

Competitive Equilibrium Fraud in the Markets for Credence-Goods

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Abstract

This is a study of the nature and prevalence of persistent fraud in competitive markets for credence-quality goods. We model the market as a dynamic game of incomplete information in which the players are customers and suppliers and analyze their equilibrium behavior. Customers characteristics, search cost and discount rate, are private information. Customers do not possess the expertise necessary to assess the service they need either ex ante or ex post. We show that there exists no fraud-free equilibrium in the markets for credence-quality goods and that fraud is a prevalent and persistent equilibrium phenomenon.

Keywords: Competitive equilibrium fraud; Credence-quality goods markets; Search with learning;

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1 Introduction

1.1 Motivation

There are markets in which customers seeking to purchase services that involve specialized knowledge might be defrauded by suppliers who prescribe unnecessary services. Examples include, medical tests and treatments, auto repairs, equipment maintenance, and taxi cab service. In these markets the service suppliers make diagnostic determinations of the service required and offer to provide it, and the customers must decide whether to purchase the prescribed service or to seek, at a cost, a second service prescription. Typically, in these situations, the customer can judge, ex post, whether or not the service provided was *sufficient* to solving the problem, but is unable to assess whether the prescribed service was also *necessary*.

Darby and Karni (1973) were the first to identify the fundamental ingredients of the problem underlying the provision of what they dubbed *credence-quality goods*. First, information asymmetry between the customer who lacks the expertise required to assess the service needed and service provider who possessed the required expertise and, second the cost saving of the joint provision of diagnosis and services. This bundling of information and service is crucial.¹ They proceeded to discuss and analyze the economic implications of transactions involving this type of asymmetric information. Specifically, Darby and Karni argued that in competitive market equilibrium for credence-quality goods there is persistent tendency of suppliers to over-prescribe services (that is, to prescribe services that are sufficient but are not

¹See Wolinsky (1993) for an analysis of the implication of separation of diagnosis and service.

necessary to solve the problem at hand). Since then, numerous studies confirm the prevalence of this phenomenon.² For medical services, especially physicians' services, over treatment, a phenomenon known in medical literature as supplier induced demand, is widely documented (see McGuire [2000], Currie, et. al [2011], Dranove [1988]). Domenighetti (1993) found that in Swiss canton of Ticino on average the population has one third more operations than medical doctors and their relatives, suggesting that greater information symmetry tends to reduce overprescription of surgical procedures. The same type of conclusion was reached by Balafoutas et. al. (2013). They report the results of a natural field experiment on taxi rides in Athens, Greece, designed to measure different types of fraud and to examine the influence of passengers' presumed information and income on the extent of fraud. Their findings indicate that passengers with inferior information about optimal routes are taken on significantly longer detours. Iizuka (2007) finds physicians drugs prescriptions are influenced by markup. Schneider (2012) reports the results of a field experiment designed to assess accuracy of service provision in the auto repair market. He finds evidences for over prescription of services as well as under prescription. Beck (2014) reports that the results of an experiment in Austria showing that car mechanics tend to supply more unnecessary services than student subjects.

The work of Darby and Karni (1973) called attention to a neglected aspect of economic interactions that results in market failure.³ Yet, their work lacks the formal

²Dulleck and Kerschbamer (2006) includes a survey of the literature and provides numerous references.

³Theirs was the first paper after the seminal work of Akerlof (1970) to discuss market failure due to asymmetric information. Unlike the asymmetry concerning hidden characteristics giving rise to adverse selection problem pointed out by Akerlof, the information asymmetry that concerns Darby and Karni has to do with the ability to assess the service needed which, in conjunction with the

structure necessary to derive more subtle implications of the concept they introduced. In this work we take a step towards a more formal analysis of competitive markets for credence-quality services. Specifically, taking a game-theoretic approach, we analyze the equilibrium behavior in a market in which two suppliers operating service stations are engaged in Bertrand competition. The suppliers post prices per service hour. We assume that Bertrand competition forces the price per service hour to be the same across suppliers.⁴ The critical aspect of the model is the information asymmetry regarding the service necessary to address the problem at hand. We assume throughout that the suppliers possess the expertise necessary to assess the required service while the customers do not. Customers heterogeneity is the consequence of idiosyncratic cost of obtaining a second prescription and the cost of waiting for service, which depends on the length of the suppliers' queues. We assume that these costs are the customers' private information and that the customers discover the lengths of the suppliers queues only when they visit their service station. The suppliers are assumed to be ex ante identical in every respect and that the only asymmetry between them is the lengths of their queues, which arises endogenously. We also assume that the market can be depicted by stationary equilibrium in which the expected demand for services is met by the current suppliers and that competitive equilibrium result in normal profits that discourage entry or exit. We show that there exists no fraud-free competitive equilibrium in the credence good market, and that the level of fraud committed by the two suppliers depends on the lengths of their queues.

provision of service, gives rise to fraud.

⁴The presumption is that the prices are posted and observed at no cost by all customers.

1.2 Related literature

Despite evidence regarding the prevalence of fraud in the market for credence goods and the distinguishing features of these markets, the theoretical literature dealing with the modeling and analysis of these markets is rather scant. The attempts, thus far, to model competitive markets for credence-quality goods offer a variety of approaches. The works that are closest to ours in terms of the questions asked, are Wolinsky (1995), Emons (1997) and Dulleck and Kerschbamer (2006). Despite the common interest in studying the prevalence of fraud in competitive equilibrium, these works are quite different from ours in the way they model the markets and, consequently, the equilibrium behavior of the customers and the suppliers.

Wolinsky (1995) proposed a model in which there are two states of disrepair, high and low. Customers do not possess the expertise necessary to determine the state and must rely on the diagnosis of the service providers. Wolinsky modeled the situation as a game in which the customers bargain with suppliers by offering a price for the repair. Suppliers have the option of rejecting the price, in which case the customers may increase their price or seek another supplier. Wolinsky showed that, in interior equilibrium, all customers who receive a prescription of the high service seek a second opinion, and the suppliers commit fraud by employing a strategy that assigns positive probability of rejecting price offers when the state diagnosed requires low service. This strategy reflects their belief that, to avoid the search cost, the customer may offer a higher price rather than seek a second opinion. Wolinsky's work is different from ours in the way the credence-goods markets are modeled and the conclusion of the analysis. To begin with, we allow for any number

of states of disrepair measured by the service hours needed to resolve the problem. More importantly, we assume that the price of service hour are fixed by the suppliers (no bargaining) and is equal among the suppliers due to competition. Suppliers are characterized by the lengths of their queue and customers are characterized by their idiosyncratic search cost and discount rate. Customers are engaged in search with learning. These differences in modeling mandate different equilibrium notions and analysis.

Emons (1997) proposed a different model of credence good market in which the suppliers must decide whether to enter the market. If a supplier enters the market he is endowed with a fixed capacity that can be allocated to diagnosis and repair service. These two functions are assumed to be priced differently. The suppliers are allowed to announce a wrong diagnosis if they find that it is more profitable, for lack of capacity, to avoid providing the needed repair. The customers are identical. Emons studies conditions under which fraud free equilibrium exists. Emons model is different from ours in the specification of the information structure and the features of the credence-good market. These differences have implications for the depiction of the product state of disrepair; the characterization of the customers and their behavior; the pricing mechanism in the market; the suppliers strategies and the penalty imposed on them for not prescribing the necessary service.

Dulleck and Kerschbamer (2006) consider a market for credence services in which the customers may experience a need for high or low levels of service. They used a game theoretic approach to study conditions under which competition will eliminate fraud. These conditions include homogeneous customer population, cost conditions

that prevent customers from seeking a second opinion and verifiability of the service provided. They discuss the implications of relaxing each of these conditions.

In the next section we describe the credence good market. The equilibrium analysis appear in sections 3. Some economic implications of our analysis appear in section 4. Section 5 includes a discussion of our results. To allow for uninterrupted reading we collected the proofs of the main results in section 6.

