shall show that, in equilibrium, the function V^j on the left-hand side of (3) is the same as that on the right-hand side.

Supplier j's objective is to choose a strategy $G \in \mathcal{G}$ that is best response to the rival's and the customer's strategies.[20] Formally, given G^-j and σ,

$$\max_{G\in\mathcal{G}} \sum_{q\in\Omega_{\omega_i}} [\int_0^\infty [v^j\left(Q^A, Q^B, \omega_i, q, t'\right) + e^{-rt'} \sum_{\omega\in\Omega} V^j\left(\omega, Q^A(t'), Q^B(t'), q\right) \mu(\omega)] dF(t')] dG(q). \tag{4}$$

3 Equilibrium Analysis

3.1 Equilibrium defined

We analyze the credence service market as Markovian sequential equilibrium of a stochastic game of incomplete information. At the start the customers learn their types which is private information. When a customer detects a problem and seek remedial service, she does not know which particular stage game $\Gamma\left(\omega, Q^A, Q^B\right)$ she initiates.

[20] Further discussion of the continuation value function appears in Section 3.2.2 below.

The suppliers observe each other's queues and when a customer calls they observe the state ω. The suppliers do not observe the customer's type. Consequently, even though at each stage game the suppliers face a single customer, not knowing the customer's type, the suppliers play strategies that are best responses against the average acceptance probability of the customer population of types induced by the customer's search strategy, σ.

To understand the customers' acceptance probabilities we need to understand the evolution of the customers beliefs. The *customer's system of beliefs* $\eta := (\mu, \upsilon, m(\omega, Q_2 \mid q_1, Q_1))$ consists of the prior belief about the stage game being played, which is determined by the prior beliefs μ on Ω and υ on I, and the updated beliefs $m(\omega, Q_2 \mid q_1, Q_1)$ on $\Omega \times [0, \bar{Q} - Q_1]$.

A strategy profile (σ, G^A, G^B), is *sequentially rational* if, given the suppliers objective functions, G^A is best response against (σ, G^A), G^B is best response against (σ, G^A), and σ is best response against (G^A, G^B), for all $(\omega, Q^A, Q^B) \in \Omega \times I$. A *Markovian equilibrium* is a strategy-profile (σ, G^A, G^B) that is sequentially rational given the system of beliefs η. Given $\omega_i \in \Omega$, a strategy G_k is *completely mixed with modulus* k if $g_k(q) \geq k^{-1}$, $k \geq n$, for all $q \in \Omega_{\omega_i}$.

Definition 1: A point $\left(\hat{V}^A, \hat{V}^B, \hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ and a system of beliefs η^* constitute a *Markovian sequential equilibrium of the stochastic game induced by the credence good market* if:

(i) The strategy profile $\left(\hat{\sigma}, \hat{G}^A, \hat{G}^B\right)$ is sequentially rational given the belief system $\eta^* = (\mu, \upsilon, m^*)$ and the value functions $\hat{V}^A, \hat{V}^B$.

(ii) There exist sequence of value functions and strategy profiles $\{(V_k^A, V_k^B, G_k^A, G_k^B, \sigma_k)\}_{k=1}^{\infty}$, where (G_k^A, G_k^B, σ_k) is a Markovian equilibrium in completely mixed strategies with modulus k of the credence good market game, with $\lim_{k\to\infty}(V_k^A, V_k^B, G_k^A, G_k^B, \sigma_k) = \left(\hat{V}^A, \hat{V}^B, \hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$, $\eta^* = \lim_{k\to\infty}\left(\mu, \upsilon, m^k\left(q_2, Q_2 \mid q_1, Q_1\right)\right)$ and $m^k\left(q_2, Q_2 \mid q_1, Q_1\right)$ derived from the prior beliefs (μ, υ) and strategy profile (G_k^A, G_k^B, σ_k) using Bayes' rule and, for $j \in \{A, B\}$,

$$V_k^j = \max_{G_k \in \mathcal{G}_k} \sum_{q \in \Omega_{\omega_i}} [\int_0^\infty [v_k^j\left(Q^A, Q^B, \omega_i, q, t'\right) + e^{-rt'} \sum_{\omega \in \Omega} V_k^j\left(\omega, Q^A(t'), Q^B\left(t'\right), q\right)\mu\left(\omega\right)]dF\left(t'\right)]d\hat{g}_k\left(q\right).$$

3.2 Equilibrium: Existence

We examine next the existence of Markovian sequential equilibrium of the credence good market game Γ. We begin by study the behavior of the customers and the suppliers assuming that when a new customer arrives, the supplier does not know whether he is the customer's first or second call. The supplier diagnoses the problem (that is, the supplier observes the state ω), prescribes a service, $q \in \Omega_\omega$, and informs the customer of the waiting time for service (which is equal to the length of the supplier's queue).

3.2.1 The customers

The customers system of beliefs: The customers prior beliefs are depicted by the distributions $\mu \in \Delta\left(\Omega\right)$ and υ on I. Moreover, in view of the ex ante symmetry of the suppliers, insofar as the customers are concerned, υ be symmetric.

Consider the state $\left(\omega, Q^A, Q^B\right)$ and let $G_k^j\left(\omega, Q^A, Q^B\right)$, $j \in \{A, B\}$, be the (mixed) strategies of the suppliers. The customers are supposed to know the strategies of the suppliers as functions of the states but not the current state $\left(\omega, Q^A, Q^B\right)$. In particular, the customers do not know which is the short-queue supplier and which is the long-queue supplier. Let (q_1, Q_1) and (q_2, Q_2) denote the prescriptions obtained and queues observed by a customer in her first and second visits, respectively.

Following her visit to the first supplier and having observed Q_1, regardless of whether it is A or B, the customer updates her beliefs about the state of disrepair, ω, and the length of the queue of the second supplier by applying Bayes' rule as follows: For all $\omega_i \leq q_1$,

$$m^k\left(\omega_i, Q_2 \mid q_1, Q_1\right) = \frac{g_{1,k}\left(\omega_i, Q_1, Q_2\right)\left(q_1\right)\mu\left(\omega_i\right)\upsilon(Q_2, Q_1)}{\int_0^{\bar{Q}}\left[\Sigma_{\omega_1 \leq \omega_i \leq q_1} g_{1,k}\left(\omega_i, Q_1, Q_2'\right)\left(q_1\right)\mu\left(\omega_i\right)\right] d\upsilon(Q_2', Q_1)}, \tag{5}$$

where $g_{1,k}\left(\omega_i, Q_1, Q_2\right)$ denotes the mixed strategy of the first supplier.[21]

The customers expected payoff and best response strategies: Given the suppliers' completely mixed strategies, G_k, modulus k, we explore next the optimal behavior of the customer in the subgame following her visit to the first supplier and the evolution of her beliefs. Having obtained the prescription q_1 and observing the length of the queue, Q_1, a customer of type (θ, β) can accept the prescription and stop the search or seek a second prescription. In the latter case the customer accepts the second supplier's prescription if $\left(1 - q_2\right)e^{-\beta Q_2} \geq \left(1 - q_1\right)e^{-\beta Q_1}$.[22] Otherwise the customer exercises the recall option and returns to the first supplier to obtain the payoff $u^\beta\left(q_1, Q_1\right) - \theta = \left(1 - q_1\right)e^{-\beta Q_1} - \theta$.

[21] Here we implicitly assume that the situation is symmetric and focus on the symmetric equilibira. Hence, from ex ante point of view, the suppliers are indistiguishable.

[22] Recall that we assumed, for simplicity, that $p = 1$.

