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Journal of Mathematical Economics xxx (xxxx) xxx

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Competitive equilibrium fraud in markets for credence-goods

Yen-Lin Chiu[1], Edi Karni*[,1]

Johns Hopkins University, United States of America

ARTICLE INFO

Article history:
Received 22 July 2020
Received in revised form 8 March 2021
Accepted 6 April 2021
Available online xxxx
Manuscript handled by Editor Andres Carvajal

Keywords:
Competitive equilibrium fraud
Credence-quality goods markets
Search with learning

ABSTRACT

This is a study of the nature and prevalence of persistent fraud in a competitive market for credence-quality goods. We model the market as a stochastic game of incomplete information in which the players are customers and suppliers and analyze their equilibrium behavior. Customers characteristics, idiosyncratic search cost and discount rate, are private information. Customers do not possess the expertise necessary to assess the service they need either ex ante or ex post. We show that there exists no fraud-free equilibrium in the markets for credence-quality goods and that fraud is a prevalent and persistent equilibrium phenomenon.

1. Introduction

Customers seeking to purchase services that require specialized knowledge are susceptible to fraud by suppliers who prescribe unnecessary services. Examples include, medical tests and treatments, auto repairs, equipment maintenance, and taxi cab service. In these markets the service suppliers make diagnostic determinations of the service required and offer to provide it, and the customers must decide whether to purchase the prescribed service or to seek, at a cost, a second service prescription. Typically in these situations, the customer can judge, ex post, whether or not the service provided was *sufficient* to solve the problem, but is unable to assess whether the prescribed service was also *necessary*.

Darby and Karni (1973) were the first to identify the fundamental ingredients of the problem underlying the provision of what they dubbed *credence-quality goods*. First, information asymmetry between the customer who lacks the expertise necessary to assess the service needed and service provider who possess the required expertise and, second, the cost saving of the joint provision of diagnosis and services.[2] They proceeded to discuss and analyze the economic implications of transactions involving this type of asymmetric information. Specifically, Darby and Karni argued that in competitive market equilibrium for credence-quality goods there is persistent tendency of suppliers to over-prescribe services (that is, to prescribe services that are sufficient but are unnecessary to solve the problem at hand).

The nature and extent of fraudulent practices depend on the specific characteristics of the credence-good market. For example, the demand for auto repair at a given service station depends on the waiting time (that is, the length of the queue of customers waiting to be served) which is not an issue when it comes to taxi cab service. It also depends on the information the customer may acquire before choosing the service provider and the cost of seeking a second opinion. For instance, in medical diagnosis that requires an invasive procedure the cost of obtaining a second opinion is prohibitively high. It is obvious, therefore, that modeling of credence-goods markets, while incorporating the fundamental ingredients of the problem – information asymmetry and the bundling of diagnosis and service – must be based on the specifics of the market under consideration. In this paper we focus on markets for the provision of services, such as auto-repair services, in which the capacity limitations may result in waiting for service. We underscore this point to avoid the impression that this is a general model of credence-good markets. We believe, however, that the game-theoretic approach invoked here is not specific to the analysis of the model we study in this paper, rather it is a natural framework for the analysis of credence-good markets in general.

Since the publication of Darby and Karni (1973), numerous studies confirm the prevalence of fraudulent behavior in the markets for credence-quality goods.[3] For medical services, especially physicians' services, over treatment, a phenomenon known in

* Corresponding author.
E-mail address: karni@jhu.edu (E. Karni).

1 We are grateful to Asher Wolinsky, Eilon Solan, Ehud Lehrer, Andrés Carvajal, and three anonymous referees for their useful comments and suggestions.

2 This bundling of information and service is crucial. See Wolinsky (1993) for an analysis of the implication of separation of diagnosis and service.

3 Dulleck and Rudolf (2006) include a survey of the literature and provide numerous references.

https://doi.org/10.1016/j.jmateco.2021.102519
0304-4068/

Please cite this article as: Y.-L. Chiu and E. Karni, Competitive equilibrium fraud in markets for credence-goods. Journal of Mathematical Economics (2021) 102519, https://doi.org/10.1016/j.jmateco.2021.102519.

medical literature as supplier induced demand, is widely documented (see Mcguire, 2000; Currie et al., 2011; Dranove, 1988). Gianfranco et al. (1993) found that in Swiss canton of Ticino on average the population has one third more operations than medical doctors and their relatives, suggesting that greater information symmetry tends to reduce overprescription of surgical procedures. The same type of conclusion was reached by Balafoutas et al. (2013). They report the results of a natural field experiment on taxi rides in Athens, Greece, designed to measure different types of fraud and to examine the influence of passengers' presumed information on the extent of fraud. Their findings indicate that passengers with inferior information about optimal routes are taken on significantly longer detours. Iizuka (2007) finds physicians drugs prescriptions are influenced by markup. Schneider (2012) reports the results of a field experiment designed to assess the accuracy of service provision in the auto repair market. He finds evidence for over prescription of services as well as under prescription. Beck et al. (2014) report that in experimental setting, car mechanics are significantly more prone to supplying unnecessary services than student subjects.

The work of Darby and Karni, while calling attention to a neglected aspect of economic interactions that results in market failure, lacks the formal structure necessary to derive more subtle implications of the concept they introduced. In this work we take a step towards a more formal analysis of markets for credence-quality services with some specific characteristics. Specifically, taking a game-theoretic approach we analyze the equilibrium behavior in a market in which two suppliers operating service stations are engaged in Bertrand competition. The suppliers are assumed to be ex ante identical in every respect. The sole asymmetry between the suppliers, which arises endogenously, is the lengths of their queues (i.e., the waiting time for service). The critical aspect of the model is the information asymmetry regarding the service that is required to address the problem at hand. The suppliers are supposed to possess the expertise necessary to assess the required service while the customers do not.

Customers heterogeneity is the consequence of idiosyncratic costs of seeking a second prescription and of waiting for service. We assume that these costs are the customers' private information. The customers are assumed to discover the lengths of the suppliers queues (that is, the waiting time) only when they visit the supplier's service outlet.

We study the market in a stationary symmetric equilibrium in which normal profits discourage entry or exit.[4] In other words, the idle time at the service stations is short enough so that no supplier loses money but is sufficiently long so as to discourage new entries or installing additional service capacity. In addition to proving its existence, we show that there exists no fraud-free equilibrium in this market, that the level of fraud committed by the two suppliers depends on the lengths of their queues, and that the short-queue supplier is more likely to overprescribe service than the long-queue supplier. These conclusions highlight the message of this work, namely, that the *fraud committed in credence good markets depends on the market's specific characteristics,* suggesting that the study of these markets, while maintaining the unifying characteristics, information asymmetry and the bundling of diagnosis and service, should proceed on a case by case basis.

In the next section we describe the credence good market. The equilibrium analysis appears in Section 3. Some economic implications of our analysis are discussed in Section 4. Section 5 includes a discussion of related literature and some concluding remarks. To allow for uninterrupted reading we collected the proofs in Section 6.

4 We confine our analysis to symmetric equilibria. The analysis of possible non-symmetric equilibria is beyond the scope of this paper.

2. The credence good market

2.1. Overview

Consider a market for credence-quality service populated by infinite number of customers and two suppliers, A and B. The information asymmetry in this market is two sided. The customers' private information consists of their idiosyncratic search cost and discount rate. The suppliers possess expertise that the customers do not have, which allows them to observe the actual state of disrepair and assess the service required to fix the problem. Let $\widetilde{\omega}$ denote a discrete random variable representing the true state of disrepair expressed as the necessary and sufficient number of service hours required to address the problem. We normalize $\widetilde{\omega}$ to take values in $\Omega := \{\omega_1, \dots, \omega_n\}$, where $0 < \omega_1 < \cdots . < \omega_n < 1$.[5] Denote the distribution of $\widetilde{\omega}$ by $\mu \in \Delta(\Omega)$, where $\Delta(\Omega)$ denotes the simplex in $\mathbb{R}^n$, and assume that μ is exogenous and commonly known.

Like the states of disrepair, the prescribed service, denoted by q, is specified in discrete quantities and, to simplify the exposition, we suppose that the prescribed service levels correspond to the states.[6] Moreover, we assume that the prescribed service must fix the problem (e.g., malfunction) or the customer refuses payment. Formally, if the state is ω_i then $q \in \Omega_{\omega_i} := \{\omega_i, \dots, \omega_n\}$.[7] The two suppliers are *identical in every respect except the lengths of their queues*, which are expressed in terms of service hours committed to serving waiting customers. We assume that the suppliers observe each other's queue and that customers only discover the length of a supplier's queue when they show up at the supplier's service station.[8] Let $Q^A(t)$ and $Q^B(t)$ denote the lengths of the suppliers queues at time t. Formally, $(Q^A(t), Q^B(t)) \in I := [0, \infty]^2$, for all $t \in \mathbb{R}_+$. Let I be endowed with the Borel σ-algebra $\mathcal{B}(I)$. A customer's arrival at the market in a state of disrepair, $\omega \in \Omega$, sets up a stage game $\Gamma\left(\omega, Q^A(t), Q^B(t)\right)$ parametrized by the triplet $\left(\omega, Q^A(t), Q^B(t)\right) \in \Omega \times I$. Let Ω be endowed with the discrete topology and I with the metric topology. Let $(\Omega \times I, \mathcal{B}(\Omega \times I))$ be a measurable space, where $\mathcal{B}(\Omega \times I)$ denote the Borel σ-algebra on $\Omega \times I$.

We assume that the customers' arrival process is stationary and is depicted by a CDF, F, that is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$ and has full support in $\mathbb{R}_{++}$. Denote by $\mathcal{V}$ the set of probability measures on $(I, \mathcal{B}(I))$.[9]

A customer's type, (θ, β), consists of idiosyncratic search cost, θ, and discount rate, β, both taking values in [0, 1]. Thus, the set of customers' types is $T = [0, 1]^2$. Let $\mathcal{B}(T)$ be the Borel σ-algebra on T and denote by ξ a continuous probability measure on the measurable type space $(T, \mathcal{B}(T))$.

When a new customer shows up at a service station, the supplier observes the state of disrepair ω and, consequently, the state

5 As will become clear later, the assumption of discrete state space has implications for the customers perception of the difference between the suppliers strategies.

6 In view of the common practice of informing the customers what are the parts that need to be fixed or replaced before the actual work begins, this assumption is realistic.

7 This assumption is dubbed liability in the literature (see Dulleck and Rudolf, 2006; Fong and Liu, 2016; Fong et al., 2020).

8 The assumption that the suppliers observe each other's queue expresses the presumption that survival in competitive markets requires the players to keep tab of their rivals positions and actions. Relaxing this assumption would require a modification of the suppliers strategies described below, and will complicate the analysis without yielding new insights.

9 An alternative formulation of the setup is to let $(Q^A(t), Q^B(t)) \in \mathbb{R}^2_+$ and assume the customers' arrival rate is such that probability of new arrival during a fixed time interval is sufficiently small so that the measures in $\mathcal{V}$ are tight. Under this assumption, by Prokhorov's theorem, $\mathcal{V}$ is sequentially compact.

$\left(\omega, Q^A(t), Q^B(t)\right)$. The suppliers do not observe the customer's type. Customers know their types but not the state ω, and they discover the length of a supplier's queue upon visiting a service station and receiving a diagnosis. In other words, a customer may discover the lengths of the suppliers queues sequentially, during the search of service process. Insofar as the customers are concerned, what matters are the lengths of the queues and not the identity of the suppliers. This assumption rules out suppliers' identity or reputation as a possible factor.[10]

Assume that the installed capacity of the two suppliers is the same, that the hourly service price is the same for the two suppliers regardless of the lengths of their queues, and is known to the customers.[11] Assume further that the price is normalized so that the profit generated by servicing customers for a fraction, x, of an hour is x.

We model the credence service market as a stochastic game of incomplete information and analyze it using the concept of *stationary Markovian symmetric equilibria.* The players in this game are the two suppliers and a finite set of potential customers. We assume that the suppliers earn normal profits, so that there is no incentive for either new suppliers to enter the market or for a current supplier to exit the market or change the level of his service capacity. A customer's arrival at the market at time t in a state of disrepair ω when the suppliers queues are $Q^A(t)$ and $Q^B(t)$ initiates a dynamic *stage game*, $\Gamma\left(\omega, Q^A(t), Q^B(t)\right)$, depicting the interaction among the customer and the two suppliers. At a state $s(t,\omega) := \left(\omega, Q^A(t), Q^B(t)\right)$ the suppliers and the customer make their decisions, after which the game proceeds to the next state as follows. If the next customer arrives at time t' in a state ω' and accepts the prescription q_A of supplier A then the new state is

$$s_A\left(t', \omega'\right) := \left(\omega', \max\{Q^A(t) - \Delta t' + q_A, 0 + q_A\},\right.$$
$$\left.\max\{Q^B(t) - \Delta t', 0\}\right),$$

where $\Delta t' := t' - t$, and if she accepts the prescription q_B of supplier B then the new state is

$$s_B\left(t', \omega'\right) := \left(\omega', \max\{Q^A(t) - \Delta t', 0\}, \max\{Q^B(t) - \Delta t' + q_B, 0 + q_B\}\right).$$

The transition probability from the state $s(t,\omega)$ to the state $s_j\left(t', \omega'\right)$ is the product of the probability that the next customer arrives at time t', the probability that the state of disrepair is ω', the probability that supplier j prescribes q_j, and the probability that the newly arrived customer accepts the prescription q_j.[12] The rest of the elements of the stochastic game, the players strategies and payoffs are described next.

2.2. The customers

Upon identifying an equipment malfunction, the customer engages in sequential search for repair service. Diagnosis of the problem and determination of the service needed to solve it requires expert knowledge, which the customer does not have.

