Competitive Equilibrium Fraud in Markets for Credence-Goods

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Abstract

This is a study of the nature and prevalence of persistent fraud in competitive markets for credence-quality goods. We model the market as a dynamic game of incomplete information in which the players are customers and suppliers and analyze their equilibrium behavior. Customers characteristics, search cost and discount rate, are private information. Customers do not possess the expertise necessary to assess the service they need either ex ante or ex post. We show that there exists no fraud-free equilibrium in the markets for credence-quality goods and that fraud is a prevalent and persistent equilibrium phenomenon.

Keywords: Competitve equilibrium fraud; Credence-quality goods markets; Search with learning;

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1 Introduction

1.1 Motivation

There are markets in which customers seeking to purchase services that involve specialized knowledge might be defrauded by suppliers who prescribe unnecessary services. Examples include, medical tests and treatments, auto repairs, equipment maintenance, and taxi cab service. In these markets the service suppliers make diagnostic determinations of the service required and offer to provide it, and the customers must decide whether to purchase the prescribed service or to seek, at a cost, a second service prescription. Typically, in these situations, the customer can judge, ex post, whether or not the service provided was *sufficient* to solving the problem, but is unable to assess whether the prescribed service was also *necessary.*

Darby and Karni (1973) were the first to identify the fundamental ingredients of the problem underlying the provision of what they dubbed *credence-quality goods.* First, information asymmetry between the customer who lacks the expertise required to assess the service needed and service provider who possessed the required expertise and, second the cost saving of the joint provision of diagnosis and services. This bundling of information and service is crucial.[1] They proceeded to discussed and analyze the economic implications of transactions involving this type of asymmetric information. Specifically, Darby and Karni argued that in competitive market equilibrium for credence-quality goods there is persistent tendency of suppliers to over-prescribe services (that is, to prescribe services that are sufficient but are not

[1] See Wolinsky (1993) for an analysis of the implication of separation of diagnosis and service.

necessary to solve the problem at hand). Since then, numerous studies confirm the prevalence of this phenomenon.[2] For medical services, especially physicians' services, over treatment, a phenomenon known in medical literature as supplier induced demand, is widely documented (see McGuire [2000], Currie, et. al [2011], Dranove [1988]). Domenighetti (1993) found that in Swiss canton of Ticino on average the population has one third more operations than medical doctors and their relatives, suggesting that greater information symmetry tends to reduce overprescription of surgical procedures. The same type of conclusion was reached by Balafoutas et. al. (2013). They report the results of a natural field experiment on taxi rides in Athens, Greece, designed to measure different types of fraud and to examine the influence of passengers' presumed information and income on the extent of fraud. Their findings indicate that passengers with inferior information about optimal routes are taken on significantly longer detours. Iizuka (2007) finds physicians drugs prescriptions are influenced by markup. Schneider (2012) reports the results of a field experiment designed to assess the accuracy of service provision in the auto repair market. He finds evidences for over prescription of services as well as under prescription. Beck (2014) reports that the results of an experiment in Austria showing that car mechanics tend to supply more unnecessary services than student subjects.

The work of Darby and Karni (1973) called attention to a neglected aspect of economic interactions that results in market failure.[3] Yet, their work lacks the formal

[2] Dulleck and Kerschbamer (2006) includes a survey of the literature and provides numerous references.

[3] Theirs was the first paper after the seminal work of Akelof (1970) to discuss market failure due to asymmetric information. Unlike the asymmetry concerning hidden characteristics giving rise to adverse selection problem pointed out by Akerlof, the information asymmetry that concerns Darby and Karni has to do with the ability to assess the service needed which, in conjunction with the

structure necessary to derive more subtle implications of the concept they introduced. In this work we take a step towards a more formal analysis of competitive markets for credence-quality services. Specifically, taking a game-theoretic approach, we analyze the equilibrium behavior in a market in which two suppliers operating service stations are engaged in Bertrand competition. The suppliers post prices per service hour. We assume that Bertrand competition forces the price per service hour to be the same across suppliers.[4] The critical aspect of the model is the information asymmetry regarding the service necessary to address the problem at hand. We assume throughout that the suppliers possess the expertise necessary to assess the required service while the customers do not. Customers heterogeneity is the consequence of idiosyncratic cost of obtaining a second prescription and the cost of waiting for service, which depends on the length of the suppliers' queues. We assume that these costs are the customers' private information and that the customers discover the lengths of the suppliers queues only when they visit their service station. The suppliers are assumed to be ex ante identical in every respect and that the only asymmetry between them is the lengths of their queues, which arises endogenously. We also assume that the market can be depicted by stationary equilibrium in which the idle time at the service stations is short enough so that no supplier loses money in the long run and must leave the market, and the queues are not long enough to induce new entries. in other words, we suppose that competitive equilibrium result in normal profits that discourage entry or exit. We show that there exists no fraud-free competitive equilibrium in the credence good market, and that the level of fraud

provision of service, gives rise to fraud.

[4] The presumption is that the prices are posted and observed at no cost by all customers.

committed by the two suppliers depends on the lengths of their queues.

1.2 Related literature

Despite evidence regarding the prevalence of fraud in the market for credence goods and the distinguishing features of these markets, the theoretical literature dealing with the modeling and analysis of these markets is rather scant. The attempts, thus far, to model competitive markets for credence-quality goods offer a variety of approaches. The works that are closest to ours in terms of the questions asked, are Wolinsky (1995), Emons (1997) and Dulleck and Kerschbamer (2006). Despite the common interest in studying the prevalence of fraud in competitive equilibrium, these works are quite different from ours in the way they model the markets and, consequently, the equilibrium behavior of the customers and the suppliers.

Wolinsky (1995) proposed a model in which there are two states of disrepair, high and low. Customers do not possess the expertise necessary to determine the state and must relay on the diagnosis of the service providers. Wolinsky modeled the situation as a game in which the customers bargain with suppliers by offering a price for the repair. Suppliers have the option of rejecting the price, in which case the customers may increase their price or seek another supplier. Wolinsky showed that, in interior equilibrium, all customers who receive a prescription of the high service seek a second opinion, and the suppliers commit fraud by employing a strategy that assigns positive probability of rejecting price offers when the state diagnosed requires low service. This strategy reflects their belief that, to avoid the search cost, the customer may offer a higher price rather than seek a second opinion.

Wolinsky's work is different from ours in the way the credence-goods markets are modeled and the conclusion of the analysis. To begin with, we allow for any number of states of disrepair measured by the service hours needed to resolve the problem. More importantly, we assume that the price of service hour are fixed by the suppliers (no bargaining) and is equal among the suppliers due to competition. Suppliers are characterized by the lengths of their queue and customers are characterized by their idiosyncratic search cost and discount rate. Customers are engaged in search with learning. These differences in modeling mandate different equilibrium notions and analysis.

Emons (1997) proposed a different model of credence good market in which the suppliers must decide whether to enter the market. If a supplier enters the market he is endowed with a fixed capacity that can be allocated to diagnosis and repair service. These two functions are assumed to be priced differently. The suppliers are allowed to announce a wrong diagnosis if they find that it is more profitable, for lack of capacity, to avoid providing the needed repair. The customers are identical. Emons studies conditions under which fraud free equilibrium exists. Emons model is different from ours in the specification of the information structure and the features of the credence-good market. These differences have implications for the depiction of the product state of disrepair; the characterization of the customers and their behavior; the pricing mechanism in the market; the suppliers strategies and the penalty imposed on them for not prescribing the necessary service.

Dulleck and Kerschbamer (2006) consider a market for credence services in which the customers may experience a need for high or low levels of service. They used a game theoretic approach to study conditions under which competition will eliminate fraud. These conditions include homogeneous customer population, cost conditions that prevent customers from seeking a second opinion and verifiability of the service provided. They discuss the implications of relaxing each of these conditions.