2 The Credence Good Market

2.1 Overview

Consider a market for a credence-quality service populated by customers and two service suppliers, say A and B . The information asymmetry in this market is two sided. The customers' private information, their type, consists of their idiosyncratic search cost and discount rate. The suppliers possess expertise, which the customers do not have, that allows them to observe the actual state of disrepair and assess the service that is needed to fix the problem. Let $\tilde{\omega}$ denote the discrete random variable representing the true state of disrepair expressed as the minimum service hours required to address the problem. We normalize $\tilde{\omega}$ to take values in $\Omega := \{\omega_1, \dots, \omega_n\}$, where $0 < \omega_1 < \dots < \omega_n < 1$.⁵ Denote the distribution of $\tilde{\omega}$ by $\mu \in \Delta(\Omega)$, where $\Delta(\Omega)$ denotes the simplex in \mathbb{R}^n . We assume that μ is exogenous and commonly known. We also assume throughout that the prescribed service must fix the problem (malfunction, malaise), otherwise the customer refuses payment. The

⁵As will become clear later, the assumption of discrete state space has implications for the customers perception of the difference between the suppliers strategies.

set of states Ω is assumed to be common knowledge and, consequently, the prescribed service must correspond to the states (that is, the prescribed service, $q \in \Omega$) and must be at least as large as the minimal service required to address the problem (that is, if the state is ω_i then $q \in \{\omega_i, \dots, \omega_n\}$). The two suppliers are identical in every respect except for the lengths of their queues, expressed in terms of service hours committed to serving customers in the queues. We assume that the suppliers observe each other's queues and that customers only discover the length (waiting time for service) of a supplier's queue upon visiting the supplier.⁶ Let Q^A and Q^B , denote the length of the suppliers queues and suppose that the market is such that the lengths of the queue are in some interval $I = [0, \bar{Q}]$. Then, at each interaction the credence-quality service market is parametrized by a state depicted by the triplet $(\omega, Q^A, Q^B) \in \Omega \times I^2$. It is important to keep in mind that, prior to visiting a supplier, the customers do not know the state. Consequently, from the customers' ex ante point of view, the two suppliers are identical.

We model the credence good market as a dynamic game of incomplete information, denoted Γ . The game begins with nature allocating each customer a type and selecting a state $\omega \in \Omega$. At each point in time, the queues of the suppliers, Q^A and Q^B , are determined by the history of the market. When a new customer shows up at their service stations, the suppliers observe the state of disrepair ω and, hence, the state (ω, Q^A, Q^B) . The suppliers do not observe the customer's type. Customer know their types but not the state ω . The customers discover the lengths of the

⁶The assumption that the suppliers observe each other queue reflect the notion that survival in a competitive market requires that the players keep tab of their rivals positions and actions. Relaxing this assumption will require a modification of the suppliers strategies described below, and will complicate the analysis without providing new insights.

suppliers queues during the process of searching for service. Insofar as the customers are concerned, what matters are the lengths of the queues and not the identity of the suppliers. This assumption rules out suppliers' identity or reputation as a possible factor.⁷ A customer arriving on the market in state of disrepair ω when the suppliers queues are Q^A and Q^B , sets up a *stage game*, $\Gamma(\omega, Q^A, Q^B)$, depicting the interaction among the customer and the two suppliers when the state is (ω, Q^A, Q^B) .

2.2 The customers

A customer's type, (θ, β) , consists of idiosyncratic search cost, θ , and discount rate, β , both taking values in $[0, 1]$. Thus, the set of customers' types is $T = [0, 1]^2$. Let $\mathcal{B}(T)$ be the Borel sigma algebra on T and denote by ξ an continuous probability measure on the measurable type space $(T, \mathcal{B}(T))$.

Strategies: Upon identifying an equipment malfunction or a sense of malaise indicating potential mechanical or health problem, the customer engages in sequential search for repair service or medical treatment. Diagnosis of the problem and determination of the service, or treatment, needed to resolve it require expert knowledge, which the customer does not possess.

We assume that the customer chooses one of the two service outlets at random with equal probabilities.⁸ Upon visiting a service outlet the customer obtains a service prescription, expressed in terms of service-hours, and the information regarding

⁷We revisit the issue of reputation in the discussion section.

⁸This assumption does not rule out customers loyalty to suppliers or that each customer visits first the supplier whose location is closest to his own. If we assume that there is a large population of customers half of which always visit first supplier A and the other half always visit first supplier B , then when a random customer shows up it is equal chance that he visits each of the two suppliers first.

the waiting time for the service delivery (that is, the length of the queue at that service station). The customer must then choose between accepting the prescribed service and waiting in the queue and rejecting it in favor of seeking a second prescription. We assume that the search is with recall so, if the customer decides to seek a second prescription and then return to the first supplier, she maintains her place in the queue and is entitled to obtain the service prescribed by the first supplier. However, returning to the first supplier after visiting the second entails a cost, expressed as utility loss. During her visit to the second supplier, the customer receives a second prescription and observes the length of the second supplier's queue. The customer must then decide whether to accept the second prescription or return to the first supplier.

Formally, a customer's search strategy is a mapping $\sigma : T \rightarrow \Sigma_1 \times \Sigma_2$, where $\Sigma_1(\theta, \beta) := \{\sigma_1 : \Omega \times I \rightarrow \{0, 1\}\}$, $\Sigma_2(\theta, \beta) := \{\sigma_2 : \Omega^2 \times I^2 \rightarrow \{0, 1\}\}$. In other words, the strategy assigns to a customer of type (θ, β) two acts depicted by the functions $\sigma_1^{(\theta, \beta)} : \Omega \times I \rightarrow \{0, 1\}$ and $\sigma_2^{(\theta, \beta)} : \Omega^2 \times I^2 \rightarrow \{0, 1\}$, where $\sigma_1^{(\theta, \beta)}(q_1, Q_1) = 1$ means that the customer accepts the prescription of the first supplier she visits and terminates the search, and $\sigma_1^{(\theta, \beta)}(q_1, Q_1) = 0$ means that she seeks a second prescription. Similarly, $\sigma_2^{(\theta, \beta)}(q_1, Q_1, q_2, Q_2) = 1$ means that the customer accepts the second supplier's prescription and $\sigma_2^{(\theta, \beta)}(q_1, Q_1, q_2, Q_2) = 0$ means that she rejects the second supplier's prescription and return to the first supplier. We denote by Σ the set of customers' strategies.

Beliefs: Since the customers do not observe the suppliers queues, at the outset the customer's information set is $\Omega \times I^2$ and her prior beliefs are captured by the

commonly known distributions μ on Ω and v on I^2 . Upon observing the length of the queue, Q_1 , and obtaining a prescription, $q_1 \in \Omega$, from the the first supplier, the customer updates her beliefs about the prescription she will receive from the second supplier and the waiting time at the second service station. In doing so, the customer is supposed to apply Bayes rule whenever possible.⁹ The updated beliefs, regarding the second supplier's queue conditional on the first supplier' prescription, q_1 and Q_1 , is represented by the conditional distribution $m(q_2, Q_2 | q_1, Q_1)$ on $\Omega \times I^2$.¹⁰

Payoffs: Accepting a prescribed service q from a supplier whose queue length is Q , the utility of a customer of type (θ, β) is: $u^\beta(q, Q) = w(q) e^{-\beta Q}$, where w is monotone decreasing and differentiable function. Without loss of generality, we assume that $w(\omega_1) = 1$ and $w(\omega_n) = 0$. Continuing the search entails a customer-specific search cost, $\theta \in [0, 1]$, expressed as utility discount. Thus, the utility of accepting the prescription, q' when the queue of the second supplier is Q' is $u^{(\theta, \beta)}(q', Q') = \theta w(q') e^{-\beta Q'}$. Returning to the first supplier after visiting the second supplier, the customer's payoff is $\theta^2 w(q) e^{-\beta Q}$.

2.3 The suppliers

There are two suppliers, A and B , operating identical service outlets. The suppliers are engaged in Bertrand competition. Assume that the cost of service is a linear function of the service hours provided q (that is, $c(q) = cq + c_F$). The profit generated by providing the service q is: $\pi(q) = q(p - c) - c_F$, where p is the price per hour of service, $c \in [0, 1]$ denotes the constant marginal cost per service hour, and $c_F > 0$

⁹This is the sense in which the search involves learning.

¹⁰We examine the updated beliefs in further details below.

denotes the fixed cost, per hour, of maintaining a service station. Note that $q = 1$ if the service station is fully occupied during a given hour. Otherwise q may take values in $[0, 1)$ meaning that the service station is idle for part of the time. Henceforth, without loss of generality, we normalize the price so that $\pi(q) = q$. In anticipation of later discussion, we note that the suppliers' expected profits depend on the expected number of service hours supplied and that the competition tends to eliminate above normal profits. Put differently, we assume that both suppliers provide, on average, sufficient number of service hours per period to earn normal profit, so that there is no incentive for new suppliers to enter the market or for the current suppliers to exit the market.

Strategies: At every point each supplier has a queue representing hours committed to serving customers that have already accepted the supplier's prescriptions. The length of the queue is determined by the history of customer arrivals, their service prescriptions, and their acceptance decisions. Hence, the lengths of the queues are determined by the realization of an exogenous stochastic process (that is, the arrival rate and the random state $\tilde{\omega}$) and the decisions of the suppliers and customers. The suppliers' *service-prescription mixed strategies* are mappings $q : \Omega \times I^2 \rightarrow \Delta(\Omega)$. We assume that $\Delta(\Omega)$ is endowed with the \mathbb{R}^n topology and denote by S the set of the suppliers' mixed strategies. Because the only asymmetry between the suppliers is due to the lengths of their queues, henceforth we shall denote by Q_s and Q_ℓ , $Q_\ell \geq Q_s$, the short and long queues, respectively. We designate the suppliers by the relative length of their queues, denoting by s the short-queue supplier and by ℓ the long-queue supplier.