The customers' strategies: Since the posted service prices are the same, the customer chooses one of the two service outlets at random with equal probabilities.[13] Upon visiting a service outlet the customer obtains a service prescription and the length of the supplier's queue, both expressed in terms of service-hours. The customer must then choose between accepting the prescribed service and waiting in the queue, and rejecting it in favor of seeking a second prescription. If she chooses the latter, the customer visits the second supplier, receives a second prescription and observes the length of the second supplier's queue. The customer must then decide between accepting the second prescription and waiting to be served and returning to the first supplier. We assume that the search is with full and costless recall. Hence, if the customer decides to seek a second prescription and then return to the first supplier, she maintains her place in the queue and is entitled to obtain the service prescribed by the first supplier.[14] Formally, a customer's search strategy is a mapping $\sigma : T \to \Sigma_1 \times \Sigma_2$, where $\Sigma_1 := \{\sigma_1 : \Omega \times [0,\infty] \to \{0,1\}\}$, $\Sigma_2 := \{\sigma_2 : \Omega^2 \times I \to \{0,1\}\}$. In other words, the strategy assigns to a customer of type (θ,β) two acts depicted by the functions $\sigma_1^{(\theta,\beta)} : \Omega \times [0,\infty] \to \{0,1\}$ and $\sigma_2^{(\theta,\beta)} : \Omega^2 \times I \to \{0,1\}$, where $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 1$ means that the customer accepts the prescription of the first supplier she visits and terminates the search, and $\sigma_1^{(\theta,\beta)}(q_1, Q_1) = 0$ means that she seeks a second prescription. Similarly, $\sigma_2^{(\theta,\beta)}(q_1, q_2, Q_1, Q_2) = 1$ means that the customer accepts the second supplier's prescription and $\sigma_2^{(\theta,\beta)}(q_1, q_2, Q_1, Q_2) = 0$ means that she rejects the second supplier's prescription and returns to the first supplier. We denote by Σ the set of customers' strategies. Let Σ denote the set of customers strategies.

The customers' beliefs: Since the customers do not observe the suppliers queues, at the outset the customer's information set is $\Omega \times I$ and her prior beliefs are captured by $\mu \in \Delta(\Omega)$ and the $\upsilon \in \mathcal{V}$. Upon observing the length of the first supplier's queue, Q_1, and obtaining a prescription, $q_1 \in \Omega$, the customer updates her beliefs about the state ω and the waiting time at the second service station. In doing so, the customer applies Bayes' rule.[15] The updated belief regarding the state ω and the second supplier's queue conditional on the first supplier' prescription, q_1, and queue length, Q_1, is represented by the conditional distribution $m(\omega, Q_2 \mid q_1, Q_1)$ on $\Omega \times [0,\infty]$.[16]

The customers' payoffs: Accepting a prescribed service q on her first visit from a supplier whose queue length is Q, the utility of a customer of type (θ,β) is: $u^{\beta}(q,Q) = (1-q)e^{-\beta Q}$.[17] Continuing the search entails a customer-specific additive search cost, $\theta \in [0,1]$.[18] Thus, the utility of accepting the prescription q' when the queue of the second supplier is Q' is $u^{(\theta,\beta)}\left(q', Q'\right) = \left(1-q'\right)e^{-\beta Q'} - \theta$. Returning to the first supplier after visiting the second supplier, the customer's payoff is $(1-q)e^{-\beta Q} - \theta$.[19] If $1-q<0$ then the customer is better off not fixing the problem. Under our assumptions $\Omega \subset [0,1]$, implicitly, this presumes that $\omega > 1$ are states of disrepair that are not worth fixing and, consequently, are not included in Ω.

10 We revisit the issue of reputation in the discussion section.

11 One interpretation is that the price is regulated (e.g., metered cab fare).

12 A detailed exposition of these probabilities and the stochastic evolution of the queues appear in Section 3.1.3.

13 This assumption does not rule out customers loyalty to suppliers or that each customer visits first the supplier whose location is closer provided that the loyalty or proximity is equally divided between the suppliers.

14 The assumption of full recall is intended to simplify the exposition. It is natural to suppose that a customer who decides to return to the first supplier may find out that, with positive probability, her place in the queue is taken. This would call for fomulating the customer decision as search with uncertain recall à la Karni and Schwartz (1977a). This would complicate the exposition without adding insight or change the results. We comment on this alternative when we define the customers' equilibrium strategies below.

15 This is the sense in which the search involves learning.

16 We examine the updated beliefs in further detail below.

17 The particular functional form is chosen to simplify the exposition. The critical feature of the customers' payoff for our analysis that is captured by this functional form is: The utility is monotonic decreasing in the recommended repairs and in the length of the queue.

18 Additive search cost is a standard assumption in the literature on optimal stopping rules.

19 This is the sense in which the recall is costless.

2.3. The suppliers

At every point in time each supplier has a queue representing hours committed to serving customers who have already accepted the supplier's prescriptions. The lengths of the queues are determined by the history of customers arrival, their service prescriptions, and their acceptance decisions. In other words, the lengths of the queues are determined by the realization of an exogenous stochastic process (that is, the arrival rate and the random state ω) and the endogenous decisions of the suppliers and customers.

The suppliers' strategies: The suppliers' *mixed prescription strategies* are mappings $G : \Omega \times I \to \mathcal{G}$, where $\mathcal{G}$ denotes the set of CDF on Ω.[20] Formally, for each $q_h \in \Omega$ and $\left(Q^j, Q^{-j}\right) \in I$, $G\left(\omega, Q^j, Q^{-j}\right)(q_h) := \Sigma_{i=1}^{h} g\left(\omega, Q^j, Q^{-j}\right)(q_i)\,\delta_{q_i}$, where $\left(g\left(\omega, Q^j, Q^{-j}\right)(q_1), \ldots, g\left(\omega, Q^j, Q^{-j}\right)(q_n)\right) \in \Delta(\Omega)$, $j \in \{A, B\}$ and δ_{q_i} denotes the distribution function that assigns the unit probability mass to q_i. Because the asymmetry between the suppliers is due solely to the lengths of their queues, the suppliers prescriptions are distinct only as a result of the difference of their queues and, in the case of mixed strategies, the randomly selected prescription.

The suppliers' payoffs: Before the start of a stage game, $\Gamma\left(\omega, Q^A, Q^B\right)$, at time, say $t = 0$, supplier j anticipates that either his or his rival's prescription be accepted and, as a result, the state of the queues transitions from the current state (Q^A, Q^B) to the state $(\hat{Q}^A, \hat{Q}^B)$. Following that there is a random waiting time $t' > 0$, before the next customer arrives and initiates the stage game $\Gamma\left(\omega, \hat{Q}^A - t', \hat{Q}^B - t'\right)$. The transition probabilities from (Q^A, Q^B) to $(\hat{Q}^A, \hat{Q}^B)$, is determined by supplier's strategies $G\left(\omega, Q^j, Q^{-j}\right)$, $j \in \{A, B\}$, and customer's acceptance rule σ.

Let $V : \Omega \times I \to \mathbb{R}_+$ be a bounded measurable function representing the suppliers' anticipated expected discounted value before the start of a stage game. We show next that the value function V exists and is unique.

Just before the start of the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$, supplier j expects to receive a cash flow from servicing the customers in his queue while waiting the arrival of the next customer, yielding a discounted value

$$\int_0^{\min(\hat{Q}^j,:t')} e^{-r\tau} d\tau,$$

and the anticipated discounted expected value from the stage game that follows given by, $e^{-rt'}\Sigma_{\omega' \in \Omega} V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t')\mu\left(\omega'\right)$. Thus, the total anticipated payoff $\hat{V}(\omega, Q^j, Q^{-j})$ from a stage game $\Gamma(\omega, Q^A, Q^B)$ that is about to be played is:

$$\hat{V}(\omega, Q^j, Q^{-j}) = \int_0^\infty \int_I [\int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau + e^{-rt'} \Sigma_{\omega' \in \Omega} V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t') \times \mu\left(\omega'\right)] \upsilon(\hat{Q}^A, \hat{Q}^B) d\left(\hat{Q}^A, \hat{Q}^B\right) dF\left(t'\right),$$

where the conditional transition probability $\upsilon(\hat{Q}^A, \hat{Q}^B)$ is a shorthand for the expression $\upsilon(\hat{Q}^A, \hat{Q}^B | Q^A, Q^B, \omega, G, \sigma)$ that makes explicit the fact that the probability of the state $\left(\hat{Q}^A, \hat{Q}^B\right)$ of the queues conditional on the state $\left(Q^A, Q^B\right)$ depends on the state of disrepair and the suppliers and customers strategies. If the players strategies (G, σ) and the anticipated value function V are all bounded and measurable, then the payoff function $\hat{V}$ is bounded measurable.

If supplier j anticipates correctly the strategies of his rival and the customers and chooses his best response, then the payoff function $\hat{V}$ coincides with the hypothesized anticipated value function V. Formally, for $j \in \{A, B\}$,

$$V(\omega, Q^j, Q^{-j}) = \max_{G \in \mathcal{G}} \int_0^\infty \int_I [\int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau + e^{-rt'} E_\Omega V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t')] \times \upsilon(\hat{Q}^A, \hat{Q}^B) d\left(\hat{Q}^A, \hat{Q}^B\right) dF\left(t'\right), \tag{1}$$

where $E_\Omega V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t') := \Sigma_{\omega' \in \Omega} V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t')\mu\left(\omega'\right)$. The expression (1) is a contraction mapping, which implies the existence and uniqueness of V. Formally,

Proposition 1. *There exists a unique value function $V : \Omega \times I \to \mathbb{R}$.*

3. Equilibrium analysis

3.1. The players' behavior and the evolution of the queues

We analyze the credence service market as Markovian sequential equilibrium of a stochastic game of incomplete information. Given $\omega \in \Omega$, a strategy G_k is *completely mixed with modulus k* if $g_k(q) \geq k^{-1}$, $k \geq n$, for all $q \in \Omega_\omega$. To start with, we study the equilibria of the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$ in completely mixed strategies with modulus k, beginning with the behavior of the customers and the suppliers.

3.1.1. The customers

The customers system of beliefs: At the start nature assigns the customers their types, which is the customers' private information. When a customer detects a problem and seeks remedial service, she does not know which particular stage game, $\Gamma\left(\omega, Q^A, Q^B\right)$, she initiates. Her prior beliefs are depicted by the distributions $\mu \in \Delta(\Omega)$ and $\upsilon \in \mathcal{V}$. In view of the ex-ante symmetry of the suppliers, insofar as the customers are concerned, υ is symmetric.[21]

Consider the state $\left(\omega, Q^A, Q^B\right)$ and let $G_k\left(\omega, Q^j, Q^{-j}\right)$, $j \in \{A, B\}$, be the suppliers completely mixed strategies with modulus k. The customers are supposed to know the strategies of the suppliers as functions of the states but not the current state $\left(\omega, Q^A, Q^B\right)$. In particular, the customers do not know which is the short-queue supplier and which is the long-queue supplier. Let (q_1, Q_1) and (q_2, Q_2) denote the prescriptions obtained and queues observed by a customer in her first and second visits, respectively.

Following her visit to the first supplier and having observed Q_1, regardless of whether it is A or B, the customer updates her beliefs about the state of disrepair, ω, and the length of the queue of the second supplier by applying Bayes' rule as follows: For all $\omega_i \leq q_1$ and $(Q_1, Q_2) \in intI$,

$$m_k(\omega_i, Q_2 \mid q_1, Q_1) = \frac{g_{1,k}(\omega_i, Q_1, Q_2)(q_1)\,\mu(\omega_i)\,\upsilon(Q_2, Q_1)}{\int_0^\infty \left[\Sigma_{\omega_1 \leq \omega_i \leq q_1} g_{1,k}\left(\omega_i, Q_1, Q_2'\right)(q_1)\,\mu(\omega_i)\right] \upsilon(Q_2', Q_1) dQ_2'}, \tag{2}$$

where $g_{1,k}(\omega_i, Q_1, Q_2)$ denotes the mixed strategy of the first supplier.[22]

The customers expected payoff and best response strategies: Given the suppliers' completely mixed strategies, G_k, with modulus k, we explore next the optimal behavior of the customer

20 We are restricting consideration to history-independent, or Markovian, symmetric strategies.

21 Because υ is part of the equilibrium, it will be shown later that this assumption is validated.

22 For more details see Section 3.1.3.

in the subgame following her visit to the first supplier and the evolution of her beliefs. Having obtained the prescription q_1 and observing the length of the queue, Q_1, a customer of type (θ, β) can accept the prescription and stop the search or seek a second prescription. In the latter case the customer accepts the second supplier's prescription if $(1 - q_2)\, e^{-\beta Q_2} \geq (1 - q_1)\, e^{-\beta Q_1}$. Otherwise the customer exercises the recall option and returns to the first supplier to obtain the payoff $(1 - q_1)\, e^{-\beta Q_1} - \theta$.

Because the customer is going to accept or reject the second offer according to whether $u^{\beta}(q_2, Q_2)$ is greater or smaller than $u^{\beta}(q_1, Q_1)$, given q_1 and Q_1 the *reservation utility* of a customer of type (θ, β), $u_{r,k}^{(\theta,\beta)}(q_1, Q_1)$, is given by

$$u_{r,k}^{(\theta,\beta)}(q_1, Q_1) + \theta = \tag{3}$$
$$\Sigma_{\omega_h \leq q_1} \Sigma_{q_2 \in \Omega_{\omega_h}} \Big[\int_0^{\infty} \max\{u^{\beta}(q_2, Q_2), u^{\beta}(q_1, Q_1)\} \Big\}$$
$$\times\, g_k(\omega_h, Q_2, Q_1)(q_2)\, m^k(\omega_h, Q_2 \mid q_1, Q_1)\, dQ_2.$$

Given her type, (θ, β), and the suppliers' strategy, G_k, the customer's expected payoff upon observing (q_1, Q_1) given the reservation utility strategy $u_{r,k}^{(\theta,\beta)}(\cdot,\cdot)$ in (3), is:

$$\bar{U}(\sigma_k(\theta,\beta), G_k) = \sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1)\, u^{\beta}(q_1, Q_1) + \left(1 - \sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1)\right) u_{r,k}^{(\theta,\beta)}(q_1, Q_1). \tag{4}$$

Hence, the customer accepts the first supplier's offer (that is, set $\sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1) = 1$) if $u^{\beta}(q_1, Q_1) \geq u_{r,k}^{(\theta,\beta)}(q_1, Q_1)$. Otherwise, the customer continues the search (that is, set $\sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1) = 0$). She accepts the second supplier's offer (that is, set $\sigma_{2,k}^{(\theta,\beta)}(q_1, q_2, Q_1, Q_2) = 1$) if $u^{\beta}(q_2, Q_2) > u^{\beta}(q_1, Q_1)$. Otherwise, she exercises the recall option (that is, set $\sigma_{2,k}^{(\theta,\beta)}(q_1, q_2, Q_1, Q_2) = 0$).[23] With this in mind we make the following definition:

Definition 1. A *reservation-utility search strategy* $\sigma_k : T \to \Sigma_1 \times \Sigma_2$ consists of two mappings $\sigma_{1,k}^{(\theta,\beta)} : \Omega \times [0,\infty] \to \{0,1\}$ and $\sigma_{2,k}^{(\theta,\beta)} : \Omega^2 \times I \to \{0,1\}$, and a function $u_{r,k}^{(\theta,\beta)} : \Omega \times [0,\infty] \to [0,1]$ such that:

(a) $\sigma_{1,k}^{(\theta,\beta)}(q, Q) = 1$ if $u^{\beta}(q, Q) \geq u_{r,k}^{(\theta,\beta)}(q, Q)$ and $\sigma_{1,k}^{(\theta,\beta)}(q, Q) = 0$, otherwise.

