Capital Account Policies and the Real Exchange Rate*

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Abstract
This paper presents a simple model of how a country can under-value its real exchange rate through a combination of foreign asset purchases by the government and controls on capital inflows. This policy combination is equivalent to a tariff on imports and a subsidy on exports. The welfare cost of such an intervention is measured by the valuation loss on the foreign assets in terms of domestic consumption. If the controls on inflows take the form of a tax on the purchase of domestic assets by foreign investors, the magnitude of the real undervaluation cannot be larger than the tax.

1 Introduction

There are debates about the extent to which emerging market and developing countries that have accumulated large amounts of foreign exchange reserves in the 2000s are doing so in order to undervalue their currency. However, we do not have a simple “textbook” model of how a country can achieve persistent real exchange rate distortions. The main purpose of this paper is

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to present such a model and to use it to answer a few questions about such a policy.

Real exchange rate undervaluation is often presented, in policy debates, as the result of a monetary operation. For example, the People’s Bank of China resists the appreciation of the renminbi by foreign exchange interventions. In an environment with perfect capital mobility, however, this policy would influence the real (as opposed to nominal) exchange rate only to the extent that there is very persistent domestic price stickiness that prevents an internal appreciation through domestic inflation, or strong portfolio effects. The available evidence does not suggest that price stickiness or portfolio effects are sufficiently strong to achieve persistent real undervaluation. In order to achieve persistent real undervaluation, thus, policy must rely on something else.

I focus in this paper on the role of controls on capital inflows. Indeed, China has strong capital account restrictions on inflows,¹ and there seems to be more generally a correlation between controls on inflows and real exchange rate undervaluation. Figure 1 plots a measure of real exchange rate undervaluation against a measure of controls on capital inflows (respectively on the horizontal axis and the vertical axis). In line with several recent papers (Rodrik, 2008; Johnson, Ostry and Subramanian, 2010) real exchange rate undervaluation is measured as the residual in a regression of the real exchange rate on GDP per capita in order to control for the Balassa-Samuelson effect. The capital account measure, which comes from Schindler (2008), is increasing with the degree of liberalization on inflows. The correlation shown in Figure 1 is not conclusive, but suggests that restrictions on capital inflows tend to be associated with real undervaluation.

In order to simplify and streamline the analysis, I use a model that is entirely real—there is no money and no monetary policy. I consider a small open economy that consumes a tradable good and a nontradable good. The government accumulates foreign assets and imposes controls on inflows (which means, in this simple model, a constraint on external borrowing by the domestic private sector). I show that this combination of policies allows the government to effectively control the level of net foreign assets for the country as a whole. The other properties of the model then follow in a

¹China limits severely financial inflows. It receives large volume of FDI inflows but even FDI is subject to controls and authorizations. The assumption here is not that the country does not receive any inflows, but that the authorities control the volume of those flows.
straightforward way. The government controls the current account balance (since it is the change in net foreign assets) and therefore the trade balance. The real exchange rate, then, has to be consistent with the trade balance. Other things equal, accumulating more net foreign assets will depreciate the real exchange rate.

I look at a simple endowment economy in which there is no investment, so that the current account balance is equal to domestic saving. Another way of looking at the real undervaluation policy, thus, is that the accumulation of foreign assets induces “forced saving” in the domestic economy. The capital controls prevent the domestic private sector from offsetting the public accumulation of foreign assets with capital inflows. The model thus provides a very simple explanation for the high saving rate in countries such as China.\(^2\)

I then use the model to look at two questions: the welfare cost of real exchange rate interventions, and the impact of price-based capital controls.

\(^2\)There are many other explanations for the high saving rate in China: see Mendola, Quadrini and Ríos-Rull (2009), Caballero, Farhi and Gourinchas (2008), Carroll and Jeanne (2009), Sandri (2010), Song, Storesletten and Zilibotti (2009) and Chamon and Prasad (2010). The model presented here explains the saving rate for the economy as a whole. It does not predict the breakdown of saving between the household and the corporate sectors.
The welfare cost of an undervaluation policy comes from the fact the country receives a low return on its foreign assets in terms of domestic consumption. The country depreciates its real exchange rate when it accumulates foreign assets and appreciates the real exchange rate when it reduces the pace of accumulation. In some sense, the country follows the opposite of the classical investor’s advice: it “buys high and sells low”. This may not be by design, but this is an equilibrium phenomenon that cannot be done away with—what one might call the “saver’s curse” in international finance.