Because in her the second visit the customer is going to accept or reject the second offer according to whether $u^{\beta}(q_2, Q_2)$ is greater or smaller than $u^{\beta}(q_1, Q_1)$, given q_1 and Q_1 the reservation utility of a customer of type (θ, β), $u_{r,k}^{(\theta,\beta)}(q_1, Q_1)$, is given by

$$u_{r,k}^{(\theta,\beta)}(q_1, Q_1) = \tag{6}$$

$$\Sigma_{\omega_1 \leq \omega_h \leq q_1} \Sigma_{\omega_h \leq q_2 \leq \omega_n} [\int_0^{\infty} \max\{u^{\beta}(q_2, Q_2), u^{\beta}(q_1, Q_1))\} g_k(\omega_h, Q_1, Q_2)(q_2) m^k(\omega_h, Q_2 \mid q_1, Q_1) dQ_2 - \theta.$$

Given her type, (θ, β), and the suppliers' strategy, G_k, the customer's expected payoff upon observing (q_1, Q_1) given the reservation utility $u_{r,k}^{(\theta,\beta)}(q_1, Q_1)$ in (6), is:

$$\bar{U}\left(\sigma_k(\theta, \beta), G_k^A, G_k^B\right) = \sigma_{1,k}^{(\theta,\beta)} u^{\beta}(q_1, Q_1) + \left(1 - \sigma_1^{(\theta,\beta)}\right) u_{r,k}^{(\theta,\beta)}(q_1, Q_1). \tag{7}$$

Hence, the customer accepts the first supplier's offer (that is, set $\sigma_{1,k}^{(\theta,\beta)} = 1$) if $u^{\beta}(q_1, Q_1) \geq u_{r,k}^{(\theta,\beta)}(q_1, Q_1)$. Otherwise, the customer continues the search (that is, set $\sigma_{1,k}^{(\theta,\beta)} = 0$). She accepts the second supplier's offer (that is, set $\sigma_{2,k}^{(\theta,\beta)} = 1$) if $u^{\beta}(q_2, Q_2) > u^{\beta}(q_1, Q_1)$. Otherwise, she exercises the recall option (that is, set $\sigma_{2,k}^{(\theta,\beta)} = 0$). With this in mind we make the following definition:

Definition 2: A *reservation-utility search strategy* $\sigma_k : T \rightarrow \Sigma_1 \times \Sigma_2$ consists of two mappings $\sigma_{1,k}^{(\theta,\beta)} : \Omega \times [0, \bar{Q}] \rightarrow \{0,1\}$ and $\sigma_{2,k}^{(\theta,\beta)} : \Omega^2 \times I \rightarrow \{0,1\}$, and a function $u_{r,k}^{(\theta,\beta)} : \Omega \times [0, \bar{Q}] \rightarrow [0,1]$ such that:

(a) $\sigma_{1,k}^{(\theta,\beta)}(q, Q) = 1$ if $u^{\beta}(q, Q) \geq u_{r,k}^{(\theta,\beta)}(q, Q)$ and $\sigma_{1,k}^{(\theta,\beta)}(q, Q) = 0$, otherwise.

(b) $\sigma_{2,k}^{(\theta,\beta)}(q_2, q_1, Q_2, Q_1) = 1$ if $\sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1) = 0$ and $u^{\beta}(q_2, Q_2) > u^{\beta}(q_1, Q_1)$ and $\sigma_{2,k}^{(\theta,\beta)}(q_2, q_1, Q_2, Q_1) = 0$, otherwise.

We summarize the above discussion in the following:

Proposition 1. *A reservation-utility strategy is the customers' unique best response to the suppliers' strategy profile* $\left(G_k^j\left(\omega, Q^j, Q^{-j}\right)\right)_{j\in\{A,B\}}$, *for all* $\left(\omega, Q^A, Q^B\right) \in \Omega \times I$.

The customer's expected payoff under the reservation-utility strategy is continuous in the suppliers strategies. Formally,

Lemma 1: *For each type* $(\theta, \beta) \in T$ *and all* $(q_1, Q_1) \in \Omega \times \left[0, \bar{Q}\right]$ *the customer's expected payoff,* $\bar{U}\left(\sigma_k(\theta, \beta), G_k^A, G_k^B\right)$, *of the reservation-utility strategy is continuous.*

The continuity of $\bar{U}$ is an immediate implication of its linearity in the strategies and the fact that $g_k^j(q) > 0$, $j \in \{A, B\}$, for all $q \in \Omega_\omega$.

3.2.2 The suppliers

Because the customer's type is private information, the suppliers must choose their strategies as best responses against the acceptance probabilities induced by the distribution of customers' types. We examine next the acceptance probabilities induced by the customers' reservation utility strategies.

For $j \in \{A, B\}$, the supplier j's utility of prescribing q is $V^i\left(Q^j + q, Q^{-j}\right)$ in the following cases: (1) j is the customer's first call and the customer accepts the prescription q immediately, (2) j is the customer's first call, the customer chooses to seek a second prescription and returns to j for the service, (3) j is the customer's second call and she accepts his prescription. We calculate the probabilities of these events.

The first-call suppliers face a distribution of acceptance rules induced by the distribution, ξ, on the set of types. Thus, for all $(q_1, Q_1) \in \Omega \times \left[0, \bar{Q}\right]$, the subset of the first callers who do not seek a second prescription when faced with the prescription q_1 and queue Q_1 is given by the subset of types $A_{1,k}(q_1, Q_1) := \{(\theta, \beta) \in T \mid u^{\beta}(q_1, Q_1) \geq u_{r,k}^{(\theta,\beta)}(q_1, Q_1)\} \in \mathcal{B}(T)$. Consequently, the average acceptance rate of first callers who, given the queue length Q_1, accepts the prescription q_1 immediately is:

$$\sigma_{1,k}(q_1, Q_1) = \int_T \sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1)\, d\xi(\theta, \beta) = \xi(A_{1,k}(q_1, Q_1)).$$

This may be interpreted as the *probabilistic demand* function of first callers.

Given the first supplier's prescription, q_1, and the length, Q_1, of his queue, the acceptance rate of a second prescription, q_2, when the length of the queue of the second supplier is Q_2, is:

$$\sigma_{2,k}(q_2, Q_2 \mid q_1, Q_1) = \int_T \sigma_{2,k}^{(\theta,\beta)}(q_2, Q_2; q_1, Q_1) d\xi(\theta, \beta).$$

The second-call supplier does not know that he is the second-call supplier. However, observing ω_i and Q_1, the second supplier can infer that *if* he is the customer's second-call then the prescription the customer obtained in her first call is a random variable $\widetilde{q}_1$ whose probability distribution is determined by the strategy of the first supplier. Specifically, if the customer first visits supplier A then q_1 was determined by the strategy $G_k^A\left(\omega_i, Q^A, Q^B\right)$ and if the customer first visits supplier B then q_1 was determined by the strategy $G_k^B\left(\omega_i, Q^B, Q^A\right)$. Moreover, given Q_1 and q_1, only customers whose type (θ, β) is such that $\sigma_{1,k}^{(\theta,\beta)}(q, Q) = 0$ (that is, customers type for whom $u^{\beta}(q_1, Q_1) < u_{r,k}^{(\theta,\beta)}(q_1, Q_1)$) seek a second prescription. Consequently, given

$\left(\omega_i, Q^A, Q^B\right)$, if j is the second supplier the customer calls upon, the probability that his prescribed service is accepted is

$$\begin{aligned}&\varsigma_{2,k}\left(q^j, \omega_i, Q^j, Q^{-j}, q_k^{-j}\right)\\ &= \Sigma_{q\in\Omega_{\omega_i}} \xi\{(\theta,\beta) \mid \sigma_{1,k}^{(\theta,\beta)}\left(q, Q\right) = 0\} \sigma_{2,k}^{(\theta,\beta)}\left(q^j, Q^j \mid q, Q^{-j}\right) g_k^{-j}\left(q\right),\ j \in \{A, B\}.\end{aligned}$$

Hence, the probability that a newly arrived customer accepts the prescription of supplier j, $j \in \{A, B\}$, is:

$$\begin{aligned}&\alpha_j^k\left(q \mid \sigma, G_k^{-j}\right) := \qquad (8)\\ &\frac{1}{2}[\sigma_{1,k}\left(q, Q^j\right) + (1 - \sigma_{1,k}\left(q, Q^j\right)\left(1 - \Sigma_{q\in\Omega_{\omega_i}} \sigma_{2,k}\left(q, Q_{-j} \mid q_j, Q_j\right) g_k^{-j}\left(q\right)\right) +\\ &\Sigma_{q\in\Omega_{\omega_i}}\left(1 - \sigma_1\left(q^{-j}, Q^{-j}\right)\right)\sigma_2\left(q, Q^j \mid q^{-j}, Q^{-j}\right) g_k^{-j}\left(q\right)].\end{aligned}$$

