Currency Wars, Trade Wars, and Global Demand

Olivier Jeanne
Johns Hopkins University*

January 2021

Abstract

This paper presents a tractable model of a global economy in which national social planners can use a broad range of policy instruments—the nominal interest rate, taxes on imports and exports, taxes on capital flows or foreign exchange interventions. Low demand may lead to unemployment because of downward nominal wage stickiness. Markov perfect equilibria with and without international cooperation are characterized in closed form. The welfare costs of trade and currency wars crucially depend on the state of global demand and on the policy instruments that are used by national social planners. National social planners have more incentives to deviate from free trade when global demand is low.

*Olivier Jeanne: Department of Economics, Johns Hopkins University, 3400 N.Charles Street, Baltimore MD 21218; email: ojeanne@jhu.edu. I thank seminar participants, especially Alberto Martin, Emmanuel Farhi, Charles Engel, Dmitry Mukhin and Oleg Itskhoki, at the NBER Conference on Capital Flows, Currency Wars and Monetary Policy (April 2018), the 2019 ASSA meeting, the Board of the US Federal Reserve System, the University of Maryland, the University of Virginia, Georgia State University, the University of Southern California, the Federal Reserve of New York, the University of Wisconsin and the Graduate Institute in Geneva for their comments.

1 Introduction

Countries have regularly accused each other of being aggressors in a currency war since the global financial crisis. Guido Mantega, Brazil's finance minister, in 2010 blamed the US for launching a currency war through quantitative easing and a lower dollar.[1] At the time Brazil itself was trying to hold its currency down by accumulating reserves and by imposing a tax on capital inflows. Many countries, including advanced economies such as Switzerland, have depreciated or resisted the appreciation of their currency by resorting to foreign exchange interventions. The term "currency war" was again used when the Japanese yen depreciated in 2013 after the Bank of Japan increased its inflation target as part of the Abenomics stimulus (and more recently when it reduced the interest rate to a negative level). Bergsten and Gagnon (2012) proposed that the US undertake countervailing currency intervention against countries that manipulate their currencies, or tax the earnings on the dollar assets of these countries. After 2016 the US administration justified the introduction of tariffs by the fact that countries such as China were manipulating their currencies.

The conventional wisdom in the official sector, echoed in Bernanke (2017) or Blanchard (2017), is that depreciations should not raise concerns as long as they are the by-product, rather than the main objective, of monetary stimulus. Other authors, e.g., Mishra and Rajan (2016), find the international spillovers from monetary and exchange rate policies less benign and advocate enhanced international coordination to limit the effects of these spillovers.

The concepts of currency war and trade war are old ones but we do not have many models to analyze these wars, separately or as concurrent phenomena (more on this in the discussion of the literature below). One feature of the real world that such a model should capture is the multiplicity of policy instruments that are used achieve apparently similar outcomes. For example, a currency can be depreciated by lowering the interest rate, by raising the inflation target, by taxing capital inflows, or by accumulating foreign exchange reserves. Similarly, the demand for home goods can be increased by depreciating the home currency, by taxing imports or by subsidizing exports. Presumably, the case for international cooperation should depend on the international spillovers associated with each of these policy instruments. Another desirable feature of a model is that it should not assume that countries are committed to policy rules. Trade and currency wars

[1] "We're in the midst of an international currency war, a general weakening of currency. This threatens us because it takes away our competitiveness." as reported by Martin Wolf in "Currencies Clash in New Age of Beggar-thy-Neighbor," Financial Times September 28, 2010.

seem to be deviations from the policy rules that are applied in normal times.

In this paper I present a simple model with these features. I consider a symmetric world with many identical countries, each one producing its own good like in Gali and Monacelli (2005). There is downward nominal stickiness in wages like in Schmitt-Grohé and Uribe (2016).[2] This assumption implies that each country is either in a classical regime with full employment and flexible wages or in a Keynesian regime with unemployment and fixed wages. I assume that each country can use four policy instruments to affect home welfare: the nominal interest rate, a tax on capital flows, a tariff on imports and a subsidy on exports. The tax on capital flows can be interpreted as foreign exchange intervention. National social planners can use all or a subset of these instruments without being able to commit to any future policy action. In particular, there is no Taylor rule and monetary policy is discretionary.

The main qualities that I look for in the model are tractability and analytical clarity. I solve for the Markov perfect equilibria in which national social planners set policies so as to maximize home welfare taking the global economic and financial conditions as given. A first-order approximation allows me to derive easily interpretable closed-form expressions for the equilibrium policies under different assumptions about the available policy instruments. The equilibrium under international policy cooperation is characterized by assuming that the policies of the representative country are set by a global social planner maximizing global welfare. The model can be used to quantify the welfare cost of currency and trade wars.

From the perspective of a small open economy, the welfare-maximizing policy mix crucially depends on whether monetary policy is or not at the zero lower bound. If monetary policy is unconstrained, there is full employment and the economy is in the classical regime. A tariff on imports is then equivalent to a tax on exports, i.e., the Lerner symmetry theorem applies. National social planners then use the trade taxes to manipulate the terms of trade like in the textbook tariff war.

The analysis is very different if the home economy is in a liquidity trap, i.e., the zero lower bound is binding and there is unemployment. Now the economy is in the Keynesian regime, and the primary objective of trade policy is to raise home employment rather than the home terms of trade. There no longer is Lerner symmetry because the social planner raises home employment by moving the trade

[2]There is considerable evidence (reviewed by these authors among others) that wages are more rigid downward than upward. The fact that wages were more rigid than prices during the Great Depression is well documented (see, e.g., Eichengreen, 1992).

taxes in opposite directions, i.e., by taxing imports or *subsidizing* exports. National social planners can also raise employment by taxing capital inflows or accumulating foreign exchange reserves, which in both cases depreciates the home currency.

At a superficial level, all these policies are beggar-thy-neighbor, in the sense that a national social planner can raise home welfare only at the cost of lowering welfare in the rest of the world. However, the model shows that the case for international cooperation crucially depends on the policy instruments and on the state of global demand.

First, whether or not there is a global liquidity trap, there is no benefit from international coordination of interest rates or inflation targets. A monetary stimulus is beggar-thy-neighbor in partial equilibrium but it is a positive sum game in general equilibrium. If there is unemployment, a global monetary stimulus, if feasible, always raises global employment and welfare.

Second, the case for coordinating trade policies depends on the policy instruments. If global demand is high and there is full employment, it is optimal to set the trade taxes to zero, like in the classical tariff war. When demand is low and the global economy is in a liquidity trap, there is a case for international cooperation to avoid a tariff war but not to prevent the use of export subsidies. Because the liquidity trap is a transitory state, the tariffs act as an intertemporal tax on consumption which further reduces demand and increases unemployment. The welfare impact of a tariff war can be substantial, possibly doubling the unemployment rate under plausible calibrations of the model. The uncoordinated use of tariffs on imports can also give rise to self-fulfilling global liquidity traps as tariffs lower the global natural rate of interest. The outcome of a trade war is quite different if countries use subsidies on exports instead of tariffs on imports. An export subsidy acts as an intertemporal subsidy on consumption and so stimulates consumption. In the Nash equilibrium with export subsidies, full employment is achieved and the benefits from international coordination, although non-zero, are much lower.

Third, setting a tax on capital flows is a zero-sum game that simply transfers welfare from the rest of the world to the country imposing capital controls. Thus a capital wars leading to a symmetric Nash equilibrium leave welfare unchanged and there are no net gains from international cooperation.

Based on this analysis, I also consider the incentives of national social planners to deviate from free trade and impose tariffs assuming that a deviation may trigger a generalized trade war. Again, the incentives to deviate from free trade crucially depend on the state of global demand. The incentives to deviate from free trade are little affected by global demand if there is full employment. By contrast, national social planners have stronger incentives to tax imports or subsidize exports if there is a global liquidity trap with unemployment. Low global demand is conducive to trade wars

Literature. There is a long line of literature on international monetary coordination—see e.g. Engel (2016) for a review. The case for international monetary cooperation in New Open Macro models was studied by Obstfeld and Rogoff (2002), Benigno and Benigno (2006), Canzoneri, Cumby and Diba (2005) among others. Obstfeld and Rogoff (2002) concluded that the welfare gains from international coordination of monetary policy were small.[3]

A more recent group of papers has explored the international spillovers associated with monetary policy when low natural rates of interest lead to insufficient global demand and liquidity traps including Eggertsson et al. (2016), Caballero, Farhi and Gourinchas (2015), Fujiwara et al. (2013), Devereux and Yetman (2014), Cook and Devereux (2013), and Acharya and Bengui (2018). Eggertsson et al. (2016) and Caballero, Farhi and Gourinchas (2015) study the international transmission of liquidity traps using a model that shares several features with this paper, in particular the downward nominal stickiness a la Schmitt-Grohé and Uribe (2016). Those papers assume Taylor rules for monetary policy and do not incorporate trade taxes to the analysis.[4] Fornaro and Romei (2019) present a model in which macroprudential policy has a negative effect on global demand when the monetary policy is at the zero lower bound. A more closely related contribution is Auray, Devereux and Eyquem (2020). These authors consider a smaller set of policy instruments than I do (their model does not include export taxes, capital controls of foreign exchange interventions) but look at the implications of fixed exchange rates, a topic that is only briefly touched upon in this paper. Auray, Devereux and Eyquem (2020) consider a two-country model without financial markets, so that trade balances are always equal to zero.

Other papers have explored whether the constraints on monetary policy resulting from a fixed exchange rate or the ZLB can be circumvented with fiscal instruments (Farhi, Gopinath and Itskhoki, 2014; Correia et al., 2013). Farhi, Gopinath and Itskhoki (2014) show that value added and payroll taxes used jointly with trade taxes can replicate the effects of nominal exchange rate devaluations across

[3] In a more recent contribution, Korinek (2016) gives a set of conditions under which international spillovers are efficient and policy coordination is uncalled for. The model in this paper does not satisfy these conditions—in particular the fact that countries do not have monopoly power.

[4] Corsetti et al. (2019) consider a partial equilibrium version of Eggertsson et al. (2016) and show that reaching full employment through a currency depreciation may, under certain conditions, decrease welfare.

a range of model specifications. Correia et al. (2013) study how fiscal instruments can be used to achieve the same allocations as if there were no ZLB on the nominal interest rate in a closed economy. By contrast, the model presented here assumes that the set of policy instruments is more limited.

This paper is related to the recent literature that looks at the macroeconomic impact of trade policy. Barbiero et al. (2019) study the macroeconomic consequences of a border adjustment tax in the context of a dynamic general equilibrium model with monetary policy conducted according to a conventional Taylor rule. Lindé and Pescatori (2019) study the robustness of the Lerner symmetry result in an open economy New Keynesian model and find that the macroeconomic costs of a trade war can be substantial. Erceg, Prestipino and Raffo (2017) and Barattieri, Cacciatore and Ghironi (2018) study the short-run macroeconomic effects of trade policies in a dynamic New Keynesian open-economy framework. Bénassy-Quéré, Bussière and Wibaux (2018) consider a model in which countries are more likely to resort to tariffs at the ZLB. These papers look at small open economies, whereas I focus on the international spillovers associated with monetary and trade policies in a general equilibrium model of the global economy.

The paper is also related to the literature that has quantified the welfare cost of trade wars in general equilibrium (Ossa, 2014). In this type of framework, Amiti, Redding and Weinstein (2019) and Fajgelbaum et al. (2019) find that the welfare cost of the 2018 trade war is moderate (less than 0.1 % of US GDP) but these papers do not take into account the global demand effects that I focus on in this paper.[5]

The presentation is structured as follows. The assumptions of the model are presented in section 2. We analyze the optimal policies from a small open economy perspective in section 3. The global equilibria are presented in sections 4 and 5 for high and low global demand respectively. Section 6 analyzes the country incentives to deviate from free trade. Section 7 presents dynamic extensions of the model.

2 Model

The model represents a world composed of a continuum of atomistic countries indexed by $j \in (0, 1)$ in infinite time $t = 1, 2, ...$ The goods structure is similar to Gali and Monacelli (2005). Each country produces its own good and has its own

[5] Freund et al. (2018) find a larger cost for the US-China trade war in a scenario where the trade war depresses investment.

currency. The nominal wage is rigid downwards as in Schmitt-Grohé and Uribe (2016). There is no uncertainty.

Preferences. Each country j is populated by a mass of identical consumers. The utility of the representative consumer can be written recursively,

$$U_{jt} = u\left(C_{jt}\right) + \beta_{jt} U_{jt+1},$$

where the utility function has a constant elasticity of intertemporal substitution ϵ_i,

$$u\left(C\right) = \frac{C^{1-1/\epsilon_i} - 1}{1 - 1/\epsilon_i}. \tag{1}$$

The time-varying discount factor will be used to model exogenous fluctuations in country j's demand.

The consumer consumes the good that is produced domestically (the home good) as well as a basket of foreign goods. The consumer cares about the Cobb-Douglas index,

$$C = \left(\frac{C_H}{\alpha_H}\right)^{\alpha_H} \left(\frac{C_F}{\alpha_F}\right)^{\alpha_F}, \tag{2}$$

(with $\alpha_H + \alpha_F = 1$) where C_H is the consumption of home good, and C_F is the consumption of foreign good.

The consumption of foreign good is a CES index of the goods produced in all the countries,

$$C_F = \left[\int_0^1 C_k^{(\epsilon_x - 1)/\epsilon_x} dk\right]^{\epsilon_x/(\epsilon_x - 1)}.$$

The composite good defined by this index will be called the "global good" in the following. The elasticity of substitution between foreign goods is assumed to be larger than one, $\epsilon_x > 1$.

Budget constraints. The consumers can invest in one-period bonds denominated in the global good that are traded internationally. The budget constraint for country j's representative consumer is,

$$P_{Fjt} \frac{B_{jt+1}}{R_t \left(1 + \tau_{jt}^b\right)} + P_{Hjt} C_{Hjt} + \left(1 + \tau_{jt}^m\right) P_{Fjt} C_{Fjt} = W_{jt} L_{jt} + Z_{jt} + P_{Fjt} B_{jt}, \tag{3}$$

where P_{Hjt} is the home-currency price of the home good, P_{Fjt} is the offshore domestic-currency price of the global good, τ_t^m is a tax on imports, τ_t^b is a tax on foreign borrowing, B_{jt} is the payoff on the consumer's holdings of real bonds in period t, R_t is the gross real interest rate in terms of the global good, L_{jt} is the quantity of labor supplied by the consumer (at nominal wage W_{jt}), and Z_{jt} is a lump-sum transfer from the government. Home currency nominal bonds can be traded but the supply of these bonds is equal to zero and they have been omitted from the budget constraints. A version of the model with money and nominal bonds is presented in Appendix A.

Production and labor market. The home good is produced with a linear production function that transforms one unit of labor into one unit of good, $Y = L$. The representative consumer is endowed with a fixed quantity of labor $\overline{L}$ and the quantity of employed labor satisfies

$$L_{jt} \leq \overline{L}. \tag{4}$$

There is full employment if this constraint is satisfied as an equality. We normalize $\overline{L}$ to 1.