Less related theoretical models of the credence good market emphasized different aspects of the efficiency loss due to the asymmetric information. Hu and Lin (2018), Fong and Liu (2016) and Fong et al. (2017) study this issue in the context of interaction between uniformed customers and a monopolistic expert service provider and Heinzel (2019) study the equilibrium of a price-regulated market in which physicians characterized by heterogeneous cost compete for servicing uniformed patients. Hu and Lin modeled repeated interaction between a customer in occasional need of maintenance service of a durable good and a monopoly supplier. The optimal service required is of credence quality. Their analysis focuses on possible deviations from the optimal level of service by prescribing undertreatment or over treatment. They show that there exist no equilibrium that supports truthful diagnosis. Fong and Liu (2016) investigated the effect of liability on the seller's incentive to maintain good reputation and its impact on market efficiency. Fong et al. (2017) focus on the use of customer service to build trust between the monopoly supplier its customers so as to mitigate the efficiency loss. Heinzel's model the interaction among physicians and patients as a game in which patients may employ mixed strategies in seeking "second opinion" when diagnosed as having a serious problem and physicians may defraud their patients by overtreating them for minor problems. Unlike in the model we present here, the distinct physicians' types is exogenous and the customer behavior is not derived from optimal search strategy.

In the next section we describe the credence good market. The equilibrium analysis appears in section 3. Some economic implications of our analysis are discussed in section 4. Section 5 includes a discussion of our results. To allow for uninterrupted reading we collected the proofs of the main results in section 6.

2 The Credence Good Market

2.1 Overview

Consider a market for a credence-quality service populated by customers and two service suppliers, say A and B. The information asymmetry in this market is two sided. The customers' private information consists of their idiosyncratic search cost and discount rate. The suppliers possess expertise that the customers do not have, which allows them to observe the actual state of disrepair and assess the service needed to fix the problem. Let $\widetilde{\omega}$ denote the discrete random variable representing the true state of disrepair expressed as the minimum service hours required to address the problem. We normalize $\widetilde{\omega}$ to take values in $\Omega := \{\omega_1, ..., \omega_n\}$, where $0 < \omega_1 < .... < \omega_n < 1$.[5] Denote the distribution of $\widetilde{\omega}$ by $\mu \in \Delta(\Omega)$, where $\Delta(\Omega)$ denotes the simplex in $\mathbb{R}^n$. We assume that μ is exogenous and commonly known. We also assume throughout that the prescribed service must fix the problem (malfunction, malaise), otherwise the customer refuses payment. The set of states Ω is assumed to be common knowledge and, consequently, the prescribed service must correspond to

[5] As will become clear later, the assumption of discrete state space has implications for the customers perception of the difference between the suppliers strategies.

the states (that is, the prescribed service, $q \in \Omega$) and must be at least as large as the minimal service required to address the problem (that is, if the state is ω_i then $q \in \{\omega_i, ..., \omega_n\}$).[6] The two suppliers are identical in every respect except for the lengths of their queues, expressed in terms of service hours committed to serving customers in the queues. We assume that the suppliers observe each other's queues and that customers only discover the length (waiting time for service) of a supplier's queue upon visiting the supplier.[7] Let $Q^A(t)$ and $Q^B(t)$ denote the length of the suppliers queues at time t and suppose that the market is such that the lengths of the queues are in some interval $I = [0, \bar{Q}]$. Then, at each interaction the credence-quality service market is parametrized by a state depicted by the triplet $\left(\omega, Q^A(t), Q^B(t)\right) \in \Omega \times I^2$. It is important to keep in mind that prior to visiting a supplier, the customers do not know the state. Consequently, from the customers' ex ante point of view, the two suppliers are identical.

We model the credence good market as a stochastic game of incomplete information, denoted Γ. We assume that new customers arrive at random intervals. A customer's arrival on the market at time t in state of disrepair ω when the suppliers queues are $Q^A(t)$ and $Q^B(t)$ initiates a dynamic *stage game*, $\Gamma\left(\omega, Q^A(t), Q^B(t)\right)$, depicting the interaction among the customer and the two suppliers. When a new customer shows up at a service station, the supplier observes the state of disrepair ω and, consequently, the state $\left(\omega, Q^A(t), Q^B(t)\right)$. The suppliers do not observe

[6] This assumption is dubbed liability in the literature (see Dulleck and Kreschbamer [2006], Fong an Liu [2016] and Fong et al. [2017]).

[7] The assumption that the suppliers observe each other's queue expresses the presumption that survival in a competitive market requires the players to keep tab of their rivals positions and actions. Relaxing this assumption would require a modification of the suppliers strategies described below, and will complicate the analysis without providing new insights.

the customer's type. Customers know their types but not the state of disrepair ω. They discover the length of a supplier's queue upon visiting a service station and receiving a diagnosis. In other words, a customer may discover the lengths of the suppliers queues sequentially, during the process of searching for service. Insofar as the customers are concerned, what matters are the lengths of the queues and not the identity of the suppliers. This assumption rules out suppliers' identity or reputation as a possible factor.[8]

At state $s(t,\omega) := \left(\omega, Q^A(t), Q^B(t)\right)$ the suppliers and customers choose there strategies and, once their choices are made the game proceeds to the next state as follows. Suppose that the next the next customer arrives at time t' in a state of disrepair ω' then if she accepts the prescription q^A of supplier A then the new state is $s_A(t',\omega') := \left(\min\{Q^A(t) - t' + q^A, 0 + q^A\}, \min\{Q^B(t) - t', 0\}, \omega'\right)$, and if she accepts the prescription q^B of supplier B then the new state is $s_B(t',\omega') := \left(\min\{Q^A(t) - t', 0\}, \min\{Q^B(t) - t' + q^B, 0 + q^B\}, \omega'\right)$. The transition probability from the state $\left(\omega, Q^A(t), Q^B(t)\right)$ to the state $\left(\min\{Q^A(t) - t' + q^A, 0 + q^A\}, \min\{Q^B(t) - t', 0\}, \omega'\right)$, denoted $\rho\left(s(t,\omega), s_j(t',\omega')\right), j \in \{A, B\}$, is the product of the probability $\mu(\omega')$ that the state of disrepair is ω'; the probability that supplier j prescribes q^j in equilibrium; the probability that the newly arrived customer, employing the equilibrium search strategy, accepts the prescription q^j. A detailed exposition of these probabilities and the induced stochastic evolution of the queues are developed in Section 3 below.

[8] We revisit the issue of reputation in the discussion section.

2.2 The customers

A customer's type, (θ, β), consists of idiosyncratic search cost, θ, and discount rate, β, both taking values in $[0,1]$. Thus, the set of customers' types is $T = [0,1]^2$. Let $\mathcal{B}(T)$ be the Borel sigma algebra on T and denote by ξ an continuous probability measure on the measurable type space $(T, \mathcal{B}(T))$.

Strategies: Upon identifying an equipment malfunction or a sense of malaise indicating, respectively, potential mechanical or health problem, the customer engages in sequential search for repair service or medical treatment. Diagnosis of the problem and determination of the service, or treatment, needed to resolve it requires expert knowledge, which the customer does not possess.

We assume that the customer chooses one of the two service outlets at random with equal probabilities.[9] Upon visiting a service outlet the customer obtains a service prescription, expressed in terms of service-hours, and the information regarding the waiting time for the service delivery (that is, the length of the queue at that service station). The customer must then choose between accepting the prescribed service and waiting in the queue and rejecting it in favor of seeking a second prescription. We assume that the search is with recall so, if the customer decides to seek a second prescription and then return to the first supplier, she maintains her place in the queue and is entitled to obtain the service prescribed by the first supplier. However, returning to the first supplier after visiting the second entails a cost, expressed as utility loss. During her visit to the second supplier, the customer receives

[9]This assumption does not rule out customers loyalty to suppliers or that each customer visits first the supplier whose location is closer provided that the loyalty or proximity are equally devided between the suppliers.

a second prescription and observes the length of the second supplier's queue. The customer must then decide whether to accept the second prescription or return to the first supplier.