Let

$$\begin{aligned}&V^j\left(\omega_i, Q^j(t'), Q^{-j}\left(t'\right), q\right) =\\ &\alpha_j^k\left(q \mid \sigma, G_k^{-j}\right) V^j\left(\omega_i, \max\{Q^j - t' + q, 0\}, \max\{Q^{-j} - t', 0\}\right) + \qquad (9)\\ &\Sigma_{q'_{\in\Omega_{\omega_i}}} V^j\left(\omega_i, \max\{Q^j - t', 0\}, \max\{Q^{-j} + q - t', 0\}\right) \times \alpha_{-j}^k\left(q' \mid \sigma, G_k^{-j}\right) g_k^{-j}\left(q'\right).\end{aligned}$$

Then, supplier j's payoff is:

$$\max_{g_k\in\Delta(\Omega)} \sum_{q\in\Omega_{\omega_i}}\left[\int_0^\infty [v^j\left(Q^A, Q^B, \omega_i, q\right) + e^{-rt'}\sum_{\omega\in\Omega} V^j\left(Q^A(t'), Q^B\left(t'\right), \omega_j\right)\mu\left(\omega\right)]dF\left(t'\right)\right] g_k\left(q\right) \tag{10}$$

Lemma 2: *For all $\omega, Q^A, Q^B \in \Omega \times I$, the expression (10) is a continuous* *function on the strategy profiles set* $\Sigma \times \Delta(\Omega)^2$.

The proof is given in Section 6.1.

Our first result establishes the existence of Markovian sequential equilibrium of the stochastic game Γ.

Theorem 1: *There exist Markovian sequential equilibrium of the stochastic game* Γ *induced by the credence good market.*

To prove the theorem we begin by restricting the suppliers' strategies to be totally mixed. Specifically, we assume for some large $k \in N$ and all $q \in \Omega_{\omega_i}$ and $j \in \{A, B\}$, $g_k^j(\omega_i, Q^j, Q^{-j})(q) \geq \frac{1}{k}$ and prove the existence of Markovian equilibrium for the stage game $\Gamma(\omega_i, Q^j, Q^{-j})$. The proof involves the following steps: First, we show that the players' objective functions are all continuous with respect to other players' strategies. Then, invoking Berge maximum theorem, we conclude that the correspondence that maps the set of value functions and strategies to itself is upper-hemicontinuous with range that is a convex-valued and compact set. Consequently, by Kakutani's fixed point theorem, the aforementioned correspondence has fixed point. Second, we verify that the strategies corresponding to fixed points constitute a stationary Markov equilibrium and that the value functions corresponding to the same fixed points are the equilibrium value functions. Third, taking the limits as k tends to infinity and invoking sequential compactness, we conclude that there exist convergent subsequence of fixed points and, hence, a limit point of fixed points. Finally, we invoke uniform continuity to show that such limit point is indeed an equilibrium point.

4 The Evolution of the Queues and the Short-Queue Advantage

4.1 The evolution of the queues

Since the lengths of the queue are finite, starting from the event that both suppliers are idle (that is, $Q^A = Q^B = 0$ and $\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}\right) = 0$) the probability, p, of returning to the same position under the equilibrium strategies is positive. Since the equilibrium is Markovian, this event is encountered infinitely often. Thus, the probability of the event "$Q^A = Q^B = 0$ infinitely often" is: $\lim_{m\to\infty} p^m > 0$. Hence, $p = 1$. In other words, starting from any state of the queues, $\left(Q^A, Q^B\right) \in I$, with probability one the queues will attain the point $Q^A = Q^B = 0$ infinitely often. From this position, the two suppliers are equally likely to become the long-queue supplier. Hence, no supplier is enjoys the short-queue advantage persistently. Therefore, the evolution of the queues under the equilibrium strategies requires that the anticipated lengths of the queues be *stochastically equal*, in the sense that the identity of the short-queue supplier is expected to change over time in such a way that the joint distribution of the queues is symmetric around its mean. We summarize this in the following:

Theorem 2. *Under the equilibrium strategies, successive stage games induce a joint distribution of the lengths of the queues that is stationary, symmetric and the two suppliers commit the same amount of fraud on average.*

4.2 The short-queue advantage and fraudulent behavior

If $Q^A \neq Q^B$ the supplier with the shorter queue enjoys a strategic advantage in the sense that, if the two suppliers prescribe the same service, the short-queue supplier is more likely to retain a new customer. The next result depicts the implications of the short-queue advantage.

Theorem 3: *If* $Q^A < Q^B$ *and* (q_A^*, q_B^*) *is a pure strategy equilibrium then* $q_A^* \geq q_B^*$, *and if* $Q^B - Q^A$ *is sufficiently large then* $q_A^* > q_B^*$.

The implication of Theorem 2 is that in pure-strategy equilibrium the short-queue supplier may exploit his advantage to overprescribe services. Put differently, if the long-queue supplier prescribes unnecessary services, then the short-queue supplier will prescribe unnecessary services, and if the long-queue supplier prescribes the necessary and sufficient level of service, the short-queue supplier may nevertheless prescribe unnecessary service. Moreover, the larger is the difference between the queues lengths the more likely it is that the short-queue supplier commit frauds by prescribing unnecessary services.

An equilibrium is said to be *fraud-free* if the equilibrium strategies are $\hat{G}(\omega_i, Q^j, Q^{-j}) = \delta_{\omega_i}$, $j \in \{A, B\}$, for all $(\omega_i, Q^j, Q^{-j}) \in \Omega \times I$. Thus, fraud-free equilibrium is a special instance of pure-strategy equilibrium. The next result asserts that fraudulent prescriptions of service is a persistent feature of competitive equilibrium in the credence good market. Formally,

Theorem 4: *If* $\bar{Q}$ *is sufficiently large, then there exists no fraud-free equilibrium in the market for credence quality services.*

One measure of the short-queue advantage is the *difference in the expected change* *of the lengths of the queues induced by equilibrium strategies.* Formally, given a stage game $\Gamma\left(\omega_i, Q^A, Q^B\right)$, if $Q^A < Q^B$ then the measure of the short-queue advantage is:

$$\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}\right) := \Sigma_{q \in Supp\hat{g}^A} \alpha^A\left(q \mid \hat{\sigma}, \hat{G}^B\right) q\hat{g}^A(q) - \Sigma_{q \in Supp\hat{g}^B} \alpha^B\left(q \mid \hat{\sigma}, \hat{G}^A\right) q\hat{g}^B(q).$$

The discussion above implies that an increase in the length of the queue of the short-queue supplier reduces its short-queue advantage. Formally, if A is the short-queue supplier then $d\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}^A, \hat{G}^B\right)/dQ^A < 0$. However, because $d\alpha^A\left(q \mid \omega_i, \hat{G}^B\right)/dQ^A < 0$ and $d\alpha^B\left(q \mid \omega_i, \hat{G}^A\right)/dQ^A > 0$, the short-queue advantage does not yield clear cut conclusions concerning its effect on the suppliers' equilibrium strategies. It is useful, therefore, to consider some simple situations whose analysis would allow us to develop further insights as to the possible nature of fraudulent behavior.

4.3 Simple examples

Suppose that $\Omega = \{\omega_L, \omega_H\}$, where $\omega_H > \omega_L$. Clearly, if the true state is ω_H then the only equilibrium is for both suppliers to prescribe the true state. The interesting situation arise when the true state is ω_L. We consider this case below.

