Currency Wars, Trade Wars, and Global Demand

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Abstract

This paper presents a tractable model of a global economy in which countries can use a broad range of policy instruments—the nominal interest rate, taxes on imports and exports, taxes on capital flows or foreign exchange interventions. Low demand may lead to unemployment because of downward nominal wage stickiness. Markov perfect equilibria with and without international cooperation are characterized in closed form. The welfare costs of trade and currency wars crucially depend on the state of global demand and on the policy instruments that are used by national policymakers. Countries have more incentives to deviate from free trade when global demand is low. Trade wars lower employment if they involve tariffs on imports but raise employment if they involve export subsidies. Tariff wars can lead to self-fulfilling global liquidity traps.

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1 Introduction

Countries have regularly accused each other of being aggressors in a currency war since the global financial crisis. Guido Mantega, Brazil's finance minister, in 2010 blamed the US for launching a currency war through quantitative easing leading to a weaker dollar.[1] At the time, Brazil itself was trying to hold its currency down with a tax on capital inflows and by accumulating reserves. Many countries, including advanced economies such as Switzerland, have resisted the appreciation of their currency by resorting to foreign exchange interventions. The term "currency war" was again used when the Japanese yen depreciated in 2013 after the Bank of Japan increased its inflation target as part of the Abenomics stimulus (and more recently when it reduced the interest rate to a negative level). Bergsten and Gagnon (2012) proposed that the US undertake countervailing currency intervention against countries that manipulate their currencies, or tax the earnings on the dollar assets of these countries. After 2016 the US administration justified the introduction of tariffs by the fact that countries such as China were manipulating their currencies.

The conventional wisdom in the official sector, echoed in Bernanke (2017) or Blanchard (2017), is that depreciations should not raise concerns as long as they are the by-product, rather than the main objective, of monetary stimulus. Other authors, e.g., Mishra and Rajan (2018), find the international spillovers from monetary and exchange rate policies less benign and advocate enhanced international coordination to limit the effects of these spillovers.

The concepts of currency war and trade war are old but we do not have many models to analyze these wars, separately or as concurrent phenomena. One feature of the real world that such a model should capture is the multiplicity of policy instruments that are used achieve similar outcomes. A currency can be depreciated by lowering the interest rate, by raising the inflation target, by taxing capital inflows, or by accumulating foreign exchange reserves. The demand for home goods can be increased by depreciating the home currency, by taxing imports or by subsidizing exports. Presumably, the international spillovers and the case for international cooperation should depend on the policy instruments. Another desirable feature of a model is that it should not assume that countries are committed to policy rules. Trade and currency wars seem to be deviations from the policy rules that are applied in normal times.

[1] "We're in the midst of an international currency war, a general weakening of currency. This threatens us because it takes away our competitiveness." as reported by Martin Wolf in "Currencies Clash in New Age of Beggar-thy-Neighbor," Financial Times September 28, 2010.

In this paper I present a simple model with these features. I consider a symmetric world with many countries, each one producing its own good like in Gali and Monacelli (2005). There is downward nominal stickiness in wages like in Schmitt-Grohé and Uribe (2016).[2] This assumption implies that each country is either in a classical regime with full employment and flexible wages or in a Keynesian regime with unemployment and fixed wages. I assume that each country can use four policy instruments: the nominal interest rate, a tax on capital flows, a tariff on imports and a subsidy on exports. The tax on capital flows can be interpreted as foreign exchange intervention. National policymakers can use all or a subset of these instruments without being able to commit to any future policy action. In particular, there is no Taylor rule and monetary policy is discretionary.

The main qualities that I look for in the model are tractability and analytical clarity. I solve for the Markov perfect equilibria in which policymakers set policies so as to maximize home welfare taking the global economic and financial conditions as given. A first-order approximation allows me to derive easily interpretable closed-form expressions for the equilibrium policies under different assumptions about the available policy instruments. The equilibrium under international policy cooperation is characterized by assuming that the policies of the representative country are set by a global social planner maximizing global welfare. Although very simple, the model can be used to quantify the welfare cost of currency and trade wars.

The model results crucially depend on whether countries are in the classical or Keynesian regime. In the classical regime, national policymakers use the trade taxes to manipulate the terms of trade in their countries' favor like in the textbook tariff war. They can do so by imposing a tax on exports or a tariff on imports (the two instruments are equivalent because of Lerner symmetry).

The analysis is very different if countries are in the Keynesian regime. Countries can be in the Keynesian regime with unemployment if the zero-lower-bound constraint is binding, which tends to happen when global demand is low. In the Keynesian regime with unemployment, increasing home welfare is in general equivalent to increasing home employment. The objective of trade policy, thus, is to raise home employment rather than the home terms of trade. To raise employment a national policymaker must move the trade taxes in opposite directions, i.e., tax imports or subsidize exports. National policymakers can also raise employment

[2]There is considerable evidence (reviewed by these authors among others) that wages are more rigid downward than upward. The fact that wages were more rigid than prices during the Great Depression is well documented (see, e.g., Eichengreen, 1992).

by taxing capital inflows or accumulating foreign exchange reserves, which in both cases depreciates the home currency. In general these policies are beggar-thy-neighbor, in the sense that welfare can be increased in one's country only at the cost of lowering welfare in the rest of the world. However, the model shows that this is not a sufficient reason for international cooperation. The case for international cooperation depends on the policy instruments and on the state of global demand in a nontrivial way.

First, there is no benefit from international coordination of interest rates or inflation targets. A monetary stimulus is beggar-thy-neighbor in partial equilibrium but it is a positive sum game in general equilibrium. If there is unemployment, a global monetary stimulus, if feasible, always raises global employment and welfare.

Second, the case for coordinating trade policies depends on the state of global demand. It is of course optimal to prevent a classical trade war under full employment. The case for preventing a Keynesian trade war is more nuanced. When demand is low and the global economy is in a liquidity trap, international cooperation should be used to avoid a tariff war, but not to prevent the use of export subsidies. A tariff war is especially costly because tariffs act as an intertemporal tax on consumption which further reduces demand and increases unemployment. The welfare impact of a tariff war can be substantial, possibly doubling the unemployment rate under plausible calibrations of the model. The uncoordinated use of tariffs on imports can also give rise to self-fulfilling global liquidity traps as tariffs lower the global natural rate of interest.

The outcome of a trade war is quite different if countries use subsidies on exports instead of tariffs on imports. A transitory export subsidy stimulates consumption in general equilibrium. In the Nash equilibrium with export subsidies, full employment is achieved and there are no benefits from international coordination in the short run.

Third, using taxes on capital flows (or foreign exchange interventions) to raise home employment is a zero-sum game that simply transfers welfare from the rest of the world to the country imposing capital controls. Thus capital wars leave welfare unchanged and there are no net gains from international cooperation.

The paper also considers the incentives of national policymakers to deviate from free trade and impose tariffs assuming that a deviation may trigger a trade war. Again, the incentives to deviate from free trade crucially depend on the state of global demand. The incentives to deviate from free trade are little affected by global demand if there is full employment. By contrast, national policymakers have much stronger incentives to tax imports or subsidize exports in the Keynesian regime with unemployment. Low global demand is conducive to trade wars

Literature. There is a long line of literature on international monetary coordination—see e.g. Engel (2016) for a review. The case for international monetary cooperation in New Open Macro models was studied by Obstfeld and Rogoff (2002), Benigno and Benigno (2006), Canzoneri, Cumby and Diba (2005) and among others. Obstfeld and Rogoff (2002) concluded that the welfare gains from international coordination of monetary policy were small. In a more recent contribution, Korinek (2016) gives a set of conditions under which international policy spillovers are efficient and international coordination is uncalled for. The model in this paper does not satisfy Korinek's conditions—in particular the fact that countries do not have monopoly power.

Another group of papers has explored the international spillovers associated with monetary policy when low natural rates of interest lead to insufficient global demand and liquidity traps including Eggertsson et al. (2016), Caballero, Farhi and Gourinchas (2015), Fujiwara et al. (2013), Devereux and Yetman (2014), Cook and Devereux (2013), Acharya and Bengui (2018) and Bianchi and Coulibaly (2021). Eggertsson et al. (2016) and Caballero, Farhi and Gourinchas (2021) study the international transmission of liquidity traps using a model that shares several features with this paper, in particular the downward nominal stickiness a la Schmitt-Grohé and Uribe (2016). Those papers generally assume Taylor rules for monetary policy and do not incorporate trade taxes to the analysis. Corsetti et al. (2019) consider a partial equilibrium version of Eggertsson et al. (2016) and show that reaching full employment through a currency depreciation may, under certain conditions, decrease welfare. Fornaro and Romei (2019) present a model in which macroprudential policy has a negative effect on global demand when the monetary policy is at the zero lower bound. A more closely related contribution is Auray, Devereux and Eyquem (2020). Auray, Devereux and Eyquem (2020) consider a two-country model without financial markets, so that trade balances are always equal to zero. These authors consider a smaller set of policy instruments than we do (their model does not include export taxes, capital controls of foreign exchange interventions) but look at the implications of fixed exchange rates, a topic that is only briefly touched upon in this paper.

Other papers have explored whether the constraints on monetary policy resulting from a fixed exchange rate or the zero lower bound can be circumvented with fiscal instruments (Farhi, Gopinath and Itskhoki, 2014; Correia et al., 2013). Farhi, Gopinath and Itskhoki (2014) show that value added and payroll taxes used jointly with trade taxes can replicate the effects of nominal exchange rate devaluations across a range of model specifications. Correia et al. (2013) study how fiscal instruments can be used to achieve the same allocations as if there were no zero lower bound on the nominal interest rate in a closed economy. By contrast, the model presented here assumes that the set of policy instruments is more limited. The policy instruments considered in this paper are second-best and do not restore the flexible-wage level of efficiency when the ZLB constraint is binding.

This paper is also related to the recent literature that looks at the macroeconomic impact of trade policy. Barbiero et al. (2019) study the macroeconomic consequences of a border adjustment tax in the context of a dynamic general equilibrium model with monetary policy conducted according to a conventional Taylor rule. Lindé and Pescatori (2019) study the robustness of the Lerner symmetry result in an open economy New Keynesian model and find that the macroeconomic costs of a trade war can be substantial. Erceg, Prestipino and Raffo (2017) and Barattieri, Cacciatore and Ghironi (2021) study the short-run macroeconomic effects of trade policies in a dynamic New Keynesian open-economy framework. Bénassy-Quéré, Bussière and Wibaux (2021) consider a model in which countries are more likely to resort to tariffs at the ZLB. These papers look at small open economies, whereas I focus on the international spillovers associated with monetary and trade policies in a general equilibrium model of the global economy. Bergin and Corsetti (2020) study the optimal monetary stabilization of tariff shocks using a New Keynesian model enriched with elements from the trade literature.

The paper is also related to the literature that has quantified the welfare cost of trade wars in general equilibrium (Ossa, 2014). In this type of framework, Amiti, Redding and Weinstein (2019) and Fajgelbaum et al. (2020) find that the welfare cost of the 2018 trade war is moderate (less than 0.1 % of US GDP) but these papers do not take into account the global demand effects that I focus on in this paper.[3]

The presentation is structured as follows. The assumptions of the model are presented in section 2. We analyze the optimal policies from a small open economy perspective in section 3. The global equilibria are presented in sections 4 and 5 for high and low global demand respectively. Section 6 analyzes the country incentives to deviate from free trade. Section 7 presents dynamic extensions of the model.

[3]Freund et al. (2018) find a larger cost for the US-China trade war in a scenario where the trade war depresses investment.

2 Model

The model represents a world composed of a continuum of atomistic countries indexed by $j \in (0,1)$ in infinite time $t = 1, 2, ...$ The goods structure is similar to Gali and Monacelli (2005). Each country produces its own good and has its own currency. The nominal wage is rigid downwards as in Schmitt-Grohé and Uribe (2016). We introduce taxes on imports, exports and capital flows, which will be the instruments of trade and capital account policies.[4] There is no uncertainty.

Preferences. Country j is populated by a mass of identical infinitely-lived consumers. The utility of the representative consumer can be written recursively,

$$U_{jt} = u\left(C_{jt}\right) + \beta_{jt} U_{jt+1},$$

where the utility function has a constant elasticity of intertemporal substitution ϵ_i,

$$u\left(C\right) = \frac{C^{1-1/\epsilon_i} - 1}{1 - 1/\epsilon_i}. \tag{1}$$

The time-varying discount factor will be used to model exogenous fluctuations in country j's demand.

The consumer consumes the good that is produced domestically (the home good) as well as a basket of foreign goods. The consumer cares about the Cobb-Douglas index,

$$C = \left(\frac{C_H}{\alpha_H}\right)^{\alpha_H} \left(\frac{C_F}{\alpha_F}\right)^{\alpha_F}, \tag{2}$$

(with $\alpha_H + \alpha_F = 1$) where C_H is the consumption of home good, and C_F is the consumption of foreign good.

The consumption of foreign good is a CES index of the goods produced in all the countries,

$$C_F = \left[\int_0^1 C_k^{(\epsilon_x-1)/\epsilon_x} dk\right]^{\epsilon_x/(\epsilon_x-1)}.$$

The elasticity of substitution between foreign goods is assumed to be larger than one, $\epsilon_x > 1$. The composite good defined by this index is the "global good" imported by all countries.

[4] We do not introduce taxes or subsidies on labor, which can be used to ensure full employment in this model.

Budget constraints. The consumers can invest in internationally traded one-period bonds denominated in the global good. The budget constraint for country j's representative consumer is,

$$P_{Fjt}\frac{B_{jt+1}}{R_t\left(1+\tau_{jt}^b\right)}+P_{Hjt}C_{Hjt}+\left(1+\tau_{jt}^m\right)P_{Fjt}C_{Fjt}=W_{jt}L_{jt}+Z_{jt}+P_{Fjt}B_{jt}, \quad (3)$$

where P_{Hjt} is the home-currency price of the home good, P_{Fjt} is the domestic-currency price of the global good before taxes, τ_{jt}^m is a tax on imports, τ_{jt}^b is a subsidy on capital outflows (or equivalently a tax on inflows), R_t is the gross real interest rate in terms of the global good, L_{jt} is the quantity of labor supplied by the consumer (at nominal wage W_{jt}), and Z_{jt} is a lump-sum transfer from the government. Home currency nominal bonds can be traded domestically but the supply of these bonds is equal to zero in equilibrium and they have been omitted from the budget constraint. A version of the model with money and nominal bonds is presented in Appendix A.