(b) $\sigma_{2,k}^{(\theta,\beta)}(q_2, q_1, Q_2, Q_1) = 1$ if $\sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1) = 0$ and $u^{\beta}(q_2, Q_2) > u^{\beta}(q_1, Q_1)$ and $\sigma_{2,k}^{(\theta,\beta)}(q_2, q_1, Q_2, Q_1) = 0$, otherwise.

We summarize the above discussion in the following:

Proposition 2. *A reservation-utility strategy is the customers' unique best response to the suppliers' strategy profile* $\left(G_k(\omega, Q^j, Q^{-j})\right)_{j\in\{A,B\}}$, *for all* $\left(\omega, Q^j, Q^{-j}\right) \in \Omega \times I$.

The customer's expected payoff under the reservation-utility strategy is continuous in the suppliers strategies. Formally,

Lemma 1. *For each type* $(\theta, \beta) \in T$ *and all* $(q_1, Q_1) \in \Omega \times [0,\infty]$ *the customer's expected payoff,* $\bar{U}(\sigma_k(\theta,\beta), G_k)$, *of the reservation-utility strategy is continuous.*

The continuity of $\bar{U}$ is an immediate implication of its linearity in the strategies and the fact that $g_k\left(\omega, Q^j, Q^{-j}\right)(q) > 0$, $j \in \{A, B\}$, for all $q \in \Omega_{\omega}$.

[23] If the full recall formulation is replaced by search with uncertain recall, the waiting time becomes a random variable, $\widetilde{Q}_1$, taking values in $[Q_1, \infty]$, whose distribution is determined by the arrival rates. Because the customer is going to accept or reject the second offer according to whether $u^{\beta}(q_2, Q_2)$ is greater or smaller than $\bar{u}^{\beta}(q_1, Q_1, Q_2) := E\left[u^{\beta}\left(q_1, \widetilde{Q}_1\right) \mid Q_1, Q_2\right]$, given q_1 and Q_1 the reservation utility of a customer of type (θ, β) is still determined by (3) with $\bar{u}^{\beta}(q_1, Q_1, Q_2)$ replacing $u^{\beta}(q_1, Q_1)$.

3.1.2. The suppliers

Because the customers types are private information, the suppliers choose their strategies as best responses against the acceptance probabilities induced by the distribution of customers' types. We examine next the acceptance probabilities induced by the customers' reservation utility strategies. Supplier j's prescription is accepted in the following cases: (1) j is the customer's first call and the customer accepts the prescription q immediately, (2) j is the customer's first call, the customer seeks a second prescription and returns to j for the service, (3) j is the customer's second call and she accepts his prescription. We calculate the probabilities of these events.

The first-call suppliers face a distribution of acceptance rules induced by the distribution, ξ, on the set of types. Thus, for all $(q_1, Q_1) \in \Omega \times [0,\infty]$, the subset of the first callers who do not seek a second prescription when faced with the prescription q_1 and queue Q_1 is given by the subset of types $A_{1,k}(q_1, Q_1) := \{(\theta,\beta) \in T \mid u^{\beta}(q_1, Q_1) \geq u_{r,k}^{(\theta,\beta)}(q_1, Q_1)\} \in \mathcal{B}(T)$. Consequently, the average acceptance rate of first callers who, given the queue length Q_1, accepts the prescription q_1 immediately is:

$$\sigma_{1,k}(q_1, Q_1) = \int_T \sigma_{1,k}^{(\theta,\beta)}(q_1, Q_1)\, d\xi(\theta,\beta) = \xi(A_{1,k}(q_1, Q_1)).$$

This may be interpreted as the *probabilistic demand* function of first callers.

Given the first supplier's prescription, q_1, and the length, Q_1, of his queue, the acceptance rate of a second prescription, q_2, when the length of the queue of the second supplier is Q_2, is:

$$\sigma_{2,k}(q_2, Q_2 \mid q_1, Q_1) = \int_T \sigma_{2,k}^{(\theta,\beta)}(q_2, Q_2; q_1, Q_1)\, d\xi(\theta,\beta).$$

The second-call supplier does not know that he is the second-call supplier. However, observing ω_i and Q_1, the second supplier can infer that *if* he is the customer's second-call then the prescription the customer obtained in her first call is a random variable $\widetilde{q}_1$ whose probability distribution is determined by the strategy of the first supplier. Specifically, if the customer first visits supplier $j \in \{A, B\}$ then q_1 was determined by the strategy $G_k\left(\omega_i, Q^j, Q^{-j}\right)$. Moreover, given Q_1 and q_1, only customers whose type (θ, β) and having obtained the prescription q and observed the queue, Q, such that $\sigma_{1,k}^{(\theta,\beta)}(q, Q) = 0$ (that is, customers type for whom $u^{\beta}(q, Q) < u_{r,k}^{(\theta,\beta)}(q, Q)$) seek a second prescription. Consequently, given $\left(\omega_i, Q^A, Q^B\right)$, if j is the second supplier the customer calls upon, the probability that his prescribed service is accepted is

$$\varsigma_{2,k}\left(q_j, \omega_i, Q^j, Q^{-j}, g_k\left(\omega_i, Q^{-j}, Q^j\right)\right)$$
$$= \Sigma_{q\in\Omega_{\omega_i}} \xi\{(\theta,\beta) \mid \sigma_{1,k}^{(\theta,\beta)}\left(q_{-j}, Q^{-j}\right) = 0\}$$
$$\times\, \sigma_{2,k}^{(\theta,\beta)}\left(q_j, Q^j \mid q, Q^{-j}\right) g_k\left(\omega_i, Q^{-j}, Q^j\right)(q),$$

$j \in \{A, B\}$. Hence, the probability that a newly arrived customer accepts the prescription of supplier j is:

$$\alpha_j^k\left(q_j \mid \sigma_k, G_k\left(\omega_i, Q^{-j}, Q^j\right)\right) := \tag{5}$$
$$\frac{1}{2}\Big[\sigma_{1,k}\left(q_j, Q^j\right) + (1 - \sigma_{1,k}\left(q_j, Q^j\right)$$
$$\times\left(1 - \Sigma_{q\in\Omega_{\omega_i}} \sigma_{2,k}\left(q, Q^{-j} \mid q_j, Q^j\right) g_k\left(\omega_i, Q^{-j}, Q^j\right)(q)\right) +$$
$$\Sigma_{q\in\Omega_{\omega_i}}\left(1 - \sigma_{1,k}\left(q, Q^{-j}\right)\right) \sigma_{2,k}\left(q_j, Q^j \mid q, Q^{-j}\right) g_k\left(\omega_i, Q^{-j}, Q^j\right)(q)\Big].$$

Given players strategies (G, σ) and current state (ω, Q^j, Q^{-j}), if a customer accepts a prescription q in a stage game, it must be either with supplier j or supplier $-j$. Moreover, before the start of a stage game, the probability that supplier j's prescription q_j will be accepted against supplier $-j$'s strategy $G_k\left(\omega_i, Q^{-j}, Q^j\right)$ is given by $\alpha_j^k\left(q_j \mid \sigma_k, G_k\left(\omega_i, Q^{-j}, Q^j\right)\right)$ in (5). Hence, after supplier j's prescription q_j, $j \in \{A, B\}$, is accepted, the conditional probability of $(\hat{Q}^j, \hat{Q}^{-j})$ is:

$$\upsilon_{k,j}(\hat{Q}^j, \hat{Q}^{-j}|Q^j, Q^{-j}) = \frac{1}{2}\alpha_j^k\left(q_j \mid \sigma_k, G_k\left(\omega_i, Q^{-j}, Q^j\right)\right) g_k\left(\omega_i, Q^j, Q^{-j}\right)(q_j),$$

if $q_j = \hat{Q}^j - Q^j \in \Omega_\omega$ and $\hat{Q}^{-j} = Q^{-j}$, and $\upsilon_j^k(\hat{Q}^j, \hat{Q}^{-j}|Q^j, Q^{-j}) = 0$, otherwise. Therefore, for any arbitrary state of the queues $(\hat{Q}^A, \hat{Q}^B)$ to occur after a stage game to occur is accepted is:

$$\upsilon_k(\hat{Q}^A, \hat{Q}^B|Q^A, Q^B) = \upsilon_{kA}(\hat{Q}^A, \hat{Q}^B|Q^A, Q^B) + \upsilon_{k,B}(\hat{Q}^A, \hat{Q}^B|Q^A, Q^B)$$

if $\hat{Q}^A - Q^A \in \Omega_\omega$ or $\hat{Q}^B - Q^B \in \Omega_\omega$, and $\upsilon((\hat{Q}^A, \hat{Q}^B)|Q^A, Q^B) = 0$, otherwise.

Lemma 2. *For all $\omega, Q^A, Q^B \in \Omega \times I$, the expression (1) is a continuous function on the strategy profiles set $\Sigma \times \Delta(\Omega)^2$.*

3.1.3. The evolution of the queues

We show next that there is a stationary distribution of υ_k on I perceived by customers. We start with an original distribution υ_k^* hypothesized by customers and trace its evolution in the wake of the end of a stage game. Suppose that υ_k^* satisfies the following properties (we will show below why we specify these requirements).

1. υ_k^* is absolutely continuous (with respect to the Lebesgue measure) except at $\left(Q^A, 0\right)$ and $\left(0, Q^B\right)$.
2. υ_k^* has full-support.
3. υ_k^* has marginal probability $\upsilon_k^*(Q^A, Q^B)$, for $(Q^A, Q^B) \geq 0$ and $(Q^A, Q^B) \neq (0, 0)$.
4. υ_k^* has probability mass $\upsilon_k^*(Q^A, Q^B)$, at $(Q^A, Q^B) = (0, 0)$.

The information that the suppliers have and the customer does not have is: (a) How long it has been since the preceding stage game ended (i.e., the waiting time t), and (b) The state (ω, Q^A, Q^B) of the previous stage game. Thus, the customer's perceived queue distribution is the unconditional expectation of $\upsilon_k(\hat{Q}^A, \hat{Q}^B|Q^A, Q^B)$.

For $(\hat{Q}^A, \hat{Q}^B) > 0$, the only possibility that $(\hat{Q}^A, \hat{Q}^B)$ obtains is that the preceding game ended with the state of the queues $(\hat{Q}^A + t, \hat{Q}^B + t)$, $t \in (0, \infty]$. Hence, conditional on the preceding stage game starting with the state (ω, Q^A, Q^B), the probability of this event is: $\Sigma_{\omega\in\Omega}\upsilon_k(\hat{Q}^A + t, \hat{Q}^B + t|\omega, Q^A, Q^B)\mu(\omega)\,\upsilon_k^*(Q^A, Q^B)$. Therefore, $\upsilon_k(\hat{Q}^A, \hat{Q}^B)$ is the unconditional expectation

$$\upsilon_k(\hat{Q}^A, \hat{Q}^B) = \int_0^\infty \int_I \left[\Sigma_{\omega\in\Omega}\upsilon_k(\hat{Q}^A + t, \hat{Q}^B + t|\omega, Q^A, Q^B)\mu(\omega)\right] \times \upsilon_k^*(Q^A, Q^B)d(Q^A \times Q^A)dF(t).$$

For $(\hat{Q}^A, \hat{Q}^B) \geq (0, 0)$ such that $\hat{Q}^j = 0$, $j \in \{A, B\}$, and $(\hat{Q}^A, \hat{Q}^B) \neq (0, 0)$, it can occur if previous game ends with either $\left(\hat{Q}^A + t, \hat{Q}^B\right)$ and some $\hat{Q}^B \leq t$, or $\left(\hat{Q}^A, \hat{Q}^B + t\right)$ and some $\hat{Q}^A \leq t$. Therefore, the probability density of, for example, $\hat{Q}^B = 0$ is the sum of the two unconditional probabilities

$$\begin{aligned}\upsilon_k(\hat{Q}^A, 0) = &\int_0^\infty \int_I \left[\Sigma_{\omega\in\Omega}\upsilon_k(\hat{Q}^A + t, 0 + t|\omega, Q^A, Q^B)\mu(\omega)\right] \times \upsilon_k^*(Q^A, Q^B)d(Q^A \times Q^B)dF(t)\\ &+ \int_0^\infty \int_{I\cap\{Q^B\leq t\}} \left[\Sigma_{\omega\in\Omega}\upsilon_k(\hat{Q}^A + t, 0|\omega, Q^A, Q^B)\mu(\omega)\right] \times \upsilon_k^*(Q^A, Q^B)d(Q^A \times Q^B)dF(t).\end{aligned}$$

For $(\hat{Q}^A, \hat{Q}^B) = (0, 0)$, it can be reached by points from the 45 degree line and also points from both axes of the triangle I. So it is the sum of the three probabilities

$$\begin{aligned}\upsilon_k(0, 0) = &\int_0^\infty [\int_{Q^A\leq t, Q^B\leq t}\left[\Sigma_{\omega\in\Omega}\upsilon_k(t, t|\omega, Q^A, Q^B)\mu(\omega)\right] \upsilon_k^*(Q^A\times Q^B)d(Q^A\times Q^B)]dF(t).\\ &+ \int_0^\infty [\int_{I\cap\{Q^A\leq t\}}\left[\Sigma_{\omega\in\Omega}\upsilon_k(0, t|\omega, Q^A, Q^B)\mu(\omega)\right] \upsilon_k^*(Q^A, Q^B)d(Q^A\times Q^B)]dF(t)\\ &+ \int_0^\infty [\int_{I\cap\{Q^B\leq t\}}\left[\Sigma_{\omega\in\Omega}\upsilon_k(t, 0|\omega, Q^A, Q^B)\mu(\omega)\right] \upsilon_k^*(Q^A, Q^B)d(Q^A\times Q^B)]dF(t).\end{aligned}$$

If the perceived distribution υ_k after a stage game coincides with the hypothesized υ_k^* then,

$$\upsilon_k^*(\hat{Q}^A, \hat{Q}^B) = \int_0^\infty [\int_I \left[\Sigma_{\omega\in\Omega}\upsilon_k(\hat{Q}^A + t, \hat{Q}^B|\omega, Q_A, Q_B)\mu(\omega)\right] \upsilon_k^*(Q^A, Q^B)d(Q^A \times Q^B)]dF(t).$$

Note that for $(\hat{Q}^A, \hat{Q}^B)$ with $\hat{Q}^j = 0$, $j \in \{A, B\}$, $\upsilon_k(\hat{Q}^A, \hat{Q}^B)$ is either a marginal probability over one axis, if only one $\hat{Q}^j = 0$, or a probability mass if $\hat{Q}^A = \hat{Q}^B = 0$.