The payoff matrix corresponding to the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$ in which A is the columns player and B is the rows player as follows:

$$\begin{array}{ccc} \downarrow B \setminus A \rightarrow & x: \ \omega_H & (1-x) \ : \omega_L \\ y: \ \omega_H & U^B_{HH}, U^A_{HH} & U^B_{HL}, U^A_{HL} \\ (1-y) \ : \ \omega_L & U^B_{LH}, U^A_{LH} & U^B_{LL}, U^A_{LL} \end{array} \tag{11}$$

where

$$U_{kk}^{j}=\alpha_{j}\left(\omega_{k},\omega_{k}\right)V^{j}\left(Q^{j}+\omega_{k},Q^{-j}\right)+\left(1-\alpha_{j}\left(\omega_{k},\omega_{k}\right)\right)V^{j}\left(Q^{j},Q^{-j}+\omega_{k}\right),$$

for $j\in\{A,B\}$, $k\in\{H,L\}$, and

$$U_{HL}^{j}=\alpha_{A}\left(\omega_{H},\omega_{L}\right)V^{j}\left(Q^{A}+\omega_{L},Q^{B}\right)+\left(1-\alpha_{A}\left(\omega_{H},\omega_{L}\right)\right)V^{j}\left(Q^{A},Q^{B}+\omega_{H}\right),$$

$$U_{LH}^{j}=\alpha_{A}\left(\omega_{L},\omega_{H}\right)V^{j}\left(Q^{A}+\omega_{H},Q^{B}\right)+\left(1-\alpha_{A}\left(\omega_{L},\omega_{H}\right)\right)V^{j}\left(Q^{A},Q^{B}+\omega_{L}\right),$$

for $j\in\{A,B\}$. Allowing for mixed strategies, x and y denote the probabilities that players A and B prescribe ω_H, respectively. Then

$$\frac{y}{1-y}=\frac{U_{LL}^{A}-U_{LH}^{A}}{U_{HH}^{A}-U_{HL}^{A}}\text{ and }\frac{x}{1-x}=\frac{U_{LL}^{B}-U_{HL}^{B}}{U_{HH}^{B}-U_{LH}^{B}}.\tag{12}$$

The primitives of the model, namely, the prior distribution on the customers' type space, T, the distribution on the possible states of disrepair, Ω, the stochastic process depicting the arrival of new customers, are quite general. This allows for wide range of values of the suppliers payoffs of the stage games which depend on the states of the queues. Consequently, the model admits a variety of equilibria, including pure strategy and mixed strategy equilibria. In the Appendix we analyze the two stage games $\Gamma\left(\omega_L,Q^A,Q^B\right)$. The first deals with the symmetric case in which the suppliers queues are of equal lengths and the second with the asymmetric case in which the suppliers' queues are of different lengths. The general conclusions that emerge are as follows:

If the suppliers queues are equal then, depending on the configurations of the signs of these expressions we may have (a) pure strategy equilibria in which either both suppliers prescribe truthfully or both commit fraud; (b) Two pure strategy equilibria in which one supplier prescribe truthfully and the other overprescribes; (c) A symmetric mixed strategy equilibrium in which each supplier overprescribes service with probability 0.5.

If the suppliers queues are of different lengths then, in mixed strategy equilibrium the short-queue supplier is more likely to commit fraud than the long-queue supplier. In other words, if the true state is w_L, and $Q^A < Q^B$ the the equilibrium mixed strategy of the supplier A first-order stochastically dominates that of supplier B in the sense that $\Pr_A\{\omega_H\} = x > y = \Pr_B\{\omega_H\}$. Moreover, $\Pr_A\{\omega_H\} > 0.5 > \Pr_B\{\omega_H\}$

5 Related Literature and Concluding Remarks

5.1 Related literature

Despite evidence regarding the prevalence of fraud in the market for credence goods and the distinguishing features of these markets, the literature dealing with the modeling and analysis of these markets is rather scant. The attempts to model competitive markets for credence-quality goods include a variety of approaches. The works that are closest to ours in terms of the questions asked, are Wolinsky (1995), Emons

(1997) and Dulleck and Kerschbamer (2006). Despite the shared interest in studying the prevalence of fraud in competitive equilibrium, these works are quite different from ours in the way they model the markets and, consequently, the equilibrium behavior of the customers and the suppliers.

Wolinsky (1995) proposed a model in which there are two states of disrepair, high and low. Customers do not possess the expertise necessary to determine the state and must relay on the diagnosis of the service providers. Wolinsky modeled the situation as a game in which the customers bargain with suppliers by offering a price for the repair. Suppliers have the option of rejecting the price, in which case the customers may increase their price or seek another supplier. Wolinsky showed that, in interior equilibrium, all customers who receive a prescription of the high service seek a second opinion, and the suppliers commit fraud by employing a strategy that assigns positive probability of rejecting price offers when the state diagnosed requires low service. This strategy reflects their belief that, to avoid the search cost, the customer may offer a higher price rather than seek a second opinion. Wolinsky's work is different from ours in the way the credence-goods markets are modeled and the conclusion of the analysis. To begin with, we allow for any number of states of disrepair measured by the service hours needed to resolve the problem. More importantly, we assume that the price of service is fixed by the suppliers (no bargaining) and is equal among the suppliers due to competition. Suppliers are characterized by the lengths of their queue and customers are characterized by their idiosyncratic search cost and discount rate. Customers are engaged in search with learning. These differences in modeling mandate different equilibrium notions and analysis.

Emons (1997) proposed a model of credence good market in which the suppliers must decide whether to enter the market. If a supplier enters the market he is endowed with a fixed capacity that can be allocated to diagnosis and repair service. These two functions are assumed to be priced differently. The suppliers are allowed to announce a wrong diagnosis if they find that it is more profitable, for lack of capacity, to avoid providing the needed repair. The customers are identical. Emons studies conditions under which fraud free equilibrium exists. Emons model is different from ours in the specification of the information structure and the features of the credence-good market. These differences have implications for the depiction of the product state of disrepair; the characterization of the customers and their behavior; the pricing mechanism in the market; the suppliers strategies and the penalty imposed on them for not prescribing the necessary service.

Dulleck and Kerschbamer (2006) consider a market for credence services in which the customers may experience a need for high or low levels of service. They used a game theoretic approach to study conditions under which competition will eliminate fraud. These conditions include homogeneous customer population, cost conditions that prevent customers from seeking a second opinion and verifiability of the service provided.

Less related theoretical models of the credence good market emphasized different aspects of the efficiency loss due to the asymmetric information. Hu and Lin (2018), Fong and Liu (2016) and Fong et. al (2017) study this issue in the context of interaction between uniformed customers and a monopolistic expert service provider.

Hu and Lin (2018) modeled repeated interaction between a customer in occasional need of maintenance service of a durable good and a monopoly supplier. Their analysis focuses on possible deviations from the optimal level of service by prescribing undertreatment or over treatment. They show that there exist no equilibrium that supports truthful diagnosis. Fong and Liu (2016) investigated the effect of liability on the seller's incentive to maintain good reputation and its impact on market efficiency. Fong et. al (2017) focus on the use of customer service to build trust between the monopoly supplier its customers so as to mitigate the efficiency loss. Heinzel (2019) study the equilibrium of a price-regulated market in which physicians characterized by heterogeneous cost compete for servicing uniformed patients. Heinzel models the interaction among physicians and patients as a game in which patients may employ mixed strategies in seeking "second opinion" when diagnosed as having a serious problem and physicians may defraud their patients by overtreating them for minor problems. Unlike in the model we present here, the distinct physicians' types is exogenous and the customer behavior is not derived from optimal search strategy.

5.2 Concluding remarks

We model a credence service market featuring two identical suppliers engaged in Bertrand competition. The customers care about the prescribed services and the waiting time. Our analysis shows that competition cannot be relayed upon to sustain fraud-free equilibrium in these kind of markets and that fraud is a persistent and prevalent phenomenon. The analysis highlights the role of the evolution of the customer's beliefs in the wake of her visit to the first supplier and the optimal stopping rule that characterizes her best response strategy and the suppliers prescription best response strategies. These aspects of our model and analysis are not specific to the two suppliers case and would show up, in a more complex form, if the number of the suppliers increase.