I then replace the quantity-based capital controls of the benchmark model with a price-based control, taking the form of a tax on the purchase of domestic assets by foreigners. There is a simple relationship between the level of the tax and the maximum real undervaluation that can be generated by domestic policy: the two are equal.

The paper is structured as follows: Section 2 presents the model, Section 3 explores its quantitative properties with a numerical illustration, Section 4 studies the welfare cost of real undervaluation, and Section 5 studies price-based capital controls.

## 2 Model

The model is deterministic and in continuous time. We consider a small open economy populated by an infinitely-lived representative consumer who consumes a tradable good and a nontradable good. The utility of the representative consumer is given by

$$U_t = \int_0^{+\infty} e^{-r^* s} u(c_{t+s}) ds,$$

where $c_t = c(c_{Tt}, c_{Nt})$ is a function of the consumption of the tradable good, $c_T$, and consumption of the nontradable good, $c_N$, which is homogeneous of degree 1. We denote by $p_t$ the price of the nontradable good in terms of the tradable good, and by $q_t$ the price of the tradable good in terms of domestic consumption. $q_t$ is the real exchange rate (an increase in $q$ is a real depreciation).

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3 Analogous to the Chilean controls or the recent measures adopted by Brazil.
The domestic consumer receives exogenous flows of nontradable and tradable goods. The budget constraint of the domestic consumer is

\[ c_t + \dot{a}_t + q_t \dot{a}^*_t = q_t(y_{Tt} + p_t y_{Nt}) + r_t a_t + q_t r^* a^*_t + z_t, \]

where \( a_t \) and \( a^*_t \) are the consumer’s assets (bonds) respectively denominated in consumption good and tradable good, \( y_{Tt} \) and \( y_{Nt} \) are the country’s endowments of the tradable good and nontradable good, and \( z_t \) is a lump-sum transfer from the government. Although the difference between \( a_t \) and \( a^*_t \) is purely the denomination of the assets, by a slight abuse of language we will sometimes call \( a_t \) and \( a^*_t \) the private sector’s holdings of “domestic assets” and “foreign assets”, respectively.

The budget constraint of the government is

\[ q_t \dot{b}^*_t + \dot{b}_t + z_t = q_t r^* b^*_t + r_t b_t, \]  

(2)

where \( b^*_t \) and \( b_t \) are the government’s holdings of bonds denominated in tradable good and consumption good respectively. We call \( b^*_t \) “international reserves.” Because we will assume that the government accumulates foreign assets by issuing domestic liabilities, it will be convenient to define the government’s domestic debt as \( d_t = -b_t \).

Government policy consists in the announcement of paths for international reserves and domestic debt, \((b^*_t, d_t)\), that satisfy the transversality condition. The impact of government policy crucially depends on the extent of capital mobility between the country and the rest of the world. With perfect capital mobility, government policy has no effect on the domestic economy and the real exchange rate. Equilibrium conditions pin down the country’s total net wealth, \( b_t + a_t + q_t (b^*_t + a^*_t) \), but not the individual components. In particular, an open market operation in which the government purchases reserves \( b^*_t \) by issuing debt \( d_t \) has no impact on the domestic economy. This is clear if the government makes the transaction with foreign investors, since in this case nothing changes for the domestic private sector (the operation does not affect the transfer \( z_t \) because of interest parity). This is also true if the government’s debt is not traded internationally and must be sold solely to the domestic private sector. In this case, the domestic private sector simply finances the purchase of domestic government debt by selling foreign assets (or issuing foreign liabilities) to foreign investors. Government policy is irrelevant if the domestic private sector is connected to the international financial
market through the frictionless trade of one asset or liability.\footnote{The various classes of assets and liabilities are perfectly substitutable, in our model, because of the absence of risk.}