Payoffs: Consider supplier's $j \in \{s, \ell\}$ problem when a new customer shows up and the state characterizing the stage game is $(\omega, Q^A, Q^B) \in \Omega \times I^2$. Let $G_j(\omega, Q^A, Q^B)(q_k) := \sum_{i=1}^k g_j(\omega, Q^A, Q^B)(q_i) \delta_{q_i}$, $j \in \{s, \ell\}$, where $(g_j(\omega, Q^A, Q^B)(q_1), \dots, g_j(\omega, Q^A, Q^B)(q_n)) \in \Delta(\Omega)$ and δ_{q_i} denotes the distribution function that assigns the unit probability mass to q_i . Henceforth, when there is no risk of misunderstanding, we suppress the state and write $g_j(\cdot)$ instead of $g_j(\omega, Q^A, Q^B)$. Let $V_j(\omega, Q^A, Q^B | \sigma, G_s, G_\ell)$ denote the value (i.e., the present value of the expected profits) in the state (ω, Q^A, Q^B) conditional on the strategies of the customers and the two suppliers. (The mixed strategies, G_s and G_ℓ , of the short and long queue suppliers are shorthand notation for the strategies $q^s(\omega, Q^A, Q^B) = G_s(\omega, Q^A, Q^B)$ and $q^\ell(\omega, Q^A, Q^B) = G_\ell(\omega, Q^A, Q^B)$, where $q^s, q^\ell \in S$). To simplify the notations, when there is not risk of confusion, we suppress the strategies and write $V_j(\omega, Q^A, Q^B)$ instead of $V_j(\omega, Q^A, Q^B | G_s, G_\ell, \sigma)$. We assume that $V_j(\omega, Q^A, Q^B)$, $j \in \{s, \ell\}$ is strictly monotonic increasing function of the supplier's the own queue length. Furthermore, given an additional service q , regardless of the length of their queues, each supplier prefers providing this service. Formally, supplier A 's payoff displays $V_j(\omega, Q^A + q, Q^B) > V_j(\omega, Q^A, Q^B + q)$, and supplier B 's payoff displays $V_j(\omega, Q^A + q, Q^B) < V_j(\omega, Q^A, Q^B + q)$, $j \in \{s, \ell\}$.¹¹

¹¹Such value function can be shown to exist if we assume that the suppliers engage in infinitely repeated game with short-lived customers in Markov equilibrium. We do not pursue this here.

3 Equilibrium Analysis

3.1 Equilibrium: definitions

We analyze the credence service market as a dynamic game of incomplete information using two notions of equilibrium: perfect Bayesian equilibrium and sequential equilibrium. At the start the customers learn their types but do not know the state ω or the lengths of the suppliers queues. Hence, they do not know which particular stage game $\Gamma(\omega, Q^A, Q^B)$ is being played. They discover the lengths of the queues sequentially during the search process. The suppliers observe each other's queue and, when a customer calls, they observe the state ω . However, the suppliers do not observe the customer's type. Consequently, even though at each stage game the suppliers face a single customer, not knowing the customer's type, the suppliers play strategies that are best responses against the average acceptance probability of the customer population of types induced by σ . To understand the customers acceptance probabilities we need to understand the evolution of the customers beliefs.

The *customer's system of beliefs* $\eta := (\mu, \nu, m(\omega, Q_2 | q_1, Q_1))$ consists of the prior belief about the stage game being played, determined by the prior beliefs μ on Ω and ν on I^2 , and the updated beliefs $m(\omega, Q_2 | q_1, Q_1)$ on $\Omega \times I^2$. A strategy profile (σ, G_s, G_ℓ) , is *sequentially rational* if \hat{G}_s best response against $(\hat{\sigma}, \hat{G}_\ell)$, \hat{G}_ℓ is best response against $(\hat{\sigma}, \hat{G}_s)$, and $\hat{\sigma}$ is best response against $(\hat{G}_s, \hat{G}_\ell)$, for all $(\omega, Q^A, Q^B) \in \Omega \times I^2$.

Definition 1: A *perfect Bayesian equilibrium of the credence service market game* Γ is a strategy profile, $(\hat{\sigma}, \hat{G}_s, \hat{G}_\ell)$, and a system of customers' beliefs, η ,

such that $(\hat{\sigma}, \hat{G}_s, \hat{G}_\ell)$ is sequentially rational and $m(\omega, Q_2 | q_1, Q_1)$ is derived from the prior beliefs (μ, ν) and strategy profile $(\hat{\sigma}, \hat{G}_s, \hat{G}_\ell)$ through Bayes' rule whenever possible.

Definition 2: A *sequential equilibrium* of the credence service market game Γ is a strategy profile (σ^*, G_A^*, G_B^*) , and a system of customers' beliefs, η^* , such that:

(i) The strategy profile (σ^*, G_A^*, G_B^*) is sequentially rational given the belief system $\eta^* = (\mu, \nu, m^*)$.

(ii) There exist a sequence of strategy profiles $\{(\sigma^k, G_A^k, G_B^k)\}_{k=1}^\infty$, where G_A^k and G_B^k are completely mixed strategies with $\lim_{k \rightarrow \infty} (\sigma^k, G_A^k, G_B^k) = (\sigma^*, G_A^*, G_B^*)$ such that $\eta^* = \lim_{k \rightarrow \infty} (\mu, \nu, m^k(\omega, Q_2 | q_1, Q_1))$ and $m^k(\omega, Q_2 | q_1, Q_1)$ derived from the prior beliefs (μ, ν) and strategy profile (σ^k, G_A^k, G_B^k) using Bayes' rule.

3.2 Equilibrium: Existence

We turn next to the study of the existence of a perfect Bayesian equilibrium and sequential equilibrium of the credence-quality market game Γ . We begin by examining the behavior of the customers and the suppliers.

Assume that the arrival time of a new customer is a random variable, \tilde{t} , taking values in $[0, \infty)$. When a new customer arrives the supplier does not know whether he is the customer's first or second call. The supplier diagnoses the problem (that is, the supplier observes the state ω) and prescribes a service, q , the supplier also informs the customer the waiting time for service, which is equals to the length of the supplier's queue, Q .

3.2.1 The customers

The customers system of beliefs: The customers prior beliefs are depicted by the distributions $\mu \in \Delta(\Omega)$ and v on I^2 , which are assumed to be commonly known. Moreover, the ex ante symmetry of the suppliers requires that v be symmetric.

Consider the state (ω, Q^A, Q^B) and let $G_s(\omega, Q^A, Q^B)$ and $G_\ell(\omega, Q^A, Q^B)$ be the (mixed) strategies of the short and long queue suppliers, respectively. The customers are supposed to know the strategies of the two suppliers but not the stage game $\Gamma(\omega, Q^A, Q^B)$ or which is the short-queue supplier and which is the long-queue supplier. Let q_1 and Q_1 denote, respectively, the prescription obtained and queue observed by customer in her first visit. Similarly, denote by q_2 and Q_2 the prescription obtained and queue observed by customer in second visit. Following her visit to the first supplier the customer is assumed to apply Bayes rule to update her beliefs about the prescription and queue of the second supplier. If the pair (q_1, Q_1) is on the equilibrium path (that is, there exists some $\omega \in \Omega, (Q^A, Q^B) \in I^2$ such that q_1 is in the support of $G_s(\omega, Q_1, Q^B)$ or $G_\ell(\omega, Q^A, Q_1)$ for some stage game $\Gamma(\omega, Q^A, Q^B)$) then the customer applies Bayes rule as follows:

Having observed Q_1 the customer knows for every Q_2 whether it is shorter or longer than Q_1 . The customer updates her beliefs about the state of disrepair ω and the length of the queue of the second supplier by applying Bayes' rule as follows: For all $\omega_i \leq q_1$,

$$m(\omega_i, Q_2 \mid q_1, Q_1) = \begin{pmatrix} \frac{g_s(\omega_i, Q^A, Q^B)(q_1)\mu(\omega_i)v(Q_2, Q_1)}{\int_{Q_1}^{\infty} [\sum_{\omega_1 \leq \omega_i \leq q_1} g_s(\omega_i, Q_1, Q)(q_1)\mu(\omega_i)] dv(Q, Q_1)} & \text{if } Q_2 \geq Q_1 \\ \frac{g_\ell(\omega_i, Q^A, Q^B)(q_1)\mu(\omega_i)v(Q_2, Q_1)}{\int_{Q_1}^{\infty} [\sum_{\omega_1 \leq \omega_i \leq q_1} g_\ell(\omega_i, Q_1, Q)(q_1)\mu(\omega_i)] dv(Q, Q_1)} & \text{if } Q_2 < Q_1 \end{pmatrix}. \quad (1)$$

If on her first call the customer's receives a prescription that, given the queue, is off the equilibrium path, the customer may still update her beliefs about the length of the rival's prescription and queue length conditional on the length of the queue she observes, using Bayes rule. However, the prescription she receives is devoid of informative value and imposes no restrictions on the customer's beliefs.