Production and labor market. The home good is produced with a linear production function that transforms one unit of labor into one unit of good, $Y=L$. The representative consumer has a fixed labor endowment $\overline{L}$ and the quantity of *employed* labor satisfies

$$L_{jt}\leq\overline{L}. \quad (4)$$

There is full employment if this constraint is satisfied as an equality.

The wage inflation rate is $\pi_{jt}\equiv\frac{W_{jt}}{W_{jt-1}}-1$. The linearity in production implies $P_{Hjt}=W_{jt}$ so that π_{jt} is the inflation rate in the price of the home good.

We assume that the nominal wage is sticky downward like in Schmitt-Grohé and Uribe (2016) or Eggertsson et al. (2016). Downward nominal stickiness in the wage is captured by the constraint,

$$\pi_{jt}\geq 0. \quad (5)$$

The economy of country j can then be in two regimes: the classical regime with full employment ($L_{jt}=\overline{L}$), or the Keynesian regime with less than full employment in which wage inflation is at its lower bound ($L_{jt}<\overline{L}$ and $\pi_{jt}=0$). This leads to a L-shaped Phillips curve where inflation can be set at any non-negative level π_{jt} once there is full employment. The constraints on the labor market can be summarized by (4), (5) and

$$\left(\overline{L}-L_{jt}\right)\pi_{jt}=0. \quad (6)$$

Demand for home labor. The period-t demand for home labor is,

$$L_{jt} = C_{Hjt} + \left[\left(1 + \tau_{jt}^{x}\right) \frac{P_{Hjt}}{P_{Fjt}}\right]^{-\epsilon_x} C_{Ft}^{W}, \tag{7}$$

where $C_F^W = \int C_{Fk} dk$ denotes global gross imports and τ_{jt}^x is country j's tax on exports. The first term on the right-hand side of (7) is the labor used to serve home demand for the home good and the second term is the labor used to produce exports.

It will be convenient to define three terms of trade,

$$S_{jt} \equiv \frac{P_{Hjt}}{P_{Fjt}}, \;\; S_{jt}^{m} \equiv \frac{S_{jt}}{1 + \tau_{jt}^{m}} \text{ and } S_{jt}^{x} \equiv \left(1 + \tau_{jt}^{x}\right) S_{jt}, \tag{8}$$

where S_{jt} denotes the undistorted terms of trade, and S_{jt}^m and S_{jt}^x are the tax-distorted terms of trade that determine the demand for imports and exports respectively.

Given the Cobb-Douglas specification (2), the home demand for the home good and for imports are respectively given by,

$$\begin{aligned} C_{Hjt} &= \alpha_H \left(S_{jt}^{m}\right)^{-\alpha_F} C_{jt}, && (9)\\ C_{Fjt} &= \alpha_F \left(S_{jt}^{m}\right)^{\alpha_H} C_{jt}, && (10)\end{aligned}$$

so that the demand for home labor can be re-written as a function of the terms of trade,

$$L_{jt} = \alpha_H \left(S_{jt}^{m}\right)^{-\alpha_F} C_{jt} + \left(S_{jt}^{x}\right)^{-\epsilon_x} C_{Ft}^{W}. \tag{11}$$

The demand for home labor increases with home or global consumption but is reduced by a loss in the price competitiveness of the home good (an increase in S_t^m or S_t^x).

Balance of payments. Using $Z_{jt} = \tau_{jt}^m P_{Fjt} C_{Fjt} + \tau_{jt}^x P_{Hjt} \left(L_{jt} - C_{Hjt}\right) - \tau_{jt}^b P_{Fjt} B_{jt+1} / \left(1 + \tau_{jt}^b\right)$, equations (7), and (10) to substitute out Z_{jt}, L_{jt}, and C_{Fjt} from the representative consumer's budget constraint (3) gives the balance of payments equation

$$\frac{B_{jt+1}}{R_t} = B_{jt} + X_{jt}, \tag{12}$$

where net exports in terms of global good are given by

$$X_{jt} = \left(S_{jt}^{x}\right)^{1-\epsilon_x} C_{Ft}^{W} - \alpha_F \left(S_{jt}^{m}\right)^{\alpha_H} C_{jt}. \tag{13}$$

The value of gross exports in terms of the global good decreases if the country loses competitiveness in foreign markets (an increase in S^x) because the export elasticity is larger than 1.

Equilibrium conditions. The first-order conditions are derived in Appendix A. In Appendix A we assume that the consumers can invest in nominal government bonds that yield a nominal interest rate i_{jt}. Two equilibrium conditions will be important for the rest of the analysis. The first condition is an interest parity relationship,

$$\frac{1+i_{jt}}{1+\pi_{jt+1}} = R_t \left(1+\tau^b_{jt}\right) \frac{S_{jt}}{S_{jt+1}}. \tag{14}$$

The left-hand side is the real interest rate in terms of country j's home good. The right-hand side is the real interest in terms of global good at home, $R_t\left(1+\tau^b_{jt}\right)$, times the gross rate of depreciation of the home good relative to the global good. This is an arbitrage relationship since there is no uncertainty.

The other relevant equilibrium condition is the Euler equation for consumption,

$$u'\left(C_{jt}\right)\left(S^m_{jt}\right)^{\alpha_F} = \beta_{jt}\frac{1+i_{jt}}{1+\pi_{jt+1}} u'\left(C_{jt+1}\right)\left(S^m_{jt+1}\right)^{\alpha_F}. \tag{15}$$

The marginal utility from consuming one unit of home good in period t, on the left-hand-side, is equal to the marginal utility from postponing that consumption to period $t+1$.

3 National Policy-Making

This section takes the perspective of a small open economy. We consider the problem of a national policymaker who tries to maximize domestic welfare taking the global economic environment as given. The policymaker can use monetary policy, trade policy, and capital account policy. The instrument of monetary policy is the nominal interest rate, which is set subject to the zero-lower-bound (ZLB) constraint $i_{jt} \geq 0$. The instruments of trade policy are the taxes on imports and exports, τ^m_{jt} and τ^x_{jt}, and the instrument of capital account policy is the tax on capital inflows, τ^b_{jt}.

The home policymaker is free to set inflation at any level if there is full employment.[5] Inflation is not welfare relevant in this model,[6] and we assume that it is set to an inflation target $\pi_j^* > 0$ if there is full employment. Inflation, thus, can be written as the following function of employment,

$$\begin{aligned} \pi_{jt} &= \pi_j^* \text{ if } L_{jt} = \overline{L}, \\ \pi_{jt} &= 0 \text{ if } L_{jt} < \overline{L}. \end{aligned}$$

The equilibrium concept that we use in the rest of the paper is that of Markov perfect equilibrium. The equilibrium allocation is a function of the state, which for country j at time t is summarized by the country's beginning-of-period foreign assets, B_{jt}, and the current and future global economic conditions $\left(C_{Ft'}^W, R_{t'}\right)_{t' \geq t}$. We denote the associated policy functions with tildes, $\widetilde{C}_{Hjt}\left(B_{jt}\right)$, $\widetilde{C}_{Fjt}\left(B_{jt}\right)$, $\widetilde{L}_{jt}\left(B_{jt}\right)$, $\widetilde{X}_{jt}\left(B_{jt}\right)$, where the dependence on the global economic conditions is subsumed in the time index. In each period t, the policymaker sets the domestic policy instruments so as to maximize home welfare, taking his own future policy functions as given. The policymaker cannot commit to future policies.

3.1 Instrument equivalence

The mapping between policies and allocations is not obvious because a given allocation can be implemented with more than one policy mix. It will be easier to characterize the equilibrium by looking for the optimal allocations rather than the optimal policies. The next section characterizes the feasible allocations and the policies that achieve them.

For a given state B_{jt}, a time-t allocation $(C_{Hjt}, C_{Fjt}, L_{jt}, X_{jt})$ is *feasible* if it can be implemented with the policy instruments available to the policymaker, taking the next-period policy functions $\widetilde{C}_{Hjt+1}(\cdot)$, $\widetilde{C}_{Fjt+1}(\cdot)$, $\widetilde{L}_{jt+1}(\cdot)$, $\widetilde{X}_{jt+1}(\cdot)$ as given. Using $L_{jt} = C_{Hjt} + \left(S_{jt}^x\right)^{-\epsilon_x} C_{Ft}^W$ to substitute out S_{jt}^x in (13), any feasible time-t allocation must satisfy,

$$X_{jt} = \left(C_{Ft}^W\right)^{1/\epsilon_x} \left(L_{jt} - C_{Hjt}\right)^{1-1/\epsilon_x} - C_{Fjt}. \tag{16}$$

[5]We assume that the policymaker sets inflation directly when there is full employment. As explained in Appendix B, the inflation rate is endogenous to money supply in an extension of the model with money in the utility function. We abstract from money supply in the baseline model for simplicity.

[6]This is a feature of models with downward nominal wage stickiness such as Schmitt-Grohé and Uribe (2016). Inflation is not welfare relevant ex post but expected inflation constrains the feasible allocations because of the ZLB constraint.

Conversely, any allocation that satisfies this condition is feasible, implying the following result.

Lemma 1 *A time-t allocation* $(C_{Hjt}, C_{Fjt}, L_{jt}, X_{jt})$ *is feasible if and only if it satisfies (16).*

Proof. See Appendix E. ■

Note that we have not included the ZLB constraint in the definition of feasibility. For the allocation to be consistent with the ZLB constraint, one needs to check that the interest rate implementing the allocation satisfies $i_{jt} \geq 0$.

It is then possible to show that a feasible allocation can be implemented by more than one policy mix $\left(i_t, \tau_t^m, \tau_t^x, \tau_t^b\right)$. The equivalence between policy instruments is characterized by the following proposition.

Proposition 2 *(Lerner Symmetry) A feasible allocation* $(C_{Hjt_t}, C_{Fjt}, L_{jt}, X_{jt})$ *that is implemented by policy* $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b)$ *can also be implemented by policy* $(i_{jt}, \widehat{\tau}_{jt}^m, \widehat{\tau}_{jt}^x, \widehat{\tau}_{jt}^b)$ *satisfying*

$$\left(1+\widehat{\tau}_{jt}^m\right)\left(1+\widehat{\tau}_{jt}^x\right) = \left(1+\tau_{jt}^m\right)\left(1+\tau_{jt}^x\right) \tag{17}$$

$$\left(1+\widehat{\tau}_{jt}^m\right)\left(1+\widehat{\tau}_{jt}^b\right) = \left(1+\tau_{jt}^m\right)\left(1+\tau_{jt}^b\right) \tag{18}$$

Proof. See Appendix E. ■

The equivalence between a tariff on imports and a tax on exports is known as Lerner's symmetry in the trade literature (Lerner, 1936).[7] To put equations (17) and (18) in words, the allocation is unchanged if the policymaker shifts the tax from exports to imports and at the same time decreases the tax on capital inflows by the same amount as the tax on exports. For the volume of gross exports to stay the same, the decrease in the export tax must be offset by an increase in the terms of trade. The real appreciation must in turn be offset by an equivalent increase in the tariff on imports to keep the volume of imports the same. The real appreciation results from a decrease in the tax on capital inflows of the same size

[7] Costinot and Werning (2019) provide a number of generalizations and qualifications of the Lerner symmetry theorem in a dynamic environment.

as the tax on exports. It cannot result from a higher interest rate because the interest rate is pinned down by the Euler condition (15).[8]

Remark 1. A long-standing question in international macroeconomics is under which conditions exchange rate manipulation can replicate the impact of tariffs (Meade, 1955). This question resurfaced in recent policy debates as the US administration justified the imposition of tariffs on Chinese exports by a claim that China was manipulating its currency.

In the context of our model, a larger set of allocations can be achieved by trade taxes under a fixed exchange rate than by a floating exchange rate without trade taxes. On one hand, a fixed exchange rate does not prevent the policymaker from spanning all the feasible allocations if she can use the trade and capital flow taxes. On the other hand, not all the feasible allocations can be implemented by varying the exchange rate when there are no trade taxes. Proposition 2 implies that the allocations achievable with trade taxes τ_t^m and τ_t^x can be replicated without trade taxes if and only if

$$(1 + \tau_t^m)(1 + \tau_t^x) = 1, \tag{19}$$

that is, if the trade taxes generate the same relative price distortion in foreign market as in the home market. An allocation achieved with trade taxes under a fixed peg can be replicated with a floating exchange rate under free trade if and only if this condition is satisfied.

Remark 2. The instrument of capital account policy could be foreign exchange interventions instead of a tax on capital flows. To show this, we consider in Appendix B a variant of the model in which the capital account is closed, i.e., only the home government can trade real bonds with foreign investors.[9] The government finances its purchases of foreign bonds by issuing domestic currency bonds to home consumers, which can be interpreted as a sterilized foreign exchange intervention. Appendix B shows that the real allocations that can be achieved in this way are the same as when the government uses a tax on capital flows. A sterilized purchase of foreign bonds is equivalent to a tax on capital inflows.

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[8]The Lerner symmetry theorem may seem counterintuitive since a tax on exports reduces the demand for the home good whereas a tax on imports increases it. Indeed, in the Keynesian regime, a tax on exports and a tax on imports have opposite effects on home employment if the other policy instruments are unchanged. Lerner symmetry, however, applies under the assumption of a constant level of employment.

[9]The assumption that there are no private capital flows is extreme but the insights remain true if frictions prevent economic agents from arbitraging the wedge between onshore and offshore interest rates.

3.2 Impact of policies in the Keynesian regime

In this section we consider the impact of changing the levels of the policy instruments in the Keynesian regime with unemployment. One can see how policies determine the allocation by looking at the equilibrium conditions (11), (13), (14) and (15). Assume for simplicity that variables dated $t+1$ are given. Then equation (14) shows how the interest rate i_{jt} and the capital flow tax τ_{jt}^b determine the terms of trade S_{jt}. Other things equal, an increase in the interest rate appreciates the home currency in real terms, whereas an increase in the tax on inflows (or a sterilized purchase of foreign bonds) has the opposite effect. The Euler equation (15) shows how consumption C_{jt} is determined by S_{jt}^m and i_{jt}. Other things equal, increasing the tariff on imports lowers home consumption because the tariff acts as an intertemporal tax on consumption. Raising the interest rate lowers consumption. Finally, equations (11) and (13) determine L_t and X_t through channels that are discussed below.