The wage inflation rate is denoted by π_{jt},

$$1 + \pi_{jt} = \frac{W_{jt}}{W_{jt-1}}.$$

The linearity in production implies $P_{Hjt} = W_{jt}$ so that π_{jt} is the inflation rate in the price of the home good.

We assume that the nominal wage is sticky downward like in Schmitt-Grohé and Uribe (2016) or Eggertsson et al. (2016). Downward nominal stickiness in the wage is captured by the constraint,[6]

$$\pi_{jt} \geq 0. \tag{5}$$

The economy can then be in two regimes: full employment ($L_{jt} = \overline{L}$), or less than full employment, in which case wage inflation is at its lower bound ($L_{jt} < \overline{L}$ and $\pi_{jt} = 0$). This leads to a L-shaped Phillips curve where inflation can be set independently of employment once there is full employment. The constraints on the labor market can be summarized by (4), (5) and

$$\left(\overline{L} - L_{jt}\right) \pi_{jt} = 0. \tag{6}$$

Demand for home labor. The period-t demand for home labor is,

$$L_{jt} = C_{Hjt} + \left[\left(1 + \tau^x_{jt}\right) \frac{P_{Hjt}}{P_{Fjt}}\right]^{-\epsilon_x} C^W_{Ft}, \tag{7}$$

[6] In general the minimum inflation rate could be negative. I assume that it is equal to zero to alleviate the notations.

where $C_F^W = \int C_{Fk} dk$ denotes global gross imports and τ_{jt}^x is the tax on exports. The first term on the right-hand side of (7) is the labor used to serve home demand for the home good and the second term is the labor used to produce exports.

It will be convenient to define three terms of trade,

$$S_{jt} \equiv \frac{P_{Hjt}}{P_{Fjt}}, \; S_{jt}^m \equiv \frac{S_{jt}}{1+\tau_{jt}^m} \text{ and } S_{jt}^x \equiv \left(1+\tau_{jt}^x\right) S_{jt}, \tag{8}$$

where S_{jt} denotes the period-t undistorted terms of trade, and S_{jt}^m and S_{jt}^x are the tax-distorted terms of trade that are relevant for imports and exports respectively.

Given the Cobb-Douglas specification (2) the home demand for the home good and for imports are respectively given by,

$$C_{Hjt} = \alpha_H \left(S_{jt}^m\right)^{-\alpha_F} C_{jt}, \tag{9}$$

$$C_{Fjt} = \alpha_F \left(S_{jt}^m\right)^{\alpha_H} C_{jt}. \tag{10}$$

The demand for home labor (7) can thus be re-written as a function of the terms of trade,

$$L_{jt} = \alpha_H \left(S_{jt}^m\right)^{-\alpha_F} C_{jt} + \left(S_{jt}^x\right)^{-\epsilon_x} C_{Ft}^W. \tag{11}$$

The demand for home labor increases with home consumption and global consumptions but is reduced by a loss in the competitiveness of the home good (an increase in S_t^m or S_t^x).

Balance of payments. Using $Z_{jt} = \tau_{jt}^m P_{Fjt} C_{Fjt} + \tau_{jt}^x P_{Hjt} \left(L_{jt} - C_{Hjt}\right) - \tau_{jt}^b P_{Fjt} B_{jt+1} / \left(1+\tau_{jt}^b\right)$, equations (7), and (10) to substitute out Z_{jt}, L_{jt}, and C_{Fjt} from the representative consumer's budget constraint (3) gives the balance of payments equation

$$\frac{B_{jt+1}}{R_t} = B_{jt} + X_{jt}, \tag{12}$$

where net exports in terms of global good are given by

$$X_{jt} = \left(S_{jt}^x\right)^{1-\epsilon_x} C_{Ft}^W - \alpha_F \left(S_{jt}^m\right)^{\alpha_H} C_{jt}. \tag{13}$$

The value of gross exports in terms of the global good decreases if the country loses competitiveness in foreign markets (an increase in S^x) because the export elasticity is larger than 1.

Equilibrium conditions. The first-order conditions for the consumer are derived in Appendix A. In the model of Appendix A we assume that the consumers can invest in nominal government bonds that yield a nominal interest rate i_{jt}. The nominal interest rate will be the instrument of monetary policy. Arbitrage between real and nominal bonds implies

$$R_t \left(1 + \tau_{jt}^b\right) = \left(1 + i_{jt}\right) \frac{P_{Fjt}}{P_{Fjt+1}}. \tag{14}$$

The left-hand-side is the global good own rate of interest at home, which is equal to the foreign level plus the tax on capital inflows. The right-hand side is the same real interest rate expressed through the Fisher relationship. Using $P_{Fjt} = P_{Hjt}/S_{jt}$, and $P_{Hjt+1}/P_{Hjt} = 1 + \pi_{jt+1}$, one obtains an expression for the first period terms of trade,

$$S_{jt} = \frac{1 + i_{jt}}{R_t \left(1 + \tau_{jt}^b\right) \left(1 + \pi_{jt+1}\right)} S_{jt+1}. \tag{15}$$

The terms of trade are increased (the currency appreciated in real terms) by an increase in the nominal interest rate or a decrease in the tax on capital flows. The interest rate and capital controls are alternative instruments of exchange rate policy.

The other relevant equilibrium condition is the Euler equation for consumption,

$$u'\left(C_{jt}\right) \left(S_{jt}^m\right)^{\alpha_F} = \beta_{jt} \frac{1 + i_{jt}}{1 + \pi_{jt+1}} u'\left(C_{jt+1}\right) \left(S_{jt+1}^m\right)^{\alpha_F}. \tag{16}$$

The marginal utility from consuming one unit of home good in period t, on the left-hand-side, is equal to the marginal utility from that investing that unit in nominal bonds to consume more home good in period $t + 1$.

3 National Policy-Making

This section continues to take the perspective of a small open economy. We consider the problem of a national social planner (NSP) who tries to maximize domestic welfare taking the global economic environment as given. The NSP can use monetary policy, trade policy, and capital account policy.[7] The instrument of monetary policy is the nominal interest rate, which is set subject to the zero-lower-bound (ZLB) constraint $i_t \geq 0$. The instruments of trade policy are the taxes on imports and exports, τ_t^m and τ_t^x, and the instrument of capital account

[7] We do not introduce taxes or subsidies on labor, which can be used to ensure full employment in this model.

policy is the tax on capital inflows, τ_t^b. As explained later capital controls can be re-interpreted as foreign exchange interventions as in (Jeanne, 2013).

What matters for home welfare is the real allocation C_{Hjt}, C_{Fjt}, L_{jt}, X_{jt}. Inflation is not welfare relevant and we assume that it is set to an inflation target $\pi_j^* > 0$ if there is full employment.[8] Inflation, thus, can be written as the following function of employment,

$$\begin{aligned}\pi_{jt} &= \pi_j^* \text{ if } L_{jt} = \overline{L}, \\ \pi_{jt} &= 0 \text{ if } L_{jt} < \overline{L}.\end{aligned}$$

Inflation is not welfare relevant ex post but expected inflation constrains the feasible allocations because of the ZLB constraint.

The equilibrium concept that we use in the rest of the paper is that of Markov perfect equilibria. The equilibrium allocation is a function of the state, which for country j at time t is summarized by the country's beginning-of-period foreign assets, B_{jt}, and the current and future global economic conditions $\left(C_{Ft'}^W, R_{t'}\right)_{t' \geq t}$. We denote the associated policy functions with tildes, $\widetilde{C}_{Hjt}(B_{jt})$, $\widetilde{C}_{Fjt}(B_{jt})$, $\widetilde{L}_{jt}(B_{jt})$, $\widetilde{X}_{jt}(B_{jt})$, where the dependence on the global economic conditions is subsumed by the time index. In each period t, the national social planner sets the domestic policy instruments so as to maximize home welfare, taking his own future policy functions as given. The NSP cannot commit to future policies.

The mapping between policies and allocations is not simple—a given allocation can be implemented with more than one policy mix. It will be easier to characterize the equilibrium by looking for the optimal allocations rather than the optimal policies. The next part of this section characterizes the feasible allocations and the equivalence between policy instruments to achieve them. We then lay out the NSP's optimization problem.

Feasible allocations. For a given state B_{jt}, a time-t allocation C_{Hjt}, C_{Fjt}, L_{jt}, X_{jt} is *feasible* if it can be implemented with the policy instruments available to the NSP, taking the next-period policy functions $\widetilde{C}_{Hjt+1}(\cdot)$, $\widetilde{C}_{Fjt+1}(\cdot)$, $\widetilde{L}_{jt+1}(\cdot)$, $\widetilde{X}_{jt+1}(\cdot)$ as given. An allocation is ZLB-feasible if it can be implemented with a non-negative nominal interest rate.

Using $L_{jt} = C_{Hjt} + \left[S_{jt}^x\right]^{-\epsilon_x} C_{Ft}^W$ to substitute out S_{jt}^x in (13), the time-t allo-

[8]As explained in Appendix B, the inflation rate is determined by money supply when there is full employment in the model with money. We abstract from money supply in the baseline model for simplicity.

cation must satisfy,

$$X_{jt} = \left(C_{Ft}^{W}\right)^{1/\epsilon_x} \left(L_{jt} - C_{Hjt}\right)^{1-1/\epsilon_x} - C_{Fjt}. \tag{17}$$

Conversely, any allocation that satisfies this condition is feasible, i.e., there exist a policy mix $\left(i_{jt}, \tau_{jt}^{m}, \tau_{jt}^{x}, \tau_{jt}^{b}\right)$ and terms of trade S_{jt} that satisfy the equilibrium conditions (9), (10), (11), (13), (15) and (16). To see this, use equations (9) and (10) to derive S_{jt}^{m} and $L_{jt} = C_{Hjt} + \left[S_{jt}^{x}\right]^{-\epsilon_x} C_{Ft}^{W}$ to derive S_{jt}^{x}. The trade taxes can then be chosen arbitrarily subject to $\left(1+\tau_{jt}^{m}\right)\left(1+\tau_{jt}^{x}\right) = S_{jt}^{x}/S_{jt}^{m}$. Observing that the next-period variables are all determined as policy functions of $B_{jt+1} = R_t\left(B_{jt} + X_{jt}\right)$, one can use equation (16) to derive i_{jt}. The capital flow tax τ_t^{b} can be chosen arbitrarily as the undistorted terms of trade S_{jt} do not matter for the real allocation (only the tax-distorted terms of trade S_{jt}^{m} and S_{jt}^{x} do). All the equilibrium conditions are satisfied, including (13) because of (17). For the allocation to be consistent with the ZLB constraint one needs to further check that the implied interest rate satisfies $i_{jt} \geq 0$.

Instrument equivalence. A feasible allocation can be implemented by more than one policy mix $\left(i_t, \tau_t^{m}, \tau_t^{x}, \tau_t^{b}\right)$. The equivalence between policy instruments is characterized by the following proposition.

Proposition 1 *(Lerner Symmetry) A feasible allocation* $(C_{Hjt_t}, C_{Fjt}, L_{jt}, X_{jt})$ *that is implemented by policy* $(i_{jt}, \tau_{jt}^{m}, \tau_{jt}^{x}, \tau_{jt}^{b})$ *can also be implemented by policy* $(i_{jt}, \widehat{\tau}_{jt}^{m}, \widehat{\tau}_{jt}^{x}, \widehat{\tau}_{jt}^{b})$ *with*

$$\left(1+\widehat{\tau}_{jt}^{m}\right)\left(1+\widehat{\tau}_{jt}^{x}\right) = \left(1+\tau_{jt}^{m}\right)\left(1+\tau_{jt}^{x}\right), \tag{18}$$

$$\left(1+\widehat{\tau}_{jt}^{m}\right)\left(1+\widehat{\tau}_{jt}^{b}\right) = \left(1+\tau_{jt}^{m}\right)\left(1+\tau_{jt}^{b}\right). \tag{19}$$

Proof. See Appendix E. ■

In words, the allocation is unchanged if the social planner shifts the tax from exports to imports and at the same time decrease the tax on capital inflows by the same amount as the tax on exports. The equivalence between a tariff on imports and a tax on exports is known as Lerner's symmetry theorem in the trade literature (Lerner, 1936).[9] Lerner symmetry may seem counterintuitive since a tax on exports makes the home good less competitive abroad whereas a tariff on imports makes it more competitive at home. However, the equivalence holds for an environment

[9] Costinot and Werning (2019) provide a number of generalizations and qualifications of the Lerner symmetry theorem in a dynamic environment.

in which the terms of trade adjust to keep home employment unchanged. For the volume of gross exports to stay the same, the decrease in the export tax must be offset by an increase in the terms of trade. The real appreciation must in turn be offset by an equivalent increase in the tariff on imports to keep the volume of imports the same. The real appreciation results from a decrease in the tax on capital inflows of the same size as the tax on exports. It cannot result from a higher interest rate because the interest rate is pinned down by the Euler condition (16).

A long-standing question in the macroeconomic and trade literature is that of the conditions under which exchange rate manipulation can replicate the impact of tariffs (Meade, 1955). This question resurfaced in recent policy debates as the US administration justified the imposition of tariffs on Chinese exports by a claim that China was manipulating its currency. In the context of our model, the question is how the allocations that are achieved by varying the trade taxes under a fixed exchange rate compare with the allocations that are achieved by varying the nominal exchange rate when there are no trade taxes.

The answer comes in two parts. On the one hand, a fixed exchange rate peg puts no constraint on the feasible allocations for a country that can use both trade taxes. To see this, let us denote by E_{jt} the period-t nominal exchange rate between the small open economy and a given foreign country, defined as the price of the foreign currency in terms of domestic currency. The law of one price implies

$$P_{Fjt} = E_{jt} P_t^*,$$

where P_t^* is the price of the global good in terms of foreign currency. The policies and allocations are consistent with the fixed exchange rate peg if they satisfy $E_{jt} = \overline{E}$. Using the definitions of S_{jt} and π_{jt}, the nominal exchange rate can be written

$$E_{jt} = \frac{1+\pi_{jt}}{S_{jt}} \frac{P_{Hjt-1}}{P_t^*}.$$

For a given inflation rate and a fixed exchange rate peg, this equation determines the level of the undistorted terms of trade S_{jt}. What matters for the real allocation, however, is not S_{jt} but the distorted terms of trade S_{jt}^m and S_{jt}^x. For given levels of E_{jt} and S_{jt} it is possible to achieve any feasible allocation by using the trade taxes τ_{jt}^m and τ_{jt}^x. Hence, all the feasible allocations can be implemented under a fixed peg if all the instruments can be used.

On the other hand, not all the feasible allocations can be implemented by varying the exchange rate when there are no trade taxes. Proposition 1 implies that the allocations achievable with trade taxes τ_t^m and τ_t^x can be replicated with no trade taxes if and only if

$$(1+\tau_t^m)(1+\tau_t^x)=1, \tag{20}$$

that is, if the trade taxes generate the same relative price distortion in foreign market as in the home market. If this condition is satisfied, any allocation achieved by a tariff under a fixed peg can be replicated by depreciating the currency with a tax on capital inflows. Importantly, the instrument of exchange rate policy must be the tax on capital flows and not the interest rate.