The model also highlight the short-queue supplier's advantage, its implications for the overprescription of service and the consequent evolution of the queues. It is worth noting that if the waiting time is not an issue (that is, the suppliers have no capacity constraints) so that each customer can be served immediately, then the analysis changes considerably. In this instance, the customers' utilities depend only on the prescribed service, and their discount rates is no longer a factor. Suppose that $\theta \in (0, 1]$ then it is easy to verify that the suppliers strategies $q^j(\omega) = \omega_n$, for all $\omega \in \Omega$ and $j \in \{A, B\}$, is an equilibrium. In other words, knowing that the equilibrium prescriptions of the two suppliers are the same, no customer is inclined to search and, consequently, the suppliers have no incentive to try and undercut each other's prescription. Maximal fraud also characterize the cab service provided to tourists in an unfamiliar city since the prescription (that is, the route taken) coincides with the service provided, leaving the customer no opportunity for seeking a second prescription. The route taken is only restricted by a tourist's conception of the reasonable length of the ride.[23]

One may think of variations on the model presented here. For instance, there are situations in which to obtain a diagnosis one has to schedule an appointment (e.g., a plumber service or medical examination). In these instances, the waiting time is

[23] See also, Stahl (1996) for a discussion of a related issue.

ahead of obtaining the diagnosis and the customer may obtain information about the waiting time at different suppliers prior to deciding which supplier to visit first. This would change the information structure and, consequently, the strategies and equilibrium of the model. The analysis of such variations is left for future research.

An important aspect of the credence good market, discussed in Darby and Karni (1973) but not touched upon in this work, is the possibility of developing a reputation for honest diagnosis and its effect on the commission of fraud. Including reputation in our model would require admitting repeated interactions in which the customers display loyalty (that is, they visit "their" supplier first) and the suppliers recognize their loyal clients. Under these conditions, the suppliers may establish what Darby and Karni dubbed client relationship. The loss of future business of, and being bad-mouthed by, a dissatisfied customer would increase the cost to the suppliers of "losing" customers, which should serve as a deterrence and, consequently, mitigate the problem of fraud. This extension of the present work requires further study.

6 Proofs

6.1 Proof of Lemma 2

For $j \in \{A, B\}$, the customer's strategy affects V_k^j through the probability α_j in (8). Since V_k^j is continuous in α_j and α_j is continuous in σ_k, V_k^j is continuous in σ_k. To show that V_k^j is continuous in $G_k\left(\omega, Q^{-j}, Q^j\right)$, it suffices to show that

$$\int_0^\infty [v_k^j\left(Q^j, Q^{-j}, \omega_i, q\right) + e^{-rt'}\left[\sum_{\omega\in\Omega} V_k^j\left(Q^j(t'), Q^{-j}\left(t'\right), \omega_j\right)\mu\left(\omega\right)\right] dF\left(t'\right) \qquad (13)$$

is continuous in $G_k\left(\omega, Q^{-j}, Q^j\right)$. By equation (9), the expression in (13) depends on $G_k\left(\omega, Q^{-j}, Q^j\right)$ through $\Sigma_{q\in\Omega_{\omega_i}} V_k^j\left(\omega_i, Q^j, Q^{-j}+q\right) g_k\left(\omega, Q^{-j}, Q^j\right)(q)$. Since the last expression is linear in the probabilities $\left(g_k\left(\omega, Q^{-j}, Q^j\right)(q)\right)_{q\in\Omega_{\omega_i}}$, it is continuous in $G_k\left(\omega, Q^{-j}, Q^j\right)$. That V_k^j is continuous in $G_k\left(\omega, Q^j, Q^{-j}\right)$ follows from its linearity in the probabilities $\left(g_k\left(\omega, Q^j, Q^{-j}\right)(q)\right)_{q\in\Omega_{\omega_i}}$. $\square$

6.2 Proof of Theorem 1

Let C_b^I denote that set of bounded and continuous real-valued functions on the compact set $I \times \Omega$. Thus, C_b^I is compact by the product topology. Denote by G_k^j or, equivalently, g_k^j the strategies $G_k\left(\omega, Q^j, Q^{-j}\right), g_k\left(\omega, Q^j, Q^{-j}\right), j \in \{A, B\}$. Given $k \in N$, define a correspondence $T_k : \left(C_b^I\right)^2 \times \Delta(\Omega)^2 \times [0,1] \rightrightarrows \left(C_b^I\right)^2 \times \Delta(\Omega)^2 \times [0,1]$ as follows:

$$T_k : \begin{pmatrix} V_k^A \\ V_k^B \\ G_k^A \\ G_k^B \\ \sigma_k \end{pmatrix} \rightrightarrows \begin{pmatrix} \bar{V}_k^A \\ \bar{V}_k^B \\ \bar{G}_k^A \\ \bar{G}_k^B \\ \bar{\sigma}_k \end{pmatrix},$$

where the elements in the range set are defined as follows: Given $(V_k^j, G_k^{-j}, \sigma_k)$, $j \in \{A, B\}$,

$$\bar{V}_k^j\left(Q^A, Q^B, \omega_i\right) = \tag{14}$$

$$\max_{g_k^j \in \Delta(\Omega)} \sum_{q \in \Omega_{\omega_i}} \left[v_k^j \left(Q^A, Q^B, \omega_i, q \right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} V_k^j \left(Q^A(t'), Q^B\left(t'\right), \omega \right) \mu\left(\omega\right) \right] dF\left(t'\right) \right] g_k^j\left(q\right),$$

and

$$\bar{G}_k^j = \arg\max_{G_k^j \in \mathcal{G}_k} \sum_{q \in \Omega_{\omega_i}} \left[v_k^j \left(Q^A, Q^B, \omega_i, q \right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} V_k^j \left(Q^A(t'), Q^B\left(t'\right), \omega \right) \mu\left(\omega\right) \right] dF\left(t'\right) \right] g_k^j\left(q\right), \tag{15}$$

where G_k^j is the CDF corresponding to g_k^j. Finally, $\bar{\sigma}_k$ is the customer's best response given $(\bar{G}_k^A, \bar{G}_k^B)$. By Lemmata 1 and 2, for all $\left(\omega, Q^A, Q^B\right) \in \Omega \times I$, the suppliers' value function $\bar{V}_k^j : \Delta\left(\Omega\right)^2 \times \Sigma \to \mathbb{R}$, $j \in \{A, B\}$, in (14) and the customer's expected payoff, $\bar{U} : \Delta\left(\Omega\right)^2 \times \Sigma \to \mathbb{R}$ in (7) are continuous functions.

The sets of the suppliers' and customers' strategies are closed and bounded in $\mathbb{R}^n$. Hence, by the Heine-Borel theorem, they are compact. By definition and Berge maximum theorem, for $j \in \{A, B\}$, $\bar{V}_k^j$ in (14) are continuous functions and the correspondences $\bar{G}_k^j$ in (15) are nonempty, compact and convex valued, and upper hemicontinuous. Moreover, by Proposition 1, the customer's strategy $\bar{\sigma}$ is single-valued and is linear in the suppliers mixed strategies. Hence it is continuous in these strategies.

Since $\bar{V}_k^A, \bar{V}_k^B \in C_b^I$ and $I \times \Omega$ is compact, they attain their maximal and minimal values. Hence, the ranges of these functions are closed and bounded intervals in $\mathbb{R}$. Moreover, $\Delta\left(\Omega\right)$ and $[0, 1]$ endowed with the Euclidean metric are compact metric spaces. Hence, the domain of the correspondence T_k is compact Housdorff space. By

Kakutani's fixed point theorem, T_k has a fixed point,

$$\begin{pmatrix} \hat{V}_k^A \\ \hat{V}_k^B \\ \hat{G}_k^A \\ \hat{G}_k^B \\ \hat{\sigma}_k \end{pmatrix} \in T_k \begin{pmatrix} \hat{V}_k^A \\ \hat{V}_k^B \\ \hat{G}_k^A \\ \hat{G}_k^B \\ \hat{\sigma}_k \end{pmatrix}.$$

By definition, a fixed point of the mapping T_k is a stationary Markov equilibrium point.