The model can be linerarized under the assumption that $1-\beta$ is first-order so that the impact of first-order changes in the countries net foreign assets have a second-order impact on flow variables. Table 1 below reports the elasticity of home output with respect to the four policy instruments $n = i$, τ^m, τ^x and τ^b,

$$\frac{1+n}{Y_H}\frac{\partial Y_H}{\partial n},$$

(see Appendix D for the derivations). The elasticities are computed in a symmetric allocation with zero trade taxes. The effects are decomposed between the expenditure-changing channel, which affects the level of home consumption C, and the expenditure-switching channel, which affects the allocation of home and foreign consumption between the home and foreign goods.

An increase in the home interest rate reduces the demand for the home good through both the expenditure-changing and the expenditure-switching channels. For the tariff rate the two channels go in opposite directions. An increase in the tariff rate reduces home consumption but at the same time shifts it toward the home good. The net effect is to stimulate the home economy if and only if the elasticity of intertemporal substitution ϵ_i is lower than the elasticity of substitution between the two goods, which is 1 because of the Cobb-Douglas assumption. The tax on exports has no impact on home demand but has an expenditure-switching effect on foreign demand. Finally, the tax on capital flows, like the tariff on imports, has expenditure-changing and -switching effects that go in opposite directions. The tax depresses home consumption through an intertemporal effect but depreciates the home currency, which shifts both home and foreign spending toward the home good. If $\epsilon_i < 1$ the tax on inflows has a positive net impact on home output.

Table 1. Elasticity of home output with respect to the policy instruments in the Keynesian regime

policy instrument n	i	τ^m	τ^x	τ^b
expenditure changing	$-\alpha_H^2\epsilon_i$	$-\alpha_H\alpha_F\epsilon_i$	0	$-\alpha_H\alpha_F\epsilon_i$
expenditure switching	$-\alpha_H\alpha_F - \alpha_F\epsilon_x$	$\alpha_H\alpha_F$	$-\alpha_F\epsilon_x$	$\alpha_H\alpha_F + \alpha_F\epsilon_x$

3.3 Optimal policies in the classical regime

We solve the policymaker's problem in the particular case where the ZLB constraint is not binding. In this case the only constraint on the time-t allocation is (16). Using the balance-of-payments equation (12) the policymaker's problem can be written in Bellman form as,

$$\mathscr{P}_{jt}\begin{cases} V_{jt}\left(B_{jt}\right) = \max_{L_{jt},C_{Hjt},C_{Fjt},B_{jt+1}}\left[u\left(C\left(C_{Hjt},C_{Fjt}\right)\right) + \beta_{jt}V_{jt+1}\left(B_{jt+1}\right)\right] \\ B_{jt+1} = R_t\left[B_{jt} + \left(C_{Ft}^W\right)^{1/\epsilon_x}\left(L_{jt} - C_{Hjt}\right)^{1-1/\epsilon_x} - C_{Fjt}\right], \\ L_{jt} \leq \overline{L}. \end{cases}$$

The solution has the following properties.

Proposition 3 *In any period t in which the ZLB constraint is not binding the national policymaker's optimal allocation features full employment ($L_{jt} = \overline{L}$) and the trade taxes satisfy*

$$\left(1+\tau_{jt}^m\right)\left(1+\tau_{jt}^x\right) = \frac{\epsilon_x}{\epsilon_x - 1}. \tag{20}$$

Proof. See Appendix E. ■

If feasible, full employment is always optimal. Full employment puts the economy in the classical regime. It is then optimal for the policymaker to use the trade taxes to manipulate the home terms of trade like in the classical textbook tariff war. The policymaker raises the home terms of trade by reducing the supply of home good to the rest of the world, which can be achieved by taxing exports or imports. The national policymaker can indifferently use the tax on exports or the tariff on imports because of Lerner's symmetry.

4 Global Equilibria

This section defines the Nash (or non-cooperative) equilibrium between national policymakers as well as the allocation chosen by a global policymaker who maximizes global welfare. We also characterize the equilibria in which the ZLB constraint is not binding.

A *Nash equilibrium* between national policymakers is a set of country allocations such that each policymaker maximizes home welfare given the global economic conditions and the global economic conditions satisfy market clearing conditions. More formally, a Nash equilibrium consists of: (i) global economic conditions $\left(C_{Ft}^{W}, R_t\right)_{t=1,\ldots,+\infty}$; (ii) net foreign asset paths $\left(B_{jt}\right)_{j\in[0,1],t=1,\ldots,+\infty}$; (iii) national value and policy functions $V_{jt}\left(\cdot\right)$, $\widetilde{C}_{Hjt}\left(\cdot\right)$, $\widetilde{C}_{Fjt}\left(\cdot\right)$, $\widetilde{L}_{jt}\left(\cdot\right)$, $\widetilde{X}_{jt}\left(\cdot\right)$, and $\widetilde{\pi}_{jt}\left(\cdot\right)$ for all countries $j \in [0,1]$ and times $t = 1, 2, \ldots$ satisfying the following conditions:

• national optimization: the national value and policy functions and net foreign asset paths solve problem $\mathscr{P}_{jt}$ subject to the ZLB constraint for all countries j and times t;

• global market clearing: net foreign assets sum up to zero and the global demand for imports is the sum of national demands for all times t,

$$\int B_{jt}dj = 0, \tag{21}$$

$$C_{Ft}^{W} = \int \widetilde{C}_{Fjt}\left(B_{jt}\right)dj. \tag{22}$$

Some properties of the Nash equilibria are easy to derive at this stage of the analysis. First, these equilibria inherit from Proposition 3 the property that in any country where the ZLB constraint is not binding, there is full employment and the policymakers set the trade taxes to manipulate the terms of trade as in equation (20). Second, integrating $X_{jt} = \left(S_{jt}^{x}\right)^{1-\epsilon_x} C_{Ft}^{W} - C_{Fjt}$ over all countries j and condition (22) implies

$$\int \left(S_{jt}^{x}\right)^{1-\epsilon_x} dj = 1. \tag{23}$$

Equation (23) reflects the fact that the terms of trade in export markets are relative prices that cannot all move in the same direction.

We restrict the attention to the symmetric case where all countries have the same discount factor and initial net foreign assets, i.e.[10]

$$\forall j,t \quad \beta_{jt} = \beta_t, \tag{24}$$

$$B_{j1} = 0. \tag{25}$$

The level of β_t determines global demand in period t. A lower β_t means higher demand and we will sometimes refer to $1/\beta_t$ as an index for global demand.

We will compare the Nash equilibrium with the equilibrium under a global policymaker who sets the allocations in all countries so as to maximize utilitarian global welfare $V_t^W = \int V_{jt} dj$ under the same constraints as national policymakers. Like its national counterparts, the global policymaker cannot commit to future policies. In the symmetric case, the global policymaker's allocation Pareto dominates the Nash allocation, and can be interpreted as the outcome of international cooperation between the national policymakers.

The following proposition states a condition for the existence of a non-cooperative equilibrium in which all countries are in the classical regime in all periods.

Proposition 4 *(Equilibria with non-binding ZLB constraint) There is a non-cooperative equilibrium in which the ZLB constraint never binds if and only if*

$$\beta_t \leq 1 + \pi^*, \tag{26}$$

in all periods t. The non-cooperative equilibrium is associated with a unique allocation.

The non-cooperative allocation is implemented by trade taxes satisfying (20) and nominal interest rates and capital flow taxes satisfying

$$1 + i_{jt} = \frac{1+\pi^*}{\beta_t}, \; 1 + \tau_{jt}^b = \frac{1+\tau_{jt+1}^m}{1+\tau_{jt}^m}, \tag{27}$$

in all countries and times.

The same allocation obtains in a non-cooperative equilibrium where the national policymakers use only the tariff on imports or the tax on exports in addition to the interest rate.

Proof. See Appendix E. ■

[10] The asymmetric case is interesting if one wants to study the international spillovers of demand shocks. In this paper we focus on the spillovers from policy actions rather than on demand shocks.

There exists a non-cooperative equilibrium in which countries always stay in the classical regime with a non-binding ZLB if and only if condition (26) is satisfied. This condition is familiar from the closed-economy literature on liquidity traps. The ZLB constraint is not binding if and only if global demand is high enough relative to the inflation target. In the non-cooperative equilibrium each national policymaker attempts to manipulate the terms of trade in his country's favor by using the trade taxes.

The non-cooperative equilibrium leads to a unique allocation but the associated policy instruments are indeterminate because of Lerner symmetry. The capital flow taxes τ_{jt}^{b} are adjusted to offset the intertemporal distortions induced by the time variation in the tariffs.

As a result of Lerner symmetry, the policymakers do not need to use all the policy instruments to bring about the non-cooperative allocation. For example, the same allocation is obtained if a tariff on imports is the only available policy instrument on top of the interest rate. In this case, the tariff rate is uniquely determined and equal to $\tau^{m} = 1/\left(\epsilon_x - 1\right)$ in all countries.

Like in the textbook tariff war, the Nash equilibrium is inefficient because it leads to under-consumption of the global good. The global policymaker's objective is to maximize the level of global consumption that can be produced with the labor endowment $\overline{L}$, or equivalently to minimize the labor needed to produce any given level of consumption. Using (9) and (10), the quantity of labor that is required to produce one unit of consumption good in a symmetric equilibria is given by

$$\ell\left(S_t^m\right) = \alpha_H \left(S_t^m\right)^{-\alpha_F} + \alpha_F (S_t^m)^{\alpha_H}, \tag{28}$$

where the terms of trade relevant for imports are

$$S_t^m = \frac{1}{\left(1+\tau_t^m\right)\left(1+\tau_t^x\right)}. \tag{29}$$

It is is easy to see that $\ell\left(S_t^m\right)$ is minimized for $S_t^m = 1$, that is if trade taxes do not distort the allocation of consumption between the home and foreign goods.

Numerical illustration. The welfare loss from a tariff war under full employment can be substantial (see for example Ossa, 2014). In the rest of the paper we will use the parameter values reported in Table 2 to illustrate the quantitative properties of the model. The elasticity of intertemporal substitution of consumption, ϵ_i, is set to 0.5, which corresponds to a risk aversion of 2, a standard value in the literature. The elasticity of substitution between foreign goods, ϵ_x, is set to 3, which is consistent with the estimates of Feenstra et al. (2018). Note in particular that the "microelasticity" between the differentiated imported goods is substantially larger than the "macroelasticity" between the home good and imports, which is equal to 1 in this model because of the Cobb-Douglas specification (2). A value of 1 for the macroelasticity is also consistent with the evidence in Feenstra et al. (2018). Finally, we assume $\alpha_H = 0.6$, i.e., home goods amount to 60 percent of total consumption.

For these values the equilibrium tariff rate amounts to $\tau^m = 50\%$ in the Nash equilibrium and the welfare loss from a tariff war amounts to 1.89% of permanent consumption.

Table 2. Baseline parameter values.

ϵ_i	ϵ_x	α_H
0.5	3	0.6

5 Global Liquidity Traps

In this section we turn our attention to the equilibria in which the ZLB constraint is binding, i.e., global liquidity traps. For simplicity, we consider liquidity traps that last one period, and leave the analysis of dynamic liquidity traps for Section 7. We interpret period 1 as the short run and the following periods as the long run. We assume that the global economy is in the full employment classical regime from period 2 onwards, which is possible by Proposition 4 if

$$\beta_t \leq 1 + \pi^* \text{ for } t \geq 2. \tag{30}$$

We assume without loss of generality that the discount factor is constant from period 2 onwards, i.e., $\beta_t = \beta_2$ for $t \geq 2$. We do not restrict the value of β_1.

We compare the equilibria under different assumptions about the policy instruments that countries can use. Since countries enjoy monetary sovereignty the interest rate is always one of the available policy instruments but we consider different policy mixes for the other instruments. This reflects the fact that in the real world not all countries use all the policy instruments all the time.[11] For simplicity we maintain the assumption of symmetry for the policy instruments, i.e., we assume that all countries can use the same instruments.

[11]For example, the use of trade taxes and capital controls can be limited by membership to international organizations such as the WTO, the EU, or the OECD.

Before proceeding with the analysis of specific equilibria, we present some preliminary considerations that will be useful across the board. In a non-cooperative equilibrium the national policymaker for each country j maximizes period-1 home welfare given by

$$U_{j1} = u\left(C_{j1}\right) + \beta_1 V_2\left(R_1 X_{j1}\right). \tag{31}$$

Period-2 welfare is a function of the country's net foreign assets $B_{j2} = R_1 X_{j1}$. This maximization problem is solved under the employment constraint $L_{j1} \leq \overline{L}$ and the ZLB constraint $i_{j1} \geq 0$. The results in this section are first-order approximations derived under the assumptions that $1 - \beta_2$ is first order, so that changes in a country's period-2 net foreign assets have a second-order impact on the steady-state flow variables. This allows us to derive closed-form expressions for the equilibrium policies in period 1.

The cooperative equilibrium is derived by solving the global policymaker's problem. The global policymaker takes into account that trade balances are equal to zero in a symmetric allocation, so that period-2 welfare is given by $V_2(0)$ independently of period-1 policies. Hence maximizing period-1 welfare is equivalent to maximizing period-1 global consumption. To see what this implies for the policy instruments, we first note that in a symmetric allocation one must have $S_t^x = 1$ and S_t^m is given by (29). It then follows from (15) for $t = 1$ that

$$C_1^W = \left[\frac{\beta_1\left(1+i_1\right)}{1+\pi^*}\right]^{-\epsilon_i}\left(\frac{S_2^m}{S_1^m}\right)^{-\alpha_F \epsilon_i} C_2^W. \tag{32}$$

Using (29), this expression shows that global consumption decreases with the total period-1 trade distortion $\left(1+\tau_1^m\right)\left(1+\tau_1^x\right)$.