To summarize, there is an equivalence between exchange rate policy and trade policy if (i) trade policy introduces the same terms of trade distortion in domestic and foreign markets; and (ii) the instrument of exchange rate policy is the tax on capital flows.

We derive two more equivalence results in Appendix B. First, we show that the instrument of monetary policy could be money supply instead of the nominal interest rate. Using money supply as the instrument of monetary policy complicates the model without changing its essential properties, but it also clarifies how the social planner can determine inflation under full employment. Second, the instrument of capital account policy could be foreign exchange interventions instead of a tax on capital flows. To show this we consider a variant of the model in which the capital account is closed, i.e., only the home government can trade real bonds with foreign investors.[10] The government finances its purchase of foreign bonds by issuing domestic currency bonds to the home consumers, which can be interpreted as a sterilized foreign exchange intervention. We show that the real allocations that can be achieved in this way are the same as when the government uses a tax on capital inflows.

The NSP problem. We conclude this section by solving the NSP problem when the ZLB constraint is not binding. In this case the only constraint on the time-t allocation is (17). Using the balance-of-payments equation (12) the NSP's problem can be written in Bellman form as,

$$P_{jt}\left\{\begin{array}{l} V_{jt}\left(B_{jt}\right)=\max_{L_{jt},C_{Hjt},C_{Fjt},B_{jt+1}}\left[u\left(C\left(C_{Hjt},C_{Fjt}\right)\right)+\beta_{jt}V_{jt+1}\left(B_{jt+1}\right)\right] \\ B_{jt+1}=R_t\left[B_{jt}+\left(C_{Ft}^W\right)^{1/\epsilon_x}\left(L_{jt}-C_{Hjt}\right)^{1-1/\epsilon_x}-C_{Fjt}\right], \\ L_{jt}\leq\overline{L}. \end{array}\right.$$

[10]The assumption that there are no private capital flows is extreme but the insights remain true if frictions prevent economic agents from arbitraging the wedge between onshore and offshore interest rates.

The solution has the following properties.

Proposition 2 . *In any period t in which the ZLB constraint is not binding the NSP allocation features full employment ($L_{jt} = \overline{L}$) and the trade taxes satisfy*

$$\left(1+\tau_{jt}^{m}\right)\left(1+\tau_{jt}^{x}\right)=\frac{\epsilon_{x}}{\epsilon_{x}-1}. \tag{21}$$

Proof. See Appendix E. ■

If feasible, full employment is always optimal because, like in Schmitt-Grohé and Uribe (2016), the marginal disutility of labor is equal to zero if there is less than full employment. Once there is full employment the economy is in the flexible price regime. It is then optimal for the NSP to use the trade taxes to manipulate the home terms of trade like in the classical textbook tariff war. The NSP raises the home terms of trade by reducing the supply of home good to the rest of the world, which can be achieved by taxing exports or imports. The optimal tax wedge is decreasing with the export elasticity. Because of Lerner symmetry it does not matter whether the tax wedge is achieved through a tariff on imports or a tax on exports.

4 Global Equilibria

This section defines the decentralized equilibrium between NSPs as well as the global social planner (GSP) allocation. It also characterizes the symmetric equilibria in which the ZLB constraint is not binding.

A *decentralized NSP equilibrium* is a set of country allocations such that each NSP maximizes home welfare given the global economic conditions and the global economic conditions satisfy market clearing conditions. More formally, a decentralized NSP equilibrium consists of: (i) global economic conditions $\left(C_{Ft}^{W}, R_t\right)_{t=1,\ldots,+\infty}$; (ii) net foreign assets $\left(B_{jt}\right)_{j\in[0,1],t=1,\ldots,+\infty}$; (iii) national value and policy functions $V_{jt}(\cdot)$, $\widetilde{C}_{Hjt}(\cdot)$, $\widetilde{C}_{Fjt}(\cdot)$, $\widetilde{L}_{jt}(\cdot)$, $\widetilde{X}_{jt}(\cdot)$, and $\widetilde{\pi}_{jt}(\cdot)$ for all countries $j \in [0,1]$ and times $t = 1, 2, \ldots$ satisfying the following conditions:

• national optimization: the national value and policy functions and net foreign assets solve problem P_{jt} subject to the ZLB constraint for all countries j and times t;

• global market clearing: net foreign assets sum up to zero and the global demand for imports is the sum of national demands for all times t

$$\begin{aligned} \int B_{jt} dj &= 0, \qquad (22) \\ C_{Ft}^{W} &= \int \widetilde{C}_{Fjt}\left(B_{jt}\right) dj. \qquad (23) \end{aligned}$$

Some properties of decentralized NSP equilibria are easy to derive at this stage of the analysis. First, these equilibria inherit from Proposition 2 the property that in any country where the ZLB constraint is not binding there is full employment and the NSP set the trade taxes to manipulate the terms of trade as in equation (21). Second, integrating $X_{jt} = \left(S_{jt}^{x}\right)^{1-\epsilon_x} C_{Ft}^{W} - C_{Fjt}$ over all countries j and condition (23) implies

$$\int \left(S_{jt}^{x}\right)^{1-\epsilon_x} dj = 1. \qquad (24)$$

The terms of trade in export markets are relative prices that cannot all move in the same direction. Some countries can become more competitive only by making the other countries less competitive.

It is difficult to derive further properties in the general case where countries differ in their demand parameters β_{jt}. We simplify the analysis by restricting the attention to the symmetric case where all countries have the same demand, i.e.[11]

$$\forall j, \beta_{jt} = \beta_t. \qquad (25)$$

The level of β_t determines global demand in period t (a lower β_t means higher demand). In the symmetric equilibrium all countries have the same policy functions.

We will compare the decentralized NSP equilibrium with the allocation chosen by a *global social planner* (GSP) who sets the allocations in all countries so as to maximize utilitarian global welfare $V_t^W = \int V_{jt} dj$ under the same constraints as national social planners. Like the NSPs, the GSP cannot commit. The GSP allocation Pareto dominates the decentralized NSP allocation in the symmetric case. In that case, the GSP allocation can be interpreted as the outcome of international cooperation between the NSPs.

The following proposition states a condition under which the decentralized NSP equilibrium features a non-binding ZLB constraint in all periods and compares it with the GSP allocation.

[11] The asymmetric case is interesting if one wants to study the international spillovers of demand shocks. In this paper we focus on the spillovers from policy actions rather than demand shocks.

Proposition 3 *(Symmetric equilibria with non-binding ZLB constraint) There is a decentralized NSP allocation in which the ZLB constraint is never binding if and only if*

$$\beta_t \leq 1 + \pi^* \tag{26}$$

in all periods t. This allocation can be implemented by policy instruments satisfying (21) and

$$1 + \tau_t^b = \frac{1 + \tau_{t+1}^m}{1 + \tau_t^m}, \quad 1 + i_t = \frac{1 + \pi^*}{\beta_t}, \tag{27}$$

in all countries.

The allocation is the same whether the NSPs can use all the policy instruments or any subset of the policy instruments containing at least the tariff on imports or the tax on exports.

The GSP allocation can be implemented by setting the trade taxes to zero.

Proof. See Appendix E. ■

Condition (26) is familiar from the closed-economy literature on liquidity traps. The ZLB constraint is not binding if and only if global demand is high enough relative to the inflation target.

In the NSP equilibrium each national social planner attempts to manipulate the terms of trade in his country's favor by using the trade taxes. Like in the textbook tariff war this is mutually destructive in equilibrium and leads to an inefficiently low level of consumption of the global good.

Using (9) and (10), the quantity of labor that is required to produce one unit of consumption good in a symmetric equilibria is given by

$$\ell\left(S_t^m\right) = \alpha_H \left(S_t^m\right)^{-\alpha_F} + \alpha_F (S_t^m)^{\alpha_H}, \tag{28}$$

where the terms of trade relevant for imports are

$$S_t^m = \frac{1}{\left(1 + \tau_t^m\right)\left(1 + \tau_t^x\right)}. \tag{29}$$

The GSP's objective is to maximize the level of global consumption that can be achieved with the labor endowment $\overline{L}$, or equivalently to minimize the labor needed to produce any given level of consumption. It is is easy to see that $\ell\left(S_t^m\right)$ is minimized if $S_t^m = 1$, that is if trade taxes do not distort the allocation of consumption between the home and foreign goods. The undistorted allocation could also be implemented by a global commitment to free trade.

The equilibrium features different levels of indeterminacy. The allocation of trade taxes between imports and exports is indeterminate because of Lerner symmetry. The real interest rate path and the capital flow taxes are indeterminate. Any path $(R_t)_{t\geq 1}$ can be replaced by an alternative path $(R'_t)_{t\geq 1}$ without changing the country allocations if all countries adjust their capital flow taxes τ^b_{jt} so as to offset the change in the real interest rate.

As a result of these indeterminacies the same allocation may obtain the NSP or the GSP use different subsets of the policy instruments. In particular, the same allocation is brought about if a tariff on imports is the only available policy instrument. In this case, the tariff rate is determinate and equal to $\tau^m = 1/(\epsilon_x - 1)$ in all countries in the decentralized equilibrium. The global social planner instead sets the tariff level to zero.

Numerical illustration. The literature has shown that the welfare loss from a tariff war under full employment can be substantial (see for example Ossa (2014)). In the rest of the paper we will use the parameter values reported in Table 1 to illustrate the quantitative properties of the model. The elasticity of intertemporal substitution of consumption, ϵ_i, is set to 0.5, which corresponds to a risk aversion of 2, a standard value in the literature. The elasticity of substitution between foreign goods, ϵ_x, is set to 3, which is consistent with the recent estimates of Feenstra et al. (2018). Note in particular that the "microelasticity" between the differentiated imported goods is substantially larger than the "macroelasticity" between the home good and imports (which is 1 because of the Cobb-Douglas specification). Finally, we assume $\alpha_H = 0.6$, i.e., home goods amount to 60 percent of total consumption.

For these values the equilibrium tariff rate amounts to $\tau^m = 50\%$ in the decentralized equilibrium and the welfare loss from a tariff war amounts to 1.89% of permanent consumption.

Table 1. Baseline parameter values.

ϵ_i	ϵ_x	α_H
0.5	3	0.6

5 Global Liquidity Trap

In this section we turn our attention to the equilibria in which the ZLB is binding, i.e., global liquidity traps. For simplicity, we consider liquidity traps that last one period, and leave the analysis of multi-period liquidity traps for the Section 7. The global economy is assumed to be in a liquidity trap in period 1 and in a steady state with a positive nominal interest rate from period 2 onwards. We can generate such an equilibrium by assuming that the discount factor is sufficiently high in the first period and sufficiently low in the following periods, that is

$$\begin{aligned} \beta_1 &> 1 + \pi^*, \\ \beta_t &= \beta_2 < 1 + \pi^* \text{ for } t \geq 2. \end{aligned}$$

We interpret period 1 as the short run and the steady state starting in period 2 as the long run.

We compare the equilibria under different assumptions about the policy instruments that countries can use. Since countries enjoy monetary sovereignty the interest rate is always one of the available policy instruments but we consider different policy mixes for the other instruments. This reflects the fact that in the real world not all countries use all the policy instruments all the time.[12] For simplicity we maintain the assumption of symmetry for the policy instruments, i.e., we assume that all countries can use the same instruments.

We assume that the steady state starting in period 2 is the decentralized equilibrium between the NSPs described in Proposition 3. To resolve the indeterminacy we assume that this equilibrium features a constant tariff rate τ_2^m and no other taxes.[13] The long-run tariff rate is thus given by

$$1 + \tau_2^m = \frac{\epsilon_x}{\epsilon_x - 1}. \tag{30}$$

The results in this section are first-order approximations derived under the assumptions that $1 - \beta_2$ is small, so that changes in a country's period-2 net foreign assets have a small impact on the steady-state flow variables. This allows us to derive closed-form expressions for the elasticities of endogenous variables with respect to policy instruments in period 1 (see Appendix D for details).

We will compare the allocations under a decentralized equilibrium between NSPs and a GSP. The GSP takes into account that trade balances are zero in a symmetric allocation and takes welfare from period 2 onwards as given. Hence maximizing period-1 welfare is equivalent to maximizing period-1 global consumption. To see what this implies for policy instruments, we first note that using (24)

[12]The use of trade taxes and capital controls can be limited by membership to international organizations such as the WTO, the EU, or the OECD.

[13]The proofs in Appendix E are derived under the more general assumption that τ_2^m is constant.

in a symmetric allocation one must have $S_t^x = 1$ and S_t^m is given by (29). It then follows from (16) for $t = 1$ that

$$C_1^W = \left[\frac{\beta_1 (1 + i_1)}{1 + \pi^*}\right]^{-\epsilon_i} \left[\frac{(1 + \tau_1^m)(1 + \tau_1^x)}{(1 + \tau_2^m)(1 + \tau_2^x)}\right]^{-\alpha_F \epsilon_i} C_2^W. \tag{31}$$

This expression shows the impact of the policy instruments on consumption in the short run. Note in particular that global consumption decreases with the total trade distortion $(1 + \tau_1^m)(1 + \tau_1^x)$. This is because this distortion acts as an intertemporal tax on consumption that makes the global good more expensive in period 1.

Another implication of equation (31) is that global consumption and welfare do not depend on the tax on capital flows τ_1^b. To understand why, observe that equation (15) and $S_t = 1/(1 + \tau_t^x)$ imply the following expression for the global real interest rate,

$$R_1 = \frac{1 + i_1}{(1 + \pi^*)\left(1 + \tau_1^b\right)} \frac{1 + \tau_1^x}{1 + \tau_2^x}. \tag{32}$$

If all countries increase their tax rate on capital inflows this is offset by an equivalent decrease in the global real rate of interest. Another implication of equation (32) is that the global real rate of interest increases with the period-1 tax on exports. An increase in the tax on exports reduce the supply of global good and the real interest rate must increase to bring demand back in line with supply.

The rest of this section is structured as follows. Section 5.1 considers the case of monetary wars where the only weapons are the nominal interest rate and the inflation targets. Section 5.2 adds tariffs and Section 5.3 adds export taxes to the policy mix. Section 5.4 considers the case where all the policy instruments can be used.

5.1 Monetary wars (i, π^*)

This section considers the case where countries use monetary policy only. From period 2 onwards the economy is in a full employment steady state with $C_2^W = \overline{L}$. The equilibrium is described in the following proposition.