Proposition 3. *The perceived measure υ_k on I is absolutely continuous with respect to the Lebesgue measure and has full support.*

Starting from the event that both suppliers are idle (i.e., $Q^A = Q^B = 0$) the probability, p, of returning to the same position under the equilibrium strategies is positive. Since the equilibrium is Markovian, this event is encountered infinitely often. Thus, the probability of the event "$Q^A = Q^B = 0$ infinitely often" is: $\lim_{m\to\infty} p^m > 0$. Hence, $p = 1$. In other words, starting from any state of finite queues, $\left(Q^A, Q^B\right)$, with probability one the queues will attain the point $Q^A = Q^B = 0$ infinitely often. From this position, the two suppliers are equally likely to become the long-queue supplier. Hence, no supplier enjoys the short-queue advantage persistently. Therefore, the evolution of the queues under the equilibrium strategies requires that the anticipated lengths of the queues be *stochastically equal*, in the sense that the identity of the short-queue supplier is expected to change over time in such a way that the joint distribution of the queues is symmetric around its mean. We summarize this in the following: *In symmetric stationary equilibrium, successive stage games induce a joint distribution of the lengths of the queues that is stationary, symmetric and the two suppliers commit the same amount of fraud on average.*

3.2. Equilibrium: Definition and existence

A *customer's system of beliefs* $\eta := (\mu, \upsilon, m(\omega, Q_2 \mid q_1, Q_1))$ consists of the prior belief about the stage game being played, which is determined by the prior beliefs $\mu \in \Delta(\Omega)$, $\upsilon \in \mathcal{V}$, and the updated beliefs $m(\omega, Q_2 \mid q_1, Q_1)$ on $\Omega \times [0, \infty]$. A strategy profile $\left(\sigma, G\left(\omega, Q^j, Q^{-j}\right)\right)$, $j \in \{A, B\}$, is *sequentially rational* if, for all $\left(\omega, Q^A, Q^B\right) \in \Omega \times I$, given the suppliers objective functions,

$G\left(\omega, Q^j, Q^{-j}\right)$ is best response against $\left(\sigma, G\left(\omega, Q^{-j}, Q^j\right)\right)$, $j \in \{A, B\}$, and, given the customer objective function, σ is best response against $\left(G\left(\omega, Q^A, Q^B\right), G\left(\omega, Q^B, Q^A\right)\right)$.

Definition 2. The strategy profile $\left(\hat{\sigma}, \hat{G}\left(\omega, Q^j, Q^{-j}\right)\right)$, $j \in \{A, B\}$, and a system of beliefs $\eta^* = (\mu, \upsilon^*, m^*(\omega, Q_2 \mid q_1, Q_1))$ constitute a symmetric *Markovian sequential equilibrium of the stochastic game induced by the credence good market* if:

(*i*) The strategy profile $\left(\hat{\sigma}, \hat{G}\left(\omega, Q^j, Q^{-j}\right)\right)$, $j \in \{A, B\}$, is sequentially rational given the belief system $\eta^* = (\mu, \upsilon^*, m^*)$, where, for $j \in \{A, B\}$, $\hat{G}(\omega, Q^j, Q^{-j})$ is in the $\arg\max_{G \in \mathcal{G}}$ of

$$\int_0^\infty \int_I [\int_0^{\min(\hat{Q}^j, t')} e^{-rs} ds + e^{-rt} \Sigma_{\omega' \in \Omega} V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t')\mu\left(\omega'\right)] \times \upsilon^*(\hat{Q}^A, \hat{Q}^B | Q^A, Q^B) d\left(\hat{Q}^A, \hat{Q}^B\right) dF\left(t'\right).$$

and, for every $(\beta, \theta) \in T$ and $(q_1, Q_1) \in \Omega \times [0, \infty]$,

$$\hat{\sigma}_1^{(\beta,\theta)}(q_1, Q_1) = \arg \max_{\sigma_1 \in \{f:(q,Q) \to [0,1]\}} [\sigma_1 u^{(\beta,\theta)}(q_1, Q_1) + (1 - \sigma_1) u_r^{(\beta,\theta)}(q_1, Q_1)],$$

and $\hat{\sigma}_2^{(\beta,\theta)}(q_1, Q_1, q_2, Q_2) = 1$ if $u^{(\beta,\theta)}(q_1, Q_1) \leq u^{(\beta,\theta)}(q_2, Q_2)$ and $\hat{\sigma}_2^{(\beta,\theta)}(q_1, Q_1, q_2, Q_2) = 0$, otherwise.

(*ii*) There exists a sequence of completely mixed G_k strategies with modulus k and strategies profiles (σ_k, G_k) that is sequentially rational given a belief system $\eta_k := (\mu, \upsilon_k, m_k(\omega, Q_2 \mid q_1, Q_1))$ with $\left(\hat{\sigma}, \hat{G}\right) = \lim_{k \to \infty}(\sigma_k, G_k)$, $\eta^* = \lim_{k \to \infty} \eta_k$, $\upsilon^* = \lim_{k \to \infty} \upsilon_k^*$, and $m_k(q_2, Q_2 \mid q_1, Q_1)$ are derived from the μ and υ_k^* and strategy profile $(\sigma_k, G_k,)$ using Bayes' rule.

Theorem 1. *There exists a symmetric Markovian sequential equilibrium of the stochastic game induced by the credence good market.*

In equilibrium, the payoff function $\hat{V}$ coincides with the anticipated value function V when every player is adopting its optimal strategy with the transition probability $\upsilon^*(\cdot, \cdot | Q^A, Q^B) = \upsilon(\cdot, \cdot | Q^A, Q^B, G^*, \sigma^*)$ based on equilibrium strategies $G^*(\omega, Q^j, Q^{-j})$, $j \in \{A, B\}$, and σ^*. Thus, the value function of supplier $j \in \{A, B\}$ is:

$$V(\omega, Q^j, Q^{-j}) = \max_{G \in \mathcal{G}} \int_0^\infty \int_I [\int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau + e^{-rt'} \Sigma_{\omega' \in \Omega} V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t')\mu\left(\omega'\right)] \upsilon^*(\hat{Q}^A, \hat{Q}^B | Q^A, Q^B) \times d\left(\hat{Q}^A, \hat{Q}^B\right) dF\left(t'\right).$$

4. The short-queue advantage and examples

4.1. The short-queue advantage and fraudulent behavior

If $Q^A \neq Q^B$ the supplier with the shorter queue enjoys a strategic advantage in the sense that, if the two suppliers prescribe the same service, the short-queue supplier is more likely to retain a new customer. However, because of the generality the primitives of the model and the fact that higher prescription entails a trade-off between the benefit of selling extra services and cost represented by lower probability of making the sale, there is no guarantee that the suppliers value functions are necessarily monotonic increasing in the suppliers' own queues. Consequently, it is not necessary that the short-queue supplier exploits his advantage by prescribing more service than the long-queue supplier. More precisely, suppose that $Q^A < Q^B$ and the long-queue supplier prescribes ω_i. If accepted, the marginal value to A of prescribing ω_{i+1} instead of ω_i is:

$$\mathbf{mv} := e^{-rQ^A} \int_{\omega_i}^{\omega_{i+1}} e^{-r\tau} d\tau + \Sigma_{\omega \in \Omega} \left[V(\omega, Q^A + \omega_{i+1}, Q^B) - V(\omega, Q^A + \omega_i, Q^B)\right] \mu(\omega).$$

If the supplier is interested in selling extra services which, presumably, is the case then this expression must be positive.

The cost to A of prescribing ω_{i+1} instead of ω_i is due to the reduction in the probability that the prescription be accepted. Formally, let $\alpha_A\left(\omega \mid Q^A, Q^B\right)$ denote the probability of supplier A's prescription ω is accepted, then the marginal cost, **mc**, is:

$$\left(\alpha_A\left(\omega_{i+1} \mid Q^A, Q^B\right) - \alpha_A\left(\omega_i \mid Q^A, Q^B\right)\right) \times \left[e^{-rQ^A} \int_0^{\omega_i} e^{-r\tau} d\tau + \Sigma_{\omega \in \Omega} \bar{V}(\omega, Q^A, Q^B)\mu(\omega)\right],$$

where

$$\bar{V}(\omega, Q^A, Q^B) = \alpha_A\left(\omega_i \mid Q^A, Q^B\right) V(\omega, Q^A + \omega_i, Q^B) + \left(1 - \alpha_A\left(\omega_i \mid Q^A, Q^B\right)\right) V(\omega, Q^A, Q^B + \omega_i).$$

Consider next the difference $\alpha_A\left(\omega_{i+1} \mid Q^A, Q^B\right) - \alpha_A\left(\omega_i \mid Q^A, Q^B\right)$. The only customers that supplier A loses by prescribing ω_{i+1} instead of ω_i are the customers that tried both suppliers and decided to accept supplier B's prescription. These are the customers whose type is in the set

$$\mathfrak{T} := \{(\beta, \theta) \in T \mid (\beta, \theta) \in T \mid (1 - \omega_{i+1}) e^{-\beta Q^A} < (1 - \omega_i) e^{-\beta Q^B}, (1 - \omega_i) e^{-\beta Q^B} - \theta < u_r^{(\theta,\beta)}(q_B, Q_B), (1 - \omega_i) e^{-\beta Q^A} - \theta < u_r^{(\theta,\beta)}(q_A, Q_A)\}.$$

Then

$$\left(\alpha_A\left(\omega_{i+1} \mid Q^A, Q^B\right) - \alpha_A\left(\omega_i \mid Q^A, Q^B\right)\right) = -\xi\{\mathfrak{T}\}.$$

Clearly, this expression decreases monotonically with Q^B and increases monotonically with Q^A. Hence, if the spread $Q^B - Q^A$ increases as a result of an increase in Q^B and decrease in Q^A then $-\xi\{\mathfrak{T}\}$decreases. But, if $V(\omega, Q^A + \omega_i, Q^B) - V(\omega, Q^A, Q^B + \omega_i) > 0$, then $\bar{V}(\omega, Q^A, Q^B)$ increases with the spread $Q^B - Q^A$ in the same way. Hence, the total effect on **mc** is ambiguous. However, since $V(\omega, Q^A + \omega_i, Q^B) - V(\omega, Q^A, Q^B + \omega_i)$ is bounded, if $Q^B - Q^A$ is sufficient large then the **mc** tends to zero and the marginal value outweighs the marginal cost. At that point, the short-queue supplier exploits his advantage be prescribing higher level of service then the short-queue supplier.

An equilibrium is said to be *fraud-free* if the equilibrium strategies are $\hat{G}\left(\omega_i, Q^j, Q^{-j}\right) = \delta_{\omega_i}$, $j \in \{A, B\}$, for all $\left(\omega_i, Q^j, Q^{-j}\right) \in \Omega \times I$. In view of the preceding discussion, the next theorem asserts that fraudulent prescriptions of service is a persistent feature of competitive equilibrium in the credence good market under consideration.

Theorem 2. *There exists no fraud-free equilibrium in the market for credence quality services.*

One measure of the short-queue advantage is the *difference in the expected change of the lengths of the queues induced by equilibrium strategies*. Formally, given a stage game $\Gamma\left(\omega_i, Q^A, Q^B\right)$, if

$Q^A < Q^B$ then the measure of the short-queue advantage is:

$$\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}\left(\omega_i, Q^A, Q^B\right), \hat{G}\left(\omega_i, Q^B, Q^A\right)\right)$$
$$:= \Sigma_{q\in\Omega_{\omega_i}} \alpha^A\left(q \mid \hat{\sigma}, \hat{G}\left(\omega_i, Q^A, Q^B\right)\right) q\hat{g}\left(\omega_i, Q^A, Q^B\right)(q) -$$
$$- \Sigma_{q\in\Omega_{\omega_i}} \alpha^B\left(q \mid \hat{\sigma}, \hat{G}\left(\omega_i, Q^B, Q^A\right)\right) q\hat{g}\left(\omega_i, Q^A, Q^B\right)(q).$$

The discussion above implies that an increase in the length of the queue of the short-queue supplier reduces its short-queue advantage. Formally, if A is the short-queue supplier then $d\Psi\left(\omega_i, Q^A, Q^B \mid \hat{\sigma}, \hat{G}\left(\omega_i, Q^A, Q^B\right), \hat{G}\left(\omega_i, Q^B, Q^A\right)\right)/dQ^A < 0$. However, because A's objective function is not necessarily monotonic increasing in Q^A, the short-queue advantage does not yield clear cut conclusions concerning its effect on the suppliers' equilibrium strategies. It is useful, therefore, to consider some simple situations whose analysis would allow us to develop further insights as to the possible nature of fraudulent behavior.

4.2. Simple examples

Suppose that $\Omega = \{\omega_L, \omega_H\}$, where $\omega_H > \omega_L$. Clearly, if the true state is ω_H then the only equilibrium is for both suppliers to prescribe the true state. The interesting situation arises when the true state is ω_L. We consider this case below.