The situation is quite different if the government is the only agent in the economy that can enter into financial relationships with the rest of the world. Let us assume that the domestic private sector must hold all of the government debt ($a_t = d_t$) and is forbidden to hold foreign assets ($a^*_t = 0$). The country’s consolidated budget constraint can then be written

$$c_{Tt} = y_{Tt} + r^* b^*_t - \dot{b}^*_t.$$ 

By setting the path for reserves ($b^*_t$), the government completely determines the path of the consumption of the tradable good ($c_{Tt}$) and of the trade balance ($y_{Tt} - c_{Tt}$). It also determines the real exchange rate, which, using the first-order condition, $q_t = \partial c_t / \partial c_{Tt}$, and the equilibrium condition in the market for the nontradable good, $c_{Nt} = y_{Nt}$, can be written in reduced form as a function of $c_{Tt}$ and $y_{Nt}$,

$$q_t = q(c_{Tt}, y_{Nt}).$$

This result is, as a matter of accounting, obvious. If the government can determine the country’s total net foreign assets, then it can also determine the current account balance (the change in the country’s net foreign assets) and the trade balance (the change in the country’s net foreign assets minus the return on those assets). In other terms, the government can induce “forced saving” in the domestic economy by forcing the private sector to buy domestic debt and by using the proceeds to buy foreign assets. The domestic private sector cannot undo this operation by selling assets to—the rest of the world.

Note that the government does not need to block both capital inflows and capital outflows at all times. If there is an excess return on domestic assets, i.e. $r_t \geq r^* + \dot{q}_t / q_t$, then it is sufficient for the government to block capital inflows. The control on outflows can be removed, such that the constraint $a^*_t = 0$ is replaced by $a^*_t \geq 0$. This does not change the equilibrium because the domestic private sector does not want to accumulate foreign assets on top of the government.\footnote{Obviously, things would be different if by liberalizing capital outflows one meant that the domestic consumer has the freedom to spend the reserves as it wishes. Then, he would spend the reserves on imports and the real exchange rate would appreciate.} However, if there is an excess return on foreign assets
(which is possible at some times), the government can control the level of
net foreign assets only if there are controls on outflows. Because, as we will
see, resisting a real appreciation tends to raise the return on domestic assets
relative to foreign assets, it requires controls on inflows.

Finally, note that the real exchange rate manipulation can similarly be
achieved by a tariff on imports and a subsidy on exports (Johnson, 1953).
The combination of tariff and subsidy raises the relative price of the tradable
good and induces a real depreciation. As shown in the appendix, if there is
perfect capital mobility but the government imposes a tariff/subsidy $\tau_t$
on imports/exports, then it is possible to implement a path for tradable good
consumption $(c_{Tt})_{t \geq 0}$, and the associated real exchange rate path, $(q_t)_{t \geq 0}$, by
imposing a tariff rate such that

$$1 + \tau_t = u'(c(c_{Tt}, y_{Nt})) \frac{\partial c(c_{Tt}, y_{Nt})}{\partial c_T}.$$ 

Thus, capital controls offer a perfect substitute to trade protectionism.

### 3 Numerical illustration

Figure 2 illustrates how a country can resist the real appreciation of its
currency by accumulating foreign assets. We consider a country in which,
starting from a steady state with zero net foreign assets, the supply of the
tradable good takes off in an unexpected way at time $t = 0$. The output of
the tradable good is given by

$$y_{Tt} = \begin{cases} 
  y_T & \text{for } t \leq 0, \\
  y_T + \Delta y_{Tt} & \text{for } t > 0,
\end{cases}$$

where the increase in tradable output relative to the old steady state, $\Delta y_{Tt}$, is
positive and nondecreasing with time. This experiment is meant to capture,
in a very stylized way, the case of countries whose economy takes off because
of the development of the export sector. For simplicity, the supply of the
nontradable good, $y_N$, remains constant. For the simulation we assume the
specification

$$\Delta y_{Tt} = (1 - e^{-\nu t}) \Delta y_T,$$ 

(3)
where $\nu$ is the speed at which the tradable output converges to its new long-run level. We also assumed that the domestic consumer has a constant relative risk aversion of $\gamma$ (i.e. $u(c) = c^{1-\gamma}/(1 - \gamma)$) and that domestic consumption is a CES index of the consumption of the tradable good and that of the nontradable good,

$$c(c_T, c_N) = \left[\eta^{1/\theta} c_T^{(\theta-1)/\theta} + (1 - \eta)^{1/\theta} c_N^{(\theta-1)/\theta}\right]^\theta/(\theta-1).$$

The values of the parameters used are given in Table 1.