Formally, a customer's search strategy is a mapping $\sigma : T \to \Sigma_1 \times \Sigma_2$, where $\Sigma_1(\theta, \beta) := \{\sigma_1 : \Omega \times I \to \{0,1\}\}$, $\Sigma_2(\theta, \beta) := \{\sigma_2 : \Omega^2 \times I^2 \to \{0,1\}\}$. In other words, the strategy assigns to a customer of type (θ, β) two acts depicted by the functions $\sigma_1^{(\theta,\beta)} : \Omega \times I \to \{0,1\}$ and $\sigma_2^{(\theta,\beta)} : \Omega^2 \times I^2 \to \{0,1\}$, where $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 1$ means that the customer accepts the prescription of the first supplier she visits and terminates the search, and $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 0$ means that she seeks a second prescription. Similarly, $\sigma_2^{(\theta,\beta)}(q_1, Q_1, q_2, Q_2) = 1$ means that the customer accepts the second supplier's prescription and $\sigma_2^{(\theta,\beta)}(q_1, Q_1, q_2, Q_2) = 0$ means that she rejects the second supplier's prescription and return to the first supplier. We denote by Σ the set of customers' strategies.

Beliefs: Since the customers do not observe the suppliers queues, at the outset the customer's information set is $\Omega \times I^2$ and her prior beliefs are captured by the commonly known distributions μ on Ω and υ on I^2. Upon observing the length of the queue, Q_1, and obtaining a prescription, $q_1 \in \Omega$, from the the first supplier, the customer updates her beliefs about the prescription she will receive from the second supplier and the waiting time at the second service station. In doing so, the customer is supposed to apply Bayes rule.[10] The updated beliefs, regarding the second supplier's queue conditional on the first supplier' prescription, q_1 and Q_1, is represented by the conditional distribution $m(q_2, Q_2 \mid q_1, Q_1)$ on $\Omega \times I^2$.[11]

[10] This is the sense in which the search involves learning.

[11] We examine the updated beliefs in further details below.

Payoffs: Accepting a prescribed service q on her first visit from a supplier whose queue length is Q, the utility of a customer of type (θ, β) is: $u^{\beta}(q, Q) = w(q) e^{-\beta Q}$, where w is monotone decreasing and differentiable function. Without loss of generality, we assume that $w(\omega_1) = 1$ and $w(\omega_n) = 0$. Continuing the search entails a customer-specific search cost, $\theta \in [0, 1]$, expressed as utility discount. Thus, the utility of accepting the prescription, q' when the queue of the second supplier is Q' is $u^{(\theta,\beta)}(q', Q') = \theta w(q') e^{-\beta Q'}$. Returning to the first supplier after visiting the second supplier, the customer's payoff is $\theta^2 w(q) e^{-\beta Q}$.

2.3 The suppliers

There are two suppliers, A and B, operating identical service outlets, engaged in Bertrand competition. Assume that the hourly cost of service is a linear function of the service provided x (that is, $c(x) = cx$). Note that $x = 1$ if the service station is fully occupied during a given hour. Otherwise x may take values in $[0, 1)$ meaning that the service station is idle for part of the time.

The profit generated by servicing customers for a fraction, x, of a fixed time period (e.g., a day) is: $\pi(q) = x(p - c)$, where p denotes the price per hour of service, $c \in [0, 1]$ denotes the constant marginal variable cost per service hour. Henceforth, without loss of generality, we normalize the price so that $\pi(x) = x$. Assuming free entry and exit, that stability of the market with two suppliers is based on the implicit assumption that both suppliers provide, on average, sufficient number of service hours per period to earn normal profit, so that there is no incentive for new suppliers to enter the market or for the current suppliers to exit the market.

Strategies: At every point each supplier has a queue representing hours committed to serving customers that have already accepted the supplier's prescriptions. The length of the queue is determined by the history of customer arrivals, their service prescriptions, and their acceptance decisions. Hence, the lengths of the queues are determined by the realization of an exogenous stochastic process (that is, the arrival rate and the random state $\widetilde{\omega}$) and the decisions of the suppliers and customers. The suppliers' *service-prescription mixed strategies* are mappings $G : \Omega \times I^2 \to \mathcal{G}$, where $\mathcal{G}$ denoted the set of CDF on Ω.[12] Formally, for each $q_k \in \Omega$, $G\left(\omega, Q^A, Q^B\right)(q_k) := \Sigma_i = 1^k g\left(\omega, Q^A, Q^B\right)(q_i)\,\delta_{qi}$, where $\left(g\left(\omega, Q^A, Q^B\right)(q_1), ..., g\left(\omega, Q^A, Q^B\right)(q_n)\right) \in \Delta(\Omega)$ and δ_{qi} denotes the distribution function that assigns the unit probability mass to q_i. Henceforth, when there is no risk of misunderstanding, we suppress the state and write $g(\cdot)$ instead of $g\left(\omega, Q^A, Q^B\right)$, and we use $G\left(\omega, Q^A, Q^B\right)$ and $g\left(\omega, Q^A, Q^B\right)$ interchangeably to designate the suppliers' strategies. We assume that $\Delta(\Omega)$ is endowed with the $\mathbb{R}^n$ topology and denote by S the set of the suppliers' mixed strategies. Since the only asymmetry between the suppliers is due to the lengths of their queues, the suppliers strategies $G^A\left(\omega, Q^A, Q^B\right)$ and $G^B\left(\omega, Q^A, Q^B\right)$ depend on the relative lengths of their queues.[13]

Payoffs: Consider the supplier j's, $j \in \{A, B\}$, problem when a new customer shows up on the market at time t in state ω when the queues are Q^A and Q^B, thereby initiating the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$. Denote by $\widetilde{t}$ the random waiting time for the arrival of the next customer taking values in $\mathbb{R}_+$. Let F denote

[12] We are restricting consideration to history-independent, or Markovian, strategies.

[13] We assume throughout that the suppliers's strategies are symmetric in the sense that, for every given pairs $\left(Q^A, Q^B\right)$, $G^A\left(\omega, Q^A, Q^B\right) = G^B\left(\omega, Q^B, Q^A\right)$.

the CDF of $\widetilde{t}$ and assume that it is time independent and have full support. Let $\alpha_j\left(q^j \mid Q^A, Q^B, \omega_i, \sigma, q^{-j}\right)$ denotes the probability that supplier j's prescription, q^j, be accepted conditional on the rival's prescription q^{-j} and customer's strategy, σ, and let $\alpha_j\left(q^j \mid Q^A, Q^B, \omega_i, G^{-j}, \sigma\right) = \Sigma_{q\in\Omega_{\omega_i}}\alpha_j\left(q^j \mid Q^A, Q^B, \omega_i, \sigma, q\right) g^{-i}(q)$ denotes the probability that supplier j's prescription, q^j, be accepted conditional on the rival's and customer's strategies, G^{-j} and σ, respectively. A more detailed and formal discussion of these probabilities appear in section 3.2 below.

Define the *interim expected value functions* $v_j : I^2 \times \Omega^2 \to \mathbb{R}$, $j \in \{A, B\}$, by

$$\begin{aligned} v^j\left(Q^A, Q^B, \omega_i, q_j\right) = \left(1 - \alpha_j\left(q^j \mid G^{-j}, \sigma\right)\right) \int_0^{Q^j} \left[\int_0^{t'} e^{-r\tau} d\tau\right] dF(t') + \\ \alpha_j\left(q^j \mid G^{-j}, \sigma\right) \int_0^{Q^j+q^j} \left[\int_{Q^j}^{t'} e^{-r\tau} d\tau\right] dF(t'), \end{aligned} \tag{1}$$

where $r > 0$ is the discount rate.

Denote by $V^j : I^2 \times \Omega \to \mathbb{R}$ supplier j's *continuation value function*, that is, the discounted expected value given the strategies (G^A, G^B, σ). Then supplier j's objective is to choose a strategy G^j that is best response to the rival's and the customer's strategies.[14] Formally,

$$\max_{G^j\in\mathcal{G}_k} \sum_{q\in\Omega_{\omega_i}} \left[v^j\left(Q^A, Q^B, \omega_i, q\right) + \int_0^{\infty} e^{-rt'} \left[\sum_{\omega\in\Omega} V^j\left(\omega_j, Q^A(t'), Q^B(t')\right)\mu(\omega)\right] dF(t')\right] g^j(q), \tag{2}$$

where $\Omega_{\omega i} := \{\omega_i, \omega_i + 1, ..., \omega_n\}$. The customer's objective is to find a acceptance rule σ that maximize his expected utility given (G^A, G^B).