The payoff matrix corresponding to the stage game $\Gamma\left(\omega, Q^A, Q^B\right)$ in which A is the columns player and B is the rows player as follows:

$$\begin{array}{ccc} \downarrow B \setminus A \rightarrow & x : \omega_H & (1-x) : \omega_L \\ y : \omega_H & U^B_{HH}, U^A_{HH} & U^B_{HL}, U^A_{HL} \\ (1-y) : \omega_L & U^B_{LH}, U^A_{LH} & U^B_{LL}, U^A_{LL} \end{array} \tag{6}$$

where

$$U^j_{\iota\iota} = \alpha_j\left(\omega_\iota, \omega_\iota\right) \Sigma_{\omega\in\Omega} V\left(\omega, Q^j + \omega_\iota, Q^{-j}\right)\mu(\omega) + \left(1 - \alpha_j\left(\omega_\iota, \omega_\iota\right)\right) \Sigma_{\omega\in\Omega} V\left(\omega, Q^j, Q^{-j} + \omega_\iota\right)\mu(\omega),$$

for $j \in \{A, B\}$, $\iota \in \{H, L\}$, and

$$U^j_{HL} = \alpha_A\left(\omega_H, \omega_L\right) \Sigma_{\omega\in\Omega} V\left(\omega, Q^A + \omega_H, Q^B\right)\mu(\omega) + \left(1 - \alpha_A\left(\omega_H, \omega_L\right)\right) \Sigma_{\omega\in\Omega} V\left(\omega, Q^A, Q^B + \omega_L\right)\mu(\omega),$$

$$U^j_{LH} = \alpha_A\left(\omega_L, \omega_H\right) \Sigma_{\omega\in\Omega} V\left(\omega, Q^A + \omega_L, Q^B\right)\mu(\omega) + \left(1 - \alpha_A\left(\omega_L, \omega_H\right)\right) \Sigma_{\omega\in\Omega} V\left(\omega, Q^A, Q^B + \omega_H\right)\mu(\omega),$$

for $j \in \{A, B\}$. Allowing for mixed strategies, x and y denote the probabilities that players A and B prescribe ω_H, respectively. Then

$$\frac{y}{1-y} = \frac{U^A_{LL} - U^A_{LH}}{U^A_{HH} - U^A_{HL}} \text{ and } \frac{x}{1-x} = \frac{U^B_{LL} - U^B_{HL}}{U^B_{HH} - U^B_{LH}}. \tag{7}$$

The primitives of the model, namely, the prior distribution on the customers' type space, T, the distribution on the possible states of disrepair, Ω, the stochastic process depicting the arrival of new customers, are quite general. This allows for wide range of values of the suppliers payoffs of the stage games which depend on the states of the queues. Consequently, the model admits a variety of equilibria, including pure strategy and mixed strategy equilibria. In the Appendix we analyze the two stage games $\Gamma\left(\omega_L, Q^A, Q^B\right)$. The first deals with the symmetric case in which the suppliers queues are of equal lengths and the second with the asymmetric case in which the suppliers' queues are of different lengths. The general conclusions that emerge are as follows:

If the suppliers queues are equal then, depending on the configurations of the signs of these expressions we may have (a) pure strategy equilibria in which either both suppliers prescribe truthfully or both commit fraud; (b) Two pure strategy equilibria in which one supplier prescribes truthfully and the other overprescribes; (c) A symmetric mixed strategy equilibrium in which each supplier overprescribes service with probability 0.5.

If the suppliers queues are of different lengths then, in mixed strategy equilibrium the short-queue supplier is more likely to commit fraud than the long-queue supplier. In other words, if the true state is w_L, and $Q^A < Q^B$ then the equilibrium mixed strategy of supplier A first-order stochastically dominates that of supplier B in the sense that $\Pr_A\{\omega_H\} = x > y = \Pr_B\{\omega_H\}$. Moreover, $\Pr_A\{\omega_H\} > 0.5 > \Pr_B\{\omega_H\}$.

5. Related literature and concluding remarks

5.1. Related literature

Shapley (1953) was the first to formulate and prove the existence of equilibrium in two-player, zero-sum, stochastic games. Extensions and review of stochastic games in more general setting are provided in Duggan (2012) and Jaskiewicz and Nowak (2018). While sharing many features of equilibrium analysis that appear in the literature on stochastic games, our model presents an important variation — no player in our model possesses perfect information. In particular, the customer in the model has only partial information about the state that parametrizes the stage game and the suppliers are ignorant of the customer's type. This variation is a contribution to the literature on stochastic games with incomplete information.

Despite evidence regarding the prevalence of fraud in the market for credence goods and the distinguishing features of these markets, the literature dealing with the modeling and analysis of these markets is rather scant. The works that are closest to ours in terms of the questions asked, are Wolinsky (1995), Emons (1997) and Dulleck and Rudolf (2006). Despite the shared interest in studying the prevalence of fraud in equilibrium, these works model markets that have distinct structures. Focusing on markets exhibiting features that are different from those of this paper, they reach different conclusions regarding the equilibrium characteristics.

Wolinsky (1995) modeled a market in which the customers bargain with suppliers by offering a price for the repair, and showed that, in interior equilibrium, suppliers commit fraud by employing a strategy that assigns positive probability of rejecting price offers when the state diagnosed requires low service. This strategy reflects the suppliers' belief that, to avoid the search cost, the customer may offer a higher price rather than seek a second opinion. Wolinsky's model is different from ours in several important respects. In addition to assuming that the price of service is fixed (no bargaining), a central feature of our model is the lengths of the suppliers' queues and the characterization of customers by their idiosyncratic search costs and discount rates. These aspects of our work are absent from Wolinsky's model. These differences in modeling mandate different equilibrium notions and analysis.

Emons (1997) depicts a credence good market with identical customers and in which the suppliers must decide whether to enter the market. If a supplier enters the market he is endowed with a fixed capacity that can be allocated between diagnosis and repair services. These two functions are priced differently. Suppliers who lack sufficient capacity, can announce a wrong diagnosis to avoid providing the needed repair. Emons studies conditions under which fraud free equilibrium exists. In addition to its focus on the entry decision, Emons' model is different from ours in the specification of the information structure, the characterization of the customers and their behavior, the pricing mechanism, and the suppliers strategies.

Dulleck and Rudolf (2006) model a market for credence services in which the customers may experience a need for a high or low level of service. Invoking a game theoretic approach to study conditions under which competition will eliminate fraud.

Hu and Lin (2018), Fong and Liu (2016) and Fong et al. (2020) study the efficiency loss due to the asymmetric information in monopolistic credence good market in which the supplier faces uninformed customers. More specifically, Hu and Lin (2018) modeled repeated interaction between a customer in occasional need of maintenance service of a durable good and a monopoly supplier. They show that there exists no equilibrium that supports truthful diagnosis. Fong and Liu (2016) investigated the effect of liability on the seller's incentive to maintain good reputation and its impact on market efficiency. Fong et al. (2020) focus on the use of customer service to build trust between the monopoly supplier and its customers so as to mitigate the efficiency loss.

Heinzel (2019) studied the equilibrium of a price-regulated market in which physicians characterized by heterogeneous cost compete for servicing uninformed patients. Heinzel models the interaction among physicians and patients as a game in which patients may employ mixed strategies in seeking "second opinion" when diagnosed as having a serious problem and physicians may defraud their patients by overtreating them for minor problems. Unlike in the model we present here, the distinct physicians' types is exogenous and the customer behavior is not derived from optimal search strategy.

5.2. Concluding remarks

We model a credence service market featuring two identical suppliers engaged in Bertrand competition. The customers care about the prescribed services and the waiting time. Our analysis shows that competition cannot be relayed upon to sustain fraud-free equilibrium and that fraud is a persistent and prevalent phenomenon. The analysis highlights the role of the evolution of the customer's beliefs in the wake of her visit to the first supplier and the optimal stopping rule that characterizes her best response strategy, and the suppliers prescription strategies. These aspects of our model and analysis are not specific to the two suppliers case and would show up, in a more complex form, if the number of the suppliers is larger.

The analysis underscores the short-queue supplier's advantage, its implications for the overprescription of service and the consequent evolution of the queues. It is worth noting that if the waiting time is not an issue (that is, the suppliers have no capacity constraints) so that each customer can be served immediately, then the analysis would change considerably. In this instance, the customers' utilities depend only on the prescribed service, and their discount rates is no longer a factor. Suppose that $\theta \in (0,1]$ then it is easy to verify that the suppliers strategies $q^j(\omega) = \omega_n$, for all $\omega \in \Omega$ and $j \in \{A, B\}$, is an equilibrium. In other words, knowing that the equilibrium prescriptions of the two suppliers are the same, no customer is inclined to search and, consequently, the suppliers have no incentive to try and undercut each other's prescription. Maximal fraud also characterizes the cab service provided to tourists in an unfamiliar city since the prescription (that is, the route taken) coincides with the service provided, leaving the customer no opportunity for seeking a second prescription. The route taken is only restricted by a tourist's conception of the reasonable length of the ride.[24]

One may think of variations on the model presented here. For instance, there are situations in which, to obtain a diagnosis, one has to schedule an appointment (e.g., a plumber service or medical examination). In these instances, the waiting time is ahead of obtaining the diagnosis and the customer may obtain information about the waiting time at different suppliers prior to deciding which supplier to visit first. This would change the information structure and, consequently, the strategies and equilibrium of the model. The analysis of such variations is left for future research.

An important aspect of the credence good market, discussed in Darby and Karni (1973) but not touched upon in this work, is the possibility of developing a reputation for honest diagnosis and its effect on the commission of fraud. Including reputation in our model would require admitting repeated interactions in which the customers display loyalty (e.g., they visit "their" supplier first) and the suppliers recognize their loyal clients. Under these conditions, the suppliers may establish what Darby and Karni dubbed client relationship. The loss of future business of, and being bad-mouthed by, a dissatisfied customer would increase the cost to the suppliers of "losing" customers, which should serve as a deterrence and, consequently, mitigate the problem of fraud.

6. Proofs

6.1. Proof of Proposition 1

To begin with, we verifying that the Blackwell sufficient conditions for contraction mapping are satisfied.[25] Define a mapping L by[26]:

$$L(V) = \max_{G\in\mathcal{G}} \int_0^\infty \int_I [\int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau + e^{-rt'} E_\Omega V(\omega', \hat{Q}^j - t', \hat{Q}^{-j} - t')] \times \upsilon^*(\hat{Q}^A, \hat{Q}^B) d\left(\hat{Q}^A, \hat{Q}^B\right) dF\left(t'\right),$$

where $\upsilon^*(\hat{Q}^A, \hat{Q}^B)$ is shorthand for the conditional probabilities $\upsilon^*(\hat{Q}^A, \hat{Q}^B | Q^A, Q^B)$.

(Monotonicity) Suppose that there are two value functions, V^{**} and V^* such that $V^{**}(\omega, Q^A, Q^B) > V^*(\omega, Q^A, Q^B)$, pointwise. Because the maximizer can always attain a higher value for V^{**} then for V^*, we have, $L(V^{**}) \geq L(V^*)$.

(Discounting) Given a value function V adding a constant a and inputting the mapping L with $V + a$ we get:

$$L(V+a) = L(V) + \max_{G\in\mathcal{G}} \int_0^\infty \int_I [e^{-rt'} \Sigma_{\omega\in\Omega} a\mu(\omega)] \upsilon^*(\hat{Q}^A, \hat{Q}^B | Q^A, Q^B) \times d\left(\hat{Q}^A, \hat{Q}^B\right) dF\left(t'\right) = L(V) + a \int_0^\infty e^{-rt'} dF\left(t'\right),$$

and $\int_0^\infty e^{-rt'} dF\left(t'\right)$ is a constant strictly lower than 1 for all $r > 0$.

By the Blackwell sufficient conditions for contraction mapping L is a contraction mapping. By the Contraction Mapping Theorem, V exists and is unique. ■

6.2. Proof of Proposition 3

To start with, observe that: (a) Since $\theta \in [0, 1]$, there is a set of positive measures of customer types who accept the prescription $q = \omega_n$. Thus, every prescription, $q \in \Omega$ has positive probability of being accepted in a stage game of modulus k given any queue distribution υ_k. (b) Because $F(t) > 0$ for all $t > 0$, the probability

24 See also, Stahl (1996) for a discussion of a related issue.

25 See Stokey and Lucas (1989).

26 Implicit in L is a fixed strategy profile $\left(G\left(\cdot, \hat{Q}^{-j}, \hat{Q}^j\right), \sigma\right)$.

of the event $(Q^A, Q^B) = (0, 0)$ is strictly positive. Hence, the state of queues $(Q^A, Q^B) = (0, 0)$ occurs with positive probability after any stage game.

Claim 1. *Given any initial probability distribution, υ_k^0 on I, for every state of the queues, (Q^A, Q^B), there is a finite $i \in \mathbb{N}$ such that the customer's perceived queue distribution υ_k^i in the ith stage game has positive probability density on (Q_A, Q_B).*

Proof. Since every point (Q^A, Q^B) can always be reached by waiting time t from a stage game that ends with $(Q^A + t, Q^B + t)$ such that $Q^A + t + Q^B + t$, our question is reduced to whether every point in the set I can be reached from a finite sequence of stage games. Because $(0, 0)$ has positive probability in the queue distribution after any stage game, the question is further reduced to whether (Q^A, Q^B) can be reached from origin point $(0, 0)$ with positive probability density. If the answer is yes, then we are done.

To begin with, note that in a stage game of modulus k, the probability that both suppliers prescribe $\omega \in \Omega$ is at least k^{-2} and it is necessary that both suppliers have positive probability to be accepted regardless whose queue is longer or shorter. Because the customer waiting time distribution $F(t)$ is absolutely continuous (with respect to the Lebesgue measure) with full support, $t \in (0, \infty]$, the points $(Q^A, 0)$ or $(0, Q^B)$ can be reached with positive marginal probability. For points in I we can make use of the full-support of $F(t)$ to first choose a proper point $(Q^A + \Delta, 0)$ and follow the sequence of m stage games, in which supplier B's prescriptions get accepted consecutively to reach the desired Q^B. Let each consecutive stage game start after a short time interval, δ, so that the total time during which the m stage games are played is $\Delta = m\delta$, so that not only Q^B is reached as desired, but also $Q^A + \Delta$ will decrease gradually to Q^A along the sequence of events. That such Δ and such a sequence of events can be constructed is implied by the fact that, for any Q^B, there is a finite set $\Omega^* = \{q | q \in \Omega\}$ such that $Q^B + 1 > \Sigma_{q\in\Omega^*} q > Q^B$. Define $\Delta = \Sigma_{q\in\Omega^*} q - Q^B$ and $\delta = \Delta/|\Omega^*|$. This completes the proof of the claim.