<table>
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<th>$y_N$</th>
<th>$y_T$</th>
<th>$\Delta y_T$</th>
<th>$r^*$</th>
<th>$\gamma$</th>
<th>$\eta$</th>
<th>$\theta$</th>
<th>$\nu$</th>
</tr>
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<td>4</td>
<td>0.05</td>
<td>2</td>
<td>0.3</td>
<td>1.5</td>
<td>0.1</td>
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</table>

The solid and dashed lines show the responses of the variables respectively under perfect capital mobility and under a policy of foreign assets accumulation that resists the real appreciation of the currency. With perfect capital mobility, the consumption of tradable good jumps to a higher level at time 0 and remains constant thereafter.\(^6\) The real exchange rate jumps down to a lower constant level (an immediate permanent appreciation). The country initially has a trade deficit that the consumer finances by borrowing against future tradable output.

The dashed lines show the same variables assuming that the government accumulates foreign assets in order to induce a continuous path of real appreciation for the domestic currency. To construct this figure we assumed that the government targets the following path for the consumption of the tradable good,

$$c_{Tt} = y_T + (1 - e^{-\sigma t})\Delta c_T, \quad (4)$$

where $\Delta c_T$ is set to the level that ensures that the transversality condition is satisfied (see the appendix for the technical details). Variable $\sigma$ is the speed at which the consumption of the tradable good converges to its long-run level. The case with perfect mobility is obtained in the limit as $\sigma$ goes to infinity. Reducing $\sigma$ means that the government is leaning more vigorously against the real appreciation. To construct Figure 2 we set $\sigma$ to 0.05.

\(^6\)Unlike in the Balassa-Samuelson model, the real appreciation is determined by the demand side, not the supply side.
The domestic interest rate \( r_t \) is higher than \( r^* \). Since, in addition, there is a real appreciation, the excess return on domestic assets expressed in terms of the tradable good, \( r_t - \dot{q}_t/q_t \), is even larger. That foreign assets yield a lower return than domestic assets is important to understanding the welfare cost of the government’s intervention (see the next section).

If the government resists real appreciation, the domestic assets must yield a higher return than foreign assets at some point in time, although not necessarily at every point in time. This is illustrated by the case in which the government intervenes to delay a discrete jump in the real exchange rate to its equilibrium value. The government maintains the real exchange rate at the old level for some time, and lets the currency appreciate to its frictionless level at some future time \( t^* > 0 \) (for example by relaxing all capital controls). The only difference with the previous case is that (4) is replaced by

\[
\begin{align*}
c_{Tt} &= y_T \text{ for } t < t^*, \\
c_{Tt} &= y_T + \Delta c_T \text{ for } t \geq t^*.
\end{align*}
\]

There is no excess return on domestic assets before or after the appreciation, that is “almost all the time”. But the excess return jumps to infinity at time \( t^* \) because the discontinuous appreciation of the real exchange rate im-
plies that an investor can make a non-infinitesimal profit over an infinitesimal time interval.

Those real exchange rate paths can be implemented with controls on inflows only. Controlling the outflows is not necessary because residents receive a higher return on domestic assets than on foreign assets. But in general, the excess return on domestic assets could be negative at some times, so that controls on outflows would also be necessary.