The customers expected payoff and best response strategies: We explore next the optimal behavior of the customer in the subgame following her visit to the first supplier and the evolution of her beliefs. Having visited the first supplier and obtaining the prescription q_1 and service waiting time Q_1 , a customer of type (θ, β) can accept the prescription and stop the search or seek a second prescription. In the latter case the customer accepts the second supplier's prescription if $u^\beta(q_2, Q_2) \geq \theta u^\beta(q_1, Q_1)$. Otherwise the customer exercises the recall option and returns to the first supplier to obtain the payoff $\theta^2 u^\beta(q_1, Q_1)$.

Because in her the second visit the customer is going to accept or reject the second offer according to whether $u^\beta(q_2, Q_2)$ is greater or smaller than $\theta u^\beta(q_1, Q_1)$, given q_1 and Q_1 , the reservation utility of a customer of type (θ, β) , $u_r^{(\theta, \beta)}(q_1, Q_1)$, is equal to

$$\Sigma_{\omega_1 \leq \omega_k \leq q_1} \Sigma_{\omega_k \leq q_2 \leq \omega_n} \int_0^\infty \max\{\theta u^\beta(q_2, Q_2), \theta^2 u^\beta(q_1, Q_1)\} g(\omega_k, Q_1, Q_2)(q_2) m(\omega_k, Q_2 | q_1, Q_1) dQ_2. \quad (2)$$

Given her type, (θ, β) and the suppliers' strategies $G_s(\omega, Q^A, Q^B)$ and $G_\ell(\omega, Q^A, Q^B)$, the customer's expected payoff upon observing (q_1, Q_1) given the reservation utility

$u_r^{(\theta, \beta)}(q_1, Q_1)$ in (2), is:

$$\bar{U}(\sigma(\theta, \beta), G_s, G_\ell) = \sigma_1^{(\theta, \beta)} u^\beta(q_1, Q_1) + (1 - \sigma_1^{(\theta, \beta)}) u_r^{(\theta, \beta)}(q_1, Q_1). \quad (3)$$

Hence, the customer accepts the first supplier's offer (that is, set $\sigma_1^{(\theta, \beta)} = 1$) if $u^\beta(q_1, Q_1) \geq u_r^{(\theta, \beta)}(q_1, Q_1)$. Otherwise, the customer continues the search (that is, $\sigma_1^{(\theta, \beta)} = 0$). She will accept the second supplier's offer (that is, set $\sigma_2^{(\theta, \beta)} = 1$) if $u^\beta(q_2, Q_2) > \theta u^\beta(q_1, Q_1)$. Otherwise, she will exercise the recall option (that is, set $\sigma_2^{(\beta, \theta)} = 0$). With this in mind we make the following definition:

Definition 3: A reservation-utility search strategy $\sigma : T \rightarrow \Sigma_1 \times \Sigma_2$ is two, type-dependent, mappings, $\sigma_1^{(\theta, \beta)} : \Omega \times I^2 \rightarrow \{0, 1\}$, $\sigma_2^{(\theta, \beta)} : \Omega^2 \times I^2 \rightarrow \{0, 1\}$, and a function $u_r^{(\theta, \beta)} : \Omega \times I^2 \rightarrow [0, 1]$ such that:

(a) $\sigma_1^{(\theta, \beta)}(q, Q) = 1$ if $u^\beta(q, Q) \geq u_r^{(\theta, \beta)}(q, Q)$ and $\sigma_1^{(\theta, \beta)}(q, Q) = 0$, otherwise.

(b) $\sigma_2^{(\theta, \beta)}(q_2, Q_2; q_1, Q_1) = 1$ if $\sigma_1^{(\theta, \beta)}(q_1, Q_1) = 0$ and $u^\beta(q_2, Q_2) > \theta u^\beta(q_1, Q_1)$ and $\sigma_2^{(\theta, \beta)}(q_2, Q_2; q_1, Q_1) = 0$, otherwise.

We summarize the above discussion in the following:

Proposition 1. *The reservation-utility strategy $\hat{\sigma}$ is the customers' unique best response strategy to the suppliers' strategy profile (G_s, G_ℓ) for all $(\omega, Q^A, Q^B) \in \Omega \times I^2$.*

In view of Proposition 1, the reservation-utility strategy based on the reservation-utility function $u_r^{(\theta, \beta)}(q_1, Q_1)$ is the best response strategy of the customers in the subgame following the visit to the first supplier regardless of whether (q_1, Q_1) is on or off the equilibrium path.

Lemma 1: *For each type $(\theta, \beta) \in T$, and all $(q_1, Q_1) \in \Omega \times \mathbb{R}$ the customer's*

expected payoff of the reservation-utility strategy, $\bar{U}(\sigma(\theta, \beta), G_s, G_\ell)$, is continuous.

The continuity of $\bar{U}(\sigma(\theta, \beta), G_s, G_\ell)$ is an immediate implication of its linearity in the strategies.

3.2.2 The suppliers

Because the customer's type is private information, the suppliers must choose their strategies as best responses against the acceptance probabilities induced by the distribution of types. We examine next the acceptance probabilities induced by reservation utility strategies.

For $j \in \{s, \ell\}$, the supplier j 's utility of subscribing q is $V_j(Q_j + q, Q_{-j})$ in the following cases: (1) the supplier j is the customer's first call and the customer accepts the prescription q immediately, (2) the supplier j is the customer's first call and the customer rejects supplier j 's prescription in the first visit seeking a second prescription and returns to supplier j , (3) the supplier j is the customer's second call and she accepts supplier j 's prescription. We calculate next the probabilities of these events

The first-call suppliers face a distribution of acceptance rules induced by the distribution, ξ , on the set of types. Thus, for all $(q, Q) \in \Omega \times I$, the subset of first callers who decline to seek a second prescription when faced with the prescription q and queue Q is given by the subset of types $A_1(q, Q) := \{(\theta, \beta) \in T \mid u^\beta(q, Q) \geq u_r^{(\theta, \beta)}(q, Q)\} \in \mathcal{B}(T)$. Consequently, the average acceptance rate of first callers who

decline to seek a second prescription is:

$$\sigma_1(q, Q) = \int_T \sigma_1^{(\theta, \beta)}(q, Q) d\xi(\theta, \beta) = \int_{A_1(q, Q)} \sigma_1^{(\theta, \beta)}(q, Q) d\xi(\theta, \beta).$$

This may be interpreted as the *probabilistic demand* function of first callers.

Let q_1 and Q_1 denote the prescription and queue length of the first supplier that the customer happened to visit. Given q_1 and Q_1 , the acceptance rate of a second prescription, q_2 , when the length of the queue of the second supplier is Q_2 , is:

$$\sigma_2(q_2, Q_2 | q_1, Q_1) = \int_T \sigma_2^{(\theta, \beta)}(q_2, Q_2; q_1, Q_1) d\xi(\theta, \beta).$$

The second-call supplier does not know that he is the second-call supplier and does not observe the first-call supplier's prescribed service. However, each supplier knows whether he is the short or the long queue supplier. Since q_1 is determined by the supplier's mixed strategy, observing ω_i and Q_1 , the second supplier can infer that, *if* he is the customer's second-call then the prescription the customer obtained in her first call is a random variable \tilde{q}_1 whose conditional probability distribution is determined by the strategy of the first supplier, namely, $G_s(\omega_i, Q^A, Q_\ell^B)$ if the second supplier is the long-queue supplier or $G_\ell(\omega_i, Q^A, Q^B)$ if he happens to be the short-queue supplier. In particular, given (ω_i, Q^A, Q^B) , the probability that the second supplier's prescribed service is accepted by a second caller depends on whether he is the long-queue or the short-queue supplier. If second supplier is the long-queue supplier then the probability that his prescribed service, q_ℓ , is accepted by a second

caller is

$$\sigma_2(q_\ell, \omega_i, Q^A, Q^B, G_s) = \sum_{q \in \{\omega_i, \dots, \omega_n\}} \sigma_2(q_\ell, Q_\ell | q, Q_s) g_s(\omega_i, Q^A, Q^B)(q)$$

and if second supplier is the short-queue supplier then the probability that his prescribed service is accepted by a second caller is

$$\sigma_2(q_s, \omega_i, Q^A, Q^B, G_\ell) = \sum_{q \in \{\omega_i, \dots, \omega_n\}} \sigma_2(q_s, Q_s | q, Q_\ell) g_\ell(\omega_i, Q^A, Q^B)(q).$$

Whereas the suppliers do not know whether they are a newly arrived customer's first or second call, each supplier knows that the probability of a customer visiting him first is 0.5. Moreover, each supplier also knows that the probability that a customer visits him second depends on the customer's type and the rival's prescription. Because the customer's type is private information, the probability that the short-queue supplier is the customer's second call is:

$$\sigma_2(\omega_i, Q^A, Q^B, G_\ell) := 0.5 \sum_{q \in \{\omega_i, \dots, \omega_n\}} (1 - \sigma_1(q, Q_\ell)) g_\ell(\omega_i, Q^A, Q^B)(q), \quad (4)$$

where $G_\ell(\omega_i, Q^A, Q^B)$ depicts the long-queue supplier's mixed prescription strategy conditional on (ω_i, Q^A, Q^B) . Similarly, the probability that the long-queue supplier is the customer's second call is:

$$\sigma_2(\omega_i, Q^A, Q^B, G_s) := 0.5 \sum_{q \in \{\omega_i, \dots, \omega_n\}} (1 - \sigma_1(q, Q_s)) g_s(\omega_i, Q^A, Q^B)(q), \quad (5)$$

where $G_\ell(\omega_i, Q^A, Q^B)$ depicts the long-queue supplier's mixed prescription strategy

conditional on (ω_i, Q^A, Q^B) .