The two trade taxes τ_t^m and τ_t^x have the same impact on global consumption but the channels through which they affect global consumption are different. The tariff on import is an intertemporal tax that reduces the consumption of the tariff-imposing country. By contrast, the tax on exports affects global consumption through the general equilibrium determination of the global good own rate of interest. To see this more formally, observe that in a symmetric allocation the global real interest rate is given by,

$$R_1 = \frac{1+i_1}{\left(1+\pi^*\right)\left(1+\tau_1^b\right)} \frac{1+\tau_1^x}{1+\tau_2^x}, \tag{33}$$

as a result of equation (14) with $S_t = 1/\left(1+\tau_t^x\right)$. The global real rate of interest increases with the ratio of period-1 to period-2 gross tax on exports. An increase in this ratio lowers the inflation rate in the global good between periods 1 and 2, which given the nominal interest rate leads to an increase in the global good own real rate of interest

Another implication of equation (32) is that global consumption and welfare do not depend on the tax on capital flows τ_1^b. As shown by (33), if all countries increase their tax on capital inflows by the same amount, this is offset by an equivalent decrease in the global real rate of interest.

The global social planner maximizes global consumption under the global employment constraint $L_1^W \leq \overline{L}$ and the ZLB constraint $i_1 \geq 0$. Let us write global employment in period 1 as global consumption times the quantity of labor it takes to produce one unit of consumption, that is, using (32) and $C_2^W = \overline{L}/\ell(S_2^m)$

$$\begin{aligned} L_1^W &= \ell(S_1^m)\, C_1^W, \\ &= \frac{\ell(S_1^m)}{\ell(S_2^m)} \left(\frac{S_1^m}{S_2^m}\right)^{\alpha_F \epsilon_i} \left[\frac{\beta_1 (1+i_1)}{1+\pi^*}\right]^{-\epsilon_i} \overline{L}. \end{aligned} \tag{34}$$

Trade taxes have an ambiguous impact on employment because they increase the quantity of labor required to produce a unit of consumption at the same time as they decrease consumption. Equation (34) shows that increasing the trade taxes by the same amount in the short run and the long run has no impact on period 1 employment. Higher tariffs lower global consumption but more labor is required to produce the same level of consumption and the two effects exactly cancel out. Raising the trade taxes in period 1 and not in period 2 has an ambiguous impact on employment. It is easy to see, by log differentiating (34), that employment decreases with the tariff rate if and only if

$$S_1^m > \frac{\alpha_H (1-\epsilon_i)}{\alpha_H + \alpha_F \epsilon_i}, \tag{35}$$

i.e., if the period-1 trade taxes are not too high.

With these considerations in mind, the analysis will proceed as follows. Section 5.1 considers the case of monetary wars where the only weapon is the nominal interest rate (and possibly also the inflation targets). Section 5.2 adds tariffs and Section 5.3 adds export taxes to the policy mix. Section 5.4 considers other combinations of instruments and Section 5.5 summarizes the key takeaways from this analysis.

5.1 Monetary wars (i, π^*)

This section considers the case where countries use monetary policy only. The equilibrium is described in the following proposition.

Proposition 5 *(Conventional monetary war) Assume that the only policy instrument available to national policymakers is the nominal interest rate. Then in the Nash equilibrium all countries set the period-t nominal interest rate to*

$$i_t = \left(\frac{1+\pi^*}{\beta_t} - 1\right)^+ .$$

There is a global liquidity trap with unemployment in period 1 *if and only if* $\beta_1 > 1 + \pi^*$. *There is no gain from international cooperation.*

Proof. See Appendix E. ■

In the Keynesian regime with unemployment, a national policymaker raises home consumption and production by lowering the interest rate. This raises home welfare irrespective of the impact of the monetary stimulus on the trade balance. The positive welfare impact of raising consumption and production dominates the negative impact of depreciating the home currency from the perspective of terms of trade manipulation.

It follows that in a symmetric non-cooperative equilibrium, all the national policymakers lower the interest rate until either there is full employment or the ZLB constraint binds in all countries. It is the latter than happens in period 1 if $\beta_1 > 1 + \pi^*$. In this case, using (32) with $i_1 = 0$, $L_1 = C_1^W$, and $\tau_1^m = \tau_1^x = 0$ gives

$$L_1 = \left(\frac{\beta_1}{1+\pi^*}\right)^{-\epsilon_i} \overline{L} < \overline{L}.$$

The global economy is in a liquidity trap with the same level of employment in all countries.

The global policymaker implements the same allocation as in the Nash equilibrium. As can be seen from (32), global welfare is maximized by setting the nominal interest rate at the lowest possible level in all countries. This is true even when a monetary stimulus has beggar-thy-neighbor effects, i.e., a decrease in i_{j1} raises welfare in country j but lowers it in the rest of the world. A monetary stimulus has beggar-thy-neighbor effects if and only if[12]

$$\epsilon_i < \left(1 + \frac{\alpha_H}{\epsilon_x}\right)\left(\epsilon_x - \alpha_H \epsilon_i - \alpha_F\right) \tag{36}$$

that is, if the expenditure-changing effect of a monetary stimulus (captured by ϵ_i) is small enough relative the expenditure-switching effect of a currency depreciation (captured by ϵ_x). This condition is satisfied by the parameter values in Table 1. Irrespective of whether condition (36) is satisfied, a global monetary stimulus is a positive-sum game because decreasing the interest rate everywhere raises employment and welfare in all countries, as it would in a closed economy (Bernanke, 2017).

The model can easily be extended to the case where national policymakers can choose their inflation targets (an "inflation target war"). Let us assume that each country j sets its inflation target π_j^* in period 1. The Nash equilibrium from that point onwards is then determined conditional on the inflation targets as before. Each policymaker sets its inflation target so as to maximize domestic welfare taking the other countries' inflation targets as given. Then we have the following result.

Proposition 6 *(Inflation target war) Assume that the national policymakers can choose their inflation targets in period* 1*. Then in a symmetric Nash equilibrium the policymakers set inflation targets* $\pi_j^* \geq \beta_1 - 1$ *and* $i_{j1} = \left(1 + \pi_j^*\right)\left(\beta_1 - 1\right.$*. There is full employment in all countries and all periods, and welfare is at the first-best level. There is no gain from international cooperation.*

Proof. See Appendix E. ■

Allowing countries to increase their inflation targets effectively allows them to relax the ZLB constraint at no cost. There is no benefit from international cooperation because the international spillovers associated with an increase in the inflation target are the same as with a decrease in the nominal interest rate. An inflation target war, thus, is a positive-sum game.

5.2 Tariff wars (i, τ^m)

We assume in this section that the national policymakers can use two instruments, the nominal interest rate and a tariff on imports. From period 2 onwards the

[12] This condition is derived in Appendix D.

economy is in the full employment equilibrium described in Proposition 4. As indicated in Proposition 4, the Nash allocation remains the same if the set of available policy instruments contains only i and τ^m. In this equilibrium the gross tariff rate is set to

$$1 + \tau_2^m = \frac{\epsilon_x}{\epsilon_x - 1}. \tag{37}$$

The Nash equilibrium in period 1 is more complicated to characterize and depends on whether the ZLB constraint is binding or not. Our first result establishes the equilibrium tariff rate in a Nash equilibrium with a binding ZLB constraint and unemployment.

Proposition 7 *Consider a Nash equilibrium in which all national policymakers use tariffs on imports in addition to the nominal interest rate. If the ZLB constraint is binding and there is less than full employment ($L_1 < \overline{L}$), the period-1 equilibrium tariff on imports is given by*

$$1 + \tau_1^m = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\left(1 + \tau_2^m\right). \tag{38}$$

The tariff rate is higher in the short run than in the long run if and only if $\epsilon_i < 1$.

Proof. See Appendix E. ■

To understand the condition under which policymakers raise tariffs in period 1, consider a symmetric allocation in which there is unemployment and the tariff rate is at the same level in the short run as in the long run. Then a national policymaker finds it beneficial to increase the period-1 tariff rate if and only if this increases home income in period 1 (the welfare cost of the intertemporal consumption distortion being second order at the margin). The terms of trade are not affected by the tariff in the Keynesian regime, hence home income is determined by employment. As discussed in section 3.2, a tariff raises employment if and only if $\epsilon_i < 1$. Hence starting from $\tau_1^m = \tau_2^m$, the national policymaker raises τ_1^m if and only if $\epsilon_i < 1$.

Increasing tariffs in all countries, however, has a deleterious impact from a multilateral perspective because this decreases global demand, as one can see from equation (32). A generalized increase in tariffs reduces welfare because the tariffs act as an intertemporal tax on period-1 consumption.

To summarize, there are two distinct welfare costs of a tariff war to consider in this model. In the long run, there is the distortionary cost of a "classical" tariff war in which all national policymakers attempt to manipulate their country's terms of trade under full employment. On top of this, there is the distortionary cost of a "Keynesian" tariff war that lowers global demand if all countries raise their tariffs because of unemployment in period 1.

A Keynesian tariff war reduces global consumption and welfare if $\epsilon_i < 1$ but has an ambiguous impact on employment. This is because, as explained at the beginning of this section, a tariff increases the quantity of labor required to produce each unit of consumption. A tariff war lowers employment if condition (35) is satisfied when S_1^m is given by (37) and (38). This is true if the export elasticity is high enough,

$$\epsilon_x \left[1 - \alpha_H \left(\frac{1}{\epsilon_i} - 1\right)\right] > 1. \tag{39}$$

This condition is satisfied for our baseline calibration.

The complete set of Nash equilibria is characterized in the following proposition in the case where $\epsilon_i < 1$ and (37) and (39) are satisfied.

Proposition 8 *(Tariff war equilibria) Assume that $\epsilon_i < 1$ and (37) and (39) are satisfied. Then there is a threshold $\beta^* < 1 + \pi^*$ such that the period-1 non-cooperative equilibrium is of the following type:*

(i) full employment, non-binding ZLB constraint and $\tau_1^m = \tau_2^m$: this equilibrium exists if and only if $\beta_1 \leq 1 + \pi^*$;

(ii) less than full employment, a binding ZLB constraint and τ_1^m given by (38): this equilibrium exists if and only if $\beta_1 \geq \beta^*$;

(iii) full employment, a binding ZLB constraint and τ_1^m between τ_2^m and the level given by (38): this equilibrium exists if and only if $\beta^* < \beta_1 < 1 + \pi^*$.

Proof. See Appendix E. ■

If β_1 is larger than a threshold β^*, there is a tariff war equilibrium with a liquidity trap and unemployment. In this equilibrium each national policymakers increases the tariff rate above the long-run level to raise home employment and welfare, leading to the opposite outcome in general equilibrium. For our baseline calibration and $\pi^* = 2\%$, the threshold β^* is equal to 0.963, implying that the tariff war equilibrium with unemployment arises for normal values of β_1.

Another point made by the proposition is that there are multiple equilibria if β_1 is in the interval $(\beta^*, 1 + \pi^*)$. To illustrate, Figure 1 shows how the equilibrium tariff rate varies with global demand $1/\beta_1$ for the parameter values in Table 1. The figure reports the amount by which the period-1 tariff rate exceeds the long-run level, $\tau_1^m - \tau_2^m$. There are multiple equilibria, labeled A, B and C, when demand takes intermediate values. Equilibrium A has the same tariff rate in period 1 as in the long run, full employment and a non-binding ZLB constraint. This equilibrium exists because $\beta_1 \leq 1 + \pi^*$, as already stated in Proposition 4. The other two equilibria have higher levels of tariff and a binding ZLB constraint. The difference between equilibria B and C is that C has less than full employment because of a higher tariff level than in B. The equilibria are Pareto-ranked: welfare is higher in equilibrium A than in B, and higher in B than in C.

Equilibrium multiplicity comes from a strategic complementarity between the national policymakers' actions. By raising home tariffs, the policymakers reduce their demand for the other countries' goods, leading those countries to lower their interest rates. Higher tariffs, thus, push the global economy towards a liquidity trap. Conversely, in a liquidity trap, each national policymaker finds it optimal to increase tariffs above the long-run level to boost domestic employment. Higher tariffs in one part of the world, thus, may lead to higher tariffs in the rest of the world.[13]

The cooperative allocation significantly differs from the Nash equilibrium. First, it follows from Proposition 4 that the cooperative allocation features zero tariffs in the long run. In period 1 the global policymaker maximizes C_1^W subject to the ZLB constraint and the labor supply constraint. Given (32) and $S_2^m = 1$ these constraints can be written

$$\begin{aligned} C_1^W &\leq \left(\frac{\beta_1}{1+\pi^*}\right)^{-\epsilon_i} (S_1^m)^{\alpha_F \epsilon_i}\, \overline{L}, \\ C_1^W \ell\left(S_1^m\right) &\leq \overline{L}. \end{aligned}$$

If $\beta_1 \leq 1 + \pi^*$, the global social planner maximizes consumption by minimizing the production distortion, i.e., by setting $S_1^m = 1$. If $\beta_1 > 1 + \pi^*$, the global social planner raises the consumption ceiling imposed by the ZLB constraint, which can be done by increasing S_1^m above 1, i.e., by subsidizing imports in period 1. The temporary import subsidy acts as an intertemporal subsidy on consumption. Home welfare is maximized by setting $S_1^m = \widehat{S}$ where $\widehat{S} < 1$ makes both the ZLB and labor constraints binding, i.e.

$$\ell\left(\widehat{S}\right)\left(\widehat{S}\right)^{\alpha_F \epsilon_i} = \left(\frac{\beta_1}{1+\pi^*}\right)^{\epsilon_i}. \tag{40}$$

[13] A tariff war leads to higher tariffs because of the assumption $\epsilon_i < 1$. If $\epsilon_i > 1$ and there is unemployment in period 1, national policymakers raise home welfare by lowering their tariff rates below the long-run level, which increases global employment. It is possible to show that in this case, the Nash equilibrium is unique.

Hence, the tariff set by the global policymaker is either equal to zero (if $\beta_1 \leq 1+\pi^*$) or negative (if $\beta_1 > 1+\pi^*$). In the latter case, the global policymaker subsidizes imports to avoid unemployment in period 1. To illustrate, the optimal tariff rate is shown with the red line on Figure 1. The difference between the Nash equilibrium and the cooperative allocation is especially stark for low levels of global demand. For $\beta_1 > 1+\pi^*$, a tariff war increases tariffs, whereas international cooperation reduces them below the long-run level.