Claim 2. *If the distribution F is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$ then the equilibrium queue distribution, υ_k^* on I, is absolutely continuous with respect to the Lebesgue measure except in $\left(Q^A, 0\right) \cup \left(0, Q^B\right)$.*

Proof. Given $\left(Q^A, Q^B\right) \in I$, that is the realization of a random variable whose distribution, υ_k, is arbitrary, the queue at the start of the next stage game is $\left(\left(\hat{Q}^A - t\right), \left(\hat{Q}^B - t\right)\right) \in I$. The transition probability from the queues $\left(Q^A, Q^B\right)$ to $\left(\left(\hat{Q}^A - t\right), \left(\hat{Q}^B - t\right)\right)$ is determined by:

(a) The customer's acceptance decision that takes $\left(Q^A, Q^B\right)$ to $\left(\hat{Q}^A, \hat{Q}^B\right)$.

(b) The waiting time for the arrival of the next customer at time $t > 0$, that takes $\left(\hat{Q}^A, \hat{Q}^B\right)$ to $\left(\left(\hat{Q}^A - t\right), \left(\hat{Q}^B - t\right)\right)$

Step (a) is fully determined by the initial distribution, υ_k on I, the distribution μ on Ω, and the suppliers and customers strategies, G and σ. Thus, without further restrictions on the distributions, because υ_k is arbitrary, the resulting random variable $\left(\hat{Q}^A, \hat{Q}^B\right)$ is arbitrarily distributed according to a probability measure υ_k'.

Define $\zeta(t) = (-t, -t)$, $t > 0$ and observe that the sum of the random variables $\widetilde{Q} := \left(\hat{Q}^A, \hat{Q}^B\right)$ and $(-t, -t)$ is distributed according to υ_k'' which is the convolution of υ_k', the measure of $\left(\hat{Q}^A, \hat{Q}^B\right)$ and $F\left(\zeta^{-1}(-t, -t)\right)$. Thus, by the Fubini–Toneli theorem, υ_k'' can be written as:

$$\upsilon_k''(\mathfrak{B}) = \upsilon_k' * \left(1 - F\left(\zeta^{-1}(-t, -t)\right)\right)(\mathfrak{B}) = \int_0^\infty \left[\int_I \mathbf{1}_{\mathfrak{B}}\left(\widetilde{Q} + \zeta(t)\right) \upsilon_k'\left(\widetilde{Q}\right) d\widetilde{Q}\right] dF(t), \tag{8}$$

for all $\mathfrak{B} \in \mathcal{B}(I)$.

Since F is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$, the measure of $\left(\left(\hat{Q}^A - t\right), \left(\hat{Q}^B - t\right)\right)$ is at the points where $\left(\hat{Q}^A - t\right) = 0$ or $\left(\hat{Q}^B - t\right) = 0$, must be absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$. By the same argument, the measure υ_k'' is non-atomic, except at $(0, 0)$.

Suppose that $\lambda(\mathfrak{B}) = 0$ and $\upsilon_k''(\mathfrak{B}) > 0$. Let

$$\mathfrak{B}^0 = \{ \left(Q^A, Q^B\right) \in I \mid \neg\left(\left(Q^A, Q^B\right) = \left(Q_l^A - t, Q_l^B - t\right)\right), \left(Q_l^A, Q_l^B\right) \in \mathfrak{B}\}.$$

Then, $\upsilon_k''\left(\mathfrak{B}^0\right) > 0$. Define $\mathfrak{B}_t^0 = \{Q^A + t, Q^B + t \in I \mid \left(Q^A, Q^B\right) \in \mathfrak{B}^0\}$, $t \in [0, \infty)$. Let $\mathcal{P} = \{P_z\}_{z\in\mathbb{N}}$ be a partition of $\mathfrak{B}^0$ such that $P_z = \{\mathfrak{B}_t^0 \mid \upsilon_k'\left(\mathfrak{B}_t^0\right) \in \left[(z+1)^{-1}, z^{-1}\right\}$. Then, at least one cell of the partition is uncountable. Let this cell be P_{z_0}, then $\upsilon_k'\left(\mathfrak{B}_t^0\right) > z_0^{-1} > 0$. Pick a countable number of elements of P_{z_0}, $\{\mathfrak{B}_{t_\ell}^0 \mid \ell \in \mathbb{N}\}$. Then,

$$\upsilon_k'\left(\cup_{\ell\in\mathbb{N}} \mathfrak{B}_{t_\ell}^0\right) = \Sigma_{\ell\in\mathbb{N}} \upsilon_k'\left(\mathfrak{B}_{t_\ell}^0\right) \geq \Sigma_{\ell\in\mathbb{N}} z_0^{-1} = \infty.$$

But υ_k' is bounded. A contradiction. Hence, υ_k'' is absolutely continuous with respect to the Lebesgue measure on I. ■

6.3. Proof of Lemma 2

The customer's strategy affects V_k through the probability α_j^k in (5). Since V_k is continuous in α_j^k which is continuous in σ_k, V_k is continuous in σ_k. To show that V_k is continuous in $G_k\left(\omega, Q^{-j}, Q^j\right)$, it suffices to show that

$$\int_0^\infty [\int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau + e^{-rt'} \Sigma_{\omega\in\Omega} V_k\left(\omega, Q^j(t'), Q^{-j}\left(t'\right)\right) \mu(\omega)] dF\left(t'\right) \tag{9}$$

is continuous in $G_k\left(\omega, Q^{-j}, Q^j\right)$. By Eq. (1), the expression in (9) depends on $G_k\left(\omega, Q^{-j}, Q^j\right)$ through $\Sigma_{q\in\Omega_{\omega_i}} V_k\left(\omega_i, Q^j, Q^{-j} + q\right) g_k\left(\omega, Q^{-j}, Q^j\right)(q)$. Since the last expression is linear in the probabilities $\left(g_k\left(\omega, Q^{-j}, Q^j\right)(q)\right)_{q\in\Omega_{\omega_i}}$, it is continuous in $G_k\left(\omega, Q^{-j}, Q^j\right)$. That V_k is continuous in $G_k\left(\omega, Q^j, Q^{-j}\right)$ follows from its linearity in the probabilities $\left(g_k\left(\omega, Q^j, Q^{-j}\right)(q)\right)_{q\in\Omega_{\omega_i}}$. ■

6.4. Proof of Theorem 1

To prove the existence of a symmetric Markovian equilibrium for the stage game $\Gamma(\omega, Q^A, Q^B)$ we first invoke Kakutani–Fan–Glicksberg fixed point theorem to prove the existence of equilibrium in strategies that are totally mixed with modulus k.[27] Following that, invoking sequential compactness and letting $k \to \infty$ and we establish the existence of a convergent subsequence of fixed points whose limit is our symmetric Markovian equilibrium.

To start with, we construct a correspondence that maps the sets of suppliers' value functions, players' strategies and the distributions on the queues into themselves. Let $M \subset \mathbb{R}_+^{\Omega\times I}$ whose

[27] See Aliprantice and Border (2006) Theorem 17.55.

elements are bounded $\mathcal{B}(\Omega\times I)$-measurable functions and suppose that the suppliers' continuation functions V_k belong to M. Assume that the supplier's strategies are in:

$$\mathcal{G}_k := \{G : \Omega\times I \to \mathcal{G} | g(\omega, Q^A, Q^B)(q) \geq \frac{1}{k}, \forall q \geq \omega\},$$

the set of totally mixed, $\mathcal{B}(\Omega\times I)$-measurable, CDF with supports Ω_ω, $\omega\in\Omega$. Trivially, $\mathcal{G}_k$ is non-empty, closed and convex set. The set of customers strategies is: $\Sigma_k := (\Sigma_1\times\Sigma_2)^T$.

Define a correspondence Υ_k from $M\times\mathcal{G}_k\times\Sigma_k\times\mathcal{V}$ to itself

$$\Upsilon_k(V_k, G_k, \sigma_k, \upsilon_k) \rightrightarrows \left(\bar{V}_k, \bar{G}_k, \bar{\sigma}_k, \bar{\upsilon}_k\right),$$

such that $\bar{V}_k$ is the suppliers maximized value function given $(G_k, \sigma_k, \upsilon_k)$, $\bar{G}_k$ is the maximizer and measurable selection (with respect to (ω, Q_A, Q_B)), $\bar{\sigma}_k$ is the customer's best response function, and $\bar{\upsilon}_k$ is the next stage distribution on I.

Claim 1. *The sets M, $\mathcal{G}_k$, Σ_k and $\mathcal{V}$ in the domain of Υ_k are all compact subsets of locally convex Hausdorff spaces.*

Proof of Claim 1. Consider the set M. Since the point-wise limit of measurable functions is measurable, we have that M is bounded and closed in $\mathbb{R}_+^{\Omega\times I}$. Hence, by Tychonoff Theorem, M is compact in the product topology and is a subset of $\mathbb{R}_+^{\Omega\times I}$, which is locally convex Hausdorff space.

To show that $\mathcal{G}_k$ are compact subsets of locally convex Hausdorff space suffices it to observe that $\mathcal{G}_k$ is a compact subset of $\mathbb{R}_+^{n\times\Omega\times I}$, which is locally convex Hausdorff space.

Consider next the set Σ_k. Since Σ_i, $i = 1, 2$ are compact sets, so is the product, $\Sigma_1\times\Sigma_2$. Hence, by Tychonoff Theorem, Σ_k is compact subset of $\mathbb{R}^{T\times\Omega^2\times I}$. Also, by Prokhorov's theorem, $\mathcal{V}\subset\mathbb{R}^{2I}$ is compact (in the topology of weak convergence). Hence, both Σ_k and $\mathcal{V}$ are compact subsets of locally convex Hausdorff spaces. $\diamond$

We show next that the set Σ_k of the customers' strategies has the required measurability properties.

Claim 2. *Given $G_k\in\mathcal{G}_k$ and $\upsilon\in\mathcal{V}$, $\sigma_{1,k}^{(\theta,\beta)}(q, Q)$ is measurable with respect to $\mathcal{B}(T)\times\mathcal{B}(\Omega\times[0,\infty])$, and $\sigma_{2,k}^{(\theta,\beta)}(q_1, Q_1) : \Omega\times[0,\infty] \to \{0, 1\}$ is measurable with respect to $\mathcal{B}(T)\times\mathcal{B}(\Omega\times[0,\infty])$.*[28] *Moreover, the mapping $\sigma_{1,k} : \Omega\times[0,\infty]\to[0, 1]$ given by*

$$\sigma_{1,k}(q, Q) = \int_T \sigma_{1,k}(q, Q)d\xi(\beta, \theta)$$

and $\sigma_{2,k}(q_1, Q_1) : \Omega\times[0,\infty]\to[0, 1]$

$$\sigma_{2,k}(q_1, Q_1)(\cdot,\cdot) = \int_T \sigma_{2,k}^{(\beta,\theta)}(q, Q)d\xi(\beta, \theta)$$

are well-defined, $\mathcal{B}(\Omega\times[0,\infty])$-measurable, functions.

Proof of Claim 2. By Definition 1 and Proposition 2, the customer's best responses to (q_1, Q_1) in the first visit, and to (q_1, Q_1, q_2, Q_2) in the second visit, are single valued functions. By Berge Maximum Theorem[29] the best response functions $\sigma_{1,k}^{(\theta,\beta)}(\cdot,\cdot)$ and $\sigma_{2,k}^{(\theta,\beta)}(q_1, Q_1)(\cdot,\cdot)$ are continuous on T. Hence, they are $\mathcal{B}(T)$-measurable. Thus, σ_k is well-defined, $\mathcal{B}(\Omega\times I)$-measurable, vector-valued function.

Next we show that σ_k is $\mathcal{B}(\Omega\times[0,\infty])$-measurable function. Since $\sigma_k^{(\beta,\theta)}(q, Q)$ is defined over the product space $T\times\Omega\times[0,\infty]$, for every given $(q, Q)\in\Omega\times[0,\infty]$, there exists a sequence $\{\sigma_{k,n}^{(\beta,\theta)}(q, Q)\}$ of simple functions

$$\sigma_{k,n}^{(\beta,\theta)}(q, Q) := \Sigma_{i=1}^n \alpha_i(q, Q)\,\mathbf{1}_{A_i}(\theta, \beta),$$

where $(A_i)_{i=1}^n$ is a partition of the customers' type space T, $\mathbf{1}_{A_i}$ are the indicator functions, and $\alpha_i(q, Q)$ are the coefficients of the simple functions that, by the Fubini–Tonelli theorem can be chosen to be $\mathcal{B}(\Omega\times[0,\infty])$-measurable, such that

$$\lim_{n\to\infty}\sigma_{k,n}^{(\beta,\theta)}(q, Q) = \sigma_k^{(\beta,\theta)}(q, Q).$$

Since T may be equipped with a finite measure and, for each $(q, Q)\in\Omega\times[0,\infty]$, $\sigma_k^{(\beta,\theta)}(q, Q)$ is $\mathcal{B}(T)$-measurable function, we have a well-defined function

$$\sigma_{1,k}(q, Q) = \int_T \sigma_k^{(\beta,\theta)}(q, Q)d\xi(\beta, \theta) = \lim_{n\to\infty}\Sigma_{i=1}^n\alpha_i(q, Q)\,\xi(A_i).$$

But the simple functions were chosen so that the respective measures $\Sigma_{i=1}^n\alpha_i(q, Q)\,\xi(A_i)$ are $\mathcal{B}(\Omega\times[0,\infty])$-measurable. Since the sum and pointwise limit of measurable functions are measurable, we get that $\sigma_{1,k}$ being the pointwise limit of sums of $\mathcal{B}(\Omega\times[0,\infty])$-measurable functions, is $\mathcal{B}(\Omega\times[0,\infty])$-measurable. By the same argument $\sigma_{2,k}(q, Q)$ is $\mathcal{B}(\Omega\times[0,\infty])$-measurable, for all $(q, Q)\in\Omega\times[0,\infty]$. $\diamond$

Claim 3. *Given any $\mathcal{B}(\Omega\times[0,\infty])$-measurable σ_k, the rival supplier's $\mathcal{B}(\Omega\times I)$-measurable strategy G'_k, a probability space $(I, \mathcal{B}(I), \upsilon)$, and $\mathcal{B}(\Omega\times I)$-measurable continuation value functions V, a supplier best response correspondence $\varphi : \Omega\times I\rightrightarrows\mathcal{G}$ is $\mathcal{B}(\Omega\times I)$-measurable that admits a measurable selector.*

Proof of Claim 3. Without loss of generality, consider supplier A's optimization problem. Since supplier A's objective function (1) is linear (hence continuous) in his own mixed strategy G_k the rival's strategy G'_k, the customer's σ_k, and υ_k and is $\mathcal{B}(\Omega\times I)$-measurable, it is a Carathéodory function.