Proposition 4 *(Conventional monetary war) Assume that the only policy instrument available to national social planners is the nominal interest rate. Then in the decentralized NSP equilibrium all countries set the nominal interest rate to zero in period* 1 *and have the same level of unemployment* ($\forall j$, $i_{j1} = 0$ *and* $L_{j1} = L_1 < \overline{L}$). *There is no gain from international coordination.*

Proof. See Appendix E. ■

If there is unemployment, lowering the interest rate unambiguously raises home welfare because this raises both home consumption and the trade balance. Hence in the decentralized equilibrium between national social planners the ZLB constraint must be binding in all countries. There is less than full employment because $\beta_1 > 1 + \pi^*$. Using (31) with $i_1 = 0$, $C_2^W = \overline{L}$ and $\tau_t^m = \tau_t^x = 0$ for $t = 1, 2$ gives

$$C_1^W = L_1 = \left(\frac{\beta_1}{1 + \pi^*} \right)^{-\epsilon_i} \overline{L} < \overline{L}.$$

The global economy is in a global liquidity trap with the same unemployment rate in all countries.

A global social planner does not deviate from the decentralized allocation. This is true even though a monetary stimulus in one country has a beggar-thy-neighbor effects—it decreases the trade balance, employment and welfare in other countries. A monetary stimulus is a positive-sum game because decreasing the interest rate everywhere raises employment and welfare in all countries, as it would in a closed economy (Bernanke, 2017).

The model can easily be extended to the case where national social planners can choose their inflation targets (an "inflation target war"). Let us assume that each country j sets its inflation target π_j^* in period 1. The Nash equilibrium from that point onwards is then determined conditional on the inflation targets as before. Each NSP sets its inflation target so as to maximize domestic welfare taking the other countries' inflation targets as given. Then we have the following result.

Proposition 5 *(Unconventional monetary war) Assume that the national social planners can choose their inflation targets in period* 1*. Then in a symmetric Nash equilibrium social planners set an inflation target* $\pi_j^* \geq \beta_1 - 1$ *and* $i_{j1} = \left(1 + \pi_j^*\right) / \beta_1 - 1$*. There is full employment in all countries and welfare is at the first-best level. There is no gain from international coordination.*

Proof. See Appendix E. ■

This result comes from the fact that the international spillovers associated with an increase in the inflation target are the same as with a decrease in the nominal interest rate. An inflation target war, thus, is a positive-sum game.

5.2 Tariff wars (i, τ^m)

In a tariff war the available policy instruments are the nominal interest rate and the tariff on imports. The equilibrium tariff is stated in the following proposition.

Proposition 6 *(Equilibrium tariff) Consider a decentralized Nash equilibrium in which all national social planners use tariffs on imports only. If the ZLB constraint is binding there is less than full employment ($L_1 < \overline{L}$) and the tariff on imports is equal to*

$$1 + \tau_1^m = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F \right) \frac{\epsilon_x}{\epsilon_x - 1}. \tag{33}$$

The tariff rate is higher in period 1 than in the long run if and only if $\epsilon_1 < 1$.

Proof. See Appendix E. ■

Increasing the tariff rate has two effects on a small open economy in the short run. First, given the level of employment this reduces consumption and increases the trade balance. Second, this may increase or decrease home employment. Raising the tariff rate shifts home consumption towards the home good but taxes consumption intertemporally. Home employment is increased by the tariff if and only if the expenditure-switching effect dominates the intertemporal effect. This is true if and only if the elasticity of intertemporal substitution is smaller than the import elasticity.

The first effect has a second-order impact on welfare if one starts from a situation without intertemporal consumption distortion, i.e., if $\tau_1^m = \tau_2^m$. In this case, the NSP raises or lowers the tariff depending on whether this raises or lowers employment. The NSP raises the tariff above the long-run level if and only if $\epsilon_i < 1$.

Tariffs have deleterious effects from a multilateral perspective though because they decrease global demand, as one can see from equation (31). In order to see if a tariff war leads to unemployment, we can write global employment in period 1 as global consumption times the quantity of labor it takes to produce one unit of consumption, that is

$$L_1^W = \ell \left(\frac{1}{1 + \tau_1^m} \right) C_1^W, \tag{34}$$

where function $\ell(\cdot)$ is defined by (28) and we use $S_1^m = 1/(1 + \tau_1^m)$. The variations of L_1^W with τ_1^m are ambiguous for positive tariffs. Raising the tariff lowers the demand for consumption but increases the quantity of labor required to produce each unit of consumption. It is easy to see, by log differentiating (31) and (34), that the consumption distortion effect dominates the production distortion effect if and only if $\tau_1^m < \frac{\epsilon_i}{\alpha_H(1-\epsilon_i)}$, that is if the tariff rate is not too high.

The tariff war decreases global employment if raising $1+\tau_1^m$ from $\frac{\epsilon_x}{\epsilon_x-1}$ to the level given by (33) lowers L_1^W. Using (31) and (34) this condition can be written,

$$\ell\left(\frac{1-1/\epsilon_x}{\alpha_H/\epsilon_i+\alpha_F}\right) < (\alpha_H/\epsilon_i+\alpha_F)^{\alpha_F\epsilon_i}\,\ell\,(1-1/\epsilon_x)\,. \tag{35}$$

This condition is satisfied if the long-run equilibrium tariff is not too high. It is satisfied for the baseline parameter values in Table 1.

A tariff war can make a liquidity trap with unemployment self-fulfilling. The strategic complementarity behind the equilibrium multiplicity is that by raising home tariffs the NSPs reduce demand for the other countries, leading them to lower the interest rate to support employment. Higher tariffs, thus, lead to lower interest rates. But conversely if $\epsilon_i < 1$ tariffs are higher if the ZLB is binding as each country increases its tariff to boost home employment. As a result there may be two equilibria, as illustrated by Figure 1. The figure shows how the global nominal interest rate and the global tariff rate are related to each other for the parameter values in Table 1 and $\beta_1 = 0.98$, $\pi^* = 2\%$. Since $\beta_1 < 1+\pi^*$ there is a full employment equilibrium in which the ZLB is not binding and the NSPs set the tariff rate at the long-run level in period 1 (point A). However there is also an equilibrium with a liquidity trap, unemployment and a higher tariff level (point B). The first equilibrium Pareto-dominates the second one.

The following proposition characterizes the tariff war equilibria.

Proposition 7 *(Tariff war equilibria) Assume (35) is satisfied. There is a threshold $\beta^* < 1+\pi^*$ such that:*

(i) if global demand is high ($\beta_1 < \beta^*$) the equilibrium is unique, the ZLB is not binding, there is full employment and the tariff rate is the same in period 1 *as in the long run ($\tau_1^m = \tau_2^m$);*

(ii) if global demand is low ($\beta_1 > 1+\pi^*$) the equilibrium is unique, the ZLB is binding, there is less than full employment and the tariff rate is higher in period 1 *than in the long run ($\tau_1^m > \tau_2^m$);*

(iii) if global demand is intermediate ($\beta^* < \beta_1 < 1+\pi^*$) the equilibria described in (i) and (ii) co-exist.

Proof. See Appendix E. ■

For the parameter values in Table 1 the threshold in the discount factor is $\beta^* = 0.963$. Thus self-fulfilling tariff wars can arise for plausible values of the discount rate. For $\beta_1 = 0.98$ a self-fulfilling tariff war increases the unemployment rate by 0.9%.

The GSP allocation differs from the decentralized NSP allocation in several ways. First, the GSP always removes the long-run distortion by setting the tariff rate to zero, $\tau_2^m = 0$. Second, the short-run policy depends on the level of demand. If demand is high enough to avoid a liquidity trap ($\beta_1 < 1 + \pi^*$), the GSP sets the tariff rate to zero also in period 1. If demand is low ($\beta_1 > 1 + \pi^*$), the GSP can raise consumption and welfare by lowering the period-1 tariff rate below the long-run level $\tau_2^m = 0$, that is by *subsidizing* imports. The GSP maximizes global welfare by achieving full employment, which using (31) and (34) with $\tau_2^m = \tau_t^x = 0$, $C_2^W = \overline{L}$ and $i_1 = 0$ implies $1 + \tau_1^m = 1/S^*$ where S^* satisfies

$$\ell(S^*)(S^*)^{\alpha_F \epsilon_i} = \left(\frac{\beta_1}{1+\pi^*}\right)^{\epsilon_i}. \tag{36}$$

If $\beta_1 > 1 + \pi^*$ there is a unique S^* larger than 1 that satisfies this equation.

Our results about the GSP allocation are summarized in the following proposition.

Proposition 8 *(GSP tariff policy) The GSP sets the tariff to zero in the long run ($\tau_2^m = 0$). The GSP tariff in the short run depends on the level of global demand as follows:*

(i) if demand is high enough ($\beta_1 \leq 1 + \pi^*$), the GSP sets the tariff to zero in the short run ($\tau_1^m = 0$);

(ii) if demand is low ($\beta_1 > 1 + \pi^*$), the GSP subsidizes imports by setting

$$\tau_1^m = 1/S^* - 1 < 0,$$

where S^* is the solution of (36) that is larger than 1.

Proof. See discussion above. ∎

5.3 Trade wars with export subsidies (i, τ^m, τ^x)

The outcome of trade wars is very different if a tax or subsidy on exports is added to the set of usable policy instruments. The Nash equilibrium with both trade taxes is characterized in the following result.

Proposition 9 *(Trade war with export subsidies) A Nash equilibrium in which national social planners use taxes/subsidies on both imports and exports has the following properties.*

(i) There is full employment in all countries irrespective of β_1*.*

(ii) The ZLB constraint is binding in period 1 *if and only if* $\beta_1 > 1 + \pi^*$*.*

(iii) If the ZLB constraint is not binding, the trade taxes are the same in period 1 *as in the long run.*

(iii) If the ZLB constraint is binding, exports are subsidized ($\tau_1^x < 0$*) and the equilibrium taxes* τ_1^m *and* τ_1^x *are respectively increasing and decreasing with* β_1*.*

Proof. See Appendix E. ■

A Nash equilibrium now features full employment. A global liquidity trap with a binding ZLB can still exist but it is no longer associated with unemployment. This is because each NSP can increase home employment and trade balance by lowering the tax (or increasing the subsidy) on exports. At the national level, the tax on exports does not affect consumption. Thus, decreasing the tax on exports unambiguously raises home welfare and all NSPs subsidize exports until there is full employment.

Figure 2 shows the variation of the period-1 equilibrium trade taxes and the nominal interest rate with global demand $1/\beta_1$. When global demand is high, countries set the trade taxes to the same levels as in the long run, which is consistent with a positive nominal interest rate.

If demand $1/\beta_1$ falls below $1/(1 + \pi^*)$ the ZLB constraint becomes binding and the NSPs lose the interest rate as an instrument. They subsidize exports to maintain full employment and increase the tariff on imports to continue manipulating the terms of trade.

A key difference between a tariff on imports and a subsidy on exports is in the congruence between partial and general equilibrium. In partial equilibrium, NSPs increase home employment by increasing τ^m or decreasing τ^x. In general equilibrium, these policy changes have an opposite impact on global demand. An increase in τ^m reduces global demand but a decrease in τ^x stimulates global demand. Export subsidies stimulate supply directly and increase demand indirectly by lowering the global real rate of interest, as can be seen from equation (32).

The GSP allocation always differs from the decentralized allocation. The GSP never distorts consumption in the long run. The GSP does not distort consumption in the short run either if the ZLB constraint is not binding. In this case the GSP allocation can be achieved with zero trade taxes as in point (i) of Proposition 8.

Like in Proposition 8, the GSP achieves full employment in period 1 by subsidizing exports or imports if the ZLB constraint is binding.

Proposition 10 *(Social planner allocation with export subsidies) If the ZLB constraint is not binding the GSP allocation is achieved with zero trade taxes. If the ZLB constraint is binding the GSP allocation is achieved by subsidizing exports or imports in period* 1*.*

Proof. See discussion above. ■

5.4 Total wars $(i, \tau^m, \tau^x, \tau^b)$

If national social planners can use all the policy instruments there is indeterminacy because of Proposition 1. Each national social planner can achieve the desired allocation with an infinity of policy combinations featuring the same wedge $(1 + \tau_t^m)(1 + \tau_t^x)$. One of these combinations involve a zero tax on capital inflows so that the Nash equilibrium is the same as in the case, already analyzed in section 5.3, where NSPs can use (i, τ^m, τ^x).

One case that we have not formally analyzed is where the NSPs can use only the tax on capital flows τ_t^b (or foreign exchange reserves interventions) in addition to the nominal interest rate. In partial equilibrium, each NSP can improve the home terms of trade by subsidizing capital inflows and can increase home employment by taxing capital inflows. Whether the Nash equilibrium results in a tax or a subsidy on capital inflows depends on the state of global demand. In either case, the use of capital account policies does not change welfare or the allocation provided that the equilibrium stays symmetric. As one can see from equation (31) global consumption is not affected by capital account policies because any change in τ_1^b is offset by a change in R_1. It is not necessarily true, however, that the Nash equilibrium in capital controls is symmetric.[14]

To conclude this section, Figure 3 shows the welfare impact of different kinds of trade and currency wars under the benchmark calibration in Table 1. The figure shows the short-run impact on period 1 welfare expressed in terms of period 1 consumption. The discount factor β_1 was set at the level that implies an unemployment rate of 5 percent in period 1 if NSPs use only monetary policy. We

[14]This is because the welfare of a country is a convex function of τ^b if the the export elasticity is high. As a result the Nash equilibrium may lead to an endogenous symmetry breakdown in which a fraction of countries impose a high tax on capital inflows to achieve full employment and a trade surplus whereas the other countries achieve the same level of welfare by financing a trade deficit at a low real interest rate. We leave a full-fledged analysis of this case for another paper.

assumed that the NSPs do not use the trade taxes in the long run (from period 2 onwards) to focus on the impact of short-run trade wars. Each bar shows the impact on period-1 welfare of letting the NSPs use the instruments reported above the bar instead of just the nominal interest rate.

The main lesson from the figure is that the welfare impact of lack of international cooperation crucially depends on which instrument is used. The worst welfare impact comes from a trade war relying on tariffs on imports because of the resulting increase in unemployment. A tariff war raises the unemployment rate from 5 percent to 12.5 percent. By contrast, a trade war involving subsidies on exports leads to full employment and raises welfare. It does not increase welfare to the first-best level (which is achieved with an inflation target war) because of the consumption distortion that results from the trade taxes.

6 Sustainability of Free Trade

This section looks into the conditions under which free trade can be sustained as a trigger-strategy equilibrium. We assume that a deviation from free trade by one country may lead to a permanent trade war between all countries. A question of interest is how the comparison between the short-run gains and the long-run cost from deviating from free trade is affected by the state of global demand.

Like in the previous section we assume that the global economy is in a full employment steady state from period 2 onwards and use β_1 to vary the level of global demand in the short run. We assume that a deviation from free trade by one country in period 1 leads to a trade war (that is, the Nash equilibrium described Proposition 3) with probability μ in period 2. A trade war starting in period 2 continues forever.[15] By Proposition 3 it does not matter which trade taxes are used in the steady state that starts in period 2 and we will assume that the NSPs use tariffs only. We assume that country j deviates from free trade by imposing a tariff on imports in period 1.[16]

If country j does not deviate from free trade, its period-1 welfare is the same as for the representative country and is given by $V_{j1}^n = u\left(C_1^W\right) + \beta_1 V_2^N(0)$ where

[15] Alternatively we could assume that the trade war lasts a finite time and use the expected duration of a trade war to vary its cost, but the equilibrium is more complicated to derive in that case. Assuming a permanent trade war keeps the analysis simple without affecting the essence of the results.