Our results about the cooperative policies and allocations are summarized in the following proposition.

Proposition 9 *(Cooperative tariff policy) The global policymaker sets the tariff to zero in the long run ($\tau_2^m = 0$). In period 1, the global policymaker sets the tariff to zero if $\beta_1 \leq 1+\pi^*$ and to a negative level*

$$\tau_1^m = 1/\widehat{S} - 1 < 0, \tag{41}$$

if $\beta_1 > 1+\pi^*$, where $\widehat{S}$ is the solution of (40) that is larger than 1. There is full employment irrespective of the state of global demand.

Proof. See discussion above. ■

5.3 Trade wars with export subsidies (i, τ^x)

The outcome of a trade war is very different if it involves taxes on exports. It follows from Proposition 4 that from period 2 onwards, there is a Nash equilibrium with full employment and in which the national policymakers set the gross export tax rate to

$$1+\tau_2^x = \frac{\epsilon_x}{\epsilon_x - 1}. \tag{42}$$

Like in the previous section we characterize the Nash equilibrium in period 1 conditional on (42). The equilibrium with export taxes is characterized in the following result.

Proposition 10 *(Trade war with export taxes) Assume that all national policymakers use export taxes in addition to the nominal interest rate and that $\epsilon_x \geq 1+\alpha_H\left(\frac{1}{\epsilon_i}-1\right)$. Then the period-1 non-cooperative equilibrium is unique. If $\beta_1 \leq 1+\pi^*$, the ZLB constraint is non-binding and policymakers impose the same export tax in the short run as in the long run, $\tau_1^x = \tau_2^x$. If $\beta_1 > 1+\pi^*$, the ZLB constraint is binding and the export tax is lower in the short run than in the long run, $\tau_1^x < \tau_2^x$. There is full employment in both cases.*

Proof. See Appendix E. ■

With export taxes there is always full employment, even when the economy is in a liquidity trap. Each policymaker can increase the home trade balance by lowering the tax on exports. The tax on exports does not affect home consumption, so that decreasing it unambiguously raises home welfare if there is unemployment. Hence, all policymakers reduce the taxation on exports until there is full employment. Full employment obtains in general equilibrium because reducing period-1 export taxes reduces the global good own real rate of interest. as explained after equation (33).

It is easy to see that with export taxes, the global policymaker's allocation is the same as in Proposition 9. This is because the variable that matters from a global perspective is $1/S_t^m = (1+\tau_t^m)(1+\tau_t^x)$, so that the same allocations can be implemented with tariffs on imports or taxes on exports. If there is a liquidity trap, the global policymaker achieves full employment in period 1 by setting $(1+\tau_1^x) = 1/\widehat{S}$ where $\widehat{S}$ satisfies (40). In this case, $\tau_1^x < 0$, i.e., the global policymaker subsidizes exports in period 1.

Observe that in a liquidity trap with export subsidies, international cooperation is beneficial in the long run but is not necessary in the short run. If there is a liquidity trap and the export tax rate is equal to zero in the long run, then the period-1 Nash equilibrium leads to the same export subsidy $\tau_1^x = 1/\widehat{S} - 1 < 0$ and the same allocation as with international cooperation.

5.4 Other policy combinations

This section discusses combinations of policy instruments other than those analyzed in the previous three sections. We first discuss the case where policymakers can use both trade taxes in addition to monetary policy. We then consider the impact of adding the tax on capital flows to the policy mix.

Trade war with both trade taxes (i, τ^m, τ^x). The allocation with both trade taxes is the same as when only the tax on exports is used. The equilibrium inherits two key properties from Proposition 10: the non-cooperative allocation is unique and features full employment. Full employment uniquely determines the total trade distortion $(1+\tau_1^m)(1+\tau_1^x)$ and so the allocation.

Furthermore the levels of τ_1^m and τ_1^x are also uniquely determined given τ_2^m and τ_2^x. This is illustrated by Figure 2, which shows the variation with global demand $1/\beta_1$ of the period-1 nominal interest rate and of the difference between period-1 and period-2 trade taxes.[14] When global demand is high, the trade taxes are set to the same levels in period 1 as in the long run, which is consistent with a positive nominal interest rate. When demand $1/\beta_1$ falls below $1/(1+\pi^*)$ the ZLB constraint becomes binding and the national policymakers lose the interest rate as an instrument. They subsidize exports to maintain full employment and increase the tariff on imports to continue manipulating the terms of trade.

The analysis of the global social planner is the same as in the previous section. The global social planner avoids distorting the economy in the long run by setting $(1+\tau_2^m)(1+\tau_2^x) = 1$ and ensures full employment in the short run by setting $(1+\tau_1^m)(1+\tau_1^x) = 1/\widehat{S}$. The global social planner does not gain anything from being able to use two trade taxes instead of one.

Capital flow taxes. One can then add the capital inflow tax τ^b to the mix and consider the Nash equilibria where all the instruments are used. Allowing the national policymakers to use the tax on capital flows leads to indeterminacy in the policy instruments because of Lerner's symmetry. Each policymaker can achieve the desired allocation with an infinity of policy combinations featuring the same wedge $(1+\tau_t^m)(1+\tau_t^x)$. One of these combinations involve a zero tax on capital inflows so that the Nash allocation is the same as in the case where policymakers do not use the tax on capital inflows.

Another case is where the policymakers use the tax on capital flows τ^b but not the trade taxes. Whether the Nash equilibrium results in a tax or a subsidy on capital inflows depends on the state of global demand. If demand is high and the ZLB constraint is not binding, each policymaker tries to improve the home terms of trade by subsidizing capital inflows like in Costinot, Lorenzoni and Werning (2014). If demand is low and the ZLB constraint binds, each policymaker can raise home employment by taxing capital inflows. Irrespective of the state of global demand, the use of capital account policies does not change welfare or the allocation in a symmetric equilibrium, since global consumption C_1^W is not affected by τ_1^b. Any change in the global level of τ_1^b is neutralized by an offseting change in the global real rate R_1. Capital wars are zero-sum games, as discussed by Korinek (2016).[15]

[14] The figure was constructed assuming $\tau_2^x = 0$. The equilibrium is formally derived in appendix E.

[15] It is not necessarily true, however, that the Nash equilibrium in capital controls is symmetric. The welfare of a country is a convex function of τ^b if the the export elasticity is above a threshold. As a result, the Nash equilibrium may lead to an endogenous symmetry breakdown in which a fraction of countries impose a higher tax on capital inflows than the rest of the world. We leave a full-fledged analysis of this case for another paper.

5.5 Conclusions

To conclude this section, Figure 3 shows the welfare impact of different kinds of trade and currency wars under the benchmark calibration in Table 1. The discount factor β_1 is set at the level that implies an unemployment rate of 5 percent in period 1 if national policymakers use only monetary policy. We assume that the policymakers do not use the trade taxes in the long run (from period 2 onwards) to measure the impact of short-run trade wars. The figure shows the impact of the trade or currency wars on period-1 consumption in percentage points of pre-war consumption. Each bar shows the impact on period-1 welfare of letting the policymakers use the instruments reported above the bar instead of just the nominal interest rate.

The main lesson from the figure is that the welfare impact of lack of international cooperation crucially depends on which policy instrument is used. The worst welfare impact comes from an import tariff war because of the resulting decrease in global demand. A tariff war raises the unemployment rate from 5 percent to 12.5 percent. By contrast, a trade war involving subsidies on exports leads to full employment and raises welfare. It does not increase welfare to the first-best level (which is achieved with an inflation target war) because of the consumption distortion that results from the trade taxes.

Another lesson from this analysis is that a case for international cooperation does not necessarily follow from the fact that a policy has beggar-thy-neighbor effects. All the policy instruments in this model have beggar-thy-neighbor effects if global demand is low,[16] but international cooperation is warranted only in the case of tariffs on imports.

6 Sustainability of Free Trade

This section looks into the conditions under which free trade can be sustained as a trigger-strategy equilibrium. We assume that a deviation from free trade by one country may lead to a permanent trade war between all countries. The risk of a generalized trade war, thus, may deter individual countries from deviating from

[16]As shown in appendix D, tariffs are always beggar-thy-neighbor, and the policy changes involving the other instruments (a currency depreciation achieved through a monetary stimulus or a tax on capital inflows, or an export subsidy) are beggar-thy-neighbor under the baseline calibration.

free trade. We use this model to explore how the temptation to deviate from free trade is affected by the state of global demand.

Like in the previous section we assume that the global economy is in a full employment steady state from period 2 onwards and use β_1 to vary the level of global demand in period 1. We assume that a deviation from free trade by one country in period 1 leads to a trade war starting in period 2 (that is, the Nash equilibrium described in Proposition 4) with probability μ. A trade war starting in period 2 continues forever.[17] Which trade taxes are used in a trade war matters only in period 1 and we assume that the national policymakers use tariffs on imports.[18]

We look at the incentives for country j to deviate assuming that it is the only one to do so. Free trade is sustainable if countries do not gain from a deviation. If country j does not deviate from free trade, its period-1 welfare is the same as for the representative country and is given by $V_{j1}^n = u\left(C_1^W\right) + \beta_1 V_2^N(0)$ where $V_2^N(0)$ is the period-2 welfare of a country with zero foreign assets if there is no trade war. If country j deviates from free trade, its period-1 welfare is

$$V_{j1}^d = u\left(C_{j1}^d\right) + \beta_1\left[(1-\mu) V_2^N\left(B_{j2}^d\right) + \mu V_2^T\left(B_{j2}^d\right)\right]$$

where C_{j1}^d and $B_{j2}^d = R_1 X_{j1}^d$ are respectively the period-1 consumption and period-2 foreign assets of country j if it deviates, and $V_2^T(\cdot)$ is period-2 welfare if there is a trade war.

The net welfare gain from deviating from free trade, $\Delta V_{j1} = V_{j1}^d - V_{j1}^n$, can thus be decomposed into two terms,

$$\Delta V_{j1} = \underbrace{u\left(C_{j1}^d\right) + \beta_1 V_2^N\left(B_{j2}^d\right) - u\left(C_1^W\right) - \beta_1 V_2^N(0)}_{\text{GAIN}} - \underbrace{\beta_1\mu\left[V_2^N\left(B_{j2}^d\right) - V_2^T\left(B_{j2}^d\right)\right]}_{\text{LOSS}}.$$

The first term is the gain that country j derives from imposing a tariff in period 1 if this does not lead to a trade war. The second term, the cost of deviating from free trade, is equal to the discounted expected welfare loss from a trade war starting in period 2. Free trade is sustainable if and only if $\Delta V_{j1} < 0$.

[17] Alternatively we could assume that the trade war lasts a finite time and use the expected duration of a trade war to vary its cost. However, the equilibrium is more complicated to derive in that case. Assuming a permanent trade war keeps the analysis simple without affecting the essence of the results.

[18] The results are similar if one assumes instead that the deviating country uses a tax or subsidy on exports. Indeed, the temptation to deviate from free trade is even stronger in that case because an export subsidy does not distort home consumption.

The left-hand side panel of Figure 4 shows how the gain and cost of imposing a tariff in period 1 vary with the state of global demand $1/\beta_1$. The figure was constructed with the parameter values in Table 1 and assuming that a deviation from free trade by one country triggers a generalized trade war with probability $\mu = 3\%$.

The gain from a deviation from free trade is not greatly affected by global demand when the economy is at full employment (i.e., when $\beta_1 \leq 1 + \pi^*$). This is because in this case $\beta_1 R_1 = 1$ so that a change in global demand is offset by a change in the global real interest rate, which leaves the benefit of deviating from free trade unchanged to a first-order of approximation. By contrast, the gains from deviating from free trade become much larger when the global economy is in a liquidity trap and any deviating country can raise its employment by imposing a tariff. Free trade, thus, tends to become less sustainable when global demand is low.

The right-hand side panel of Figure 4 shows the variation of the equilibrium tariff rate, interest rate and trade balance for a deviating country. When global demand is high, the deviating country imposes a tariff of about 27% to increase its terms of trade. The period-1 equilibrium tariff rate is lower than in a generalized trade war (where it is equal to $1/(\epsilon_x - 1) = 50\%$ by equation (37)) because the tariff, being temporary if it is not followed by a trade war, has a larger distortionary effect on the deviating country's consumption than in steady state. The deviating country offsets the stimulative impact of the tariff on home demand for the home good by raising its nominal interest rate. The deviating country thus falls in a liquidity trap for a lower level of global demand than the rest of the world. When it does fall in a liquidity trap, the deviating country raises the tariff rate to much higher levels in order to preserve full employment at home. The tariff-imposing country always increases its trade balance whether the global economy is in a liquidity trap or not.

7 Dynamic Trade and Currency Wars

We generalize our analysis in this section by considering the case where a global liquidity trap can last for several periods. We assume that the economy is a steady state with full employment starting in a period, denoted by T, that can be arbitrarily large. The Nash equilibrium is still defined as in section 4. The analysis presented so far was about the special case $T = 2$. We generalize (30) by assuming

$$\beta_t = \beta_T \leq 1 + \pi^* \text{ for } t \geq T. \tag{43}$$

This ensures the existence of a steady state equilibrium with full employment after period T.[19]

First, let us assume that the national policymakers can use only the nominal interest rate as a policy instrument. Going through the same steps as in Section 5, it is easy to see that a global liquidity trap with unemployment arises in period $T-1$ if and only if $\beta_{T-1} > 1+\pi^*$. If the global economy is in a liquidity trap with unemployment in all periods before period T, it follows from $L_t^W = C_t^W$, equation (15) with $i_t = 0$, $S_t^m = 1$, and $L_T^W = \overline{L}$, that the level of global employment in any period $t < T$ satisfies

$$u'\left(L_t^W\right) = \frac{\beta_t \beta_{t+1} \cdots \beta_{T-1}}{1+\pi^*} u'\left(\overline{L}\right).$$

We have used the fact that inflation is equal to π^* in period T and equal to 0 in all periods before period T. It follows that there is unemployment in all periods before time T if and only if

$$\prod_{s=t}^{T-1} \beta_s > 1+\pi^*, \quad \forall t < T. \tag{44}$$

We assume this condition to be satisfied in the following.