Hence, the probability that a newly arrived customer accepts the prescription of the short-queue supplier is:

$$\alpha_s(q | \omega_i, Q^A, Q^B, \sigma, G_\ell) := \quad (6)$$

$$\frac{1}{2}[\sigma_1(q, Q_s) - (1 - \sigma_1(q, Q_s))(1 - \sigma_2(\omega_i, Q^A, Q^B, G_\ell)) + \sigma_2(\omega_i, Q^A, Q^B, G_\ell) \sigma_2(q_s, \omega_i, Q^A, Q^B, G_\ell)].$$

Similarly, the probability that a newly arrived customer accepts prescription of the long-queue suppliers:

$$\alpha_\ell(q | \omega_i, Q^A, Q^B, \sigma, G_s) := \quad (7)$$

$$\frac{1}{2}[\sigma_1(q, Q_\ell) - (1 - \sigma_1(q, Q_\ell))(1 - \sigma_2(q, \omega_i, Q_s, Q_\ell, G_s)) + \sigma_2(\omega_i, Q_s, Q_\ell, G_s) \sigma_2(q_\ell, \omega_i, Q_s, Q_\ell, G_s)].$$

Facing the probabilistic demand function $\alpha_s(q | Q^A, Q^B, G_\ell)$, the short-queue supplier's, expected payoff is:

$$W_s(q | \omega_i, Q^A, Q^B, \sigma, G_\ell) := \alpha_s(q | \omega_i, Q^A, Q^B, G_\ell) V_s(\omega_i, Q_s + q, Q_\ell) \quad (8)$$

$$+ (1 - \alpha_s(q | \omega_i, Q^A, Q^B, G_\ell)) \sum_{q_\ell \in \{\omega_i, \dots, \omega_n\}} V_s(\omega_i, Q_s, Q_\ell + q_\ell) g_\ell(\omega_i, Q^A, Q^B)(q_\ell).$$

Similarly, facing the probabilistic demand function $\alpha_\ell(q_\ell | Q^A, Q^B, G_s)$, the long-queue supplier's, expected payoff is:

$$W_\ell(q \mid \omega_i, Q^A, Q^B, G_s) := \alpha_\ell(q \mid \omega_i, Q^A, Q^B, G_s) V_\ell(\omega_i, Q_s, Q_\ell + q) \quad (9)$$

$$+ (1 - \alpha_\ell(q \mid \omega_i, Q^A, Q^B, G_s)) \sum_{q_s \in \{\omega_i, \dots, \omega_n\}} V_\ell(\omega_i, Q_s + q_s, Q_\ell) g_s(\omega_i, Q^A, Q^B)(q_s).$$

For each $\omega_i \in \Omega$ and $(Q^A, Q^B) \in I^2$ define $\bar{W}_j(\cdot, \cdot, \cdot \mid \omega_i, Q^A, Q^B) : \Sigma \times \Delta(\Omega)^2 \rightarrow \mathbb{R}$, $j \in \{s, \ell\}$, as follows:

$$\bar{W}_s(\sigma, G_s, G_\ell \mid \omega_i, Q^A, Q^B) = \sum_{q_s \in \{\omega_i, \dots, \omega_n\}} W_s(q \mid \omega_i, Q^A, Q^B, G_\ell) g_s(\omega_i, Q^A, Q^B)(q_s). \quad (10)$$

and

$$\bar{W}_\ell(\sigma, G_s, G_\ell \mid \omega_i, Q^A, Q^B) = \sum_{q_\ell \in \{\omega_i, \dots, \omega_n\}} W_\ell(q \mid \omega_i, Q^A, Q^B, G_s) g_\ell(\omega_i, Q^A, Q^B)(q_\ell). \quad (11)$$

Lemma 2: For all $\omega, Q^A, Q^B \in \Omega \times I^2$, $\bar{W}_j(\cdot, \cdot, \cdot \mid \omega, Q^A, Q^B) : \Sigma \times \Delta(\Omega)^2 \rightarrow \mathbb{R}$, $j \in \{s, \ell\}$, is a continuous function.

With this in mind, our first result establishes the existence of a perfect Bayesian and sequential equilibria of the credence-quality market game Γ .

Theorem 1: *There exist perfect Bayesian equilibrium of the credence-good market game Γ .*

Theorem 2: *There exist sequential equilibrium of the credence-good market game Γ*

4 Fraudulent Behavior and Short-Queue Advantage

4.1 Fraud-free equilibrium

An equilibrium is said to be *fraud-free* if the equilibrium strategies, $\hat{G}_j(\omega_i, Q^A, Q^B) = \delta_{\omega_i}$, for $j \in \{s, \ell\}$ and for all $(\omega_i, Q^A, Q^B) \in \Omega \times I^2$. The next result is that fraudulent prescriptions of service is a persistent feature of competitive equilibrium in the credence good market. Formally,

Theorem 3: *There exists no fraud-free equilibrium in the market for credence quality services.*

While there exists no fraud-free equilibrium, the level of fraud committed depends on the stage game. In particular, since the supports of the equilibrium strategies is contained in $\{\omega_i, \dots, \omega_n\}$, the larger is ω , the less room there is for fraudulent service overprescription.

4.2 Short-queue advantage and the evolution of the queues

The sole element of asymmetry between the suppliers in our model is the length of their queues. Hence, if $Q^A \neq Q^B$ the supplier with a shorter queue enjoys a strategic advantage in the sense that, if the two suppliers prescribe the same service, the supplier with the shorter queue is more likely to attract a new customer. More generally, the short-queue advantage is measured by the difference in the expected change of the lengths of the queues induced by equilibrium strategies $(\hat{\sigma}, \hat{G}_s, \hat{G}_\ell)$. Formally,

given a stage game $\Gamma(\omega_i, Q^A, Q^B)$, the measure of the short-queue advantage is:

$$\begin{aligned} \Psi(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}_s, \hat{G}_\ell) := \\ \sum_{q_s \in \text{Supp} \hat{G}_s} \alpha_s(q_s \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}_\ell) q_s \hat{g}_s(\omega_i, Q^A, Q^B)(q_s) \\ - \sum_{q_\ell \in \text{Supp} \hat{G}_\ell} \alpha_\ell(q_\ell \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}_s) q_\ell \hat{g}_\ell(\omega_i, Q^A, Q^B)(q_\ell). \end{aligned}$$

In the stationary setting of our model, the capacity of serving the flow of customers matches the demand for service, competitive behavior exerts pressure that tends to eliminate above normal profits. Since profit increases with the amount of accepted service prescription, if the equilibrium strategies of both suppliers generate the same expected present value of profits, it must be the case that no supplier enjoys the short-queue advantage persistently. In other words, the evolution of the queues under the equilibrium strategies in successive stage games requires that the lengths of the queues be *stochastically equal*, in the sense that the identity of the short queue supplier changes over time in such a way that the joint distribution of the queues is symmetric around its mean. Furthermore, the mean lengths of the queues, and the idle time it implies, induce normal profits for the two suppliers. Hence, equilibrium behavior tends to “compress” the lengths of the queues so the probability of divergence tends to zero. We summarize this in the following:

Proposition 2. *Under the equilibrium strategies, successive stage games induce a joint distribution of the lengths of the queues that is symmetric and the two suppliers commit the same amount of fraud on average.*

The discussion above implies that an increase in the length of the queue of the

short-queue supplier reduces its short-queue advantage. Formally, $d\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}_s, \hat{G}_\ell\right) / dQ_s < 0$. However, because $d\alpha_s\left(q_s \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}_\ell\right) / dQ_s < 0$ and $d\alpha_\ell\left(q_\ell \mid \omega_i, Q^A, Q^B, \hat{\sigma}, \hat{G}_s\right) / dQ_s > 0$, the short-queue advantage does not yield clear cut conclusions concerning its effect on the suppliers' equilibrium strategies.