[16] The results are similar if one assumes instead that country j uses a tax or subsidy on exports, and the temptation to deviate from free trade is even stronger in that case.

$V_2^N(0)$ is the period-2 welfare of a country with zero foreign assets if there is not trade war. If country j deviates from free trade, its period-1 welfare is

$$V_{j1}^d = u\left(C_{j1}^d\right) + \beta_1\left[(1-\mu)\, V_2^N\left(B_{j2}^d\right) + \mu V_2^T\left(B_{j2}^d\right)\right],$$

where C_{j1}^d and $B_{j2}^d = R_1 X_{j1}^d$ are respectively the period-1 consumption and period-2 foreign assets of country j if it deviates, and $V_2^T(\cdot)$ is period-2 welfare if there is a trade war.

The net welfare gain from deviating from free trade, $\Delta V_{j1} = V_{j1}^d - V_{j1}^n$, can thus be decomposed into two terms,

$$\Delta V_{j1} = \underbrace{u\left(C_{j1}^d\right) + \beta_1 V_2^N\left(B_{j2}^d\right) - u\left(C_1^W\right) - \beta_1 V_2^N(0)}_{\text{GAIN}} - \underbrace{\beta_1\mu\left[V_2^N\left(B_{j2}^d\right) - V_2^T\left(B_{j2}^d\right)\right]}_{\text{LOSS}}.$$

The first term is the gain that country j derives from imposing a tariff in period 1 if this does not lead to a trade war. This gain is necessarily positive since the country could always choose to impose a zero tariff when it deviates. The second term, the cost of deviating from free trade, is equal to the discounted expected welfare loss from a trade war starting in period 2. This loss is positive because the welfare of all countries is reduced by a trade war.

The left-hand side panel of Figure 4 shows how the gain and cost of imposing a tariff in period 1 vary with the state of global demand $1/\beta_1$. The figure was constructed with the parameter values in Table 1 and assuming that a deviation from free trade by one country triggers a generalized trade war with probability $\mu = 3\%$.

The gain from a deviation from free trade is not greatly affected by global demand when the economy is at full employment (i.e., when $\beta_1 < 1 + \pi^*$). This is because in this case $\beta_1 R_1 = 1$ so that a change in global demand is offset by a change in the global real interest rate, which leaves the benefit of deviating from free trade unchanged to a first-order of approximation. By contrast, the gains from deviating from free trade become much larger when the global economy is in a liquidity trap, because then any deviating country can raise its employment by imposing a tariff. The cost of deviating from free trade increases as global demand falls because consumers discount the expected loss from a trade war at a lower rate. For the value of μ that we have assumed in Figure 4, the gain from imposing a tariff is lower than the cost if and only if global demand is high enough. Free trade, thus, tends to become less sustainable when global demand is low.

The right-hand side panel of Figure 4 shows the variation of the equilibrium tariff rate, interest rate and trade balance for a deviating country. When global demand is high, the deviating country imposes a tariff of about 27% to increase its terms of trade. The period-1 equilibrium tariff rate is lower than in a generalized trade war (where it is equal to $1/(\epsilon_x - 1) = 50\%$ by equation (30)) because the tariff, being temporary if it is not followed by a trade war, has a larger distortionary effect on the deviating country's consumption than in steady state. The deviating country offsets the stimulative impact of the tariff on home demand for the home good by raising its nominal interest rate. The deviating country thus falls in a liquidity trap for a lower level of global demand than the rest of the world. When it does fall in a liquidity trap, the deviating country raises the tariff rate to much higher levels in order to preserve full employment at home. The tariff-imposing country always increases its trade balance whether the global economy is in a liquidity trap or not.

7 Dynamic Trade and Currency Wars

We generalize our analysis in this section by considering the case where a global liquidity trap can last for several periods. We assume that the economy is a steady state with full employment starting in a period, denoted by T, that can be arbitrarily large. The decentralized equilibrium is still defined as in section 4. This section generalizes the analysis presented in the previous section, which was about the special case $T = 2$.

First, let us assume that the NSPs can use only the nominal interest rate. By Proposition 3 a steady state with full employment from T onwards exists if and only if $\beta_t = \beta_T < 1 + \pi^*$ for $t \geq T$. Going through the same steps as in the previous section, it is easy to see that a global liquidity trap arises in period $T-1$ if $\beta_{T-1} > 1 + \pi^*$. If the global economy is in a liquidity trap with unemployment before period T, it follows from $L_t^W = C_t^W$, equation (16) with $i_t = 0$, $S_t^m = 1$, and $L_T^W = \overline{L}$, that the level of global employment in any period $t < T$ satisfies

$$u'\left(L_t^W\right) = \frac{\beta_t \beta_{t+1} \cdots \beta_{T-1}}{1 + \pi^*} u'\left(\overline{L}\right).$$

We have used the fact that inflation is equal to to π^* in period T and equal to 0 before period T because of unemployment. The ZLB is indeed binding before time T if $L_t^W < \overline{L}$, that is if

$$\prod_{s=t}^{T-1} \beta_s > 1 + \pi^*, \quad \forall t < T. \tag{37}$$

We assume this condition to be satisfied in the following. Observe that this condition does not require the discount factor to be larger than $1+\pi^*$ before period $T-1$. For example one could have $\beta_{T-1} > 1+\pi^*$ and $\beta_t = 1$ for $t < T-1$. This is because unemployment lowers inflation to 0, which raises the real interest rate when the nominal interest rate is at the ZLB. The expectation of a liquidity trap in one period tends to pull the economy into a liquidity trap in the previous periods.[17]

Second, let us assume that condition (37) being satisfied, the NSPs can use trade taxes. By Proposition 3 the steady state allocation after period T does not not depend on which trade taxes are available. For simplicity, we assume that the NSPs use only tariffs on imports. Hence there is a constant tariff on imports τ_T^m given by (30) starting in period T.[18]

The Nash equilibrium can then be solved by backward induction from period T. The equilibrium before time T depends on which trade taxes are available. If the NSPs can use only tariffs on imports, the equilibrium in period $T-1$ can be constructed like for period 1 in section 5.2 and τ_{T-1}^m is given by (30). The equilibrium tariff in the previous periods can be derived by further iterating backwards. It is possible to show that the equilibrium tariff rate is the same in all the periods before time T (see Proposition 11).

The social planner allocation can be solved for by generalizing the analysis in sections 5.2 and 5.3. Iterating on (16) with $i_{jt} = 0$ and , $\pi_{jT} = \pi^*$ and $\pi_{jt} = 0$ for $t < T$ gives the following expression for global consumption and employment,

$$C_t^W = \left[\frac{\prod_{s=t}^{T-1} \beta_s}{1+\pi^*}\right]^{-\epsilon_i} \left(\frac{S_T^m}{S_t^m}\right)^{-\alpha_F \epsilon_i} C_T^W, \tag{38}$$

$$L_t^W = \ell\left(S_t^m\right) C_t^W. \tag{39}$$

These expressions generalize (31) and (34). It remains true that the GSP maximizes welfare by setting the trade taxes to zero in the long run and by subsidizing imports (and/or exports if the export tax is available) so as to achieve full employ-

[17]This mechanism may lead to self-fulfilling liquidity traps that last forever. We rule out this type of equilibria here by assuming that the economy is in full employment after some finite period T.

[18]If $\beta_T > \beta^*$ where β^* is the threshold defined in Proposition 7, there could also be a self-fulfilling global liquidity trap in any period after time T. We rule out this type of multiplicity here as it has been already analyzed in section 5.2.

ment. This implies that the GSP sets the trade taxes such that $(1+\tau_t^m)(1+\tau_t^x) = 1/S_t^*$ where S_t^* satisfies equation (36) with β_1 replaced by $\prod_{s=t}^{T-1} \beta_s$.

Our main results are summarized in the following proposition.

Proposition 11 *(Multi-period trade and currency wars) Assume that the economy is in a global liquidity trap before period T if NSPs use monetary policy only. Then*

(i) in the decentralized equilibrium where NSPs use tariffs on imports only, the equilibrium tariff τ_t^m is given by (33) in all periods $t < T$;

(ii) in the decentralized equilibrium where NSPs use export taxes only, in all periods $t < T$ the NSPs tax exports at a lower rate in the short run than in the long run and there is full employment;

(ii) the GSP implements zero trade taxes in the long run and subsidizes exports or imports to achieve full employment before period T.

Proof. See Appendix E. ■

The dynamics of a multi-period trade war are illustrated by Figure 5. To construct this figure we assumed that $\beta_t = 1.03$ for four periods before decreasing to its long-run level of 0.98 in period $T = 5$. The left-hand side panel compares the dynamics of unemployment under free trade, under a tariff war and when the national social planners use export taxes. The right-hand side panel shows the variation of the trade taxes over time. We assume that the national social planners do not use both trade taxes at the same time.

In the long run (from period T onward), the national social planners attempt to manipulate the terms of trade and by Lerner symmetry the tax rate is at the same level $\tau^m = \tau^x = 1/(\epsilon_x - 1)$ whether it is applied to imports or exports. Before period T the outcome of a trade war is very different depending on whether the national social planners tax imports or exports. In the former case they impose a large constant tariff on imports to increase home employment, which depresses global demand and results in an increase in unemployment. In the latter case, they succeed in reducing unemployment to zero by reducing the tax on exports. The tax on exports increases over time, which stimulates global supply. In this example the tax on exports stays positive (the national social planners do not subsidize exports) but it could turn negative if the global liquidity trap lasted longer.

8 Conclusions

We have analyzed a tractable model in which countries use trade taxes and capital controls to maximize home welfare. When global demand is high the trade taxes are used to manipulate the terms of trade like in a textbook tariff war. When global demand is low and the ZLB constraint is binding, countries use the same instruments to raise home employment. The analysis suggests that there is one case where uncoordinated policies lead to large welfare losses: when global demand is low and countries use tariffs on imports. The uncoordinated use of all the other instruments we have looked at (interest rate, inflation target, export subsidy and capital controls) is Pareto optimal or neutral in the short run. However, tariffs seem to be the instrument of choice in the real world. One interesting question is why revealed preferences favor tariffs over other instruments such as export subsidies. A possible explanation is that subsidies on exports are financed with distortionary taxation.

The paper opens several directions for further research. Making the model less symmetric would allow us to look at questions that have not been analyzed in this paper. For example, assuming that countries differ in their time preferences would make it possible to examine how a "global savings glut" in one part of the world may affect the benefits of international policy coordination. Another relevant source of asymmetry is if countries have access to different policy instruments. In the real world subsets of countries are committed not to use certain policy instruments.

The structure of production could also be enriched. In particular there could be international trade in production inputs and not only in final consumption goods.

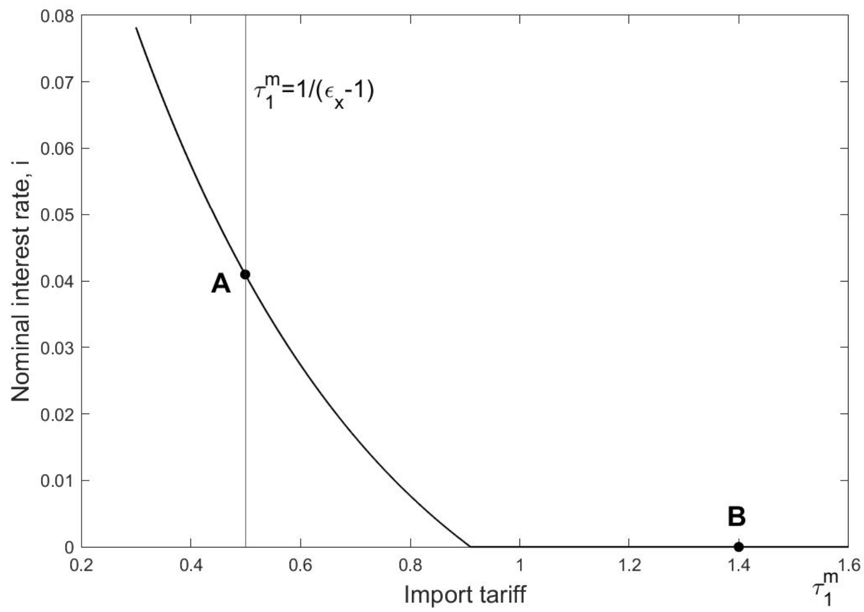

Figure 1: Tariff rate (x-axis) and nominal interest rate (y-axis) in Nash equilibrium

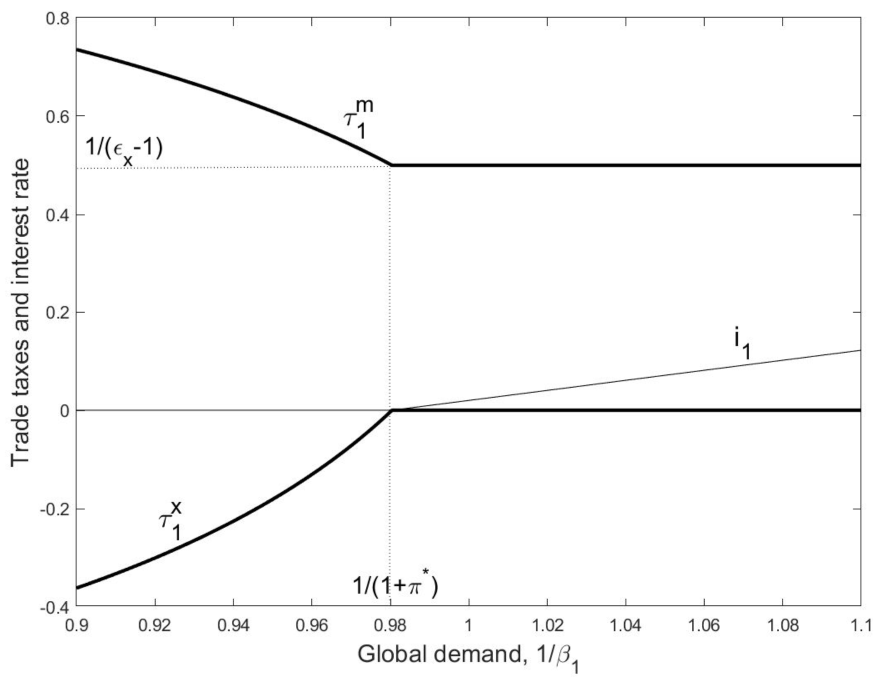

Figure 2: Variation with demand $1/\beta_1$ of trade taxes and nominal interest rate in Nash equilibrium