4 Welfare cost of intervention

In this model, resisting the real appreciation of the currency reduces domestic welfare (it might increase welfare if, for example, growth were endogenous to the real exchange rate—see Korinek and Serven (2010)—but under our assumptions, the first welfare theorem applies and an undervaluation policy unambiguously reduces welfare). What is the welfare cost of the undervaluation? In policy discussions, this cost is sometimes identified with the valuation loss to the central bank when the currency finally appreciates. But it has also been argued that the intervention might in fact be costless since the revaluation does not reduce the purchasing power of the reserves in terms of the tradable good.

We study the welfare impact of such a policy by looking at a small change in the accumulation of foreign assets. Let us assume that the government changes the path \((b^*_t)_{t\geq 0}\) by an infinitesimal (first order) amount \((\delta b^*_t)_{t\geq 0}\). If \(\delta b^*_t\) is continuously differentiable (so that \(c_{Tt}\) and \(q_t\) are continuous), then the impact on welfare can be written

\[
\delta U_0 = \int_0^{+\infty} u'(c_t) \frac{\partial c_t}{\partial c_{Tt}} \delta c_{Tt} e^{-r^* t} dt.
\]

Then using \(\delta c_{Tt} = r^* \delta b^*_t - \dot{b}^*_t\), \(\partial c_t/\partial c_{Tt} = q_t\), \(\dot{\lambda}_t/\lambda_t = r^* - r_t\) (where \(\lambda_t = u'(c_t)\)) and integrating by parts gives

\[
\delta U_0 = -\int_0^{+\infty} u'(c_t) q_t \left( r_t - r^* \right) \delta b^*_t e^{-r^* t} dt.
\]

The term in parenthesis is the excess return on domestic assets. Multiplied by \(\delta b^*_t\) it is the flow opportunity cost of financing the stock of foreign bonds.
by issuing domestic debt. The impact of undervaluation on welfare is the present discounted value of this flow cost weighted by the marginal utility of consumption.

This formula can be extended to the case where the real exchange rate is discontinuous. Let us assume that \( c_t, c_{Tt} \) and \( q_t \) are discontinuous at time \( t^* \), and denote with a superscript “−” (“+”) the levels of the variables just before (after) that time. The variation of the welfare is then given by

\[
\delta U_0 = -\int_{t\geq 0, t\neq t^*} u'(c_t)q_t \left( r_t - r^* - \frac{\dot{q}_t}{q_t} \right) \delta b_t^* e^{-r^*t} dt - \left[ u'(c_t^-)q_t^- - u'(c_{t^*})q_{t^*}^+ \right] \delta b_t^*.
\]

If consumption were continuous, \( u'(c_t^-) = u'(c_{t^*}) \), the term that is added by the revaluation would reduce to

\[
-u'(c_t^-) \left( q_t^- - q_{t^*}^+ \right) \delta b_t^*,
\]

which is the valuation loss on the foreign assets held at time \( t^* \), \( \delta b_t^* \), weighted by the marginal utility of consumption. The fact that consumption jumps magnifies the welfare loss, which can be written,

\[
-u'(c_t^-) \left( q_t^- - q_{t^*}^+ \right) \delta b_t^* - \left[ u'(c_t^-) - u'(c_{t^*}) \right] q_t^+ \delta b_t^*.
\]

## 5 Price-based capital controls

Many real world controls are price-based, i.e., they take the form of a tax on capital inflows. It is interesting to know how much these controls can “buy” in terms of real exchange rate manipulation. Let us assume that the domestic government levies a tax, \( \tau_t \), on the purchase of domestic assets by foreigners. What is the tax that guarantees no capital inflows?

We assume that a foreign investor can buy \( (1 - \tau)q \) units of domestic bonds in exchange of one unit of the tradable good. He can sell the bonds and repatriate the proceeds at any time (there are no controls on outflows). By investing in foreign bonds between time \( t \) and time \( t' > t \) the foreign investor receives a gross return,

\[
\exp \left[ r^*(t' - t) \right].
\]
To prevent foreign capital from flowing to the domestic economy, this should be larger (for any time $t'$) than the gross return on an investment in domestic bonds, which, net of tax, is given by

$$(1 - \tau_t) \exp \left[ \int_t^{t'} r_s ds \right] \frac{q_t}{q_{t'}}.$$

This gives an upper bound on the real exchange rate,

$$q_t \leq \frac{1}{1 - \tau_t} \min_{t'} \left\{ \exp \left[ \int_t^{t'} (r^* - r_s) ds \right] q_{t'} \right\}.$$