Second, let us assume that condition (44) being satisfied, the national policymakers can use trade taxes. In the long run (from period T onward), the national policymakers attempt to manipulate the terms of trade and set the trade tax to $1/\left(\epsilon_x - 1\right)$. Like in the previous section, the outcome of a trade war before period T is very different depending on whether the national policymakers tax imports or exports. Consider first the case of tariffs on imports. The equilibrium in period $T-1$ can be constructed like in section 5.2 and τ_{T-1}^m is given by (38). In fact, it is possible to show that the equilibrium tariff rate is given by (38) in all the periods before time T (see Proposition 11).

If the national policymakers use tax on exports, they achieve full employment by lowering the tax on exports before period T.

Our main results are summarized in the following proposition.

Proposition 11 *(Dynamic trade and currency wars) Assume that the economy is in a global liquidity trap before period T if national policymakers use monetary policy only. Then*

[19] If $\beta_T > \beta^*$ where β^* is the threshold defined in Proposition 8, there could also be a self-fulfilling global liquidity trap in any period after time T. We rule out this type of multiplicity here as it has been already analyzed in Section 5.2.

(i) in the Nash equilibrium where policymakers use tariffs on imports only, the equilibrium tariff τ_t^m is given by (38) in all periods $t < T$;

(ii) in the Nash equilibrium where policymakers use export taxes only, the policymakers tax exports at a lower rate before time T than after time T and there is full employment.

Proof. See Appendix E. ∎

The dynamics of a multi-period trade war are illustrated by Figure 5. To construct this figure, we assumed that $\beta_t = 1.03$ for four periods before decreasing to its long-run level of 0.98 in period $T = 5$. The left-hand side panel compares the dynamics of consumption under free trade, under a tariff war and when the policymakers use export taxes. The right-hand side panel shows the variation of the policy instruments over time.

Similarly, the global policymaker's allocation can be solved for by generalizing the analysis in sections 5.2 and 5.3. Iterating on (15) with $i_{jt} = 0$ and , $\pi_{jT} = \pi^*$ and $\pi_{jt} = 0$ for $t < T$ gives the following expression for global consumption and employment,

$$C_t^W = \left(\frac{\prod_{s=t}^{T-1} \beta_s}{1 + \pi^*} \right)^{-\epsilon_i} \left(\frac{S_T^m}{S_t^m} \right)^{-\alpha_F \epsilon_i} C_T^W, \tag{45}$$

$$L_t^W = \ell \left(S_t^m \right) C_t^W. \tag{46}$$

These expressions generalize (32) and (34). It remains true that the global policymaker maximizes welfare by setting the trade taxes to zero in the long run (after time T) and by subsidizing imports and/or exports so as to achieve full employment before time T. This implies that the global policymaker sets the trade taxes such that $(1 + \tau_t^m)(1 + \tau_t^x) = 1/\widehat{S}_t$ where $\widehat{S}_t$ satisfies equation (40) with β_1 replaced by $\prod_{s=t}^{T-1} \beta_s$.

8 Conclusions

We have analyzed a tractable model in which countries use monetary policy, trade taxes and capital controls to maximize home welfare. When global demand is high the trade taxes are used to manipulate the terms of trade like in a textbook tariff war. When global demand is low and the ZLB constraint is binding, countries use the trade taxes to raise home employment. The analysis suggests that there is one case where uncoordinated policies lead to large welfare losses: when global demand is low and countries use tariffs on imports. The uncoordinated use of all the other instruments that we have looked at (interest rate, inflation target, export subsidy and capital controls) is Pareto-improving or -neutral in the short run. However, tariffs seem to be the instrument of choice in the real world. One interesting question is why real world policymakers favor tariffs over other instruments such as export subsidies. A possible explanation is that subsidies on exports may have to be financed with distortionary taxation.

The paper opens several directions for further research. The assumptions about price stickiness could be changed to look at the implications of local currency pricing or dominant currency pricing.[20] The structure of production could also be enriched. In particular there could be international trade in production inputs and not only in final consumption goods. Making the model less symmetric would allow us to explore questions that have not been analyzed in this paper. For example, assuming that countries differ in their time preferences would make it possible to examine how a "savings glut" in one part of the world affects the benefits of international policy coordination. Another relevant source of asymmetry is if countries have access to different policy instruments.

[20] Egorov and Mukhin (2021) and Auray, Devereux and Eyquem (2020) discuss the benefits of international policy coordination in an environment with dollar pricing.

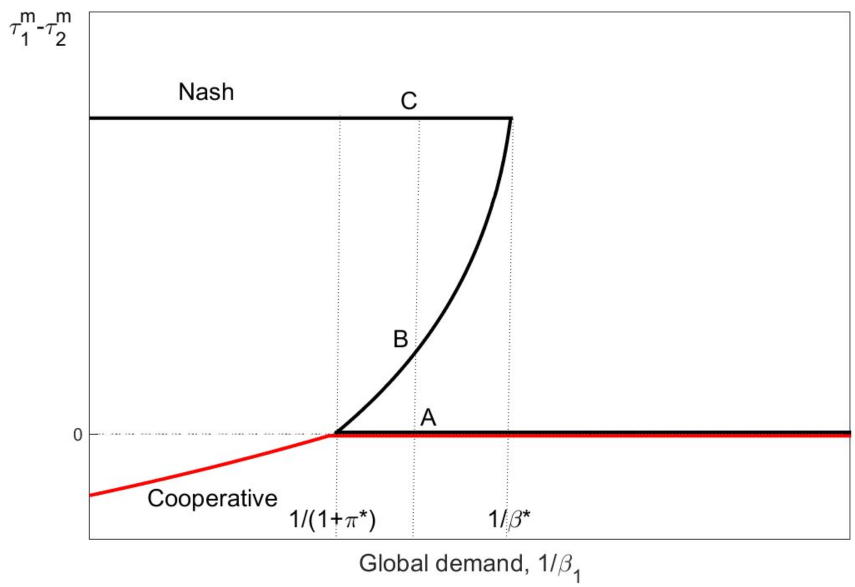

Figure 1: Variation of $\tau_1^m - \tau_2^m$ with demand $1/\beta_1$: Nash equilibrium (black) and cooperative equilibrium (red)

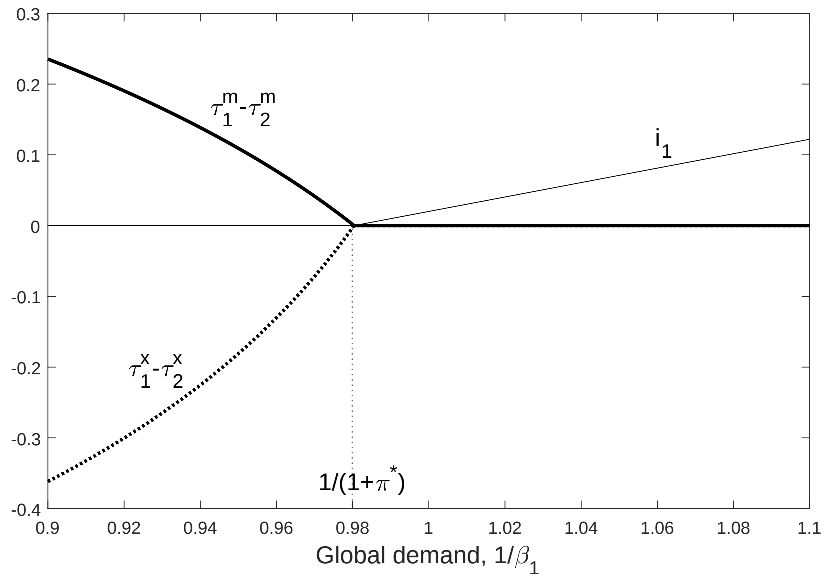

Figure 2: Variation with demand $1/\beta_1$ of trade taxes and nominal interest rate in Nash equilibrium

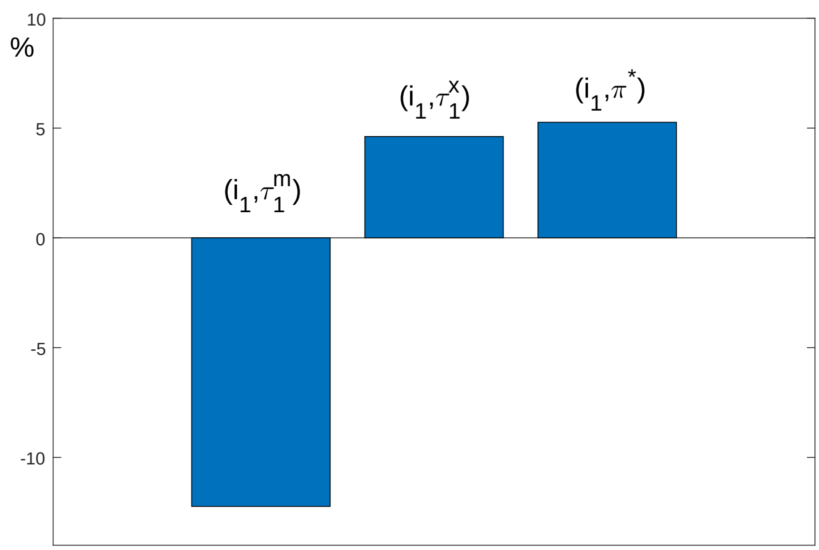

Figure 3: Impact of trade and currency wars on period-1 consumption (percentage points of pre-war consumption)

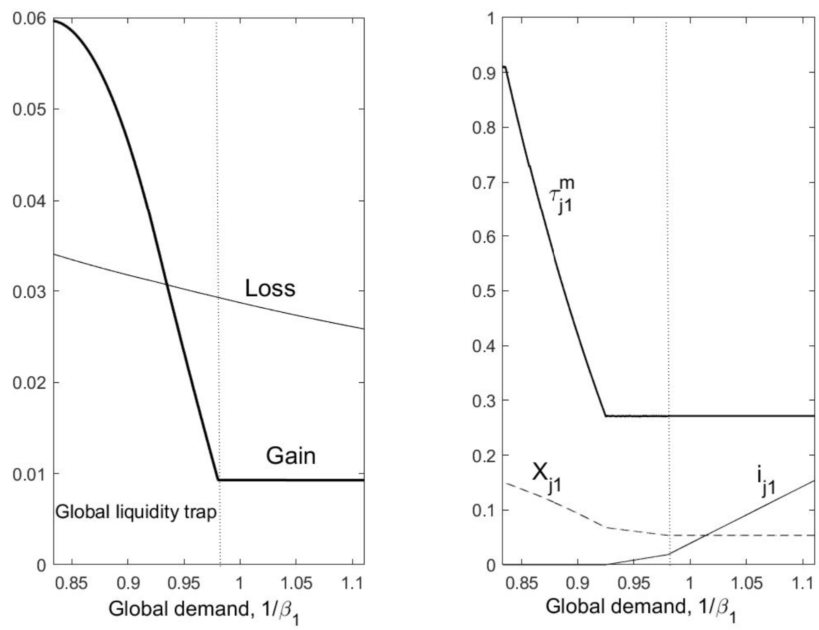

Figure 4: Deviation from free trade. The l.h.s. panel shows the variation with global demand of the gain and cost of deviating from imposing a tariff. The r.h.s. panel shows the equilibrium tariff, interest rate and trade balance for a tariff-imposing country.

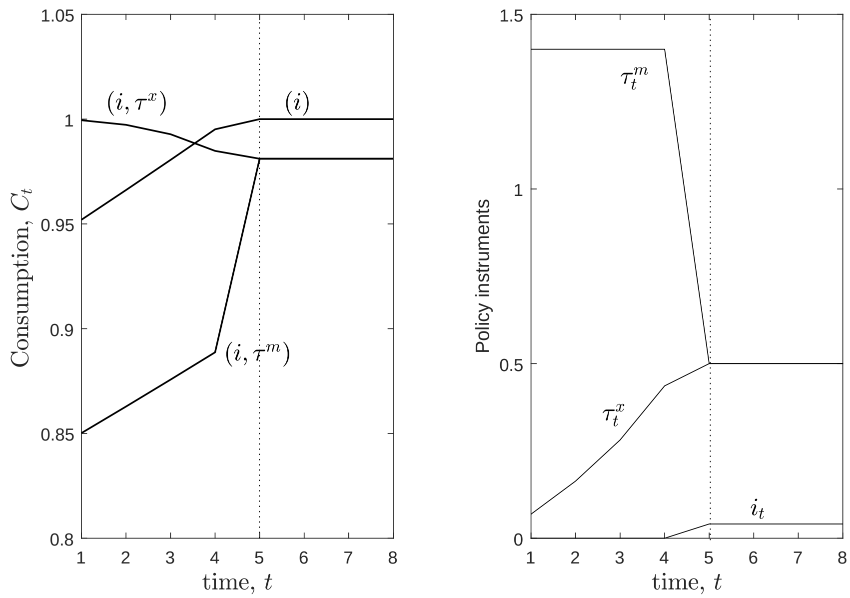

Figure 5: Consumption and policy instruments in a dynamic trade war

APPENDICES

APPENDIX A. FIRST-ORDER CONDITIONS

This appendix derives the first-order conditions in the model augmented to include money and nominal bonds. The consumer derives utility from real money balances. We omit the country subscript j to alleviate the notations. The consumer's problem in Bellman form is

$$V_t\left(B_t + \frac{M_t + B_t^n}{P_{Ft}}\right) = \max_{C_t, B_{t+1}, B_{t+1}^n, M_{t+1}} \left[u\left(C_t\right) + v\left(\frac{M_{t+1}}{P_{Ht}}\right) + \beta_t V_{t+1}\left(B_{t+1} + \frac{M_{t+1} + B_{t+1}^n}{P_{Ft+1}}\right)\right]$$

subject to

$$P_{Ft}\frac{B_{t+1}}{R_t\left(1+\tau_t^b\right)} + \frac{B_{t+1}^n}{1+i_t} + M_{t+1} + P_t^c C_t = P_{Ht}L_t + Z_t + P_{Ft}B_t + B_t^n + M_t,$$

where $P_t^c = \left(P_{Ht}\right)^{\alpha_H}\left[\left(1+\tau_t^m\right)P_{Ft}\right]^{\alpha_F}$ is the consumption price index, B_t^n is the payoff on nominal bonds, M_t is the consumer's money holding at the beginning of period t, and $v\left(\cdot\right)$ is the utility from real money balances. The government supplies zero nominal bonds and injects newly printed money through a lump-sum transfer to the consumer, so that $Z_t = \tau_t^m P_{Ft}C_{Ft} + \tau_t^x P_{Ht}\left(L_t - C_{Ht}\right) - \tau_t^b P_{Ft}B_{t+1}/\left(1+\tau_t^b\right) + M_{t+1} - M_t$. Period-$t$ money supply bears the time subscript $t+1$ to be consistent with the notations for bonds.