Note that $\mathcal{G}_k$ is a separable metrizable space. Define the constant correspondence $\zeta : \Omega\times I\rightrightarrows\mathcal{G}$ by $\zeta\left(\omega, Q^A, Q^B\right) = \mathcal{G}_k$, for all $\left(\omega, Q^A, Q^B\right)\in\Omega\times I$. Then ζ is weakly measurable correspondence with non-empty compact and convex value. Denote by $V_k^*\left(\omega, Q^j, Q^{-j}\right)$ the solution to (1) for supplier $j\in\{A, B\}$. Then, by the Measurable Maximum Theorem[30] the argmax correspondence $\varphi : \Omega\times I\rightrightarrows\mathcal{G}_k$ defined by

$$\varphi\left(\omega, Q^A, Q^B\right) = \{G_k\in\mathcal{G}_k \mid V_k\left(\omega, Q^j, Q^{-j}\right) = V_k^*\left(\omega, Q^j, Q^{-j}\right)\}$$

is weakly measurable correspondence with non-empty compact values. Hence, supplier A's point-wise maximization problem has a solution, the maximized value function V_k is $\mathcal{B}(\Omega\times I)$-measurable, and the correspondence φ maximizer admits a measurable selector. $\diamond$

Claim 4. *The correspondence Υ_k is non-empty, compact and convex valued.*

Proof of Claim 4. Because $\Delta(\Omega)$ and $[0, 1]$ are both compact sets (in the $\mathbb{R}^n$ topology), by Tychonoff Theorem, the spaces $\Delta(\Omega)^{\Omega\times I}$ and $[0, 1]^{\Omega\times[0,\infty]}$ are compact in the product topology. Moreover, because $\mathcal{G}_k$ is closed subset of $\Delta(\Omega)^{\Omega\times I}$ it is compact in product topology.

The suppliers objective function, V_k, is linear and, hence, continuous, in G_k, σ_k and υ_k, and υ_k is linear and, hence, continuous, in G_k and σ_k. Moreover, because the domain of Υ_k is a product of measurable spaces, the objective function is $\mathcal{B}(\Omega\times I)$-measurable, and υ_k is $\mathcal{B}(I)$-measurable, they are Carathéodory functions. Hence, by the Measurable Maximum Theorem, for every given $(\omega, Q^A, Q^B)\in\Omega\times I$, the suppliers' maximization problems (1) has a solution in $\mathcal{G}_k$ and the correspondence

28 $\mathcal{B}(\Omega\times[0,\infty])$ is the restriction of $\mathcal{B}(\Omega\times I)$ to $\Omega\times[0,\infty]$.

29 See Aliprantis and Border (2006) Theorem 17.31.

30 See Aliprantis and Border (2006) Theorem 18.19.

$\vartheta : \mathcal{G}_k \times \mathcal{G}_k \times \Sigma_k \rightrightarrows M \times \Delta(I)$ has non-empty single values. Given $(q, Q) \in \Omega \times [0, \infty]$, the customers' optimization problem has a solution in $\sigma_k(q, Q) \in [0, 1]$ and, since the suppliers strategies are in $\mathcal{G}_k$, there is no prescription with probability zero. Thus, the existence of $\bar{V}_k$, $\bar{G}_k$ and $\bar{\sigma}_k$ is implied by the Measurable Maximum Theorem. Moreover, by the same theorem, $\bar{V}_k$, is $\mathcal{B}(\Omega \times I)$-measurable, $\bar{\sigma}_k$ is $\mathcal{B}(\Omega \times [0, \infty])$-measurable, and we can always select measurable maximizers $\bar{G}_k$.

Since $\bar{V}_k$, $\bar{\sigma}_k$ and $\bar{\upsilon}_k$ are single-valued, they are trivially convex and compact. The convexity of $\bar{G}_k$, is an immediate implication of the linearity of the suppliers' and customers' objective function in the maximizers distributions which implies that any convex combination of the maximizers is a maximizer. Furthermore, since the measurable selectors are the intersection of the set of maximizers which, by the Measurable Maximum Theorem are compact valued, and the space $\mathcal{G}_k^{\Omega \times I}$ of measurable functions, $\bar{G}_k$ is measurable and compact and convex valued. ◇

Claim 5. *The mapping Υ_k has a closed graph.*

Proof of Claim 5. Since $\mathcal{G}_k$ is a compact set and the set $\bar{G}_k = \mathcal{G}_k \cap \Lambda_k$, where

$$\Lambda_k := \arg \max_{\{G \in \mathcal{G} | g(\omega, Q^A, Q^B)(q) \geq \frac{1}{k}\}} \int_0^\infty [\int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau + e^{-rt'} \left(\sum_{\omega \in \Omega} V_k \left(Q^A(t'), Q^B\left(t'\right), \omega_j \right) \mu(\omega) \right)] dF\left(t'\right),$$

is the intersection of all the suppliers maximizers. By the Measurable Maximum Theorem Λ_k is compact and forms a closed graph. In addition, $\mathcal{G}_k$ also forms a closed graph. But the graph of the projection of the correspondence Υ_k on $\mathcal{G}_k$ constitutes of the intersection of the closed graphs formed by Λ_k and $\mathcal{G}_k$. Moreover, since the projections of Υ_k on $M \times \Sigma_k \times \Delta(I^2)$ are continuous functions, their graph is closed. Thus, Υ_k has a closed graph. ◇

Proof of Theorem 1. By Claims 1–3 and the Kakutani–Fan–Glicksberg fixed point theorem, the set of fixed points of Υ_k is non-empty and compact. Since the product set of compact sets is compact, we can construct a sequence $\left(\hat{V}_k, \hat{G}_k, \hat{\sigma}_k, \hat{\upsilon}_k\right)_{k=1}^\infty$ that has a convergent subsequence. Denote by $\left(\hat{V}, \hat{G}, \hat{\sigma}, \hat{\upsilon}\right)$ the subsequential limit point. Since it is the limit point of measurable functions, it is measurable.

Let $\{k_n \mid n = 1, 2 \ldots\}$ be a convergent subsequence and consider supplier j. Given $\left(\hat{V}_{k_n}, \hat{G}_{k_n}, \hat{\sigma}_{k_n}\right)$, for all $G_{k_n}\left(\omega, Q^j, Q^{-j}\right)$ we have

$$\begin{aligned}
\Phi_{k_n}^j &:= \int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau \\
&+ \Sigma_{q \in \Omega_{\omega_i}} \int_0^\infty e^{-rt'} \left[\Sigma_{\omega \in \Omega} \hat{V}_{k_n} \left(Q^j(t'), Q^{-j}\left(t'\right), \omega \right) \mu(\omega) \right] \\
&\times dF\left(t'\right) \hat{g}_{k_n}\left(\omega, Q^j, Q^{-j}\right)(q) \\
&\geq \Sigma_{q \in \Omega_{\omega_i}} \int_0^\infty e^{-rt'} \left[\Sigma_{\omega \in \Omega} \hat{V}_{k_n} \left(Q^j(t'), Q^{-j}\left(t'\right), \omega \right) \mu(\omega) \right] \\
&\times dF\left(t'\right) g_{k_n}\left(\omega, Q^j, Q^{-j}\right)(q) \\
&+ \int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau := \phi_{k_n}^j
\end{aligned}$$

for all k_n, $n \in \mathbb{N}$. Hence, $\lim_{n\to\infty} \Phi_{k_n}^j \geq \lim_{n\to\infty} \phi_{k_n}^j$. Let $\lim_{n\to\infty} \hat{G}_{k_n} = \hat{G}$, $\lim_{n\to\infty} \hat{\sigma}_{k_n} = \hat{\sigma}$. Then,

$$\begin{aligned}
\lim_{n\to\infty} \Phi_{k_n}^j &= \int_0^{\min(\hat{Q}^j, t')} e^{-r\tau} d\tau \\
&+ \Sigma_{q \in \Omega_{\omega_i}} \int_0^\infty e^{-rt'} \left[\Sigma_{\omega \in \Omega} \hat{V} \left(Q^j(t'), Q^{-j}\left(t'\right), \omega \right) \mu(\omega) \right] \\
&\times dF\left(t'\right) \hat{g}\left(\omega, Q^j, Q^{-j}\right)(q),
\end{aligned}$$

where $\hat{V}\left(Q^j(t'), Q^{-j}\left(t'\right), \omega\right) = \hat{V}\left(\hat{G}\left(\omega, Q^j, Q^{-j}\right), \hat{G}\left(\omega, Q^{-j}, Q^j\right), \hat{\sigma} \mid Q^j\left(t'\right), Q^{-j}\left(t'\right), \omega\right)$. Note also that $\lim_{n\to\infty} \phi_{k_n}^j$ is the value function of player j of the strategy $G\left(\omega, Q^j, Q^{-j}\right)$ when player $-j$ and the customer play the limit strategies $\hat{G}\left(\omega, Q^{-j}, Q^j\right)$ and $\hat{\sigma}$, respectively, given the continuation function $\hat{V}^A$. Thus, $\lim_{n\to\infty} \Phi_{k_n}^A \geq \lim_{n\to\infty} \phi_{k_n}^A$ implies that $\hat{G}^A$ is best response to $\hat{G}^B$ and $\hat{\sigma}$, given the continuation function $\hat{V}$. That $\hat{\sigma}$ is best response to $\hat{G}\left(\omega, Q^j, Q^{-j}\right)$, $j \in \{A, B\}$ is obvious.

Finally, by definition, $\hat{V}$ is the value function corresponding to the strategies $\left(\hat{G}, \hat{\sigma}\right)$. ■

6.5. Proof of Theorem 2

We need to show that, for some stage game $\Gamma\left(\omega, Q^A, Q^B\right)$, $\hat{G}\left(\omega, Q^j, Q^{-j}\right) = \delta_\omega$ is not a best response to $\hat{G}\left(\omega, Q^{-j}, Q^j\right) = \delta_\omega$, for some $j \in \{A, B\}$.

Suppose that $Q^A = 0 < Q^B$. In fraud-free equilibrium the customers believe that both suppliers prescribe the necessary service truthfully. Hence, the only reason to obtain a second prescription is the expectation that the second supplier has a sufficiently shorter queue that would justify bearing the cost of obtaining a second prescription. Let the state be ω_i and suppose that the long-queue supplier prescribes truthfully, (that is, $q_B = \omega_i$). We show that if Q^B is sufficiently large then prescribing ω_i is not a best response of the short-queue supplier.

To begin with, observe that if the customer visits the short-queue supplier first then, because supplier B's queue cannot possibly be shorter than $Q^A = 0$, the customer never seeks a second prescription.

The probability of a new customer accepting the prescription ω_i of the long-queue supplier is as follows: If the long-queue supplier (that is, supplier B) is the customer's first call then the probability of acceptance is:

$$p_1\left(Q^B\right) := \xi\{(\theta, \beta) \in T \mid e^{-\beta Q^B} \geq E\left[e^{-\beta Q} \mid Q^B\right] - \theta\},$$

where $E\left[e^{-\beta Q} \mid Q^B\right] = \int_0^\infty e^{-\beta Q} \upsilon^*\left(Q \mid Q^B\right) dQ$ and $\upsilon^*\left(Q \mid Q^B\right)$ is the equilibrium distribution of supplier A's queue conditional on Q^B. Note that $p_1\left(Q^B\right)$ is independent of the prescription, q_A, of the short-queue supplier.

Suppose that the customer visits the long-queue supplier first and decides to seek a second prescription. Suppose further that the short-queue supplier prescribes $q_A \in \Omega_{\omega_i}$. The customer will return to the long-queue supplier if and only if

$$p_2\left(q_A, \omega_i, Q^B\right) := \Pr\{\beta \in [0, 1] \mid 1 - q_A < (1 - \omega_i) e^{-\beta Q^B}\}.$$

Define $p_B\left(q_A, \omega_i, Q^B\right) = p_1\left(Q^B\right) + p_2\left(q_A, \omega_i, Q^B\right)$. Since $1 - \omega_i > (1 - \omega_i) e^{-\beta Q^B}$ for all $\beta \in (0, 1]$, $q_A = \omega_i$ implies that $p_2\left(q_A, \omega_i, Q^B\right) = 0$. Thus, $p_B\left(q_A, \omega_i, Q^B\right) = p_1\left(Q^B\right)$. Hence, the short-queue supplier's payoff if he prescribes $q_A = \omega_i$ is:

$$R(\omega_i) = \left(1 - p_1\left(Q^B\right)\right) V\left(\omega_i, 0, Q^B\right) + p_1\left(Q^B\right) V\left(\omega_i, 0, Q^B + \omega_i\right)$$

and if he prescribes $q_A = \omega_{i+1}$ the short-queue supplier's payoff is:

$$\begin{aligned}
R(\omega_{i+1}) &= \left(1 - p_B\left(\omega_{i+1}, \omega_i, Q^B\right)\right) V\left(\omega_{i+1}, 0, Q^B\right) \\
&+ p_B\left(\omega_{i+1}, \omega_i, Q^B\right) V\left(\omega_i, 0, Q^B + \omega_i\right).
\end{aligned}$$

Hence,

$$R(\omega_{i+1}) - R(\omega_i) =$$
$$\left(1-p_1\left(Q^B\right)\right)\left(V\left(\omega_{i+1},0,Q^B\right)-V\left(\omega_i,0,Q^B\right)\right)-$$
$$p_2\left(\omega_{i+1},\omega_i,Q^B\right)\left(V\left(\omega_{i+1},0,Q^B\right)-V\left(0,0,Q^B+\omega_i\right)\right)$$
$$=\left(1-p_1\left(Q^B\right)\right)\left(1-F\left(\omega_{i+1}\right)\right)\int_{\omega_i}^{\omega_{i+1}}e^{-r\tau}d\tau$$
$$+\int_0^{\omega_{i+1}}e^{-rt'}\Sigma_{\omega'\in\Omega}[V(\omega',\omega_{i+1}-t',Q^B-t')$$
$$-V(\omega',\omega_i-t',Q^B-t')]\mu\left(\omega'\right)dF\left(t'\right)$$
$$-\ p_2\left(\omega_{i+1},\omega_i,Q^B\right)\left(V\left(\omega_{i+1},0,Q^B\right)-V\left(\omega_i,0,Q^B+\omega_i\right)\right).$$

But $\lim_{Q^B\to\infty}p_2\left(\omega_{i+1},\omega_i,Q^B\right)=0$ and $\lim_{Q^B\to\infty}p_1\left(Q^B\right)=0$. Hence, in the limit as $Q^B\to\infty$, $R(\omega_{i+1})-R(\omega_i)>0$. Thus, there is N such that, for all $Q^B>N$, $\hat{G}\left(\omega_i,0,Q^B\right)=\delta_{\omega_i}$ is not best response to $\hat{G}\left(\omega_i,Q^B,0\right)=\delta_{\omega_i}$.■

Appendix

Let $\Omega=\{\omega_L,\omega_H\}$, where $\omega_H>\omega_L$, and consider situations in which the true state is ω_L. Let (6) depict the payoff matrix corresponding to the stage game $\Gamma\left(\omega,Q^A,Q^B\right)$.