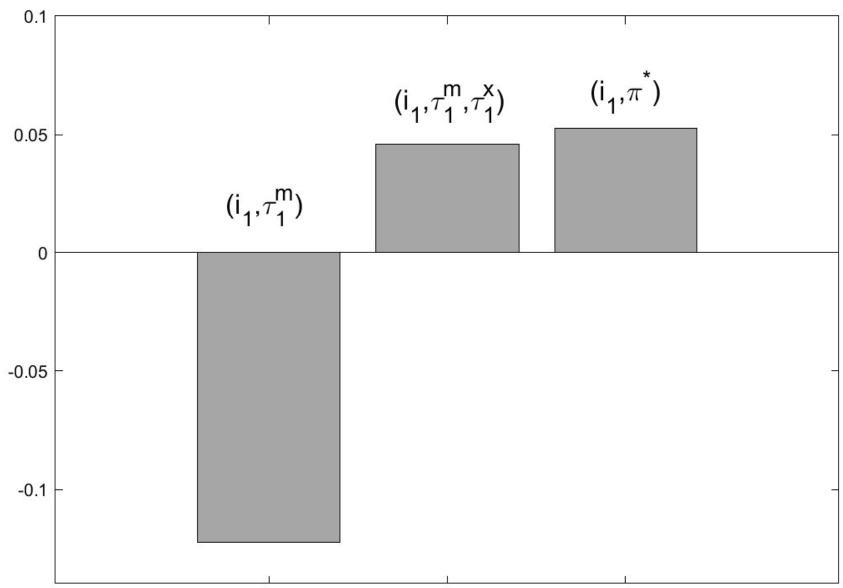

Figure 3: Impact of trade and currency wars on period-1 welfare

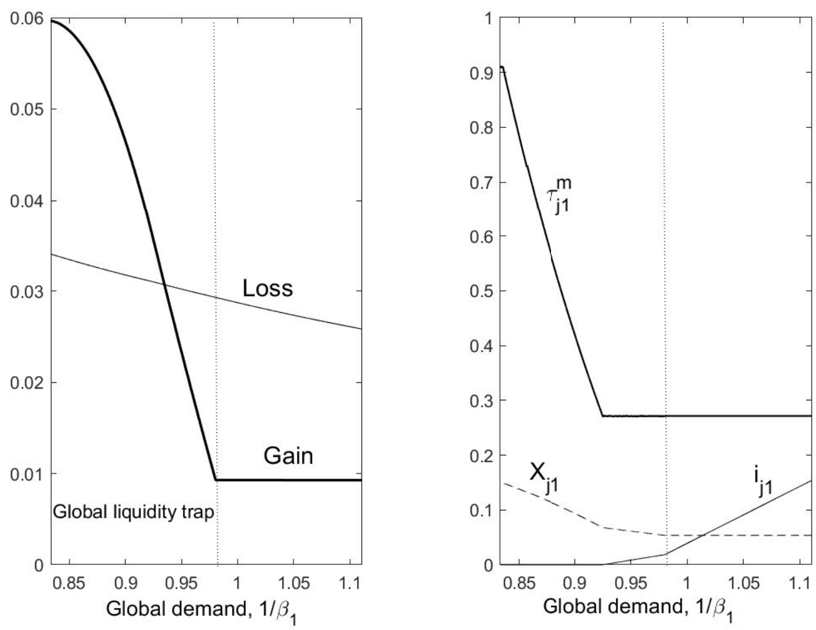

Figure 4: Deviation from free trade. The l.h.s. panel shows the variation with global demand of the gain and cost of deviating from imposing a tariff. The r.h.s. panel shows the equilibrium tariff, interest rate and trade balance for a tariff-imposing country.

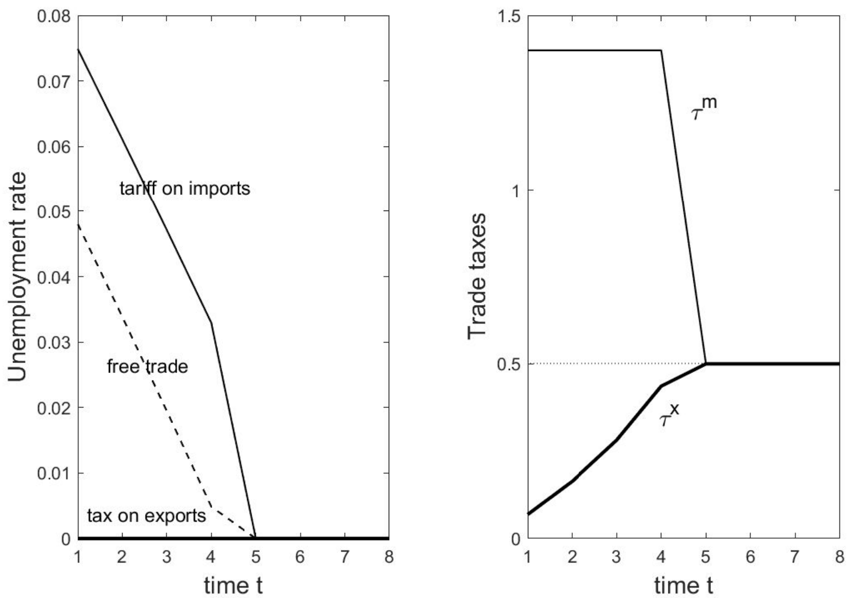

Figure 5: Unemployment rate in a dynamic trade war

APPENDICES

APPENDIX A. CONSUMER'S FIRST-ORDER CONDITIONS

This appendix derives the consumer's first-order conditions in the model augmented to include money and nominal bonds. The consumer derives utility from real money balances. We omit the country j to alleviate the notations. The consumer's problem in Bellman form is

$$V_t\left(B_t + \frac{M_t + B_t^n}{P_{Ft}}\right) = \max_{C_t, B_{t+1}, B_{t+1}^n, M_{t+1}} \left[u\left(C_t\right) + v\left(\frac{M_{t+1}}{P_{Ht}}\right) + \beta_t V_{t+1}\left(B_{t+1} + \frac{M_{t+1} + B_{t+1}^n}{P_{Ft+1}}\right)\right],$$

subject to

$$P_{Ft}\frac{B_{t+1}}{R_t\left(1+\tau_t^b\right)} + \frac{B_{t+1}^n}{1+i_t} + M_{t+1} + P_t^c C_t = P_{Ht}L_t + Z_t + P_{Ft}B_t + B_t^n + M_t,$$

where $P_t^c = \left(P_{Ht}\right)^{\alpha_H}\left(\left(1+\tau_t^m\right)P_{Ft}\right)^{\alpha_F}$ is the price index for consumption, B_t^n and M_t are the consumer's holdings of government nominal bonds and money at the beginning of period t, and $v\left(\cdot\right)$ is the utility from real money balances. The government supplies zero nominal bonds and injects newly printed money through a lump-sum transfer to the consumer, so that $Z_t = \tau_t^m P_{Ft}C_{Ft} + \tau_t^x P_{Ht}\left(L_t - C_{Ht}\right) - \tau_t^b P_{Ft}B_{t+1}/\left(1+\tau_t^b\right) + M_{t+1} - M_t$. Note that period-$t$ money supply bears the time subscript $t+1$ to be consistent with our notations for bonds.

The first-order conditions for B_{t+1} and B_{t+1}^n imply equation (14). The first-order condition for C_t and the envelope condition give the Euler condition,

$$u'\left(C_t\right)\frac{P_{Ft}}{P_t^c} = \beta_t R_t\left(1+\tau_t^b\right)u'\left(C_{t+1}\right)\frac{P_{Ft+1}}{P_{t+1}^c}.$$

Then using (14) to substitute out $R_t\left(1+\tau_t^b\right)$ from this equation and $P_t^c = P_{Ht}\left(S_t^m\right)^{-\alpha_F}$, one can rewrite the Euler equation as (16).

The first-order condition for M_{t+1} and the envelope condition imply,

$$v'\left(\frac{M_{t+1}}{P_{Ht}}\right) = \frac{P_{Ht}}{P_t^c}u'\left(C_t\right)\left(1 - \frac{1}{1+i_t}\right). \tag{40}$$

APPENDIX B. ALTERNATIVE POLICY INSTRUMENTS

This appendix studies the two alternative policy instruments mentioned in section 3, money suuply and foreign exchange interventions.

Money supply. We go back to the model with money in the utility function presented in Appendix A. Using (16) to substitute out $u'(C_t)$ from equation (40) and $(S_t^m)^{\alpha_F} = P_{Ht}/P_t^c$ gives the following equation for money demand,

$$v'\left(\frac{M_{t+1}}{P_{Ht}}\right) = \beta_t i_t \frac{u'(C_{t+1})\left(S_{t+1}^m\right)^{\alpha_F}}{1+\pi_{t+1}}. \tag{41}$$

Nominal stickiness sets a lower bound on the nominal price of the home good, $P_{Ht} \geq W_{t-1}$. Figure 6 shows how P_{Ht} and L_t vary with period-t money supply M_{t+1}, assuming that next-period variables are constant (a first-order approximation). There is unemployment if money supply is lower than a threshold. In this range, P_{Ht} is fixed and an increase in money supply lowers the nominal interest rate by equation (41). This depreciates the home currency and raises consumption and the demand for home labor by equations (15), (16) and (11). When the demand for home labor reaches $\overline{L}$ the economy transitions to the flexible wage regime where further increases in money supply raise the nominal wage and have no impact on real variables. The social planner sets inflation at the target level by choosing the appropriate level of money supply, which corresponds to point A in Figure 6.

Observe however that it is not always possible to raise L_t to the full employment level by increasing money supply. The nominal interest rate goes to zero as money supply goes to infinity or reaches the satiation level. It is not always the case that the level of labor demand corresponding to $i = 0$ is larger than labor supply $\overline{L}$. If the maximum level of labor demand is smaller than $\overline{L}$ the economy is in a liquidity trap.

Foreign exchange interventions. We now assume that the capital account is closed, i.e., the only home agent that can trade real bonds with foreign investors is the government (the home consumers cannot). The government finances its purchase of foreign bonds by issuing domestic currency bonds to the home consumers. This can be interpreted as a sterilized foreign exchange interventions in which the central bank buys dollars. The budget constraints of the home consumer and the government are respectively given by

$$\frac{B_{t+1}^n}{1+i_t} + M_{t+1} + P_t^c C_t = P_{Ht} L_t + Z_t + B_t^n + M_t,$$

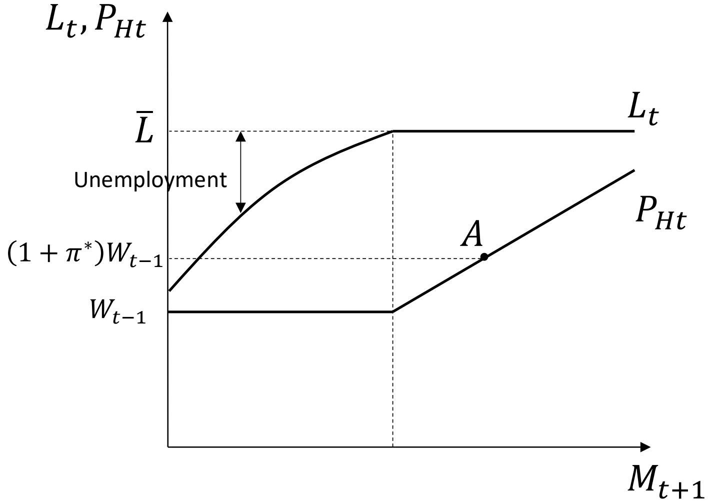

Figure 6: Money supply, price level and demand for labor

and

$$Z_t = \tau_t^m P_{Ft} C_{Ft} + \tau_t^x P_{Ht} \left(L_t - C_{Ht}\right) + P_{Ft} B_t - P_{Ft} \frac{B_{t+1}}{R_t} + \frac{B_{t+1}^n}{1+i_t} - B_t^n + M_{t+1} - M_t,$$

where the net supply of nominal bonds B_t^n is no longer equal to zero. Using the second expression to substitute out Z_t in the first expression still gives the Balance-of-Payments equation (12).

The real allocation can be derived as follows. Because of (12), the government determines the trade balance by setting the amount of reserves B_{t+1}. The period-$t+1$ allocation is also determined by B_{t+1} through the policy functions. Given X_t and the policy instruments i_t, τ_t^m and τ_t^x, equations (13), (16) and $S_t^x / S_t^m = (1+\tau_t^m)(1+\tau_t^x)$ is a system of three equations that can be solved for C_t, S_t^x and S_t^m. One can then derive C_{Ht} and C_{Ft} using (9) and (10). The terms of trade S_t can be derived from S_t^m and τ_t^m.

Equation (15) no longer applies since the home consumer can no longer arbitrage between home currency bonds and foreign bonds. However the allocation is the same as when the capital account is open and τ_t^b is set to the level satisfying (15). The same allocations, thus, can be implemented with foreign exchange interventions or with a tax on on capital flows.

APPENDIX C. STEADY STATES

This appendix analyzes the steady states in which one atomistic country has a non-zero level of foreign assets. We denote with subscript j the country with a non-zero level of foreign assets, and with superscript W the representative country in the rest of the world. We omit the time index since all variables are constant over time.

We consider steady states with no other taxes than a constant tariff rate τ^m that is the same for all countries. This is true in a decentralized equilibrium between NSPs even if countries have different levels of foreign assets. Proposition 2 shows that in a decentralized equilibrium with no export taxes each social planner applies the same import tariff $\tau^m = 1/(\epsilon_x - 1)$ independently of the country's level of net foreign assets.

Outside of country j, it follows from $S = S^x = 1$ that $S^m = 1/(1+\tau^m)$ and

$$\frac{C_H^W}{C_F^W} = \frac{\alpha_H}{\alpha_F}(1+\tau^m).$$

Together with this equation, the resource constraint that home production is consumer either at home or abroad, $\overline{L} = C_H^W + C_F^W$, implies

$$\begin{aligned} C_H^W &= \alpha_H \frac{1+\tau^m}{1+\alpha_H \tau^m}\overline{L}, \\ C_F^W &= \frac{\alpha_F}{1+\alpha_H\tau^m}\overline{L}, && (42) \\ C^W &= \frac{(1+\tau^m)^{\alpha_H}}{1+\alpha_H\tau^m}\overline{L} = \frac{\overline{L}}{\ell\left(1/\left(1+\tau^m\right)\right)}. && (43) \end{aligned}$$

As for country j, equation (11) and $L_j = \overline{L}$ imply

$$\overline{L} = \alpha_H \left(\frac{S_j}{1+\tau^m}\right)^{-\alpha_F} C_j + C_F^W S_j^{-\epsilon_x}.$$

Equation (12), together with $\beta R = 1$ and (13), implies

$$(1-\beta) B_j = \alpha_F \left(\frac{S_j}{1+\tau^m}\right)^{\alpha_H} C_j - C_F^W S_j^{1-\epsilon_x}.$$

These two equations determine C_j and S_j for any given B_j. For $B_j = 0$ the solution is $S_j = 1$ and $C_j = C^W$. Differentiating these equations with respect to

B_j in $B_j = 0$ and using (42) to substitute out C_F^W gives the derivatives of the policy functions,

$$\begin{aligned}\frac{\partial C_j}{\partial B_j} &= (1-\beta)\left[1+\frac{\alpha_H}{\epsilon_x}(1+\tau^m)\right]\frac{C^W}{\overline{L}}, \\ \frac{\partial S_j}{\partial B_j} &= (1-\beta)\frac{\alpha_H}{\alpha_F \epsilon_x}\frac{1+\tau^m}{\overline{L}}.\end{aligned}$$

The welfare of country j's representative consumer is given by

$$V(B_j) = \frac{u(C_j)}{1-\beta}.$$

Differentiating this equation in $B_j = 0$ gives the marginal utility of external wealth,

$$V'(0) = u'\left(C^W\right)\left[1+\frac{\alpha_H}{\epsilon_x}(1+\tau^m)\right]\frac{C^W}{\overline{L}}. \tag{44}$$

If the tariff is at the decentralized NSP level, $1+\tau^m = \epsilon_x/(\epsilon_x - 1)$, then using (43) to substitute out C^W in (44) gives

$$V'(0) = u'\left(C^W\right)\left(\frac{\epsilon_x}{\epsilon_{x-1}}\right)^{\alpha_H}. \tag{45}$$

APPENDIX D. ELASTICITIES

This appendix derives the elasticities of macroeconomic variables with respect to the policy instruments, which will be used to prove our main results in Appendix E. We assume that the economy is in the full employment steady state from period 2 onwards so that $\pi_2 = \pi^*$.