5 Discussion

Our analysis shows that not only it is impossible for competition to sustain fraud-free equilibrium in markets for credence-quality goods, in fact fraud is a persistent and prevalent phenomenon. Moreover, we argue that the supplier with a shorter queue enjoys a short-queue advantage in the sense that the expected increase in the length of his queue is larger.

The model in this paper envisages two suppliers engaged in Bertrand competition. The analysis highlights the role of the evolution of the customer's beliefs in the wake of her visit to the first supplier and the optimal stopping rule that characterized her best response strategy. The analysis also indicates the manner in which the suppliers formulate their best response strategies given the information they possess. These aspects of our model and analysis are not specific to the two suppliers case and would show up, in a more complex form, if the number of the suppliers increase. Hence, there seems to be no essential loss of generality in so far as the main insights are concerned and much is gained by the relative simplicity the consideration of two suppliers affords.

It is worth noting that if the queue is not an issue and each customer can be

served immediately, then the analysis changes considerably.¹² In this instance, the customers utilities depend only on the prescribed service, and their discount rates is no longer a factor (That is, the customers' types are their idiosyncratic search cost, θ). Suppose that $\theta \in (0, 1]$ then it is easy to verify that suppliers strategies $q^j(\omega_i) = \omega_n$, for all ω_i and $j \in \{s, \ell\}$, is an equilibrium. In other words, knowing that the prescription is the same, no customer is inclined to search and, consequently, the suppliers have no incentive to try and undercut each other's prescription. This, maximal fraud, situations may characterize the cab service provided to tourists in an unfamiliar city. The route taken is only restricted by a tourist's conception of the reasonable length of the route. However, if some customers are "searchers", that is, they incur not cost of search (that is, the measure of $\theta = 0$ is sufficiently large) then their presence induces the suppliers to undercut each other, so no pure strategy equilibrium exists.

One aspect of the credence market good discussed in Darby and Karni (1973) that is not touched upon in this work is the possibility of developing a reputation and its effect on the commission of fraud. Including reputation in our model would require admitting repeated interactions in which the customers display loyalty (that is, they visit "their" supplier first) and the suppliers recognize their loyal clients. Under these conditions, the suppliers may establish what Darby and Karni dubbed client relationship. The loss of future business of, and being bad-mouthed by, a dissatisfied customer would increase the cost to the suppliers of "losing" customers, which should serve as a deterrence and, consequently, mitigate the problem of fraud.

¹²This may be a good depiction of taxi cab service. See also, Stahl (1996) for a discussion of a related issue.

This extension of the present work requires further study.

6 Proofs

6.1 Proof of lemma 2

The customer's strategy affects $\bar{W}_s(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ through the probabilities α_j , $j \in \{s, \ell\}$. But $\bar{W}_j(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ is continuous in α_j and these probabilities are continuous in σ . Hence, $\bar{W}_j(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ is continuous in σ .

To show that $\bar{W}_s(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ is continuous in G_ℓ , it suffices to show that $W_s(q \mid \omega_i, Q_s, Q_\ell, \sigma, G_\ell)$ is continuous in G_ℓ . By equation (8), $W_s(q \mid \omega_i, Q_s, Q_\ell, \sigma, G_\ell)$ depends on G_ℓ through $\sum_{q_\ell \in \{\omega_i, \dots, \omega_n\}} V_s(\omega_i, Q_s, Q_\ell + q_\ell) g_\ell(\omega_i, Q_s, Q_\ell)(q_\ell)$. But this expression is linear in the probabilities $(g_\ell(\omega_i, Q_s, Q_\ell)(q_\ell))_{q_\ell \in \{\omega_i, \dots, \omega_n\}}$. Hence, it is continuous in G_ℓ . By the same argument $\bar{W}_\ell(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ is continuous in G_s .

That $\bar{W}_s(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ is continuous in G_s follows from the linearity of $\bar{W}_s(\sigma, G_s, G_\ell \mid \omega_i, Q_s, Q_\ell)$ in the probabilities $(g_s(\omega_i, Q_s, Q_\ell)(q_s))_{q_s \in \{\omega_i, \dots, \omega_n\}}$. \square

6.2 Proof of theorem 1

Define customers, the short-queue and the long-queue best response set functions, respectively, as follows:

$$B_c(G_s, G_\ell) = \{\sigma \in \Sigma \mid \bar{U}(\sigma(\theta, \beta), G_s, G_\ell) \geq \bar{U}(\bar{\sigma}(\theta, \beta), G_s, G_\ell), \forall \bar{\sigma} \in \Sigma, (\theta, \beta) \in T\},$$

$$B_s(\sigma, G_\ell) = \{G_s \in \Delta(\Omega) \mid \bar{W}_s(\sigma, G_s, G_\ell \mid Q^A, Q^B) \geq \bar{W}_s(\sigma, \bar{G}_s, G_\ell \mid Q^A, Q^B), \forall \bar{G}_s \in \Delta(\Omega)\}$$

and

$$B_\ell(\sigma, G_s) = \{G_\ell \in \Delta(\Omega) \mid \bar{W}_\ell(\sigma, G_s, G_\ell \mid Q^A, Q^B) \geq \bar{W}_\ell(\sigma, G_s, \bar{G}_\ell \mid Q^A, Q^B), \forall \bar{G}_\ell \in \Delta(\Omega)\}.$$

Define the correspondence $\varphi : \Sigma \times \Delta(\Omega)^2 \rightrightarrows \Sigma \times \Delta(\Omega)^2$ by: $\varphi(\sigma, G_s, G_\ell) = B_c(G_s, G_\ell) \times B_s(\sigma, G_\ell) \times B_\ell(\sigma, G_s)$.

Claim 1: The set of strategy profiles $\Sigma \times \Delta(\Omega)^2$ is a nonempty convex and compact subset of a locally convex Housdorff space.

Proof. For all $\sigma, \hat{\sigma} \in \Sigma$ and $(\theta, \beta) \in T$, let $\sigma(\theta, \beta) = u_r^{(\theta, \beta)}$ and $\hat{\sigma}(\theta, \beta) = \hat{u}_r^{(\theta, \beta)}$. For each $\lambda \in [0, 1]$ define $\lambda\sigma + (1 - \lambda)\hat{\sigma}$ to be the reservation-utility strategy $(\lambda\sigma + (1 - \lambda)\hat{\sigma})(\theta, \beta) = \lambda u_r^{(\theta, \beta)} + (1 - \lambda)\hat{u}_r^{(\theta, \beta)}$, $(\theta, \beta) \in T$. Clearly, $\lambda\sigma + (1 - \lambda)\hat{\sigma} \in \Sigma$. Hence, Σ is a convex set.

To show that Σ is compact, we note that $\sigma(\theta, \beta) = u_r^{(\theta, \beta)}$, can be any value in the range $\{w(q) e^{-\beta Q} \mid q, Q \in \Omega \times I\}$, of the customer utility. Without loss of generality, let $w(\omega_1) = 1$ and $w(\omega_n) = 0$ then $\{w(q) e^{-\beta Q} \mid (q, Q) \in \Omega \times I\} = [0, 1]$. Thus, $\Sigma = [0, 1]^T$ and, by Tychonoff's theorem it is compact in the product topology.

For all $G_j(\omega, Q^A, Q^B), \bar{G}_j(\omega, Q^A, Q^B) \in \Delta(\Omega)$, $j \in \{s, \ell\}$, and $\lambda \in [0, 1]$, define $\lambda G_j(\omega, Q^A, Q^B) + (1 - \lambda)\bar{G}_j(\omega, Q^A, Q^B)$ by

$$(\lambda G_j + (1 - \lambda)\bar{G}_j)(\omega, Q^A, Q^B)(x) = \lambda G_j(\omega, Q^A, Q^B)(x) + (1 - \lambda)\bar{G}_j(\omega, Q^A, Q^B)(x), \forall x \in \Omega.$$

Thus, $\lambda G_j(\omega, Q^A, Q^B)(x) + (1 - \lambda)\bar{G}_j(\omega, Q^A, Q^B)(x) \in \Delta(\Omega)$. Hence, $j \in \{s, \ell\}$

is a convex set.

Since $\Delta(\Omega)$ is compact, $\Sigma \times \Delta(\Omega)^2$ is compact in the product topology. Consequently, $\Sigma \times \Delta(\Omega)^2$ is locally convex Hausdorff space.¹³ \triangle

Given the credence good market game Γ , let the system of beliefs, η , be depicted by $(\mu, \nu, m(q_2, Q_2 | q_1, Q_1))$, where $m(q_2, Q_2 | q_1, Q_1)$, given by (??), is derived from the prior beliefs (μ, ν) and strategy profile (G_s, G_ℓ) through Bayes' rule whenever possible. Let $U(\sigma^*(\beta, \theta), G_s, G_\ell)$ be given by (3). Then, by Proposition 1, for all $(\beta, \theta) \in T$, $\sigma^*(\beta, \theta)$ is the unique best response to the suppliers strategies G_s and G_ℓ .