By compactness of the domain, every sequence, $\left(\hat{V}_k^A, \hat{V}_k^B, \hat{G}_k^A, \hat{G}_k^B, \hat{\sigma}_k\right)_{k \in N}$ of fixed points has convergent subsequence. Denote by $\left(\hat{V}^A, \hat{V}^B, \hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ a subsequential limit point. We show next that $\left(\hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ constitutes an equilibrium with respect to the value functions $\hat{V}^A$ and $\hat{V}^B$ and that $\left(\hat{V}^A, \hat{V}^B\right)$ are the value functions corresponding to the strategies $\left(\hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$.

Let $\{k_n \mid n = 1, 2...\}$ be a convergent subsequence and consider supplier A. Given $\left(\hat{V}_{k_n}^A, \hat{V}_{k_n}^B, \hat{G}_{k_n}^A, \hat{G}_{k_n}^B, \hat{\sigma}_{k_n}\right)$, for all $G_{k_n}^A = \left(g_{k_n}^A(q)\right)_{q \in \Omega}$, we have

$$\Phi_{k_n}^A := \sum_{q \in \Omega_{\omega_i}} \left[\hat{v}_{k_n}^A \left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} \hat{V}_{k_n}^A \left(Q^A(t'), Q^B(t'), \omega\right) \mu(\omega)\right] dF(t')\right] \hat{g}_{k_n}^A(q) \geq$$

$$\sum_{q \in \Omega_{\omega_i}} \left[\hat{v}_{k_n}^A \left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} \hat{V}_{k_n}^A \left(Q^A(t'), Q^B(t'), \omega\right) \mu(\omega)\right] dF(t')\right] g^A(q) := \phi_{k_n}^A,$$

for all k_n, $n \in N$. Hence, $\lim_{n\to\infty} \Phi_{k_n}^A \geq \lim_{n\to\infty} \phi_{k_n}^A$. Let $\lim_{n\to\infty} \hat{G}_{k_n}^A = \hat{G}^A$,

$\lim_{n\to\infty} \hat{G}^B_{k_n} = \hat{G}^B$, $\lim_{n\to\infty} \hat{\sigma}_{k_n} = \hat{\sigma}$. Then,

$$\lim_{n\to\infty} \Phi^A_{k_n} = \sum_{q\in\Omega_{\omega_i}} \left[\hat{v}^A\left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega\in\Omega} \hat{V}^A\left(Q^A(t'), Q^B(t'), \omega\right)\mu(\omega)\right] dF(t')\right] \hat{g}^A(q),$$

where $\hat{V}^A\left(Q^A(t'), Q^B(t'), \omega_j\right) = \hat{V}^A\left(\hat{G}^A, \hat{G}^B, \hat{\sigma} \mid Q^A(t'), Q^B(t'), \omega\right)$. Note also that $\lim_{n\to\infty} \phi^A_{k_n}$ is the value function of player A of the strategy G^A when player B and the customer play the limit strategies $\hat{G}^B$ and $\hat{\sigma}$, respectively, given the continuation function $\hat{V}^A$. Thus, $\lim_{n\to\infty} \Phi^A_{k_n} \geq \lim_{n\to\infty} \phi^A_{k_n}$ implies that $\hat{G}^A$ is best response to $\hat{G}^B$ and $\hat{\sigma}$, given the continuation function $\hat{V}^A$. Repeating the same argument for supplier B we conclude that $\hat{G}^B$ is best response to $\hat{G}^A$ and $\hat{\sigma}$, given the continuation function $\hat{V}^B$. That $\hat{\sigma}$ is best response to $\hat{G}^A$ and $\hat{G}^B$ is obvious.

Finally, since $\hat{V}^j\left(Q^A, Q^B, \omega\right) = \hat{V}^j\left(\hat{G}^A, \hat{G}^B, \hat{\sigma} \mid Q^A, Q^B, \omega\right)$, $j \in \{A, B\}$, $\hat{V}^A$ and $\hat{V}^B$ are the value functions corresponding to the strategies $\left(\hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$. $\square$

6.3 Proof of theorem 3

Let $Q^B > Q^A$ and suppose, by way of negation, that $q^*_A < q^*_B = \omega_i$. Then $(1 - q^*_A)\, e^{-\beta Q^A} > (1 - q^*_B)\, e^{-\beta Q^B}$. Consequently, the only customers who accept the prescription of supplier B are customers who visits B first and choose not to seek a second prescription. The probability of this event is: $\alpha_B\left(\omega_i, Q^B\right) = 0.5\xi\{(\theta, \beta) \in T \mid (1 - \omega_i)\, e^{-\beta Q^B} > u_r^{(\theta,\beta)}\left(\omega_i, Q^B\right)\}$ which is independent of q^*_A. Thus, $q^*_A \leq \omega_i$ implies that supplier A's payoff is:

$$R_A(q^*_A) = \left(1 - \alpha_B\left(\omega_i, Q^B\right)\right) V\left(Q^A + q^*_A, Q^B\right) + \alpha_B\left(\omega_i, Q^B\right) V\left(Q^A, Q^B + \omega_i\right).$$

If $q_A^* = \omega_j < \omega_i$ then $\hat{\alpha}_B\left(\omega_i, Q^B\right) = \alpha_B\left(\omega_i, Q^B\right) + p_B\left(\omega_i, Q^B\right)$, where

$$p_B\left(q_A^*, \omega_i, Q^B\right) := \Pr\{\beta \in [0,1] \mid \frac{(1-q_A^*)}{(1-\omega_i)} < e^{-\beta(Q^A - Q^B)}\}.$$

But $q_A^* < \omega_i$ implies that $p_B\left(q_A^*, \omega_i, Q^B\right) = 0$. Hence,

$$R\left(q_A^*\right) - R\left(\omega_i\right) = \left(1 - \alpha_B\left(\omega_i, Q^B\right)\right)\left(V\left(Q^A + q_A^*, Q^B\right) - V\left(Q^A + \omega_i, Q^B\right)\right) < 0.$$

Thus, $q_A^* \geq q_B$.

If $q_A^* = \omega_{i+1}$,

$$R_A\left(\omega_{i+1}\right) = \left(1 - \hat{\alpha}_B\left(\omega_i, Q^B\right)\right) V\left(Q^A + \omega_{i+1}, Q^B\right) + \hat{\alpha}_B\left(\omega_i, Q^B\right) V\left(Q^A, Q^B + \omega_i\right).$$

Since $V\left(Q^A + \omega_{i+1}, Q^B\right) - V\left(Q^A + \omega_i, Q^B\right) > 0$ and, for sufficiently large $Q^B - Q^A$, $p_B\left(q_A^*, \omega_i, Q^B\right)$ is arbitrarily small. Thus,

$$R\left(\omega_{i+1}\right) - R\left(\omega_i\right)$$
$$= \left(1 - \alpha_B\left(\omega_i; Q^B\right)\right)\left(V\left(Q^A + \omega_{i+1}, Q^B\right) - V\left(Q^A + \omega_i, Q^B\right)\right) - p_B\left(\omega_{i+1}, \omega_i, Q^B\right)\left(V\left(Q^A + \omega_{i+1}, Q^B\right)\right.$$

Hence, $q_A^* \geq \omega_{i+1} > \omega_i = q_B$. □

6.4 Proof of theorem 4

We need to show that, for some stage game $\Gamma\left(\omega, Q^A, Q^B\right)$, $\hat{G}\left(\omega, Q^j, Q^{-j}\right) = \delta_\omega$ is not a best response to $\hat{G}\left(\omega, Q^{-j}, Q^j\right) = \delta_\omega$, for some $j \in \{A, B\}$.