We use a first-order approximation in the sense that the dependence of period-2 variables on the country's net foreign assets B_{j2} is omitted. As shown in Appendix C this approximation is legitimate if $1-\beta_1$ is small of the first order. Hence (15) and (16) for $t = 1$ can be written

$$S_{j1} = \frac{1+i_{j1}}{R_1\left(1+\tau_{j1}^b\right)(1+\pi^*)}S_2. \tag{46}$$

and

$$u'(C_{j1})\left(S_{j1}^m\right)^{\alpha_F} = \beta_1 \frac{1+i_{j1}}{1+\pi^*} u'(C_2)\left(S_2^m\right)^{\alpha_F}, \tag{47}$$

where the period 2 variables S_2, S_2^m and C_2 will be taken as invariant to the period 1 policies of country j.

We denote by $e(\bullet, n)$ the elasticity of variable $\bullet = S, C, L, X$ with respect to instrument $n = i, \tau^m, \tau^x$ and τ^b defined as follows,

$$\begin{aligned} e(S,n) &= \frac{1+n}{S}\frac{\partial S}{\partial n},\ e(C,n) = \frac{1+n}{C}\frac{\partial C}{\partial n}, \\ e(L,n) &= \frac{1+n}{C}\frac{\partial L}{\partial n},\ e(X,n) = \frac{1+n}{C}\frac{\partial X}{\partial n}. \end{aligned}$$

The elasticities are computed in a symmetric allocation assuming less than full employment. They are reported in Table D1.

Table D1. Elasticities in a symmetric allocation with unemployment

	i	τ^m	τ^x	τ^b
S	1	0	0	-1
C	$-\alpha_H \epsilon_i$	$-\alpha_F \epsilon_i$	0	$-\alpha_F \epsilon_i$
L	$-(\alpha_H \epsilon_i + \alpha_F)\frac{C_H}{C} - \epsilon_x \frac{C_F}{C}$	$\alpha_F(1-\epsilon_i)\frac{C_H}{C}$	$-\epsilon_x \frac{C_F}{C}$	$\alpha_F(1-\epsilon_i)\frac{C_H}{C} + \epsilon_x \frac{C_F}{C}$
X	$-(\epsilon_x - \alpha_H \epsilon_i - \alpha_F)\frac{C_F}{C}$	$(\alpha_H + \alpha_F \epsilon_i)\frac{C_F}{C}$	$-(\epsilon_x - 1)\frac{C_F}{C}$	$[\epsilon_x - \alpha_F(1-\epsilon_i)]\frac{C_F}{C}$

The values of $e(S,n)$ reported in the top two rows of Table D1 directly follows from (46) and (47).

Differentiating (11) and (13) for $t=1$ and using $C_{F1}^W = C_{F1}$ in a symmetric equilibrium we obtain

$$\begin{aligned} e(L,n) &= [e(C,n) - \alpha_F e(S^m,n)]\frac{C_H}{C} - \epsilon_x e(S^x,n)\frac{C_F}{C}, && (48) \\ e(X,n) &= -[(\epsilon_x - 1)e(S^x,n) + e(C,n) + \alpha_H e(S^m,n)]\frac{C_F}{C}. && (49) \end{aligned}$$

Using the elasticities for C and S given in the first two rows of Table D1 we can use (48) and (49) to derive the expressions in the bottom two rows of the table.

APPENDIX E. PROOFS

Proof of Proposition 1. Assume that the allocation $(C_{Hjt}, C_{Fjt}, L_{jt}, X_{jt})$ stays the same. This implies that B_{jt+1} and so all the time-$t+1$ variables (which are policy functions of B_{jt+1}) stay the same. It then follows from (9), (10) that S_{jt}^m and S_{jt}^x must stay the same. Since $S_{jt}^x / S_{jt}^m = \left(1 + \tau_{jt}^m\right)\left(1 + \tau_{jt}^x\right)$ this implies (18). The fact that C_{jt} and S_{jt}^m stay the same in equation (16) implies that i_{jt} must also stay the same. Then equation (15) and the fact that S_{jt}^m must stay the same imply (19). Conversely, (18) and (19) imply that the allocation is not changed by the alternative policy mix.

Proof of Proposition 2. We omit the country and time index and denote next-period variables with a prime to alleviate notations. The Bellman form of the NSP problem is

$$
\begin{aligned}
V(B) \;=\; & \max_{C_H, C_F, L, B'} u\left(C\left(C_H, C_F\right)\right) + \beta V\left(B'\right) + \\
& \lambda\left[B + \left(C_F^W\right)^{1/\epsilon_x}\left(L - C_H\right)^{1-1/\epsilon_x} - C_F - B'/R\right] + \mu\left(\overline{L} - L\right).
\end{aligned}
$$

The first-order conditions are

$$
\begin{aligned}
u'(C)\frac{\partial C}{\partial C_F} &= \lambda, \\
u'(C)\frac{\partial C}{\partial C_H} &= \lambda\left(1 - \frac{1}{\epsilon_x}\right)\left(C_F^W\right)^{1/\epsilon_x}\left(L - C_H\right)^{-1/\epsilon_x}, \\
\mu &= \lambda\left(1 - \frac{1}{\epsilon_x}\right)\left(C_F^W\right)^{1/\epsilon_x}\left(L - C_H\right)^{-1/\epsilon_x}, \\
\lambda &= \beta R V'\left(B'\right).
\end{aligned}
$$

This implies that λ and μ are strictly positive, so that the constraint $L \leq \overline{L}$ is binding. Dividing the first-order condition for C_H by the first-order condition for C_F gives

$$
\frac{C_F}{C_H} = \frac{\alpha_F}{\alpha_H}\left(1 - \frac{1}{\epsilon_x}\right)\left(\frac{C_F^W}{L - C_H}\right)^{1/\epsilon_x} = \frac{\alpha_F}{\alpha_H}\left(1 - \frac{1}{\epsilon_x}\right)S^x.
$$

Since $C_F/C_H = \alpha_F/\alpha_H S^m$ and $S^x/S^m = \left(1 + \tau^m\right)\left(1 + \tau^x\right)$ this implies (21).

Proof of Proposition 3. Consider a decentralized NSP equilibrium with full employment. The expression for the demand for labor then implies $C_H + (S^x)^{-\epsilon_x} C_F^W = \overline{L}$. Hence all countries have the same level of S^x, which together with (24) and $C_F = C_F^W$ implies $S^x = 1$ and

$$C_H + C_F = \overline{L}. \tag{50}$$

Using equation (21), $S^x/S^m = (1+\tau^m)(1+\tau^x)$, and $S^x = 1$ gives $S^m = 1 - 1/\epsilon_x$. Then the equation for the intratemporal allocation of consumption $\frac{C_F}{C_H} = \frac{\alpha_F}{\alpha_H} S^m$ implies

$$\frac{C_F}{C_H} = \frac{\alpha_F}{\alpha_H}\left(1 - 1/\epsilon_x\right). \tag{51}$$

Equations (50) and (51) then imply

$$\begin{aligned} C_H &= \frac{\alpha_H}{1 - \alpha_F/\epsilon_x}, \\ C_F &= \frac{\alpha_F\left(1 - 1/\epsilon_x\right)}{1 - \alpha_F/\epsilon_x}. \end{aligned}$$

The global social planner maximizes the welfare of the representative country under the allocation constraint $C_H + C_F = L$ and $X = 0$. The GSP problem is

$$\begin{aligned} V^{GSP}(B) = & \max_{C_H, C_F, L} u\left(C\left(C_H, C_F\right)\right) + \\ & \lambda\left[L - C_H - C_F\right] + \mu\left(\overline{L} - L\right). \end{aligned}$$

The first-order conditions for C_H and C_F imply $C_H/C_F = \alpha_H/\alpha_F$ and $L = \overline{L}$, which, together with $C_H + C_F = L$, implies $C_H = \alpha_H$ and $C_F = \alpha_F$. This allocation requires $S^m = 1$ and the trade taxes to satisfy $(1+\tau^m)(1+\tau^x) = 0$. This condition is satisfied if $\tau^m = \tau^x = 0$.

Proof of Proposition 4. As shown in Table D1 both C_1 and X_1 decrease with i_1. Hence welfare U_1 is increased by an interest rate reduction. It follows that in a symmetric decentralized NSP equilibrium countries lower their interest rates until the ZLB constraint is binding in all countries.

It follows from equation (31) that lowering i_1 in all countries raises global consumption and employment $L_1 = C_1^W$. Hence, the global social planner also lowers the interest rate to zero in all countries.

Proof of Proposition 5. There cannot be unemployment in a symmetric Nash equilibrium, otherwise any national social planner could increase domestic welfare by raising the domestic inflation target π_j^*. Hence all social planners set an inflation target such that $\beta_1 \leq 1 + \pi_j^*$, leading to full employment. The inflation target is indeterminate as long as it satisfies this condition.

Proof of Proposition 6. The social planner maximizes domestic welfare over the policy instruments i and τ^m. In a global liquidity trap with unemployment we can ignore the constraint $L_1 \leq \overline{L}$ and the NSP's period-1 problem becomes

$$\max_{\tau_1^m} u\left(C_{j1}\right) + \beta_1 V_2\left(R_1 X_{j1}\right).$$

Using symmetry ($C_{j1} = C_1^W$ and $X_{j1} = 0$) the first-order condition of this problem can be written

$$u'\left(C_1^W\right) e\left(C, \tau^m\right) + \beta_1 R_1 V_2'\left(0\right) e\left(X, \tau^m\right) = 0. \tag{52}$$

Equation (15) with $S_1 = S_2 = 1$, $i_1 = 0$ and $\tau_1^b = 0$ implies $R_1 = 1/\left(1+\pi^*\right)$. Then equation (16) for $t = 1$ can be written

$$u'\left(C_1^W\right)\left(S_1^m\right)^{\alpha_F} = \beta_1 R_1 u'\left(C_2^W\right)\left(S_2^m\right)^{\alpha_F}.$$

Equation (44) implies

$$V_2'\left(0\right) = u'\left(C_2^W\right)\left[1 + \frac{\alpha_H}{\epsilon_x}\left(1+\tau_2^m\right)\right]\frac{C_2^W}{\overline{L}}.$$

Using the two previous equations to substitute out $u'\left(C_1^W\right)$ and $V_2'\left(0\right)$, and the expressions in Table D1 to substitute out the elasticities $e\left(C, \tau^m\right)$ and $e\left(X, \tau^m\right)$ in equation (52), we obtain

$$\alpha_F \frac{C_1^W}{C_{F1}^W}\left(S_1^m\right)^{-\alpha_F} = \left[1 + \frac{\alpha_H}{\epsilon_x}\left(1+\tau_2^m\right)\right]\left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\frac{C_2^W}{\overline{L}}\left(S_1^m\right)^{-\alpha_F}.$$

By (10) the l.h.s. of this equation is equal to $1/S_1^m = 1+\tau_1^m$. Using $C_2^W\left(S_1^m\right)^{-\alpha_F}/\overline{L} = \left(1+\tau_2^m\right)/\left(1+\alpha_H\tau_2^m\right)$ this equation can thus be written as expression for the period-1 tariff rate

$$1+\tau_1^m = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\left[1 + \frac{\alpha_H}{\epsilon_x}\left(1+\tau_2^m\right)\right]\frac{1+\tau_2^m}{1+\alpha_H\tau_2^m}. \tag{53}$$

In the case where the decentralized equilibrium prevails from period 2 onwards, $1+\tau_2^m$ is given by (30), which implies equation (33).

We have $\tau_1^m > \tau_2^m$ if and only if the elasticity of intertemporal substitution is lower than 1 ($\epsilon_i < 1$), as can be easily seen from comparing equations (30) and (33).

Proof of Proposition 7 There is an equilibrium with less than full employment if

$$L_1^W = \ell\left(S_1^m\right) C_1^W < \overline{L}$$

where C_1^W is given by (31). Using (43) to substitute out C_2^W in (31), the condition above can be written,

$$\left(\frac{\beta_1}{1+\pi^*}\right)^{-\epsilon_i} \left(\frac{S_2^m}{S_1^m}\right)^{-\alpha_F \epsilon_i} \frac{\ell\left(S_1^m\right)}{\ell\left(S_2^m\right)} < 1,$$

where $S_t^m = 1/\left(1+\tau_t^m\right)$ and τ_2^m and τ_1^m are respectively given by (30) and (33). Using $\ell\left(1+\tau_2^m\right) = \left(1+\alpha_H \tau_2^m\right)\left(1+\tau_2^m\right)^{-\alpha_H}$ this gives,

$$\beta_1 > \beta^* \equiv \left[\frac{\ell\left(S_1^m\right)}{\ell\left(S_2^m\right)}\right]^{1/\epsilon_i} \left(\frac{S_2^m}{S_1^m}\right)^{-\alpha_F} \left(1+\pi^*\right).$$

Condition (35) implies that $\beta^* < 1+\pi^*$.

Proof of Proposition 9 As shown in Table D1 any NSP can increase net exports X_{j1} without changing consumption C_{j1}, and thus increase welfare, by lowering τ_{j1}^x in a symmetric allocation with unemployment. Hence there cannot be unemployment in a decentralized symmetric equilibrium.

If the ZLB constraint is not binding in period 1 the equilibrium is the same as described in Proposition 3. (As noted after that Proposition the allocation does not depend on whether the NSPs can use capital controls.) Since there is no capital control ($\tau_1^b = 0$) it follows from (27) that $\tau_1^m = \tau_2^m$ and it follows from (21) and (30) that $\tau_2^x = 0$. It follows from (16) for $t = 1$ with $C_1^W = C_2^W$ and $S_1^m = S_2^m$ that $1 + i_1 = \left(1+\pi^*\right)/\beta_1$. Hence, the ZLB is indeed not binding if and only if β_1 is larger than $1+\pi^*$.