Claim 2: φ is upper hemicontinuous with nonempty compact and convex values.

Proof. Define the correspondences $\varphi_c : \Delta(\Omega)^2 \rightarrow \Sigma$ and $\varphi_j : \Sigma \times \Delta(\Omega) \rightarrow \Delta(\Omega)$, $j \in \{s, \ell\}$, by $\varphi_c(G_s, G_\ell) = B_c(G_s, G_\ell)$ and $\varphi_j(\sigma, G_{-j}) = B_j(\sigma, G_{-j})$, $j \in \{s, \ell\}$. By Lemmata 1 and 2, these are continuous correspondences. Moreover, let Gr denote the graph of a correspondence then, by Lemmata 1 and 2, $U : Gr\varphi_c \rightarrow \mathbb{R}$ and $W_j : Gr\varphi_j \rightarrow \mathbb{R}$, $j \in \{s, \ell\}$, are continuous functions.

For every given $(G_s, G_\ell) \in \Delta(\Omega)^2$ and $(\beta, \theta) \in \Sigma$ define

$$U_c^*(G_s, G_\ell) = \bar{U}(\sigma^*(\theta, \beta), G_s, G_\ell).$$

For every given $(\sigma, G_\ell) \in \Sigma \times \Delta(\Omega)$ define the short-queue supplier's value function

$$W_s^*(\sigma, G_\ell | \omega, Q^A, Q^B) = \max_{G_s \in \Delta(\Omega)} \bar{W}_s(\sigma, G_s, G_\ell | \omega, Q^A, Q^B)$$

¹³See Aliprantis and Border (2006) Lemma 5.74.

and, for every given $(\sigma, G_s) \in \Sigma \times \Delta(\Omega)$ define the long-queue supplier's value function

$$W_\ell^*(\sigma, G_s \mid \omega, Q^A, Q^B) = \max_{G_\ell \in \Delta(\Omega)} \bar{W}_\ell(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B).$$

Then,

$$B_c(G_s, G_\ell) = \{\sigma \in \Sigma \mid \bar{U}(\sigma(\theta, \beta), G_s, G_\ell) = U_c^*(G_s, G_\ell), \forall (\theta, \beta) \in T\},$$

$$B_s(\sigma, G_\ell) = \{G \in \Delta(\Omega) \mid \bar{W}_s(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B) = W_s^*(\sigma, G_\ell \mid \omega, Q^A, Q^B)\}$$

and

$$B_\ell(\sigma, G_s) = \{G \in \Delta(\Omega) \mid \bar{W}_\ell(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B) = W_\ell^*(\sigma, G_s \mid \omega, Q^A, Q^B)\}.$$

But $\Sigma \times \Delta(\Omega)^2$ is a Housdorff space. Hence, by Berge maximum theorem, φ_c and φ_j , $j \in \{s, \ell\}$, have nonempty compact values and, since $\Delta(\Omega)^2$ and $\Sigma \times \Delta(\Omega)$ are Housdorff, φ_c and φ_j , $j \in \{s, \ell\}$ are upper hemicontinuous.¹⁴ But $\varphi(\sigma, G_s, G_\ell) = (\varphi_c(G_s, G_\ell), \varphi_s(\sigma, G_\ell), \varphi_\ell(\sigma, G_s))$. Consequently, φ has nonempty compact values and is upper hemicontinuous.

To see that correspondence φ is convex valued let $\varphi(\tilde{\sigma}, \tilde{G}_s, \tilde{G}_\ell) = B_c(\tilde{G}_s, \tilde{G}_\ell) \times B_s(\tilde{\sigma}, \tilde{G}_\ell) \times B_\ell(\tilde{\sigma}, \tilde{G}_\ell)$, for some $\tilde{\sigma}, \tilde{G}_s, \tilde{G}_\ell \in \Sigma \times \Delta(\Omega)^2$. But, for all $(\sigma, G_s, G_\ell), (\bar{\sigma}, \bar{G}_s, \bar{G}_\ell) \in$

¹⁴See Aliprantis and Border (2006), Theorem 17.31.

$\varphi(\tilde{\sigma}, \tilde{G}_s, \tilde{G}_\ell)$ and $\gamma \in (0, 1)$,

$$\gamma(\sigma, G_s, G_\ell) + (1 - \gamma)(\bar{\sigma}, \bar{G}_s, \bar{G}_\ell) = (\gamma\sigma + (1 - \gamma)\bar{\sigma}, \gamma G_s + (1 - \gamma)\bar{G}_s, \gamma G_\ell + (1 - \gamma)\bar{G}_\ell).$$

But, $\sigma, \bar{\sigma} \in B_c(G_s, G_\ell)$ implies that

$$U(\sigma(\beta, \theta), G_s, G_\ell) = U(\bar{\sigma}(\beta, \theta), G_s, G_\ell) = U_{(\beta, \theta)}^*(G_s, G_\ell), \quad \forall (\beta, \theta) \in T,$$

$G_s, \bar{G}_s \in B_s(\sigma, G_\ell)$ implies that

$$\bar{W}_s(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B) = \bar{W}_s(\bar{G}_s, G_\ell \mid Q^A, Q^B) = W_s^*(\sigma, G_\ell \mid \omega, Q^A, Q^B),$$

and $G_\ell, \bar{G}_\ell \in B_\ell(\sigma, G_s)$

$$\bar{W}_\ell(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B) = \bar{W}_\ell(\sigma, G_s, \bar{G}_\ell \mid Q^A, Q^B) = W_\ell^*(\sigma, G_s \mid \omega, Q^A, Q^B).$$

By Proposition 1, $\sigma, \bar{\sigma} \in B_c(G_s, G_\ell)$ implies that $\sigma = \bar{\sigma}$. Hence,

$$U(\gamma\sigma(\beta, \theta) + (1 - \gamma)\bar{\sigma}(\beta, \theta), G_s, G_\ell) = U_{(\beta, \theta)}^*(G_s, G_\ell), \quad \forall (\beta, \theta) \in T.$$

By the linearity of $\bar{W}_s(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B)$ in G_s , and $\bar{W}_\ell(\sigma, G_s, G_\ell \mid \omega, Q^A, Q^B)$ in G_ℓ , we get:

$$\bar{W}_s(\sigma, \gamma G_s + (1 - \gamma)\bar{G}_s, G_\ell \mid Q^A, Q^B) = W_s^*(\sigma, G_\ell \mid \omega, Q^A, Q^B)$$

and

$$\bar{W}_\ell(\sigma, G_s, \gamma G_\ell + (1 - \gamma) \bar{G}_\ell \mid Q^A, Q^B) = W_\ell^*(\sigma, G_s \mid \omega, Q^A, Q^B).$$

Hence, φ is convex valued. \triangle

By the Kakutani-Fan-Glicksberg theorem, φ has nonempty, compact, set of fixed points.¹⁵ Each of these points is a perfect Bayesian equilibrium of the stage game $\Gamma(\omega, Q^A, Q^B)$. \blacksquare

6.3 Proof of theorem 2

By subgame perfection, the customer's decision in second visit must be $\sigma_2^{\theta, \beta}(q_2, Q_2, q_1, Q_1) = 1$ if $\theta u(q_1, Q_1) \geq u(q_2, Q_2)$ and $\sigma_2^{\theta, \beta}(q_2, Q_2, q_1, Q_1) = 0$ otherwise. Hence, we need to consider only the customer's decision in her first visit.

For each $\omega_i \in \Omega$ define $\Omega_i = \{\omega_i, \dots, \omega_n\}$. Define a completely mixed strategy set $\mathcal{G}_k = \{G \mid g(\omega, Q^A, Q^B) \in \Delta(\Omega), g(\omega, Q^A, Q^B)(q_i) \geq \frac{1}{k} \forall q_i \geq \Omega_i, g(\omega, Q^A, Q^B)(q_i) = 0 \forall q_i < \omega, \forall (\omega, Q^A, Q^B)\}$.

Claim 1: For any given $k \in N$, $k \geq |\Omega|$, the completely mixed strategy set, \mathcal{G}_k , is compact.

Proof. For each $(\omega, Q^A, Q^B) \in \Omega \times I^2$, let $\mathcal{G}_k(\omega, Q^A, Q^B) =: \{g(\omega, Q^A, Q^B) \in \Delta(\Omega) \mid g(\omega, Q^A, Q^B)(q_i) \geq \frac{1}{k} \forall q_i \in \Omega_i, g(\omega, Q^A, Q^B)(q_i) = 0 \forall q_i < \omega\}$. The constraints $g(\omega, Q^A, Q^B)(q_i) \geq \frac{1}{k} \forall q_i \in \Omega_i, g(\omega, Q^A, Q^B)(q_i) = 0 \forall q_i < \omega$ imply that the set $\mathcal{G}_k(\omega, Q^A, Q^B) \subset \Delta(\Omega)$ is closed and bounded subset of \mathbb{R}^n and, by the Heine-Borel theorem, is compact. By definition, $\mathcal{G}_k = \times_{(\omega, Q^A, Q^B) \in \Omega \times I^2} \mathcal{G}_k(\omega, Q^A, Q^B)$. Hence, by Tichonoff's theorem, \mathcal{G}_k is compact. \triangle

¹⁵See Aliprantis and Border and (2006) Corllary 17.55.