Using the first-order condition $r^* - r_s = \dot{\lambda}_s / \lambda_s$, where $\lambda_s = u'(c_s)$, this can be rewritten

$$q_t \leq \frac{1}{1 - \tau_t} \min_{t'} \left\{ \frac{u'(c_{t'})}{u'(c_t)} q_{t'} \right\}. \quad (7)$$

Let us denote by $\tilde{q}_t$ the “shadow” real exchange rate that one would observe at time $t$ if the government stopped intervening at that time. (The term is analogous with the shadow exchange rate in the theory of speculative attacks; see Flood and Garber.) Similarly, we define $\tilde{c}_t$ as the domestic consumption level under no intervention. That the government intervenes to prevent the appreciation of the currency means that the shadow exchange rate is always below (and the shadow consumption level above) the observed level; that is

$$\tilde{q}_t < q_t, \quad \tilde{c}_t > c_t.$$

It will be useful to define not only the time-$t$ shadow exchange rate but the whole subsequent path. We denote by $\tilde{q}_t(t')$ the shadow real exchange rate that one would observe at any time $t' \geq t$ if the government stopped intervening at time $t$ (with a similar notation for consumption). The paths $(\tilde{q}_t(t'))_{t' \geq t}$ and $(q_{t'})_{t' \geq t}$ both satisfy the intertemporal budget constraint with the same level of initial foreign wealth. The fact that $\tilde{q}_t < q_t$ implies that there must be a future time $t'$ such that $\tilde{q}_t(t') = q_{t'}$ and $\tilde{c}_t(t') = c_{t'}$. Then equation (7) implies

$$q_t \leq \frac{1}{1 - \tau_t} \frac{u'(\tilde{c}_t(t'))}{u'(c_t)} \tilde{q}_t(t') < \frac{1}{1 - \tau_t} \frac{u'(\tilde{c}_t(t'))}{u'(\tilde{c}_t)} \tilde{q}_t(t') = \frac{1}{1 - \tau_t} \tilde{q}_t.$$

This result is summarized in the following proposition.
Proposition 1 Assume that the government taxes the purchase of domestic assets by foreigners at rate $\tau_t$. Then by accumulating foreign assets the government can undervalue the real exchange rate, i.e., raise the real exchange rate $q_t$ above the shadow level $\tilde{q}_t$ that one would observe if the capital account were liberalized at time $t$. The extent of the undervaluation is bounded by the tax rate, i.e.,

$$\frac{q_t - \tilde{q}_t}{q_t} \leq \tau_t.$$  \hspace{1cm} (8)

Proof. See discussion above. ■

If the government tried to devalue the currency by more than the amount allowed by the tax, its attempts would fail because the purchase of reserves would be offset by capital inflows to the private sector.

Another way of deriving condition (8) is by envisaging a speculative attack against the real exchange rate peg $q_t$. The speculative attack is associated with large capital inflows leading to the revaluation of the currency, along the lines of Grilli (1986). Assume that the government has an upper limit on the amount of foreign assets that it holds

$$b^*_t \leq \tilde{b}_t^*.$$ 

Rather than letting foreign assets exceed that limit, the government prefers to let the real exchange rate “float”, i.e., go to the shadow level $\tilde{q}_t$. Then foreign speculators do not strictly benefit from mounting an attack (buying domestic assets to resell them after the revaluation) if and only if condition (8) is satisfied. Note the analogy with the Tobin tax: the interest rate wedge is larger for “round trip” investments of shorter maturity.