[14]Further discussion of the continuation value function appears in Section 3.2.2 below.

To simplify the notations, when there is not risk of confusion, we suppress the strategies and write $V^j\left(\omega, Q^A, Q^B\right)$ instead of $V^j\left(\omega, Q^A, Q^B \mid G^A, G^B, \sigma\right)$. Clearly, $V^j\left(\omega, Q^A, Q^B\right)$, $j \in \{A, B\}$ is strictly monotonic increasing function of the supplier's own queue length. Furthermore, given an additional service q, regardless of the length of their queues, each supplier prefers providing this service. Formally, regardless of whether he is the short or the long queue supplier A's payoff satisfies $V^A\left(\omega, Q^A + q, Q^B\right) > V^A\left(\omega, Q^A, Q^B + q\right)$. Similarly, $\acute{B}$'s payoff satisfies $V^B\left(\omega, Q^A + q, Q^B\right) < V^B\left(\omega, Q^A, Q^B + q\right)$.

3 Equilibrium Analysis

3.1 Equilibrium: definitions

We analyze the credence service market as a stochastic game of incomplete information using sequential equilibrium as our equilibrium concept. At the start the customers learn their types. However, when customers detect a problems and seek remedial service, they do not know which particular stage game $\Gamma\left(\omega, Q^A, Q^B\right)$ is being played.

The suppliers observe each other's queue and, when a customer calls, they observe the state ω. However, the suppliers do not observe the customer's type. Consequently, even though at each stage game the suppliers face a single customer, not knowing the customer's type, the suppliers play strategies that are best responses against the average acceptance probability of the customer population of types induced by σ. To understand the customers' acceptance probabilities we need to understand the evolution of the customers beliefs.

The *customer's system of beliefs* $\eta := (\mu, \upsilon, m(\omega, Q_2 \mid q_1, Q_1))$ consists of the prior belief about the stage game being played, determined by the prior beliefs μ on Ω and υ on I^2, and the updated beliefs $m(\omega, Q_2 \mid q_1, Q_1)$ on $\Omega \times I^2$. A strategy profile $\left(\sigma, G^A, G^B\right)$, is *sequentially rational* if, given V_A and V_B, G^A is best response against $\left(\sigma, G^B\right)$, G^B is best response against $\left(\sigma, G^A\right)$, and σ is best response against $\left(G^A, G^B\right)$, for all $\left(\omega, Q^A, Q^B\right) \in \Omega \times I^2$. Given $\omega_i \in \Omega$, a strategy G_k is *completely mixed with modulus* k if $g_k(q) \geq k^{-1}$, for all $q \in \Omega_{\omega_i}$.

Definition 1: An point $\left(\hat{V}^A, \hat{V}^B, \hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ and a system of beliefs η^* constitute a *Markovian sequential equilibrium of the stochastic game induced by the credence good market* if:

(i) The strategy profile $\left(\hat{\sigma}, \hat{G}^A, \hat{G}^B\right)$ is sequentially rational given the belief system $\eta^* = (\mu, \upsilon, m^*)$ and the value functions $\hat{V}^A, \hat{V}^B$.

(ii) There exist sequence of value functions and strategy profiles $\{(V_k^A, V_k^B, G_k^A, G_k^B, \sigma_k)\}_{k=1}^{\infty}$, where G_k^A and , G_k^B are completely mixed strategies with $\lim_{k\to\infty}(V_k^A, V_k^B, G_k^A, G_k^B, \sigma_k) = \left(V^A, V^B, \hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ such that $\eta^* = \lim_{k\to\infty}\left(\mu, \upsilon, m^k(\omega, Q_2 \mid q_1, Q_1)\right)$ and $m^k(\omega, Q_2 \mid q_1, Q_1)$ derived from the prior beliefs (μ, υ) and strategy profile (G_k^A, G_k^B, σ_k) using Bayes' rule and, for $j \in \{A, B\}$,

$$V_k^j = \max_{G_k^j \in \mathcal{G}_k} \sum_{q \in \Omega_{\omega_i}} \left[v_k^j\left(Q^A, Q^B, \omega_i, q\right) + \int_0^{\infty} e^{-rt'} \left[\sum_{\omega \in \Omega} V_k^j\left(Q^A(t'), Q^B(t'), \omega_j\right) \mu(\omega)\right] dF(t')\right] \hat{g}_k^j(q).$$

3.2 Equilibrium: Existence

We turn next to the study of the existence of sequential equilibrium of the credence-quality market game Γ. We begin by examining the behavior of the customers and the suppliers. When a new customer arrives the supplier does not know whether he is the customer's first or second call. The supplier diagnoses the problem (that is, the supplier observes the state ω) and prescribes a service, $q \in \Omega_\omega$, the supplier also informs the customer the waiting time for service, which is equals to the length of the supplier's queue, Q.

3.2.1 The customers

The customers system of beliefs: The customers prior beliefs are depicted by the distributions $\mu \in \Delta(\Omega)$ and υ on I^2, which are assumed to be commonly known. Moreover, the ex ante symmetry of the suppliers requires that υ be symmetric.

Consider the state $\left(\omega, Q^A, Q^B\right)$ and let $G^j\left(\omega, Q^A, Q^B\right)$, $j \in \{A.B\}$, be the (mixed) strategies of the two suppliers. The customers are supposed to know the strategies of the two suppliers as functions of the states but not the current state $\left(\omega, Q^A, Q^B\right)$ and stage game $\Gamma\left(\omega, Q^A, Q^B\right)$. Moreover, the customers do not know which is the short-queue supplier and which is the long-queue supplier. Let q_1 and Q_1 denote, respectively, the prescription obtained and queue observed by customer in her first visit. Similarly, denote by q_2 and Q_2 the prescription obtained and queue observed by customer in second visit.

Following her visit to the first supplier, regardless of whether it is A or B, and having observed Q_1 the customer deduces, for every Q_2, whether it is shorter or longer than Q_1. The customer updates her beliefs about the state of disrepair ω and the length of the queue of the second supplier by applying Bayes' rule as follows: For all $\omega_i \leq q_1$,

$$m\left(\omega_i, Q_2 \mid q_1, Q_1\right) = \begin{pmatrix} \frac{g_k(\omega_i, Q_1, Q_2)(q_1)\mu(\omega_i)\upsilon(Q_2, Q_1)}{\int_{Q_1}^{\infty}\left[\Sigma_{\omega_1 \leq \omega_i \leq q_1} g_k(\omega_i, Q_1, Q)(q_1)\mu(\omega_i)\right] d\upsilon(Q, Q_1)} \text{ if } Q_2 \geq Q_1 \\ \frac{g_k(\omega_i, Q_1, Q_2)(q_1)\mu(\omega_i)\upsilon(Q_2, Q_1)}{\int_{0}^{Q_1}\left[\Sigma_{\omega_1 \leq \omega_i \leq q_1} g_k(\omega_i, Q_1, Q)(q_1)\mu(\omega_i)\right] d\upsilon(Q, Q_1)} \text{ if } Q_2 < Q_1 \end{pmatrix}, \tag{3}$$

where $g_k\left(\omega_i, Q_1, Q_2\right) := 0.5 g_k^A\left(\omega_i, Q_1, Q_2\right) + 0.5 g_k^B\left(\omega_i, Q_2, Q_1\right)$ is the mixed strategies of the second supplier.[15]

The customers expected payoff and best response strategies: We explore next the optimal behavior of the customer in the subgame following her visit to the first supplier and the evolution of her beliefs. Having obtained the prescription q_1 and service waiting time Q_1, a customer of type (θ, β) can accept the prescription and stop the search or seek a second prescription. In the latter case the customer accepts the second supplier's prescription if $u^{\beta}\left(q_2, Q_2\right) \geq \theta u^{\beta}\left(q_1, Q_1\right)$. Otherwise the customer exercises the recall option and returns to the first supplier to obtain the payoff $\theta^2 u^{\beta}\left(q_1, Q_1\right)$.