The first-order conditions for B_{t+1} and B_{t+1}^n imply equation (14). The first-order condition for C_t and the envelope condition give the Euler condition,

$$u'\left(C_t\right)\frac{P_{Ft}}{P_t^c} = \beta_t R_t\left(1+\tau_t^b\right)u'\left(C_{t+1}\right)\frac{P_{Ft+1}}{P_{t+1}^c}.$$

or

$$u'\left(C_t\right)\frac{\left(S_t^m\right)^{-\alpha_H}}{1+\tau_t^m} = \beta_t R_t\left(1+\tau_t^b\right)u'\left(C_{t+1}\right)\frac{\left(S_{t+1}^m\right)^{-\alpha_H}}{1+\tau_{t+1}^m}. \tag{47}$$

Then using (14) to substitute out $R_t\left(1+\tau_t^b\right)$ from this equation, one can rewrite the Euler equation as (15).

The first-order condition for M_{t+1} and the envelope condition imply,

$$v'\left(\frac{M_{t+1}}{P_{Ht}}\right) = \frac{P_{Ht}}{P_t^c}u'\left(C_t\right)\left(1 - \frac{1}{1+i_t}\right) \tag{48}$$

APPENDIX B. ALTERNATIVE POLICY INSTRUMENTS

This appendix studies the two alternative policy instruments mentioned in section 3, money supply and foreign exchange interventions.

Money supply. We go back to the model with money in the utility function presented in Appendix A. Using (15) to substitute out $u'(C_t)$ from equation (48) and $(S_t^m)^{\alpha_F} = P_{Ht}/P_t^c$ gives the following equation for money demand,

$$v'\left(\frac{M_{t+1}}{P_{Ht}}\right) = \beta_t i_t \frac{u'(C_{t+1})\left(S_{t+1}^m\right)^{\alpha_F}}{1+\pi_{t+1}}. \tag{49}$$

Nominal stickiness sets a lower bound on the nominal price of the home good, $P_{Ht} \geq W_{t-1}$.

Figure 6 shows how P_{Ht} and L_t vary with period-t money supply M_{t+1}, assuming that next-period variables are constant (a first-order approximation as explained in Appendix D). The economy is in the Keynesian regime with unemployment if money supply is lower than a threshold. In this regime, P_{Ht} is fixed and an increase in money supply lowers the nominal interest rate by equation (49). This depreciates the home currency and raises consumption and the demand for home labor by equations (14), (15) and (11). When the demand for home labor reaches $\overline{L}$ the economy transitions to the classical regime where further increases in money supply raise the nominal wage and have no impact on real variables. The policymaker sets inflation at the target level by choosing the appropriate level of money supply, which corresponds to point A in Figure 6.

Observe however that it is not always possible to raise L_t to the full employment level by increasing money supply. The nominal interest rate goes to zero as money supply goes to infinity or reaches the satiation level. If the level of labor demand corresponding to $i = 0$ is lower than labor supply $\overline{L}$, the economy is in a liquidity trap.

Foreign exchange interventions. We now assume that the capital account is closed, i.e., the only home agent who can trade real bonds with foreign investors is the government. The government finances its purchase of foreign bonds by selling home currency bonds to the home consumers. This can be interpreted as a sterilized foreign exchange interventions in which the central bank buys dollars. The budget constraints of the home consumer and the government are respectively given by

$$\frac{B_{t+1}^n}{1+i_t} + M_{t+1} + P_t^c C_t = P_{Ht} L_t + Z_t + B_t^n + M_t,$$

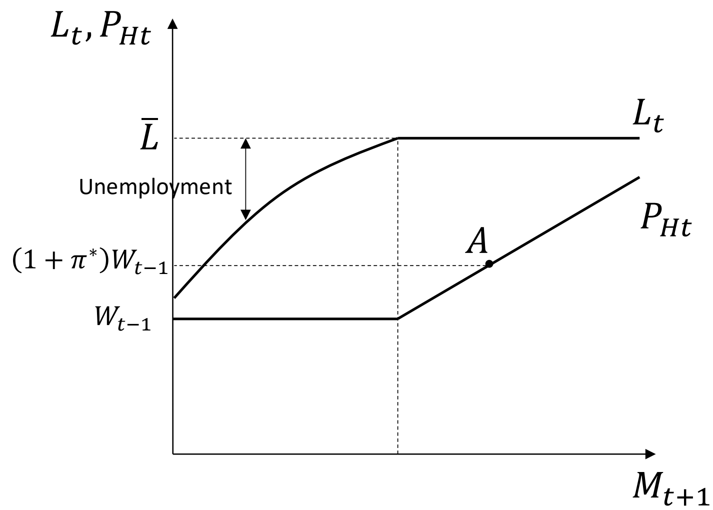

Figure 6: Money supply, price level and demand for labor

and

$$Z_t = \tau_t^m P_{Ft} C_{Ft} + \tau_t^x P_{Ht} (L_t - C_{Ht}) + P_{Ft} B_t - P_{Ft} \frac{B_{t+1}}{R_t} + \frac{B_{t+1}^n}{1+i_t} - B_t^n + M_{t+1} - M_t,$$

where the net supply of nominal bonds B_t^n is no longer equal to zero. Using the second expression to substitute out Z_t in the first expression still gives the balance-of-payments equation (12).

The real allocation can be derived from policies as follows. Because of (12), the government determines the trade balance by setting the amount of reserves B_{t+1}. The period-$t+1$ allocation is also determined by B_{t+1} through the next-period policy functions. Given X_t and the policy instruments i_t, τ_t^m and τ_t^x, equations (13), (15) and $S_t^x/S_t^m = (1+\tau_t^m)(1+\tau_t^x)$ is a system of three equations that can be solved for C_t, S_t^x and S_t^m. One can then derive C_{Ht} and C_{Ft} using (9) and (10). The terms of trade S_t can be derived from S_t^m and τ_t^m and L_t results from (11).

Equation (14) no longer applies since the home consumer no longer arbitrages between home currency bonds and foreign bonds. However the allocation is the same as when the capital account was open and τ_t^b was set to the level satisfying (14). The same allocations, thus, can be implemented with foreign exchange interventions under a closed capital account or with a tax on capital flows under an open capital account.

APPENDIX C. STEADY STATES

This appendix analyzes the full employment steady states in which all countries have the same level of trade taxes τ^m and τ^x and one atomistic country (denoted by j) has a non-zero level of foreign assets. We denote with superscript W the representative country in the rest of the world. We omit the time index since all variables are constant over time.

In such steady states the tariff-adjusted terms of trade are given by

$$S^m = \frac{1}{(1+\tau^m)(1+\tau^x)},$$

in all countries. We do not assume a particular value for S^m. The Nash equilibrium is a special case with $S^m = 1 - 1/\epsilon_x$ and the global social planner allocation corresponds to $S^m = 1$.

We have

$$\frac{C_F^W}{C_H^W} = \frac{\alpha_F}{\alpha_H} S^m.$$

Together with this equation, the resource constraint that home production is consumer either at home or abroad, $\overline{L} = C_H^W + C_F^W$, implies

$$\begin{aligned} C_H^W &= \frac{\alpha_H}{\alpha_H + \alpha_F S^m}\overline{L}, \\ C_F^W &= \frac{\alpha_F S^m}{\alpha_H + \alpha_F S^m}\overline{L}. \end{aligned} \tag{50}$$

As for country j, equation (11) with $L_j = \overline{L}$ and $S_j^x = S_j(1+\tau^x) = S_j^m(1+\tau^m)(1+\tau^x) = S_j^m/S^m$ imply

$$\overline{L} = \alpha_H \left(S_j^m\right)^{-\alpha_F} C_j + C_F^W \left(\frac{S_j^m}{S^m}\right)^{-\epsilon_x}. \tag{51}$$

Equation (12), together with $\beta R = 1$ and (13), implies

$$(1-\beta) B_j = \alpha_F \left(S_j^m\right)^{\alpha_H} C_j - C_F^W \left(\frac{S_j^m}{S^m}\right)^{1-\epsilon_x}, \tag{52}$$

where $C_F^W = \alpha_F (S^m)^{\alpha_H} C^W = \alpha_F (S^m)^{\alpha_H} \overline{L}/\ell(S^m)$. Equations (51) and (52) determine C_j and S_j^m for any given B_j. For $B_j = 0$ the solution is $S_j^m = S^m$ and

$C_j = C^W$. Differentiating (51) and (52) with respect to B_j in $B_j = 0$ and using (50) to substitute out C_F^W gives the derivatives of the policy functions,

$$\frac{\partial C_j}{\partial B_j} = (1-\beta)\left(1+\frac{\alpha_H}{\epsilon_x S^m}\right)\frac{1}{\ell(S^m)}, \tag{53}$$

$$\frac{\partial S_j^m}{\partial B_j} = (1-\beta)\frac{\alpha_H}{\alpha_F \epsilon_x \overline{L}}. \tag{54}$$

The welfare of country j's representative consumer is given by

$$V(B_j) = \frac{u(C_j)}{1-\beta}.$$

Differentiating this equation with respect to B_j in $B_j = 0$ gives the marginal utility of the country's external wealth, or social marginal utility of external wealth

$$V'(0) = \frac{u'(C_j)}{1-\beta}\frac{\partial C_j}{\partial B_j} = u'\left(C^W\right)\left(1+\frac{\alpha_H}{\epsilon_x S^m}\right)\frac{1}{\ell(S^m)}. \tag{55}$$

Using (55) and (28) it is easy to see that the social marginal utility of external wealth $V'(0)$ is larger than the private marginal utility, $u'\left(C^W\right)(S^m)^{-\alpha_H}$ if and only if $S^m > 1 - 1/\epsilon_x$. The difference between the two marginal utilities comes from the fact that individual consumers do not internalize the impact of external wealth on the home terms of trade. In the Nash equilibrium with $S^m = 1 - 1/\epsilon_x$ equation (55) becomes

$$V'(0) = u'\left(C^W\right)(S^m)^{-\alpha_H}. \tag{56}$$

APPENDIX D. ELASTICITIES AND SPILLOVERS

Elasticities. We derive the elasticities of macroeconomic variables with respect to the policy instruments, which will be used to prove our main results in Appendix E. We assume that the economy is in the full employment steady state analyzed in appendix C from period 2 onwards so that $\pi_2 = \pi^*$.

We assume that $1 - \beta_2$ is small of the first order. If B_{j2} is first-order, it then follows from (53) and (54) that the impact of country j's period-1 policies on S_{j2} and C_{j2} is second-order and can be omitted to a first order of approximation. Hence (14) and (15) for $t = 1$ can be written

$$S_{j1} \simeq \frac{1 + i_{j1}}{R_1 \left(1 + \tau^b_{j1}\right)\left(1 + \pi^*\right)} S_2, \tag{57}$$

and

$$u'\left(C_{j1}\right)\left(S^m_{j1}\right)^{\alpha_F} \simeq \beta_1 \frac{1 + i_{j1}}{1 + \pi^*} u'\left(C_2\right)\left(S^m_2\right)^{\alpha_F}, \tag{58}$$

where the period 2 variables S_2, S^m_2 and C_2 will be taken as invariant to the period-1 policies of country j.

We denote by $e(\bullet, n)$ the elasticity of variable $\bullet = S, C, L, X$ with respect to instrument $n = i, \tau^m, \tau^x$ and τ^b. The elasticities are defined as follows,

$$e\left(S, n\right) = \frac{1+n}{S}\frac{\partial S}{\partial n},\ e\left(C, n\right) = \frac{1+n}{C}\frac{\partial C}{\partial n}, \tag{59}$$

$$e\left(L, n\right) = \frac{1+n}{C}\frac{\partial L}{\partial n},\ e\left(X, n\right) = \frac{1+n}{C_F}\frac{\partial X}{\partial n}. \tag{60}$$

Observe that the elasticities for labor and net exports are scaled by C and C_F respectively. The elasticities are computed in a symmetric allocation assuming less than full employment. The elasticities are reported in Table D1.

Table D1. Elasticities in a symmetric allocation with unemployment

	i	τ^m	τ^x	τ^b
S	1	0	0	-1
C	$-\alpha_H \epsilon_i$	$-\alpha_F \epsilon_i$	0	$-\alpha_F \epsilon_i$
L	$-\left(\alpha_H \epsilon_i + \alpha_F\right)\frac{C_H}{C} - \epsilon_x \frac{C_F}{C}$	$\alpha_F\left(1 - \epsilon_i\right)\frac{C_H}{C}$	$-\epsilon_x \frac{C_F}{C}$	$\alpha_F\left(1 - \epsilon_i\right)\frac{C_H}{C} + \epsilon_x \frac{C_F}{C}$
X	$-\left(\epsilon_x - \alpha_H \epsilon_i - \alpha_F\right)$	$\alpha_H + \alpha_F \epsilon_i$	$-\left(\epsilon_x - 1\right)$	$\epsilon_x - \alpha_F\left(1 - \epsilon_i\right)$

The values of $e(S,n)$ reported in the top two rows of Table D1 directly follows from (57) and (58).

Differentiating (11) and (13) for $t = 1$ and using the fact that $C^W_{F1} = C_{F1}$, $S^x_1 = 1$ in a symmetric equilibrium, as well as (9) and (10) we obtain

$$
\begin{aligned}
e(L,n) &= [e(C,n) - \alpha_F e(S^m,n)] \frac{C_H}{C} - \epsilon_x e(S^x,n) \frac{C_F}{C}, && (61)\\
e(X,n) &= -[(\epsilon_x - 1) e(S^x,n) + e(C,n) + \alpha_H e(S^m,n)]. && (62)
\end{aligned}
$$

The elasticities in the bottom two rows of the table are derived using (61) and (62) and the expressions in the first two rows of Table D1.