Claim 2: For all $(q, Q) \in \Omega \times \mathbb{R}$, the customer's objective function conditional on (q_1, Q_1) is continuous in (σ, G_A, G_B) for $G_A, G_B \in \mathcal{G}_k$.

Proof. Since, for $g_j(\omega, Q^A, Q^B) \in \mathcal{G}_k(\omega, Q^A, Q^B)$, $i \in \{s, \ell\}$, $m(\omega_i, Q_2 \mid q_1, Q_1)$ is continuous with respect to g_j in the \mathbb{R}^n topology, the claim is an immediate implication of Lemma 1. \triangle

Claim 3: For any $(\omega, Q^A, Q^B) \in \Omega \times I^2$, the supplier's objective function conditional on (ω, Q^A, Q^B) is continuous in (σ, G_A, G_B) , for $G_A, G_B \in \mathcal{G}_k$.

Proof. This is an immediate implication of Lemma 2.

Next consider the best response correspondences of the customer and the two suppliers as defined in the proof of Theorem 1, with $G_A, G_B \in \mathcal{G}_k$. Sequential rationality implies that the best response correspondence of the customer is the product of the best responses conditional on $(q_1, Q_1) \in \Omega \times I$ and the best response correspondences of the suppliers are the products of the corresponding best responses conditional on $(\omega, Q^A, Q^B) \in \Omega \times I^2$.

Claim 4: For each $k \in N$, the correspondence $\varphi : \Sigma \times \mathcal{G}_k^2 \rightarrow \Sigma \times \mathcal{G}_k^2$ given by $\varphi(\sigma, G_A, G_B) = (\varphi_c(G_A, G_B), \varphi_A(\sigma, G_B), \varphi_B(\sigma, G_A))$ with the corresponding posterior customer's belief $m(\cdot, \cdot \mid q_1, Q_1)$ derived from Bayes rule has fixed point.

Proof. By Berge's theorem, the customer's best response correspondence, $\varphi_c(G_A, G_B)$, and the suppliers best response correspondences $\varphi_A(\sigma, G_B)$ and $\varphi_B(\sigma, G_A)$, are compact-valued upper hemicontinuous. Hence, the existence of fixed point is implied by the Kakutani-Fan-Glicksberg theorem. \triangle

Claim 5: A sequential equilibrium exists.

Proof. Let $M := (\Omega \times I)^{\Omega \times I}$ then, because $\Omega \times I$ is compact, by Tichonoff's the-

orem, M is compact and, consequently, the set $\Sigma \times \mathcal{G}_k^2 \times M$ is compact in the product topology. Since $\{(\sigma^k, G_A^k, G_B^k)\}_{k=1}^\infty \subset \Sigma \times \mathcal{G}_k^2 \times M$ there is a convergent subsequence of equilibria, $\{(\sigma^{k_j}, G_A^{k_j}, G_B^{k_j})\}_{j=1}^\infty$ with limit $(\sigma^*, G_A^*, G_B^*) = \lim_{j \rightarrow \infty} (\sigma^{k_j}, G_A^{k_j}, G_B^{k_j})$.

The customer's best response is a pure strategy and her belief system, m is continuous in (G_A, G_B) in the product topology.¹⁶ Hence, the customer's best response strategies, σ_{k_j} , are continuous in m_{k_j} . Thus, letting $m^* = \lim_{j \rightarrow \infty} m^{k_j}$ the customer's best response $\sigma^* = \lim_{j \rightarrow \infty} \sigma^{k_j}$. By definition of upper hemi-continuity, G_A^* and G_B^* are best response to (σ^*, G_B^*) and (σ^*, G_A^*) , respectively.

Since σ_{k_j} and the completely mixed strategies $(G_A, G_B)_{k_j}$ are sequentially rational and the customer's beliefs m_{k_j} are consistent with $(\sigma^{k_j}, G_A^{k_j}, G_B^{k_j})$, the strategy profile (σ^*, G_A^*, G_B^*) is sequentially rational and the belief system $\eta^* = (\mu, \nu, m^*)$ is derived from (σ^*, G_A^*, G_B^*) by Bayes rule. Hence, (σ^*, G_A^*, G_B^*) and η^* constitute a sequential equilibrium. ■

6.4 Proof of theorem 3

Fraud-free equilibrium in the credence-good market requires that, $\hat{G}_s(\omega, Q^A, Q^B) = \hat{G}_\ell(\omega, Q^A, Q^B) = \delta_\omega$, for every stage game $\Gamma(\omega, Q^A, Q^B)$. To prove that there is no fraud-free equilibrium we need to show that, for some stage game $\Gamma(\omega, Q^A, Q^B)$, $\hat{G}_s(\omega, Q^A, Q^B) = \delta_\omega$ is not a best response to $\hat{G}_\ell(\omega, Q^A, Q^B) = \delta_\omega$.

Suppose that there is fraud-free equilibrium and let $\hat{G}_\ell(\omega, Q^A, Q^B) = \delta_\omega$, for all Q^A and Q^B . In fraud-free equilibrium the customers believe that both suppliers prescribe the necessary service truthfully. Hence, the only reason to obtain a second

¹⁶See (1).

prescription is the expectations that the second supplier has a sufficiently shorter queue that justifies bearing the cost of obtaining a second prescription. Thus, the probability of a new customer accepting the prescription ω from the long-queue supplier is as follows.

If the long queue supplier is the customer's first call then the probability of acceptance is:

$$p_1(Q_\ell) := \xi\{(\theta, \beta) \in T \mid e^{-\beta Q_\ell} > \theta E[e^{-\beta Q} \mid Q_\ell]\},$$

where $E[e^{-\beta Q} \mid Q_\ell] = \int_0^\infty e^{-\beta Q} v(Q \mid Q_\ell) dQ$ and $v(Q \mid Q_\ell)$ is the distribution of the second supplier's queue conditional on Q_ℓ .

If the customer's visits the short-queue supplier first, the probability that he accepts eventually the prescription ω of the long-queue supplier requires that his type (θ, β) satisfies $e^{-\beta Q_s} < \theta E[e^{-\beta Q} \mid Q_s]$ and $w(q_s) e^{-\beta Q_s} < \theta w(\omega) e^{-\beta Q_\ell}$. The probability of this event is

$$p_2(\omega, Q^A, Q^B, q_s) := \xi\{(\theta, \beta) \in T \mid e^{-\beta Q_s} < \theta E[e^{-\beta Q} \mid Q_s] \text{ and } w(q_s) e^{-\beta Q_s} < \theta w(\omega) e^{-\beta Q_\ell}\}.$$

Define $p_\ell(\omega, Q^A, Q^B, q_s) = p_1(Q_\ell) + p_2(\omega, Q^A, Q^B, q_s)$. Then the short-queue supplier's expected profit is given by:

$$\max_{q_s \geq \omega} [(1 - p_\ell(\omega, Q^A, Q^B, q_s)) V(Q_s + q_s, Q_\ell) + p_\ell(\omega, Q^A, Q^B, q_s) V(Q_s, Q_\ell + \omega)].$$

Let $Q_s = 0$ then, for all $\beta > 0$, $e^{-\beta Q_s} > \theta e^{-\beta Q_\ell}$ and, $e^{-\beta Q_s} > \theta E[e^{-\beta Q} \mid 0]$.

Hence, $p_2(\omega_i, Q^A, Q^B, \omega_{i+1}) = 0$ and, since $p_1(Q_\ell)$ is independent of q_s , $p_\ell(\omega_i, Q^A, Q^B, \omega_{i+1}) = p_\ell(\omega_i, Q^A, Q^B, \omega_i) = 0$.

If $\beta = 0$ then $e^{-\beta Q_\ell} \geq \theta E[e^{-\beta Q_s} | Q_\ell]$, for all $\theta \in \Theta$. Hence, $p_1(Q_\ell) = \xi\{(\Theta, 0)\}$ and $p_2(\omega, Q^A, Q^B, q_s) = 0$, for all $q_s \geq \omega_i$. Hence, $p_\ell(\omega, Q^A, Q^B, q_s) = p_1(Q_\ell)$ is independent of q_s .

But $V(Q_s + \omega_{i+1}, Q_\ell) - V(Q_s + \omega_i, Q_\ell) > 0$. Hence, $\hat{G}_s(\omega_i, Q^A, Q^B) = \delta_{\omega_i}$ is not best response to $\hat{G}_\ell(\omega_i, Q^A, Q^B) = \delta_{\omega_i}$. ■

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