If $\beta_1 > 1+\pi^*$ the ZLB constraint is binding. Using equations (31), $S_t = 1/\left(1+\tau_t^m\right)\left(1+\tau_t^x\right)$, $\overline{L} = C_t^W \ell\left(S_t\right)$ with $i_1 = 0$, $1+\tau_2^m = \epsilon_x/\left(\epsilon_x - 1\right)$, $\tau_2^x = 0$ and $C_2^W = \overline{L}/\ell\left(\frac{\epsilon_x}{\epsilon_x - 1}\right)$, there is full employment in period 1 if and only if

$$\left(1+\tau_1^m\right)\left(1+\tau_1^x\right) = 1/\widehat{S},$$

where $\widehat{S}$ is the solution to

$$\ell\left(\widehat{S}\right)\left(\widehat{S}\right)^{\alpha_F\epsilon_i}=\left(\frac{\beta_1}{1+\pi^*}\right)^{\epsilon_i}\ell\left(1-\frac{1}{\epsilon_x}\right)\left(1-\frac{1}{\epsilon_x}\right)^{\alpha_F\epsilon_i} \tag{54}$$

that is larger than $\epsilon_x/\left(\epsilon_x-1\right)$.

The nominal interest rate being at the ZLB, the NSP sets the trade taxes so as to solve the following problem

$$\max_{\tau_{11}^m,\tau_{11}^x} u\left(C_{j1}\right)+\beta_1 V_2\left(R_1 X_{j1}\right)+\lambda\left(\overline{L}-L_{j1}\right),$$

In a symmetric equilibrium, the first-order condition for instrument $n=\tau^m,\tau^x$

$$u'\left(C_1^W\right)e\left(C,n\right)+\beta_1 R_1 V_2'\left(0\right)e\left(X,n\right)-\lambda e\left(L,n\right)=0.$$

Using the elasticities reported in Table D1 this gives,

$$\begin{aligned} u'\left(C_1^W\right) &= \beta_1 R_1 V_2'\left(0\right)\left(1+\frac{\alpha_H}{\alpha_F\epsilon_i}\right)\frac{C_{F1}^W}{C_1^W}-\lambda\left(\frac{1}{\epsilon_i}-1\right)\frac{C_{H1}^W}{C_1^W}. \\ \lambda &= \beta_1 R_1 V_2'\left(0\right)\frac{\epsilon_x-1}{\epsilon_x}. \end{aligned}$$

Using the second expression to substitute out λ from the first one, and $C_{F1}^W/C_1^W=\alpha_F\widehat{S}^{\alpha_H}$ and $C_{H1}^W/C_1^W=\alpha_H\widehat{S}^{-\alpha_F}$ from equations (9) and (10), one obtains,

$$u'\left(C_1^W\right)=\beta_1 R_1 V_2'\left(0\right)\left[\left(1+\frac{\alpha_H}{\alpha_F\epsilon_i}\right)\widehat{S}-\left(1-\frac{1}{\epsilon_x}\right)\left(\frac{1}{\epsilon_i}-1\right)\right]\widehat{S}^{-\alpha_F}.$$

We then use (47) with $i_1=0$, $S_1^m=\widehat{S}$ and $S_2^m=1-1/\epsilon_x$ to substitute out $u'\left(C_1^W\right)$; equation (32) with $i_1=\tau_1^b=\tau_2^x=0$ to substitute out R_1; and equation (45) to substitute out $V_2'\left(0\right)$. This gives, after some manipulations

$$1+\tau_1^x=\left[\left(\frac{\alpha_H}{\epsilon_i}+\alpha_F\right)\left(\frac{\widehat{S}}{1-1/\epsilon_x}-1\right)+1\right]^{-1}. \tag{55}$$

In the limit case where the ZLB constraint is not binding ($\beta_1=1+\pi^*$) equation (54) implies $\widehat{S}=1-1/\epsilon_x$ so that $\tau_1^x=0$. Raising β_1 above $1+\pi^*$ increases $\widehat{S}$ above $1-1/\epsilon_x$ by equation (54) and lowers τ_1^x below zero. Hence, the NSPs subsidize exports in a global liquidity trap and the rate of subsidy increases as global demand falls.

To see the impact of global demand on the equilibrium level of tariffs one can use $(1+\tau_1^m)(1+\tau_1^x) = 1/\widehat{S}$ and equation (55) to obtain

$$1+\tau_1^m = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right) \frac{1}{1-1/\epsilon_x} - \alpha_H \left(\frac{1}{\epsilon_i} - 1\right) \frac{1}{\widehat{S}}.$$

Hence an increase in β_1 that raises $\widehat{S}$ also increases the tariff rate τ_1^m.

Proof of Proposition 11 Point (i). We assume that the economy is in a global liquidity trap before time T and that the NSPs can use tariffs. For any period $t < T$ the representative NSP's value function can be written,

$$V_t(B_t) = \max_{\tau_t^m} u(C_t) + \beta_t V_{t+1}(R_t(X_t + B_t)). \tag{56}$$

The equilibrium C_t and X_t are functions of B_t. If $1-\beta$ is first order, the partial derivatives $\partial C_t/\partial B_t$ and $\partial X_t/\partial B_t$ are second-order and can be neglected to a first order of approximation. Thus we have

$$V_t'(0) = \beta_t R_t V_{t+1}'(0).$$

Iterating on this equation gives

$$V_t'(0) = \prod_{s=t}^{T-1} \beta_s R_s V_T'(0). \tag{57}$$

The first-order condition for the NSP problem (56) can be written (in a symmetric equilibrium)

$$u'\left(C_t^W\right) e(C, \tau^m) + \beta_t R_t V_{t+1}'(0)\, e(X, \tau^m) = 0$$

where the elasticities are given in Table D1. Using the expressions in that table to substitute out the elasticities and $C_{Ft}^W / C_t^W = \alpha_F (S_t^m)^{\alpha_H}$ and (57) one obtains

$$\begin{aligned} u'\left(C_t^W\right)(S_t^m)^{-\alpha_H} &= \beta_t R_t V_{t+1}'(0)\left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right), \\ &= \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\left(\prod_{s=t}^{T-1} \beta_s R_s\right) V_T'(0). \end{aligned} \tag{58}$$

Using the Euler equation (16) with $i_t = 0$ and $R_t = 1/(1+\pi_{t+1})$ from (15) with $\tau_t^b = i_t = 0$ and $S_t = S_{t+1} = 1$ (in a symmetric allocation without export taxes) one gets

$$\begin{aligned} u'\left(C_t^W\right)\left(S_t^m\right)^{\alpha_F} &= \beta_t R_t u'\left(C_{t+1}^W\right)\left(S_{t+1}^m\right)^{\alpha_F}, \\ &= \left(\prod_{s=t}^{T-1} \beta_s R_s\right) u'\left(C_T^W\right)\left(S_T^m\right)^{\alpha_F}. \end{aligned} \tag{59}$$

Dividing (59) by (58) and using $V_T'(0) = u'\left(C_T^W\right)\left(1 + \frac{\alpha_H}{\epsilon_x}\frac{1}{S_T^m}\right)/\ell\left(S_T^m\right)$ (from equations (44) and (43)) gives

$$1 + \tau_t^m = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\left[1 + \frac{\alpha_H}{\epsilon_x}\left(1 + \tau_T^m\right)\right]\frac{1 + \tau_T^m}{1 + \alpha_H \tau_T^m}.$$

This generalizes equation (53), obtained in the case $T = 2$. Hence the tariff rate is the same as in Proposition 6.

Point (ii) can be proven like for Propositions 9 and 10. The NSPs can increase the trade balance without distorting consumption by reducing τ^x as long as there is unemployment, implying that there must be full employment in the decentralized equilibrium.

In the long run, the NSPs tax exports at rate $\tau^x = 1/(\epsilon_x - 1)$ and consumption is given by $C_T^W = \overline{L}/\ell(1 - 1/\epsilon^x)$. Iterating over equation (16) with $i_t = 0$ and $\pi_t = \pi^*$ for $t = T$ and $\pi_t = 0$ for $t < T$ implies

$$u'\left(C_t^W\right) S_t^{\alpha_F} = \frac{\prod_{s=t}^{T-1} \beta_s}{1+\pi^*} u'\left(C_T^W\right) S_T^{\alpha_F}.$$

Then using $C_t^W = \overline{L}/\ell(S_t)$ and $S_T = 1 - 1/\epsilon_x$ implies $S_t = \widehat{S}_t$ defined by

$$\ell\left(\widehat{S}_t\right)\left(\widehat{S}_t\right)^{\alpha_F \epsilon_i} = \left(\frac{\prod_{s=t}^{T-1} \beta_s}{1+\pi^*}\right)^{\epsilon_i} \ell\left(1 - \frac{1}{\epsilon_x}\right)\left(1 - \frac{1}{\epsilon_x}\right)^{\alpha_F \epsilon_i},$$

which generalizes (54).

To prove point (iii), note that the GSP sets the trade taxes to zero after period T by Proposition 3. Before that the GSP subsidizes exports or imports to achieve full employment like in Proposition 10.

References

Acharya, Sushant, and Julien Bengui. 2018. "Liquidity Traps, Capital Flows." *Journal of International Economics*, 114: 276–298.

Amiti, Mary, Stephen Redding, and David Weinstein. 2019. "The Impact of the 2018 Trade War on U.S. Prices and Welfare." *NBER Working Paper 25672*.

Auray, Stéphane, Michael B. Devereux, and Aurélien Eyquem. 2020. "Trade Wars, Currency Wars." *NBER Working Paper 27460*.

Barattieri, Alessandro, Matteo Cacciatore, and Fabio Ghironi. 2018. "Protectionism and the Business Cycle." *NBER Working Paper 24353*.

Barbiero, Omar, Emmanuel Farhi, Gita Gopinath, and Oleg Itskhoki. 2019. "The Macroeconomics of Border Taxes." *NBER Macroeconomics Annual*, 33(1): 395–457.

Bénassy-Quéré, Agnès, Matthieu Bussière, and Pauline Wibaux. 2018. "Trade and Currency Weapons." *CESifo Working Paper 7112*.

Benigno, Gianluca, and Pierpaolo Benigno. 2006. "Designing Targeting Rules for International Monetary Policy Cooperation." *Journal of Monetary Economics*, 53(3): 473–506.

Bergsten, C. Fred, and Joseph E. Gagnon. 2012. "Currency Manipulation, the US economy and the Global Economic Order." *Peterson Institute for International Economics Policy Brief 12-25*.

Bernanke, Ben S. 2017. "Federal Reserve Policy in an International Context." *IMF Economic Review*, 65(1): 1–32.

Blanchard, Olivier. 2017. "Currency Wars, Coordination and Capital Controls." *International Journal of Central Banking*, 13(2): 283–308.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas. 2015. "Global Imbalances and Currency Wars at the ZLB." *NBER Working Paper 21670*.

Canzoneri, Matthew B, Robert E Cumby, and Behzad T Diba. 2005. "The Need for International Policy Coordination: What's Old, What's New, What's Yet to Come?" *Journal of International Economics*, 66(2): 363–384.

Cook, David, and Michael B Devereux. 2013. "Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap." *American economic Journal: macroeconomics*, 5(3): 190–228.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles. 2013. "Unconventional Fiscal Policy at the Zero Bound." *American Economic Review*, 103(4): 1172–1211.

Corsetti, Giancarlo, Eleonora Mavroeidi, Gregory Thwaites, and Martin Wolf. 2019. "Step Away from the Zero Lower Bound: Small Open Economies in a World of Secular Stagnation." *Journal of International Economics*, 116: 88–102.

Costinot, Arnaud, and Iván Werning. 2019. "Lerner Symmetry: A Modern Treatment." *American Economic Review: Insights*, 1(1): 13–26.

Devereux, Michael B, and James Yetman. 2014. "Capital controls, Global Liquidity Traps, and the International Policy Trilemma." *The Scandinavian Journal of Economics*, 116(1): 158–189.

Eggertsson, Gauti B, Neil R Mehrotra, Sanjay R Singh, and Lawrence H Summers. 2016. "A Contagious Malady? Open Economy Dimensions of Secular Stagnation." *IMF Economic Review*, 64(4): 581–634.

Eichengreen, Barry J. 1992. *Golden Fetters: The Gold Standard and the Great Depression, 1919-1939.* Oxford University Press.

Engel, Charles. 2016. "International Coordination of Central Bank Policy." *Journal of International Money and Finance*, 67: 13–24.

Erceg, C, A Prestipino, and A Raffo. 2017. "The Macroeconomic Effects of Trade Policy." *mimeo, Board of Governors of the Federal Reserve System.*

Fajgelbaum, Pablo D., Pinelopi K. Goldberg, Patrick J. Kennedy, and Amit K. Khandelwal. 2019. "The Return to Protectionism." *NBER Working Paper 25638*.

Farhi, Emmanuel, Gita Gopinath, and Oleg Itskhoki. 2014. "Fiscal Devaluations." *Review of Economic Studies*, 81(2): 725–760.

Feenstra, Robert, Philip Luck, Maurice Obstfeld, and Katheryn Russ. 2018. "In Search of the Armington Elasticity." *Review of Economics and Statistics*, 100(1): 135–150.

Fornaro, Luca, and Federica Romei. 2019. "The Paradox of Global Thrift." *American Economic Review*, 109(11): 3745–79.

Freund, Caroline, Michael Ferrantino, Maryla Maliszewska, and Michele Ruta. 2018. "Impacts on Global Trade and Income of Current Trade Disputes." *Macroeconomics, Investment and Trade Practice Note 2, World Bank*.

Fujiwara, Ippei, Tomoyuki Nakajima, Nao Sudo, and Yuki Teranishi. 2013. "Global Liquidity Trap." *Journal of Monetary Economics*, 60(3): 936–949.

Gali, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *The Review of Economic Studies*, 72(3): 707–734.

Jeanne, Olivier. 2013. "Capital Account Policies and the Real Exchange Rate." *NBER International Seminar on Macroeconomics 2012 (ISOM)*, , ed. Francesco Giavazzi and Kenneth D. West, 7–42. University of Chicago Press.

Korinek, Anton. 2016. "Currency Wars or Efficient Spillovers? A General Theory of International Policy Cooperation." *NBER Working Paper 23004*.

Lerner, Abba P. 1936. "The Symmetry Between Import and Export Taxes." *Economica*, 3(11): 306–313.

Lindé, Jesper, and Andrea Pescatori. 2019. "The Macroeconomic Effects of Trade Tariffs: Revisiting the Lerner Symmetry Result." *Journal of International Money and Finance*, 95: 52–69.

Meade, James Edward. 1955. *Trade and welfare.* Vol. 2, Oxford University Press.

Mishra, Prashi, and Raghuram Rajan. 2016. "Rules of the Monetary Game." *Reserve Bank of India, Manuscript.*

Obstfeld, Maurice, and Kenneth Rogoff. 2002. "Global Implications of Self-Oriented National Monetary Rules." *The Quarterly Journal of Economics*, 117(2): 503–535.

Ossa, Ralph. 2014. "Trade Wars and Trade Talks with Data." *American Economic Review*, 104(12): 4104–46.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2016. "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment." *Journal of Political Economy*, 124: 1466–1514.