Because in her the second visit the customer is going to accept or reject the second offer according to whether $u^{\beta}\left(q_2, Q_2\right)$ is greater or smaller than $\theta u^{\beta}\left(q_1, Q_1\right)$, given q_1 and Q_1 the reservation utility of a customer of type (θ, β), $u_r^{(\theta,\beta)}\left(q_1, Q_1\right)$, is given by

$$u_r^{(\theta,\beta)}\left(q_1, Q_1\right) =$$

[15] This assumption entails, implicitly, that the suppliers' names are uniformative (that is to say, from an ex ante point of view they are symmetric).

$$\Sigma_{\omega_1 \leq \omega_h \leq q_1} \Sigma_{\omega_h \leq q_2 \leq \omega_n} [\int_0^{Q_1} \max\{\theta u^\beta (q_2, Q_2), \theta^2 u^\beta (q_1, Q_1))\} g_k (q_2) m (\omega_h, Q_2 \mid q_1, Q_1) dQ_2 \tag{4}$$

$$+ \int_{Q_1}^{\infty} \max\{\theta u^\beta (q_2, Q_2), \theta^2 u^\beta (q_1, Q_1))\} g_k (q_2) m (\omega_h, Q_2 \mid q_1, Q_1) dQ_2], \tag{5}$$

where $g_k(\cdot) = g_k(\omega_i, Q_1, Q_2)(\cdot)$.

Given her type, (θ, β), and the suppliers' strategies G_k^A and G_k^B, the customer's expected payoff upon observing (q_1, Q_1) given the reservation utility $u_r^{(\theta,\beta)}(q_1, Q_1)$ in (4), is:

$$\bar{U}\left(\sigma(\theta,\beta), G_k^A, G_k^B\right) = \sigma_1^{(\theta,\beta)} u^\beta (q_1, Q_1) + \left(1 - \sigma_1^{(\theta,\beta)}\right) u_r^{(\theta,\beta)} (q_1, Q_1). \tag{6}$$

Hence, the customer accepts the first supplier's offer (that is, set $\sigma_1^{(\theta,\beta)} = 1$) if $u^\beta(q_1, Q_1) \geq u_r^{(\theta,\beta)}(q_1, Q_1)$. Otherwise, the customer continues the search (that is, $\sigma_1^{(\theta,\beta)} = 0$). She accepts the second supplier's offer (that is, set $\sigma_2^{(\theta,\beta)} = 1$) if $u^\beta(q_2, Q_2) > \theta u^\beta(q_1, Q_1)$. Otherwise, she exercises the recall option (that is, set $\sigma_2^{(\beta,\theta)} = 0$). With this in mind we make the following definition:

Definition 2: A *reservation-utility search strategy* $\sigma : T \to \Sigma_1 \times \Sigma_2$ is two, type-dependent, mappings, $\sigma_1^{(\theta,\beta)} : \Omega \times I \to \{0,1\}, \sigma_2^{(\theta,\beta)} : \Omega^2 \times I^2 \to \{0,1\}$, and a function $u_r^{(\theta,\beta)} : \Omega \times I \to [0,1]$ such that:

(a) $\sigma_1^{(\theta,\beta)}(q, Q) = 1$ if $u^\beta(q, Q) \geq u_r^{(\theta,\beta)}(q, Q)$ and $\sigma_1^{(\theta,\beta)}(q, Q) = 0$, otherwise.

(b) $\sigma_2^{(\theta,\beta)}(q_2, Q_2; q_1, Q_1) = 1$ if $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 0$ and $u^\beta(q_2, Q_2) > \theta u^\beta(q_1, Q_1)$ and $\sigma_2^{(\theta,\beta)}(q_2, Q_2; q_1, Q_1) = 0$, otherwise.

We summarize the above discussion in the following:

Proposition 1. *The reservation-utility strategy* $\hat{\sigma}$ *is the customers' unique best response to the suppliers' strategy profile* $\left(g_k^A\left(\omega, Q^A, Q^B\right), g_k^B\left(\omega, Q^A, Q^B\right)\right)$, *for all* $\left(\omega, Q^A, Q^B\right) \in \Omega \times I^2$.

In view of Proposition 1, the reservation-utility strategy based on the reservation-utility function $u_r^{(\theta,\beta)}\left(q_1, Q_1\right)$ is the best response strategy of the customers in the subgame following the visit to the first supplier.

Lemma 1: *For each type* $(\theta, \beta) \in T$ *and all* $(q_1, Q_1) \in \Omega \times I$ *the customer's expected payoff,* $\bar{U}\left(\sigma(\theta, \beta), G_k^A, G_k^B\right)$, *of the reservation-utility strategy is continuous.*

The continuity of $\bar{U}\left(\sigma(\theta, \beta), G_k^A, G_k^B\right)$ is an immediate implication of its linearity in the strategies and the fact that $g_k^j(q) > 0$, $j \in \{A, B\}$, for all $q \in \Omega_\omega$.

3.2.2 The suppliers

Because the customer's type is private information, the suppliers must choose their strategies as best responses against the acceptance probabilities induced by the distribution of customers' types. We examine next the acceptance probabilities induced by reservation utility strategies.

For $j \in \{A, B\}$, the supplier j's utility of subscribing q is $V^i\left(Q^j + q, Q^{-j}\right)$ in the following cases: (1) the supplier j is the customer's first call and the customer accepts the prescription q immediately, (2) the supplier j is the customer's first call, the customer rejects supplier j's prescription in the first visit seeking a second prescription and returns to supplier j, (3) the supplier j is the customer's second call and she accepts supplier j's prescription. We calculate next the probabilities of these events.

The first-call suppliers face a distribution of acceptance rules induced by the distribution, ξ, on the set of types. Thus, for all $(q_1, Q_1) \in \Omega \times I$, the subset of the first callers who do not seek a second prescription when faced with the prescription q_1 and queue Q_1 is given by the subset of types $A_1(q_1, Q_1) := \{(\theta, \beta) \in T \mid u^{\beta}(q_1, Q_1) \geq u_r^{(\theta,\beta)}(q_1, Q_1)\} \in \mathcal{B}(T)$. Consequently, the average acceptance rate of first callers who, given the queue length Q, accepts the prescription q immediately is:

$$\sigma_1(q_1, Q_1) = \int_T \sigma_1^{(\theta,\beta)}(q_1, Q_1)\, d\xi(\theta, \beta) = \xi(A_1(q_1, Q_1)).$$

This may be interpreted as the *probabilistic demand* function of first callers.

Given the prescription q_1 and queue length Q_1 of the first supplier that the customer happened to visit, the acceptance rate of a second prescription, q_2, when the length of the queue of the second supplier is Q_2, is:

$$\sigma_2(q_2, Q_2 \mid q_1, Q_1) = \int_T \sigma_2^{(\theta,\beta)}(q_2, Q_2; q_1, Q_1) d\xi(\theta, \beta).$$

The second-call supplier does not know that he is the second-call supplier and does not observe the first-call supplier's prescribed service. However, since q_1 is determined by the supplier's mixed strategy, observing ω_i and Q_1, the second supplier can infer that, *if* he is the customer's second-call then the prescription the customer obtained in her first call is a random variable $\widetilde{q}_1$ whose conditional probability distribution is determined by the strategy of the first supplier. Specifically, if the customer first visit is to supplier is A then q_1 was determined by the strategy $G_k^A\left(\omega_i, Q^A, Q^B\right)$ and if the customer first visit is to supplier B then q_1 was determined by the strategy

$G_k^B\left(\omega_i, Q^A, Q^B\right)$.