Suppose that there is fraud-free equilibrium and consider the case $Q^A = 0 < Q^B$. In fraud-free equilibrium the customers believe that both suppliers prescribe the necessary service truthfully. Hence, the only reason to obtain a second prescription is the expectations that the second supplier has a sufficiently shorter queue that would justify bearing the cost of obtaining a second prescription. Suppose that the state is ω_i and that the long-queue supplier prescribes truthfully, (that is, $q_B = \omega_i$). We show that if Q^B is sufficiently large then prescribing ω_i is not a best response of the short-queue.

To begin with, observe that if the customer visits the short-queue supplier first then, because supplier B's queue cannot possibly be shorter than $Q^A = 0$, the customer will never seek a second prescription.

The probability of a new customer accepting the prescription ω_i from the long-queue supplier is as follows:[24] If the long-queue supplier (that is, supplier B) is the customer's first call then the probability of acceptance is:

$$p_1\left(Q^B\right) := \xi\{(\theta, \beta) \in T \mid e^{-\beta Q^B} > E\left[e^{-\beta Q} \mid Q^B\right] - \theta\},$$

where $E\left[e^{-\beta Q} \mid Q^B\right] = \int_0^\infty e^{-\beta Q} \upsilon\left(Q \mid Q^B\right) dQ$ and $\upsilon\left(Q \mid Q^B\right)$ is the distribution of supplier A's queue conditional on Q^B. Note that $p_1\left(Q^B\right)$ is independent of the prescription, q_A, of the short-queue supplier.

Suppose that the customer visits the long-queue supplier first and decides to seek a second prescription. Suppose further that the short-queue supplier prescribes

[24] Recall that in fraud-free equilibrium the customer expects that $q_A = \omega_i$.

$q_A \in \Omega_{\omega_i}$. The customer will return to the long-queue supplier if and only if

$$p_2\left(q_A, \omega_i, Q^B\right) := \Pr\{\beta \in [0,1] \mid 1 - q_A < (1-\omega_i)\, e^{-\beta Q^B}\}.$$

Define $p_B\left(q_A, \omega_i, Q^B\right) = p_1\left(Q^B\right) + p_2\left(q_A, \omega_i, Q^B\right)$. Since $1-\omega_i > (1-\omega_i)\, e^{-\beta Q^B}$ for all $\beta \in (0,1]$, $q_A = \omega_i$ implies that $p_2\left(q_A, \omega_i, Q^B\right) = 0$. Thus, $p_B\left(q_A, \omega_i, Q^B\right) = p_1\left(Q^B\right)$. Hence, the short-queue supplier's payoff if he prescribes $q_A = \omega_i$ is:

$$R(\omega_i) = \left(1 - p_B\left(\omega_i, Q^B\right)\right) V\left(\omega_i, Q^B\right) + p_B\left(\omega_i, \omega_i, Q^B\right) V\left(0, Q^B + \omega_i\right)$$

and if he prescribes $q_A = \omega_{i+1}$ the short-queue supplier's payoff is:

$$R(\omega_{i+1}) = \left(1 - p_B\left(\omega_{i+1}, Q^B\right)\right) V\left(\omega_{i+1}, Q^B\right) + p_B\left(\omega_{i+1}, \omega_i, Q^B\right) V\left(0, Q^B + \omega_i\right).$$

Since $p_2\left(\omega_i, \omega_i, Q^B\right) = 0$, $p_2\left(\omega_{i+1}, \omega_i, Q^B\right) - p_2\left(\omega_i, \omega_i, Q^B\right) = p_2\left(\omega_{i+1}, \omega_i, Q^B\right)$. For Q^B sufficiently large, $p_2\left(\omega_{i+1}, \omega_i, Q^B\right)$ is small and consequently,

$$R(\omega_{i+1}) - R(\omega_i) =$$

$$\left(1 - p_1\left(Q^B\right)\right)\left(V\left(\omega_{i+1}, Q^B\right) - V\left(\omega_i, Q^B\right)\right) - p_2\left(\omega_{i+1}, \omega_i, Q^B\right)\left(V\left(\omega_{i+1}, Q^B\right) - V\left(0, Q^B + \omega_i\right)\right) > 0.$$

Thus, $\hat{G}\left(\omega_i, 0, Q^B\right) = \delta_{\omega_i}$ is not best response to δ_{ω_i}. $\square$

APPENDIX

Let $\Omega = \{\omega_L, \omega_H\}$, where $\omega_H > \omega_L$, and consider situations in which the true state is ω_L. Let (11) depict the payoff matrix corresponding to the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$.

Example 1: Consider the symmetric stage game $\Gamma\left(\omega_L, Q^A, Q^B\right)$, where $Q^A = Q^B = 0$. In this case $\alpha_j\left(\omega_k, \omega_k\right) = 1/2$, and $U^A_{kk} = U^B_{kk}$, $j \in \{A, B\}$, $k \in \{H, L\}$. Moreover, $U^B_{HH} - U^B_{LH} = U^A_{HH} - U^A_{HL}$ and $U^B_{LL} - U^B_{HL} = U^A_{LL} - U^A_{LH}$.

$$U^B_{HH} - U^B_{LH} = \left(\alpha_A\left(\omega_H, \omega_H\right) - \alpha_A\left(\omega_L, \omega_H\right)\right)\left(V^B\left(\omega_H, 0\right) - V^B\left(0, \omega_H\right)\right) + \tag{16}$$

$$\left(V^B\left(0, \omega_H\right) - V^B\left(0, \omega_L\right)\right)\left(1 - \alpha_A\left(\omega_L, \omega_H\right)\right).$$

Since $V^B\left(\omega_H, 0\right) - V^B\left(0, \omega_H\right) < 0$, $\alpha_A\left(\omega_H, \omega_H\right) - \alpha_A\left(\omega_L, \omega_H\right) > 0$, and $V^B\left(0, \omega_H\right) - V^B\left(0, \omega_L\right) > 0$, the sign of the first term is negative and that of the second term is positive. Thus, in general, the sign of $U^B_{HH} - U^B_{LH}$ is ambiguous. More specifically, since $\alpha_A\left(\omega_H, \omega_H\right) = 0.5$,

$$U^B_{HH} - U^B_{LH} \geq (<)\, 0 \Longleftrightarrow \frac{0.5 - \alpha_A\left(\omega_L, \omega_H\right)}{1 - \alpha_A\left(\omega_L, \omega_H\right)} \leq (>) \frac{V^B\left(0, \omega_H\right) - V^B\left(0, \omega_L\right)}{V^B\left(0, \omega_H\right) - V^B\left(\omega_H, 0\right)}.$$

Consider next

$$U^B_{LL} - U^B_{HL} = \left(\alpha_A\left(\omega_L, \omega_L\right) - \alpha_A\left(\omega_H, \omega_L\right)\right)\left(V^B\left(\omega_L, 0\right) - V^B\left(0, \omega_L\right)\right) + \tag{17}$$

$$\left(V^B\left(0,\omega_L\right)-V^B\left(0,\omega_H\right)\right)\left(1-\alpha_A\left(\omega_H,\omega_L\right)\right).$$

Since $V^B\left(\omega_L,0\right)-V^B\left(0,\omega_L\right)<0$, $\alpha_A\left(\omega_L,\omega_L\right)-\alpha_A\left(\omega_H,\omega_L\right)<0$ and $V^B\left(0,\omega_L\right)-V^B\left(0,\omega_H\right)<0$, the sign of the first term is positive and that of the second term is negative. Thus, the sign of $U_{LL}^B-U_{HL}^B$ is ambiguous. More specifically,

$$U_{LL}^B-U_{HL}^B\geq(<)\,0\Longleftrightarrow\frac{\alpha_A\left(\omega_H,\omega_L\right)-0.5}{1-\alpha_A\left(\omega_H,\omega_L\right)}\leq(>)\,\frac{V^B\left(0,\omega_H\right)-V^B\left(0,\omega_L\right)}{V^B\left(0,\omega_L\right)-V^B\left(\omega_L,0\right)}.$$

Observe that if B prescribes ω_L and A prescribes ω_H then A will only get the customers that visit him first and do not seek a second prescription. Thus, $\alpha_A\left(\omega_L,\omega_H\right)=\xi\{(\theta,\beta)\in T\mid(1-\omega_H)>u_r^{(\theta,\beta)}\left(\omega_H,Q^A\right)-\theta\}$. By the same logic, if B prescribes ω_H and A prescribes ω_L then A will get the customers that visit him first and all the customers that visit B first and seek a second prescription. Thus, $\alpha_A\left(\omega_H,\omega_L\right)=0.5+\xi\{(\theta,\beta)\in T\mid(1-\omega_H)>u_r^{(\theta,\beta)}\left(\omega_H,Q^B\right)-\theta\}$. Since $Q^A=Q^B$, we get that $0.5-\alpha_A\left(\omega_L,\omega_H\right)=\alpha_A\left(\omega_H,\omega_L\right)-0.5$, or $\alpha_A\left(\omega_L,\omega_H\right)+\alpha_A\left(\omega_H,\omega_L\right)=1$.