Example 1. Consider the symmetric stage game $\Gamma\left(\omega_L,Q^A,Q^B\right)$, where $Q^A=Q^B=0$. In this case $\alpha_j\left(\omega_k,\omega_k\right)=1/2$, and $U^A_{kk}=U^B_{kk}$, $j\in\{A,B\}$, $k\in\{H,L\}$. Moreover, $U^B_{HH}-U^B_{LH}=U^A_{HH}-U^A_{HL}$ and $U^B_{LL}-U^B_{HL}=U^A_{LL}-U^A_{LH}$.

$$U^B_{HH}-U^B_{LH}=\left(\alpha_A\left(\omega_H,\omega_H\right)-\alpha_A\left(\omega_L,\omega_H\right)\right)\left(\bar{V}\left(\omega_H,0\right)-\bar{V}\left(0,\omega_H\right)\right)+ \tag{10}$$
$$\left(1-\alpha_A\left(\omega_L,\omega_H\right)\right)\left(\bar{V}\left(0,\omega_H\right)-\bar{V}\left(0,\omega_L\right)\right),$$

where $\bar{V}\left(Q^A,Q^B\right):=\Sigma_{\omega\in\Omega}\left(V\left(\omega,Q^A,Q^B\right)\right)\mu\left(\omega\right)$.

Since $\Sigma_{\omega\in\Omega}\left(V\left(\omega,\omega_H,0\right)-V\left(\omega,0,\omega_H\right)\right)\mu\left(\omega\right)<0$, $\alpha_A\left(\omega_H,\omega_H\right)-\alpha_A\left(\omega_L,\omega_H\right)>0$, and $\Sigma_{\omega\in\Omega}\left(V\left(\omega,0,\omega_H\right)-V\left(\omega,0,\omega_L\right)\right)\mu\left(\omega\right)>0$, the sign of the first term is negative and that of the second term is positive. Thus, in general, the sign of $U^B_{HH}-U^B_{LH}$ is ambiguous. More specifically, since $\alpha_A\left(\omega_H,\omega_H\right)=0.5$,

$$U^B_{HH}-U^B_{LH}\geq(<)\,0\Longleftrightarrow\frac{0.5-\alpha_A\left(\omega_L,\omega_H\right)}{1-\alpha_A\left(\omega_L,\omega_H\right)}\leq(>)\,\frac{\bar{V}\left(0,\omega_H\right)-\bar{V}\left(0,\omega_L\right)}{\bar{V}\left(\omega_H,0\right)-\bar{V}\left(0,\omega_H\right)}.$$

Consider next

$$U^B_{LL}-U^B_{HL}=\left(\alpha_A\left(\omega_L,\omega_L\right)-\alpha_A\left(\omega_H,\omega_L\right)\right)\left(\bar{V}\left(\omega_L,0\right)-\bar{V}\left(0,\omega_L\right)\right)+ \tag{11}$$
$$\left(1-\alpha_A\left(\omega_H,\omega_L\right)\right)\left(\bar{V}\left(0,\omega_L\right)-\bar{V}\left(0,\omega_H\right)\right).$$

Since $\Sigma_{\omega\in\Omega}\left(V\left(\omega,\omega_L,0\right)-V\left(\omega,0,\omega_L\right)\right)\mu\left(\omega\right)<0$, $\alpha_A\left(\omega_L,\omega_L\right)-\alpha_A\left(\omega_H,\omega_L\right)<0$ and $\Sigma_{\omega\in\Omega}\left(V\left(\omega,0,\omega_L\right)-V\left(\omega,0,\omega_H\right)\right)\mu\left(\omega\right)<0$, the sign of the first term is positive and that of the second term is negative. Thus, the sign of $U^B_{LL}-U^B_{HL}$ is ambiguous. More specifically,

$$U^B_{LL}-U^B_{HL}\geq(<)\,0\Longleftrightarrow\frac{\alpha_A\left(\omega_H,\omega_L\right)-0.5}{1-\alpha_A\left(\omega_H,\omega_L\right)}\leq(>)\,\frac{\bar{V}\left(0,\omega_L\right)-\bar{V}\left(0,\omega_H\right)}{\bar{V}\left(\omega_L,0\right)-\bar{V}\left(0,\omega_L\right)}.$$

Observe that if B prescribes ω_L and A prescribes ω_H then A will only get the customers that visit him first and do not seek a second prescription. Thus, $\alpha_A\left(\omega_L,\omega_H\right)=\xi\{(\theta,\beta)\in T\mid(1-\omega_H)>u_r^{(\theta,\beta)}\left(\omega_H,Q^A\right)-\theta\}$. By the same logic, if B prescribes ω_H and A prescribes ω_L then A will get the customers that visit him first and all the customers that visit B first and seek a second prescription. Thus, $\alpha_A\left(\omega_H,\omega_L\right)=0.5+\xi\{(\theta,\beta)\in T\mid(1-\omega_H)>u_r^{(\theta,\beta)}\left(\omega_H,Q^B\right)-\theta\}$. Since $Q^A=Q^B$, we get that $0.5-\alpha_A\left(\omega_L,\omega_H\right)=\alpha_A\left(\omega_H,\omega_L\right)-0.5$, or $\alpha_A\left(\omega_L,\omega_H\right)+\alpha_A\left(\omega_H,\omega_L\right)=1$.

Consequently, depending on the configurations of the signs of these expressions we may have the following equilibria.

If $U^B_{LL}-U^B_{HL}=U^A_{LL}-U^A_{LH}>0$ and $U^B_{HH}-U^B_{LH}=U^A_{HH}-U^A_{HL}>0$ then $\left(q^*_A=\omega_L,q^*_B=\omega_L\right)$ and $\left(q^*_A=\omega_H,q^*_B=\omega_H\right)$ are pure strategy equilibria in which either both suppliers prescribe truthfully or both commit fraud.

If $U^B_{HH}-U^B_{LH}=U^A_{HH}-U^A_{HL}>0$ and $U^B_{LL}-U^B_{HL}=U^A_{LL}-U^A_{LH}<0$ then $\left(q^*_A=\omega_H,q^*_B=\omega_H\right)$ is the unique, pure strategy, equilibrium in which both suppliers commit fraud.

If $U^B_{LL}-U^B_{HL}=U^A_{LL}-U^A_{LH}>0$ and $U^B_{HH}-U^B_{LH}=U^A_{HH}-U^A_{HL}<0$ then there is a unique, pure strategy, equilibrium $\left(q^*_A=\omega_H,q^*_B=\omega_H\right)$ in which the two suppliers prescribe truthfully.

If $U^B_{LL}-U^B_{HL}=U^A_{LL}-U^A_{LH}<0$ and $U^B_{HH}-U^B_{LH}=U^A_{HH}-U^A_{HL}<0$ then there are two pure strategy equilibria $\left(q^*_A=\omega_H,q^*_B=\omega_L\right)$ and $\left(q^*_A=\omega_L,q^*_B=\omega_H\right)$ in which one supplier prescribes truthfully and the other overprescribes.

If $\left(U^B_{LL}-U^B_{HL}\right)/\left(U^B_{HH}-U^B_{LH}\right)\in(0,1)$ then there is a symmetric mixed strategy equilibrium in which each supplier overprescribes service with probability 0.5.

The same logic applies to all symmetric situations (that is, for all $Q^A=Q^B$).

Example 2. Consider the asymmetric case where the state is $\left(\omega_L,Q^A,Q^B\right)$, where $Q^A<Q^B$. By Theorem 2, in pure-strategy equilibria, $q^*_A\geq q^*_B$. Hence, in pure-strategy equilibrium the following case may arise: both suppliers overprescribe services, both suppliers prescribe truthfully, the long-queue supplier prescribes truthfully and the short queue supplier prescribes unnecessary service.

A mixed strategy equilibrium requires that $\left(U^j_{LL}-U^j_{HL}\right)/\left(U^j_{HH}-U^j_{LH}\right)\in(0,1)$, $j\in\{A,B\}$. Thus, $U^j_{LL}-U^j_{HL}$ and $U^j_{HH}-U^j_{LH}$ must be of the same sign. Moreover, letting $\Delta\bar{V}(Q^A,Q^B)\mu\left(\omega\right):=\bar{V}\left(Q^A+\omega_H,Q^B\right)-\bar{V}\left(Q^A+\omega_L,Q^B\right)$

$$\frac{U^A_{LL}-U^A_{LH}}{U^A_{HH}-U^A_{HL}}=\frac{\left[\alpha_A\left(\omega_L,\omega_L\right)-\alpha_A\left(\omega_L,\omega_H\right)\right]}{\left[\alpha_A\left(\omega_H,\omega_H\right)-\alpha_A\left(\omega_H,\omega_L\right)\right]}\times$$
$$\frac{\left[\bar{V}\left(Q^A+\omega_L,Q^B\right)-\bar{V}\left(Q^A,Q^B+\omega_L\right)\right]-\alpha_A\left(\omega_L,\omega_H\right)\Delta V(Q^A,Q^B)}{\left[\bar{V}\left(Q^A+\omega_H,Q^B\right)-\bar{V}\left(Q^A,Q^B+\omega_H\right)\right]+\alpha_A\left(\omega_H,\omega_L\right)\Delta V(Q^A,Q^B)}$$

and letting $\Delta\bar{V}^B\left(Q^A,Q^B\right):=\bar{V}\left(Q^A,Q^B+\omega_H\right)-\bar{V}\left(Q^A,Q^B+\omega_L\right)$

$$\frac{U^B_{LL}-U^B_{HL}}{U^B_{HH}-U^B_{LH}}=\frac{\left[\alpha_B\left(\omega_L,\omega_L\right)-\alpha_B\left(\omega_H,\omega_L\right)\right]}{\left[\alpha_B\left(\omega_H,\omega_H\right)-\alpha_B\left(\omega_L,\omega_H\right)\right]}\times$$
$$\frac{\left[\bar{V}\left(Q^A,Q^B+\omega_L\right)-\bar{V}\left(Q^A+\omega_L,Q^B\right)\right]-\alpha_B\left(\omega_H,\omega_L\right)\Delta\bar{V}(Q^A,Q^B)}{\left[\bar{V}\left(Q^A,Q^B+\omega_H\right)-\bar{V}\left(Q^A+\omega_H,Q^B\right)\right]+\alpha_B\left(\omega_L,\omega_H\right)\Delta\bar{V}(Q^A,Q^B)}.$$

If $Q^A<Q^B$ then, by decreasing marginal value of the queues,

$$\bar{V}\left(Q^A+\omega_H,Q^B\right)-\bar{V}\left(Q^A,Q^B+\omega_H\right)$$
$$>\bar{V}\left(Q^A+\omega_H,Q^B\right)-\bar{V}\left(Q^A,Q^B+\omega_L\right)$$
$$\bar{V}\left(Q^A+\omega_L,Q^B\right)-\bar{V}\left(Q^A,Q^B+\omega_L\right)$$
$$>\bar{V}\left(Q^A,Q^B+\omega_L\right)-\bar{V}\left(Q^A+\omega_L,Q^B\right)$$
$$\bar{V}\left(Q^A+\omega_H,Q^B\right)-\bar{V}\left(Q^A,Q^B+\omega_H\right)$$
$$>\bar{V}\left(Q^A,Q^B+\omega_H\right)-\bar{V}\left(Q^A+\omega_H,Q^B\right).$$

Furthermore, $\alpha_A(\omega_L, \omega_H) > \alpha_B(\omega_H, \omega_L)$.

$\alpha_A(\omega_L, \omega_L) - \alpha_A(\omega_L, \omega_H) > \alpha_B(\omega_L, \omega_L) - \alpha_B(\omega_H, \omega_L) > 0$

and

$\alpha_A(\omega_H, \omega_H) - \alpha_A(\omega_H, \omega_L) < \alpha_B(\omega_H, \omega_H) - \alpha_B(\omega_L, \omega_H) < 0.$

Thus, if $U^j_{LL} - U^j_{HL}$ and $U^j_{HH} - U^j_{LH}$, $j \in \{A, B\}$ are of the same sign, then

$$\frac{U^A_{LL} - U^A_{LH}}{U^A_{HH} - U^A_{HL}} < \frac{U^B_{LL} - U^B_{HL}}{U^B_{HH} - U^B_{LH}}.$$

Hence, $x > 0.5 > y$. This means that the mixed strategy of the short-queue supplier first-order stochastically dominates that of the long-queue supplier. Thus, in mixed strategy equilibrium the short-queue supplier is more likely to commit fraud than the long-queue supplier.

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