Given $\left(\omega_i, Q^A, Q^B\right)$ if j is the second supplier the customer calls upon, the probability that his prescribed service is accepted is

$$\varsigma_2\left(q^j, \omega_i, Q^A, Q^B, G_k^{-j}\right) = \Sigma_{q\in\Omega_{\omega_i}} \sigma_2\left(q^j, Q^j \mid q, Q^{-j}\right) g_k^{-j}\left(\omega_i, Q^A, Q^B\right)(q), \ j \in \{A, B\}.$$

Hence, the probability that a newly arrived customer accepts the prescription of supplier j, $j \in \{A, B\}$, is:

$$\alpha_j\left(q \mid \omega_i, Q^A, Q^B, \sigma, G_k^{-j}\right) := \tag{7}$$

$$\frac{1}{2}[\sigma_1\left(q, Q^j\right) + \left(1 - \sigma_1\left(q, Q^j\right)\right)(1 - \varsigma_2\left(q, \omega_i, Q^A, Q^B, G_k^{-j}\right)) +$$

$$\Sigma_{q\in\Omega_{\omega_i}}\left(1 - \sigma_1\left(q, Q^{-j}\right)\right)\sigma_2\left(q^j, Q^j \mid q, Q^{-j}\right) g_k^{-j}\left(\omega_i, Q^A, Q^B\right)(q)],$$

Then the supplier j's payoff is:

$$\max_{g_k^j\in\cdot(\Omega)} \sum_{q\in\Omega_{\omega_i}}\left[v^j\left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'}\left[\sum_{\omega\in\Omega} V^j\left(Q^A(t'), Q^B(t'), \omega_j\right)\mu(\omega)\right] dF(t')\right] g_k^j(q) \tag{8}$$

where

$$V^j\left(\omega_i, Q^j(t'), Q^{-j}(t'), q^j\right) =$$

$$\alpha_j\left(q^j \mid Q^A, Q^B, \omega_i, G^{-j}, \sigma\right) V^j\left(\omega_i, \max\{Q^j - t' + q_j, 0\}, \max\{Q^{-j} - t', 0\}\right) + \tag{9}$$

$$\Sigma_{q\in\Omega_{\omega_i}} V^j\left(\omega_i, \max\{Q^j - t', 0\}, \min\{Q^{-j} + q - t', 0\}\right)\alpha_j\left(q^j \mid Q^A, Q^B, \omega_i, \sigma, q\right) g_{-j}\left(q \mid Q^A(t'), Q^B(t'), \omega_i\right.$$

Lemma 2: *For all $\omega, Q^A, Q^B \in \Omega\times I^2$, the expression (8) is a continuous function* *on the strategy profiles set* $\Sigma \times \cdot (\Omega)^2$.

The proof appears in Section 6.1.

Our first result establishes the existence of Markovian sequential equilibrium of the stochastic game Γ.

Theorem 1: *There exist Markovian sequential equilibrium of the stochastic game induced by the credence good market.*

To prove the theorem we begin by restricting the suppliers' strategies to be totally mixed. Specifically, we assume for some large $k \in N$ and all $q \in \Omega_{\omega_i}$, $g^j(Q^A, Q^B, \omega_i)(q) \geq \frac{1}{k}$ and prove the existence Markov equilibrium. We then proceed as follows. First, we prove the players' objective functions are all continuous in respect to other players' strategies. Invoking Berge maximum theorem, we conclude that the range of the correspondence that maps the set of value functions and strategies to itself is upper-semi continuous and that its range is a convex-valued and compact set. Then, invoking Kakutani's theorem we conclude that the aforementioned correspondence has fixed point. Second, we verify that the strategies corresponding to fixed points constitute a stationary Markov equilibrium and the value functions corresponding to the same fixed points are the equilibrium value functions. Third, taking the limits as k tends to infinity and using the sequential compactness, we conclude that there exist convergent subsequence of fixed points and, hence, a limit point of fixed points. Finally, we invoke uniform continuity to show that such limit point is indeed an equilibrium point.

4 Fraudulent Behavior and Short-Queue Advantage

4.1 Fraud-free equilibrium

An equilibrium is said to be *fraud-free* if the equilibrium strategies, $\hat{G}^j\left(\omega_i, Q^A, Q^B\right) = \delta_{\omega_i}$, for $j \in \{A, B\}$ for all $\left(\omega_i, Q^A, Q^B\right) \in \Omega \times I^2$. The next result asserts that fraudulent prescriptions of service is a persistent feature of competitive equilibrium in the credence good market. Formally,

Theorem 2: *There exists no fraud-free equilibrium in the market for credence quality services.*

While there exists no fraud-free equilibrium, the level of fraud committed depends on the stage game. In particular, since the supports of the equilibrium strategies is contained in $\{\omega_i, ..., \omega_n\}$, the larger is ω, the less room there is for fraudulent service overprescription.

4.2 Short-queue advantage and the evolution of the queues

The sole element of asymmetry between the suppliers in our model is the lengths of their queues. If $Q^A \neq Q^B$ the supplier with a shorter queue enjoys a strategic advantage in the sense that, if the two suppliers prescribe the same service, the supplier with the shorter queue is more likely to attract and retain a new customer. More generally, the short-queue advantage is measured by the *difference in the expected change of the lengths of the queues induced by equilibrium strategies.* Formally, given a stage game $\Gamma\left(\omega_i, Q^A, Q^B\right)$, if $Q^A < Q^B$ then the measure of the short-queue advantage is:

$$\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}^A, \hat{G}^B\right) :=$$

$$\Sigma_{q \in Supp \hat{g}^A} \alpha\left(q \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}^B\right) q \hat{g}^A\left(\omega_i, Q^A, Q^B\right)(q)$$

$$-\Sigma_{q \in Supp \hat{g}^B} \alpha\left(q \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}^A\right) q \hat{g}^B\left(\omega_i, Q^A, Q^B\right)(q).$$

In the stationary setting of our model, in which no new suppliers enter and no exiting existing supplier exits the market, the competition between results in normal expected profits. The capacity of serving the flow of customers matches the demand for service in the sense that the lengths of the queues is finite. Moreover, we assume that probability distribution of the random waiting time for the arrival of the next customer is time independent and have full support. Hence, starting from the event that both suppliers are idle (that is, $Q^A = Q^B = 0$) the probability, p, of returning to the same position of the queues under the equilibrium strategies is positive. Since the equilibrium is Markovian, this event is encountered infinitely often. Thus, the probability of the event "$Q^A = Q^B = 0$ infinitely often" is: $\lim_{m \to \infty} p^m > 0$. Hence, $p = 1$. In other words, starting from any state of the queues, $\left(Q^A, Q^B\right) \in I^2$, with probability one the queues will be reduced to $Q^A = Q^B = 0$ infinitely often. From this position, the two suppliers are equally likely to become the long queue supplier. Hence, no supplier is *expected* to enjoys the short-queue advantage persistently. In other words, the evolution of the queues under the equilibrium strategies requires that the anticipated lengths of the queues be *stochastically equal*, in the sense that the identity of the short queue supplier is expected to change over time in such a way that the joint distribution of the queues is symmetric around its mean. We summarize this in the following:

Proposition 2. *Under the equilibrium strategies, successive stage games induce a joint distribution of the lengths of the queues that is stationary, symmetric and the two suppliers commit the same amount of fraud on average.*

The discussion above implies that an increase in the length of the queue of the short-queue supplier reduces its short-queue advantage. Formally, if A is the short queue supplier then $d\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}^A, \hat{G}^B\right)/dQ^A < 0$. However, because $d\alpha\left(q \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}^B\right)/dQ^A < 0$ and $d\alpha\left(q \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}^A\right)/dQ^A > 0$, the short-queue advantage does not yield clear cut conclusions concerning its effect on the suppliers' equilibrium strategies.

5 Discussion

Our analysis shows that not only it is impossible for competition to sustain fraud-free equilibrium in markets for credence-quality goods but that, in fact, fraud is a persistent and prevalent phenomenon. Moreover, we argue that the supplier with a shorter queue enjoys a short-queue advantage in the sense that the expected increase in the length of his queue is larger.

The model in this paper envisages two suppliers engaged in Bertrand competition. The analysis highlights the role of the evolution of the customer's beliefs in the wake of her visit to the first supplier and the optimal stopping rule that characterized her best response strategy. The analysis also depicts the manner according to which the suppliers formulate their best response strategies given the information they possess. These aspects of our model and analysis are not specific to the two suppliers case and would show up, in a more complex form, if the number of the suppliers increase. Hence, there seems to be no essential loss of generality in so far as the main insights are concerned and much is gained by the relative simplicity afforded by considering two suppliers.