Consequently, depending on the configurations of the signs of these expressions we may have the following equilibria.

If $U_{LL}^B-U_{HL}^B=U_{LL}^A-U_{LH}^A>0$ and $U_{HH}^B-U_{LH}^B=U_{HH}^A-U_{HL}^A>0$ then $(q_A^*=\omega_L,q_B^*=\omega_L)$ and $(q_A^*=\omega_H,q_B^*=\omega_H)$ are pure strategy equilibria in which either both suppliers prescribe truthfully or both commit fraud.

If $U_{HH}^B-U_{LH}^B=U_{HH}^A-U_{HL}^A>0$ and $U_{LL}^B-U_{HL}^B=U_{LL}^A-U_{LH}^A<0$ then $(q_A^*=\omega_H,q_B^*=\omega_H)$ is the unique, pure strategy, equilibrium in which both suppliers commit fraud.

If $U_{LL}^B-U_{HL}^B=U_{LL}^A-U_{LH}^A>0$ and $U_{HH}^B-U_{LH}^B=U_{HH}^A-U_{HL}^A<0$ then there is a unique, pure strategy, equilibrium $(q_A^* = \omega_H, q_B^* = \omega_H)$ in which the two suppliers prescribe truthfully.

If $U_{LL}^B - U_{HL}^B = U_{LL}^A - U_{LH}^A < 0$ and $U_{HH}^B - U_{LH}^B = U_{HH}^A - U_{HL}^A < 0$ then there is are two pure strategy equilibria $(q_A^* = \omega_H, q_B^* = \omega_L)$ and $(q_A^* = \omega_L, q_B^* = \omega_H)$ in which one supplier prescribe truthfully and the other overprescribes .

If $\left(U_{LL}^B - U_{HL}^B\right) / \left(U_{HH}^B - U_{LH}^B\right) \in (0,1)$ then there is a symmetric mixed strategy equilibrium in which each supplier overprescribes service with probability 0.5.

The same logic applies to all symmetric situations (that is, for all $Q^A = Q^B$).

Example 2: Consider the asymmetric case where the state is $\left(\omega_L, Q^A, Q^B\right)$, where $Q^A < Q^B$. By Theorem 2, in pure-strategy equilibria, $q_A^* \geq q_B^*$. Hence, in pure-strategy equilibrium the following case may arise: both suppliers overprescribe services, both suppliers prescribe truthfully, the long-queue supplier prescribes truthfully and the short queue supplier prescribes unnecessary service.

A mixed strategy equilibrium requires that $\left(U_{LL}^j - U_{HL}^j\right) / \left(U_{HH}^j - U_{LH}^j\right) \in (0,1)$, $j \in \{A, B\}$.Thus, $U_{LL}^j - U_{HL}^j$ and $U_{HH}^j - U_{LH}^j$ must be of the same sign. Moreover, letting $\Delta V^A\left(Q^A, Q^B\right) := V^A\left(Q^A + \omega_H, Q^B\right) - V^A\left(Q^A + \omega_L, Q^B\right)$

$$\frac{U_{LL}^A - U_{LH}^A}{U_{HH}^A - U_{HL}^A} =$$
$$\frac{\left[\alpha_A\left(\omega_L, \omega_L\right) - \alpha_A\left(\omega_L, \omega_H\right)\right]\left[V^A\left(Q^A + \omega_L, Q^B\right) - V^A\left(Q^A, Q^B + \omega_L\right)\right] - \alpha_A\left(\omega_L, \omega_H\right))\Delta V^A(Q^A, Q^B)}{\left[\alpha_A\left(\omega_H, \omega_H\right) - \alpha_A\left(\omega_H, \omega_L\right)\right]\left[V^A\left(Q^A + \omega_H, Q^B\right) - V^A\left(Q^A, Q^B + \omega_H\right)\right] + \alpha_A\left(\omega_H, \omega_L\right))\Delta V^A(Q^A, Q^B)}$$

and letting $\Delta V^B\left(Q^A, Q^B\right) := V^B\left(Q^A, Q^B + \omega_H\right) - V^B\left(Q^A, Q^B + \omega_L\right)$

$$\frac{U^B_{LL} - U^B_{HL}}{U^B_{HH} - U^B_{LH}} =$$
$$\frac{\left[\alpha_B\left(\omega_L, \omega_L\right) - \alpha_B\left(\omega_H, \omega_L\right)\right]\left[V^B\left(Q^A, Q^B + \omega_L\right) - V^B\left(Q^A + \omega_L, Q^B\right)\right] - \alpha_B\left(\omega_H, \omega_L\right)\Delta V^A(Q^A, Q^B)}{\left[\alpha_B\left(\omega_H, \omega_H\right) - \alpha_B\left(\omega_L, \omega_H\right)\right]\left[V^B\left(Q^A, Q^B + \omega_H\right) - V^B\left(Q^A + \omega_H, Q^B\right)\right] + \alpha_B\left(\omega_L, \omega_H\right)\Delta V^A(Q^A, Q^B)}.$$

If $Q^A < Q^B$ then, by decreasing marginal value of the queues,

$$V^A\left(Q^A + \omega_H, Q^B\right) - V^A\left(Q^A + \omega_L, Q^B\right) > V^B\left(Q^A, Q^B + \omega_H\right) - V^B\left(Q^A, Q^B + \omega_L\right)$$
$$V^A\left(Q^A + \omega_L, Q^B\right) - V^A\left(Q^A, Q^B + \omega_L\right) > V^B\left(Q^A, Q^B + \omega_L\right) - V^B\left(Q^A + \omega_L, Q^B\right)$$
$$V^A\left(Q^A + \omega_H, Q^B\right) - V^A\left(Q^A, Q^B + \omega_H\right) > V^B\left(Q^A, Q^B + \omega_H\right) - V^B\left(Q^A + \omega_H, Q^B\right).$$

Furthermore, $\alpha_A\left(\omega_L, \omega_H\right) > \alpha_B\left(\omega_H, \omega_L\right)$.

$$\alpha_A\left(\omega_L, \omega_L\right) - \alpha_A\left(\omega_L, \omega_H\right) > \alpha_B\left(\omega_L, \omega_L\right) - \alpha_B\left(\omega_H, \omega_L\right) > 0$$

and

$$\alpha_A\left(\omega_H, \omega_H\right) - \alpha_A\left(\omega_H, \omega_L\right) < \alpha_B\left(\omega_H, \omega_H\right) - \alpha_B\left(\omega_L, \omega_H\right) < 0.$$

Thus, if $U^j_{LL} - U^j_{HL}$ and $U^j_{HH} - U^j_{LH}$, $j \in \{A, B\}$ are of the same sign, then

$$\frac{U^A_{LL} - U^A_{LH}}{U^A_{HH} - U^A_{HL}} < \frac{U^B_{LL} - U^B_{HL}}{U^B_{HH} - U^B_{LH}}.$$

Hence, $x > 0.5 > y$. This means that the mixed strategy of the short-queue supplier first-order stochastically dominates that of the long-queue supplier. Thus, in mixed strategy equilibrium the short-queue supplier is more likely to commit fraud than the long-queue supplier.

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