6 Conclusions

There are several directions in which the analysis could be extended. First, it would be interesting to provide a welfare analysis of an undervaluation policy that has some benefits in terms of growth. Second, one could also look at a two-country model in order to assess the general equilibrium effects of capital controls. This is left for further research.
APPENDIX

A tariff

Let us assume that capital is perfectly mobile and that imports (exports) of the tradable good are taxed (subsidized) at rate \( \tau_t \). The real exchange rate \( q_t \) is the price of the tradable good before tax in terms of the consumption good. Then using \( c_{Nt} = y_{Nt} \) and assuming (without loss of generality) that \( a_t = 0 \), the consumer’s budget constraint is,

\[
(1 + \tau_t)c_{Tt} + a^*_t = (1 + \tau_t)y_{Tt} + r^*a^*_t + \frac{z_t}{q_t}.
\]

Maximizing welfare (1) under this constraint gives the first-order condition,

\[
u'(c_t) \frac{\partial c_t}{\partial c_{Tt}} = \mu(1 + \tau_t),
\]

where \( \mu \) is the (constant) shadow cost of the budget constraint. A given consumption path \( (c_{Tt})_{t \geq 0} \) can be implemented with a tariff path \( (\tau_t)_{t \geq 0} \) satisfying this equation. The shadow cost \( \mu \) (and so the tariff rate path) is not uniquely determined.

Model of Section 3: closed-form solutions

The economy starts with zero foreign assets, so that the intertemporal budget constraint can be written,

\[
\int_0^{+\infty} c_{Tt} e^{-r^*t} dt = \int_0^{+\infty} y_{Tt} e^{-r^*t} dt.
\]

Using equation (3) to compute the r.h.s. we have

\[
\int_0^{+\infty} c_{Tt} e^{-r^*t} dt = \frac{1}{r^*} \left[ y_T + \frac{\nu}{r^* + \nu} \Delta y_T \right].
\]

If there is no government intervention, the consumption of the tradable good is constant, and given by

\[
\bar{c}_T = y_T + \frac{\nu}{r^* + \nu} \Delta y_T.
\]
The foreign assets are given by,
\[ b_t^* = \int_0^t e^{r^*(t-s)}(y_{Ts} - \bar{c}_T)ds = -\Delta y_T \frac{1 - e^{-\nu t}}{r^* + \nu}. \]

By contrast, if the government targets the consumption of tradable good according to (4), we have
\[ \int_0^{+\infty} c_T e^{-r^* t} dt = \frac{1}{r^*} \left[ y_T + \frac{\sigma}{r^* + \sigma} \Delta c_T \right], \]
which implies
\[ \Delta c_T = \frac{1 + r^*/\sigma}{1 + r^*/\nu} \Delta y_T. \]

The real exchange rate can be derived using the equation,
\[ q_t = \left( \frac{\eta c_t}{c_T t} \right)^{1/\theta}. \]

The trade surplus is given by
\[ y_{Tt} - c_{Tt} = \left[ 1 - e^{-\nu t} - (1 - e^{-\sigma t}) \frac{1 + r^*/\sigma}{1 + r^*/\nu} \right] \Delta y_T. \]

The net foreign assets are given by,
\[ b_t^* = \int_0^t e^{r^*(t-s)}(y_{Ts} - c_{Ts})ds, \]
\[ = \frac{\Delta y_T}{r^* + \nu} \left[ \frac{\nu}{\sigma} (1 - e^{-\sigma t}) - (1 - e^{-\nu t}) \right]. \]

Assume now that the consumption target jumps at time \( t^* \) according to (5) and (6). The foreign assets are given by,
\[ b_t^* = \int_0^t e^{r^*(t-s)}(y_{Ts} - c_{Ts})ds, \]
\[ = \int_0^t e^{r^*(t-s)}(1 - e^{-\nu s})ds \Delta y_T, \]
\[ = \left( \frac{e^{r^* t} - 1}{r^*} - \frac{e^{r^* t} - e^{-\nu t}}{r^* + \nu} \right) \Delta y_T. \]
After the capital account is liberalized, the consumption of the tradable good is constant and given by

\begin{align*}
    c_{T^*} &= r^* \left[ b^*_t + \int_{t^*}^{+\infty} e^{-r^*(s-t^*)} y_{Ts} ds \right], \\
        &= y_T + \frac{\nu}{r^* + \nu} e^{r^* t^*} \Delta y_T.
\end{align*}
References


IMF Staff Papers, 56(1): 222–238.