It is worth noting that if the queue is not an issue and each customer can be served immediately, then the analysis changes considerably.[16] In this instance, the customers' utilities depend only on the prescribed service, and their discount rates is no longer a factor (that is, the customers' types are their idiosyncratic search cost, θ). Suppose that $\theta \in (0,1]$ then it is easy to verify that suppliers strategies $q^j(\omega_i) = \omega_n$, for all ω_i and $j \in \{A, B\}$, is an equilibrium. In other words, knowing that the prescription is the same, no customer is inclined to search and, consequently, the suppliers have no incentive to try and undercut each other's prescription. This, maximal fraud, situations may characterize the cab service provided to tourists in an unfamiliar city. The route taken is only restricted by a tourist's conception of the reasonable length of the ride. However, if sufficiently large number of customers are "searchers", (that is, the measure customers of type $\theta = 0$ is sufficiently large) then their presence induces the suppliers to undercut each other, so no pure strategy equilibrium exists.

One aspect of the credence market good discussed in Darby and Karni (1973), not touched upon in this work, is the possibility of developing a reputation and

[16] This may be a good depiction of taxi cab service. See also, Stahl (1996) for a discussion of a related issue.

its effect on the commission of fraud. Including reputation in our model would require admitting repeated interactions in which the customers display loyalty (that is, they visit “their” supplier first) and the suppliers recognize their loyal clients. Under these conditions, the suppliers may establish what Darby and Karni dubbed client relationship. The loss of future business of, and being bad-mouthed by, a dissatisfied customer would increase the cost to the suppliers of “losing” customers, which should serve as a deterrence and, consequently, mitigate the problem of fraud. This extension of the present work requires further study.

6 Proofs

6.1 Proof of Lemma 2

The customer’s strategy affects V_k^j through the probabilities α_j, $j \in \{A, B\}$, in (7). Since V_k^j is continuous in α_j and α_j is continuous in σ_k, V_k^j is continuous in σ_k. To show that $V_k^j\left(\cdot,\cdot,\cdot \mid \omega, Q^A, Q^B\right)$, $j \in \{A, B\}$ is continuous in G_k^{-j}, it suffices to show that

$$v_k^j\left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'}\left[\sum_{\omega\in\Omega} V_k^j\left(Q^A(t'), Q^B\left(t'\right), \omega_j\right)\mu\left(\omega\right)\right] dF\left(t'\right) \qquad (10)$$

is continuous in G_k^{-j}. By equation (9), the expression in (10) depends on G_k^{-j} through $\Sigma_{q\in\Omega_{\omega_i}} V_k^A\left(\omega_i, , Q_s, Q_\ell + q\right) g_k^{-j}\left(q\right)$. Since the last expression is linear in the probabilities $\left(g_k^j\left(q\right)\right)_{q\in\Omega_{\omega_i}}$, it is continuous in G_k^j. That $V_k^j\left(\cdot,\cdot,\cdot \mid \omega, Q^A, Q^B\right)$ is continuous in G_k^j follows from its linearity in the probabilities $\left(g_k^j\left(q\right)\right)_{q\in\Omega_{\omega_i}}$. $\square$

6.2 Proof of Theorem 1

Let $C_b^{I\times I}$ denote that set of bounded and continuous real-valued functions on $I\times I\times\Omega$. Given $k\in N$, define a correspondence $T_k:\left(C_b^{I\times I}\right)^2\times\Delta\left(\Omega\right)^2\times[0,1]\rightrightarrows\left(C_b^{I\times I}\right)^2\times\Delta\left(\Omega\right)^2\times[0,1]$ as follows:

$$T_k:\begin{pmatrix} V_k^A \\ V_k^B \\ G_k^A \\ G_k^B \\ \sigma_k \end{pmatrix}\rightrightarrows\begin{pmatrix} \bar{V}_k^A \\ \bar{V}_k^B \\ \bar{G}_k^A \\ \bar{G}_k^B \\ \bar{\sigma}_k \end{pmatrix}$$

where the elements in the range set are defined as follows: Given $(V_k^j, G_k^{-j}, \sigma_k)$, $j\in\{A,B\}$,

$$\bar{V}_k^j\left(Q^A,Q^B,\omega_i\right)= \tag{11}$$

$$\max_{g_k^j\in\Delta(\Omega)}\sum_{q\in\Omega_{\omega_i}}\left[v_k^j\left(Q^A,Q^B,\omega_i,q\right)+\int_0^\infty e^{-rt'}\left[\sum_{\omega\in\Omega}V_k^j\left(Q^A(t'),Q^B\left(t'\right),\omega_j\right)\mu\left(\omega\right)\right]dF\left(t'\right)\right]g_k^j\left(q\right),$$

$$\bar{G}_k^j=\arg\max_{G_k^j\in\mathcal{G}_k}\sum_{q\in\Omega_{\omega_i}}\left[v_k^j\left(Q^A,Q^B,\omega_i,q\right)+\int_0^\infty e^{-rt'}\left[\sum_{\omega\in\Omega}V_k^j\left(Q^A(t'),Q^B\left(t'\right),\omega_j\right)\mu\left(\omega\right)\right]dF\left(t'\right)\right]g_k^j\left(q\right), \tag{12}$$

where G_k^j is the CDF corresponding to g_k^j, and $\bar{\sigma}_k$ is the customer's best response given (G_k^A, G_k^B). By Lemmata 1 and 2, for all $\omega, Q^A, Q^B\in\Omega\times I^2$, the function $\bar{V}_k^j\left(\cdot,\cdot,\cdot\mid Q^A,Q^B,\omega,\right):\Delta\left(\Omega\right)^2\times\Sigma\to\mathbb{R}$, $j\in\{A,B\}$, in (11) and the customer's expected payoff, $\bar{U}(\cdot,\cdot,\cdot) : \Delta(\Omega)^2 \times \Sigma \to \mathbb{R}$ in (6) are continuous functions.

The sets of strategies of the suppliers and the customers are closed and bounded in $\mathbb{R}^n$. Hence, by the Heine-Borel theorem, they are compact. By definition and Berge maximum theorem, for $j \in \{A, B\}$, $\bar{V}_k^j$ in (11) are continuous functions and the correspondences $\bar{G}_k^j$ in (12) are nonempty, compact and convex valued, and upper hemicontinuous. Moreover, by Proposition 1, the customer's strategy $\bar{\sigma}$ is single-valued and is linear in the suppliers mixed strategies. Hence it is continuous in these strategies.

Since $\bar{V}_k^A, \bar{V}_k^B \in C_b^{I\times I}$ and $I \times I \times \Omega$ is compact, they attain their maximal and minimal values. Hence, the ranges of these functions are closed and bounded intervals in $\mathbb{R}$. Consequently, $\left(C_b^{I\times I}\right)^2$ is compact in the product topology. Moreover, $\Delta(\Omega)$ and $[0, 1]$ endowed with the Euclidean metric are compact metric spaces. Hence, the domain of the correspondence T_k is compact Housdorff space. Thus, by Kakutani's fixed point theorem, T_k has fixed point,

$$T_k : \begin{pmatrix} \hat{V}_k^A \\ \hat{V}_k^B \\ \hat{G}_k^A \\ \hat{G}_k^B \\ \hat{\sigma}_k \end{pmatrix} = \begin{pmatrix} \hat{V}_k^A \\ \hat{V}_k^B \\ \hat{G}_k^A \\ \hat{G}_k^B \\ \hat{\sigma}_k \end{pmatrix}.$$

By definition, a fixed point of the mapping T_k is a stationary Markov equilibrium point.

By compactness of the domain, every sequence, $\left(\hat{V}_k^A, \hat{V}_k^B, \hat{G}_k^A, \hat{G}_k^B, \hat{\sigma}_k\right)_{k\in N}$ of fixed points has convergent subsequence. Denote by $\left(\hat{V}^A, \hat{V}^B, \hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ a subsequential limit point. We show next that $\left(\hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$ constitutes an equilibrium with respect to the value functions $\hat{V}^A$ and $\hat{V}^B$ and that $\left(\hat{V}^A, \hat{V}^B\right)$ are the value functions corresponding to the strategies $\left(\hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$.

Let $\{k_n \mid n = 1, 2...\}$ be a convergent subsequence and consider supplier A. Given $\left(\hat{V}^A_{k_n}, \hat{V}^B_{k_n}, \hat{G}^A_{k_n}, \hat{G}^B_{k_n}, \hat{\sigma}_{k_n}\right)$, for all $G^A_{k_n} = \left(g^A_{k_n}(q)\right)_{q \in \Omega}$, we have

$$\Phi^A_{k_n} := \sum_{q \in \Omega_{\omega_i}} \left[\hat{v}^A_{k_n}\left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} \hat{V}^A_{k_n}\left(Q^A(t'), Q^B(t'), \omega_j\right) \mu(\omega)\right] dF(t')\right] \hat{g}^A_{k_n}(q) \geq$$

$$\sum_{q \in \Omega_{\omega_i}} \left[\hat{v}^A_{k_n}\left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} \hat{V}^A_{k_n}\left(Q^A(t'), Q^B(t'), \omega_j\right) \mu(\omega)\right] dF(t')\right] g^A(q) := \phi^A_{k_n},$$

for all k_n, $n \in N$. Hence, $\lim_{n\to\infty} \Phi^A_{k_n} \geq \lim_{n\to\infty} \phi^A_{k_n}$. Let $\lim_{n\to\infty} \hat{G}^A_{k_n} = \hat{G}^A$, $\lim_{n\to\infty} \hat{G}^B_{k_n} = \hat{G}^B$, $\lim_{n\to\infty} \hat{\sigma}_{k_n} = \hat{\sigma}$ denote the limit strategies. Then,

$$\lim_{n\to\infty} \Phi^A_{k_n} = \sum_{q \in \Omega_{\omega_i}} \left[\hat{v}^A\left(Q^A, Q^B, \omega_i, q\right) + \int_0^\infty e^{-rt'} \left[\sum_{\omega \in \Omega} \hat{V}^A\left(Q^A(t'), Q^B(t'), \omega_j\right) \mu(\omega)\right] dF(t')\right] \hat{g}^A(q),$$

where $\hat{V}^A\left(Q^A(t'), Q^B(t'), \omega_j\right) = \hat{V}^A\left(\hat{G}^A, \hat{G}^B, \hat{\sigma} \mid Q^A(t'), Q^B(t'), \omega\right)$. Note also that $\lim_{n\to\infty} \phi^A_{k_n}$ is the value function of player A of the strategy G^A when player B and the customer play the limit strategies $\hat{G}^B$ and $\hat{\sigma}$, respectively, given the continuation function $\hat{V}^A$. Thus, $\lim_{n\to\infty} \Phi^A_{k_n} \geq \lim_{n\to\infty} \phi^A_{k_n}$ implies that $\hat{G}^A$ is best response to $\hat{G}^B$ and $\hat{\sigma}$, given the continuation function $\hat{V}^A$. Repeating the same argument for supplier B we conclude that $\hat{G}^B$ is best response to $\hat{G}^A$ and $\hat{\sigma}$, given the continuation function $\hat{V}^B$. That $\hat{\sigma}$ is best response to $\hat{G}^A$ and $\hat{G}^B$ is obvious.

Finally, since $\hat{V}^j\left(Q^A, Q^B, \omega_j\right) = \hat{V}^j\left(\hat{G}^A, \hat{G}^B, \hat{\sigma} \mid Q^A, Q^B, \omega\right)$, $j \in \{A, B\}$, $\hat{V}^A$ and $\hat{V}^B$ are the value functions corresponding to the strategies $\left(\hat{G}^A, \hat{G}^B, \hat{\sigma}\right)$. $\square$

6.3 Proof of theorem 2

Fraud-free equilibrium in the credence-good market requires that, $\hat{G}^A\left(\omega, Q^A, Q^B\right) = \hat{G}^B\left(\omega, Q^A, Q^B\right) = \delta_\omega$, for all $\left(\omega, Q^A, Q^B\right) \in \Omega \times I^2$. To prove that there is no fraud-free equilibrium we need to show that, for some stage game $\Gamma\left(\omega, Q^A, Q^B\right)$, $\hat{G}^j\left(\omega, Q^A, Q^B\right) = \delta_\omega$ is not a best response to $\hat{G}^{-j}\left(\omega, Q^A, Q^B\right) = \delta_\omega$, for some $j \in \{A, B\}$.

Suppose that there is fraud-free equilibrium and, without loss of generality, let Q^A be the short-queue supplier. Let $\hat{G}^B\left(\omega, Q^A, Q^B\right) = \delta_\omega$, for all Q^A and Q^B. In fraud-free equilibrium the customers believe that both suppliers prescribe the necessary service truthfully. Hence, the only reason to obtain a second prescription is the expectations that the second supplier has a sufficiently shorter queue that justifies bearing the cost of obtaining a second prescription. Thus, the probability of a new customer accepting the prescription ω from the long-queue supplier is as follows.

If the long-queue supplier (that is, supplier B) is the customer's first call then the probability of acceptance is:

$$p_1\left(Q^B\right) := \xi\{(\theta, \beta) \in T \mid e^{-\beta Q^B} > \theta E\left[e^{-\beta Q} \mid Q^B\right]\},$$

where $E\left[e^{-\beta Q} \mid Q^B\right] = \int_0^\infty e^{-\beta Q} \upsilon\left(Q \mid Q^B\right) dQ$ and $\upsilon\left(Q \mid Q^B\right)$ is the distribution of supplier A's queue conditional on Q^B.

If the customer's visits the short-queue supplier (that is, supplier A) first, the probability that he accepts eventually the prescription ω of the long-queue supplier requires that his type (θ, β) satisfies $e^{-\beta Q^A} < \theta E\left[e^{-\beta Q} \mid Q^A\right]$ and $w(q_A) e^{-\beta Q^A} < \theta w(\omega) e^{-\beta Q^B}$. The probability of this event is

$$p_2\left(\omega, Q^A, Q^B, q_A\right) := \xi\{(\theta, \beta) \in T \mid e^{-\beta Q^A} < \theta E\left[e^{-\beta Q} \mid Q^A\right] \text{ and } w(q_A) e^{-\beta Q^A} < \theta w(\omega) e^{-\beta Q^B}\}.$$

Define $p_B\left(\omega, Q,^A Q^B, q_A\right) = p_1\left(Q^B\right) + p_2\left(\omega, Q^A, Q^B, q_A\right)$. Then the short-queue supplier's expected profit is given by:

$$\max_{q_A \geq \omega}\left[\left(1 - p_B\left(\omega, Q^A, Q^B, q_A\right)\right) V\left(Q^A + q_A, Q^B\right) + p_B\left(\omega, Q^A, Q^B, q_A\right) V\left(Q^A, Q^B + \omega\right)\right].$$

Let $Q^A = 0$ then $e^{-\beta Q^A} > \theta e^{-\beta Q^B}$ and $e^{-\beta Q^A} > \theta E\left[e^{-\beta Q} \mid 0\right]$, for all $\beta > 0$. Hence, $p_2\left(\omega_i, Q^A, Q^B, \omega_{i+1}\right) = 0$ and, since $p_1\left(Q^B\right)$ is independent of q_A, $p_B\left(\omega_i, Q^A, Q^B, \omega_{i+1}\right) = p_B\left(\omega_i, Q^A, Q^B, \omega_i\right)$.

If $\beta = 0$ then $e^{-\beta Q^B} \geq \theta E\left[e^{-\beta Q^A} \mid Q^B\right]$, for all $\theta \in \Theta$. Hence, $p_1\left(Q^B\right) = \xi\{(\Theta, 0)\}$ and $p_2\left(\omega, Q^A, Q^B, q_A\right) = 0$, for all $q_A \geq \omega_i$. Hence, $p_B\left(\omega, Q^A, Q^B, q_A\right) = p_1\left(Q^B\right)$ is independent of q_A.

But $V\left(Q^A + \omega_{i+1}, Q^B\right) - V\left(Q^A + \omega_i, Q^B\right) > 0$. Hence, $\hat{G}^A\left(\omega_i, Q^A, Q^B\right) = \delta_{\omega_i}$ is not best response to $\hat{G}^B\left(\omega_i, Q^A, Q^B\right) = \delta_{\omega_i}$. □

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