The expressions reported in Table 1 in the main text can be derived using the expressions for $e(L,n)$ in Table D1 and the fact that $C_H/C = \alpha_H$ and $C_F/C = \alpha_F$ in an equilibrium with zero trade taxes.

International spillovers. The international spillovers associated with the different policies are measured as follows. We consider a symmetric allocation with unemployment in period 1 and with zero trade or capital flow taxes in all periods ($\tau^m_t = \tau^x_t = \tau^b_t = 0$ for all t). We still assume that the economy is in a full employment steady state from period 2 onwards. This implies $S^m_1 = S^m_2 = 1$, $C_H = \alpha_H C$ and $C_F = \alpha_F C$. We then look at the impact of a small group of countries j of mass ε marginally changing one policy instrument $n_{j1} = i_{j1}, \tau^m_{j1}, \tau^x_{j1}, \tau^b_{j1}$ on their own welfare, U_{j1}, on the welfare of the rest of the world, U_{-j1}, and on global welfare,

$$U^W_1 = \varepsilon U_{j1} + (1 - \varepsilon) U_{-j1}.$$

where U_{j1} and U_{-j1} are given by (31).

To find the impact of the policy change on the welfare of countries j, we differentiate (31)

$$
\begin{aligned}
\frac{\partial U_{j1}}{\partial n_{j1}} &= u'(C_1) \frac{\partial C_{j1}}{\partial n_{j1}} + \beta_1 R_1 V_2'(0) \frac{\partial X_{j1}}{\partial n_{j1}},\\
&= \frac{u'(C_1) C_1}{1 + n_1} \left[e(C,n) + \alpha_F \left(1 + \frac{\alpha_H}{\epsilon_x}\right) e(X,n) \right] \Bigg(
\end{aligned}
$$

To derive the second line we used $u'(C_1) = \beta_1 R_1 u'(C_2)$ from (47) with $\tau^m_1 = \tau^m_2 = \tau^b_1 = 0$ and $S^m_1 = S^m_2 = 1$ and substituted out $V_2'(0)$ using (55) with $S^m_2 = 1$ and $C_F/C = \alpha_F$. The first line of Table D2 reports the welfare elasticity $\frac{1+n_1}{u'(C_1)C_1} \frac{\partial U_{j1}}{\partial n_{j1}}$

computed with the equation above and the elasticites reported in Table D1. For the sake of alleviating the algebra we defined the two auxiliary parameters

$$\gamma \equiv \alpha_F \left(1 + \frac{\alpha_H}{\epsilon_x}\right), \quad \eta \equiv \epsilon_x - \alpha_H \epsilon_i - \alpha_F.$$

Note that $\eta > 0$ because $\epsilon_x > 1$.

The impact on the rest of the world's welfare is

$$\begin{aligned} \frac{\partial U_{-j1}}{\partial n_{j1}} &= u'\left(C_1\right) \frac{\partial C_{-j1}}{\partial n_{j1}} + \beta_1 R_1 V_2'\left(0\right) \frac{\partial X_{-j1}}{\partial n_{j1}}, \\ &= \frac{u'\left(C_1\right) C_1}{1+n_{j1}} \left[\frac{1+n_{j1}}{C_1} \frac{\partial C_{-j1}}{\partial n_{j1}} + \alpha_F \left(1 + \frac{\alpha_H}{\epsilon_x}\right) \frac{1+n_{j1}}{C_{F1}} \frac{\partial X_{-j1}}{\partial n_{j1}}\right] \end{aligned} \tag{63}$$

The fact that trade balances sum up to zero implies

$$\left(1-\varepsilon\right) \frac{\partial X_{-j1}}{\partial n_{j1}} + \varepsilon \frac{\partial X_{j1}}{\partial n_{j1}} = 0,$$

so that

$$\frac{\partial X_{-j1}}{\partial n_{j1}} = -\frac{\varepsilon}{1-\varepsilon} \frac{\partial X_{j1}}{\partial n_{j1}} \simeq -\varepsilon e\left(X, n\right) \frac{C_{F1}}{1+n_1}. \tag{64}$$

The rest of the world's consumption is given by

$$\begin{aligned} C_{-j1} &= \left(\beta_1 \frac{1+i_{-j1}}{1+\pi^*}\right)^{-\epsilon_i} \left(S^m_{-j1}\right)^{\alpha_F \epsilon_i} \overline{L}, \\ &= \left(\beta_1 \frac{1+i_{-j1}}{1+\pi^*}\right)^{-\epsilon_i} \left(\frac{1}{R_1} \frac{1+i_{-j1}}{1+\pi^*}\right)^{\alpha_F \epsilon_i} \overline{L}, \end{aligned}$$

where we have used (15) with $C_{-j2} = \overline{L}$ for the first line, and $S^m_{-j1} = S_{-j1}$, (14) with $\tau^b_{-j1} = 0$ and $S_{-j2} = 1$ to derive the second equality. This equation shows that the policies of countries j affect consumption in the rest of the world only through the real rate of interest R_1, i.e.

$$\frac{1+n_1}{C_1} \frac{\partial C_{-j1}}{\partial n_{j1}} = -\alpha_F \epsilon_i \frac{1+n_1}{R_1} \frac{\partial R_1}{\partial n_{j1}}. \tag{65}$$

Equation (23) can be written

$$\left(1-\varepsilon\right) \left(S^x_{-j1}\right)^{1-\epsilon_x} + \varepsilon \left(S^x_{j1}\right)^{1-\epsilon_x} = 1.$$

Using (14) to substitute out $S^x_{-j1} = S_{-j1}$ and $S^x_{j1} = S_{j1}\left(1+\tau^x_{j1}\right)$ with $S_{j2} = S_{-j2} = 1$ then gives

$$R_1^{1-\epsilon_x} = (1-\varepsilon)\left(\frac{1+i_{-j1}}{1+\pi^*}\right)^{1-\epsilon_x} + \varepsilon\left[\frac{\left(1+i_{j1}\right)\left(1+\tau^x_{j1}\right)}{\left(1+\pi^*\right)\left(1+\tau^b_{j1}\right)}\right]^{1-\epsilon_x}.$$

Differentiating then gives

$$\frac{1+n_{j1}}{R_1}\frac{\partial R_1}{\partial n_{j1}} = \varepsilon e\left(R,n\right), \tag{66}$$

where

$$e\left(R,i\right) = e\left(R,\tau^x\right) = 1,\ e\left(R,\tau^m\right) = 0,\ e\left(R,\tau^b\right) = -1. \tag{67}$$

Using (64), (65), and (66), equation (63) can be re-written

$$\frac{1+n_1}{\varepsilon u'\left(C_1\right)C_1}\frac{\partial U_{-j1}}{\partial n_{j1}} = -\alpha_F\left[\epsilon_i e\left(R,n\right) + \left(1+\frac{\alpha_H}{\epsilon_x}\right)e\left(X,n\right)\right]$$

Then substituting out the elasticities $e\left(R,n\right)$ and $e\left(X,n\right)$ using (67) and Table D1 gives the expressions in the second line of Table D2. The third line is obtained by summing the first two lines.

Table D2. International spillovers

	i_j	τ^m_j	τ^x_j	τ^b_j
U_j	$-\alpha_H\epsilon_i - \gamma\eta$	$-\alpha_F\epsilon_i + \gamma\left(\alpha_H + \alpha_F\epsilon_i\right)$	$-\gamma\left(\epsilon_x - 1\right)$	$-\alpha_F\epsilon_i + \gamma\left(\eta + \epsilon_i\right)$
U_{-j}	$-\alpha_F\epsilon_i + \gamma\eta$	$-\gamma\left(\alpha_H + \alpha_F\epsilon_i\right)$	$-\alpha_F\epsilon_i + \gamma\left(\epsilon_x - 1\right)$	$\alpha_F\epsilon_i - \gamma\left(\eta + \epsilon_i\right)$
U^W	$-\epsilon_i$	$-\alpha_F\epsilon_i$	$-\alpha_F\epsilon_i$	0

Based on the expressions reported in Table D2 we can draw the following conclusions on the impact of policy changes on home and foreign welfare in the Keynesian regime.

A monetary stimulus (a decrease in i_j) raises the country's own welfare. Using the expressions for γ and η one can shows that the elasticity of country j's own welfare with respect to i_j is given by

$$-\alpha_H\epsilon_i - \gamma\eta = -\left(1 - \alpha_F/\epsilon_x\right)\left(\alpha_F\epsilon_x + \alpha_H\right) < 0.$$

A monetary stimulus is beggar-thy-neighbor (i.e., it decreases foreign welfare) if and only if $-\alpha_F \epsilon_i + \gamma\eta > 0$, i.e.

$$\epsilon_i < \left(1 + \frac{\alpha_H}{\epsilon_x}\right)\left(\epsilon_x - \alpha_H \epsilon_i - \alpha_F\right).$$

This condition is satisfied under our baseline calibration.

A tariff increase raises country j's own welfare if and only if $-\alpha_F \epsilon_i + \gamma\left(\alpha_H + \alpha_F \epsilon_i\right) > 0$, that is

$$\epsilon_i < \left(1 + \frac{\alpha_H}{\epsilon_x}\right)\left(\alpha_H + \alpha_F \epsilon_i\right).$$

This condition is necessarily satisfied if $\epsilon_i < 1$ (that is, if a tariff increase raises home employment), which is the case under our baseline calibration. Tariffs are always beggar-thy-neighbor.

An export subsidy (a decrease in τ_j^x) always raises country j's own welfare because $\epsilon_x > 1$. It is beggar-thy-neighbor if $-\alpha_F \epsilon_i + \gamma\left(\epsilon_x - 1\right) > 0$, that is

$$\epsilon_i < \left(1 + \frac{\alpha_H}{\epsilon_x}\right)\left(\epsilon_x - 1\right),$$

which is true under our benchmark calibration.

An increase in the tax on capital inflows raises the country's own welfare if and only if $-\alpha_F \epsilon_i + \gamma\left(\eta + \epsilon_i\right) > 0$, that is

$$\epsilon_i < \left(1 + \frac{\alpha_H}{\epsilon_x}\right)\left(\epsilon_x + \alpha_F \epsilon_i - \alpha_F\right).$$

This is true under the baseline calibration. This is also the condition for this policy to be beggar-thy-neighbor since the tax on capital flows does not affect global welfare at the margin.

APPENDIX E. PROOFS

Proof of Lemma 1. We need to show that if condition (16) is satisfied there exists a policy mix $\left(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b\right)$ and terms of trade S_{jt} that satisfy the equilibrium conditions (9), (10), (11), (13), (14) and (15). To see this, use equations (9) and (10) to derive S_{jt}^m, and $L_{jt} = C_{Hjt} + \left(S_{jt}^x\right)^{-\epsilon_x} C_{Ft}^W$ to derive S_{jt}^x. The trade taxes can then be chosen arbitrarily subject to $\left(1 + \tau_{jt}^m\right)\left(1 + \tau_{jt}^x\right) = S_{jt}^x / S_{jt}^m$. Observing that the next-period variables are all determined as policy functions of $B_{jt+1} = R_t \left(B_{jt} + X_{jt}\right)$, one can use equation (15) to derive i_{jt}. The capital flow tax τ_t^b is determined by (47). All the equilibrium conditions are satisfied, including (13) because of (16).

Proof of Proposition 2. Assume that a given allocation $(C_{Hjt}, C_{Fjt}, L_{jt}, X_{jt})$ is consistent with two policy mixes $(i_{jt}, \tau_{jt}^m, \tau_{jt}^x, \tau_{jt}^b)$ and $(\widehat{i}_{jt}, \widehat{\tau}_{jt}^m, \widehat{\tau}_{jt}^x, \widehat{\tau}_{jt}^b)$. As shown in the proof of Lemma 1, the allocation determines S_{jt}^x and S_{jt}^m satisfying $S_{jt}^x / S_{jt}^m = \left(1 + \tau_{jt}^m\right)\left(1 + \tau_{jt}^x\right) = \left(1 + \widehat{\tau}_{jt}^m\right)\left(1 + \widehat{\tau}_{jt}^x\right)$. This implies (17). Equation (15) implies that $i_{jt} = \widehat{i}_{jt}$. Let us denote by $\widehat{S}_{jt}$ the value of the undistorted terms of trade with the alternative policy mix. The fact that S_{jt}^m is the same with the two policy mixes implies

$$\widehat{S}_{jt} = \frac{1 + \widehat{\tau}_{jt}^m}{1 + \tau_{jt}^m} S_{jt}.$$

Then equation (14) and $i_{jt} = \widehat{i}_{jt}$ implies (18).

Proof of Proposition 3. We omit the country and time index and denote next-period variables with a prime to alleviate notations. The national policy-maker's problem can be written

$$\begin{aligned} V(B) \quad = \quad & \max_{C_H, C_F, L, B'} u\left(C\left(C_H, C_F\right)\right) + \beta V\left(B'\right) + \\ & \lambda \left[B + \left(C_F^W\right)^{1/\epsilon_x} \left(L - C_H\right)^{1 - 1/\epsilon_x} - C_F - B'/R\right] + \mu\left(\overline{L} - L\right). \end{aligned}$$

The first-order conditions are

$$
\begin{aligned}
u'(C)\frac{\partial C}{\partial C_F} &= \lambda, \\
u'(C)\frac{\partial C}{\partial C_H} &= \lambda\left(1-\frac{1}{\epsilon_x}\right)\left(C_F^W\right)^{1/\epsilon_x}(L-C_H)^{-1/\epsilon_x}, \\
\mu &= \lambda\left(1-\frac{1}{\epsilon_x}\right)\left(C_F^W\right)^{1/\epsilon_x}(L-C_H)^{-1/\epsilon_x}, \\
\lambda &= \beta R V'(B').
\end{aligned}
$$

The envelope condition is $V'(B) = \lambda$, which gives the national policymaker's Euler equation,

$$V'(B) = \beta R V'(B').$$

The first-order conditions for C_F and L imply that λ and μ are strictly positive, so that the constraint $L \leq \overline{L}$ is binding. Using the first-order condition for C_F to substitute out λ in the first-order condition for C_H gives

$$\frac{C_F}{C_H} = \frac{\alpha_F}{\alpha_H}\left(1-\frac{1}{\epsilon_x}\right)\left(\frac{C_F^W}{L-C_H}\right)^{1/\epsilon_x} = \frac{\alpha_F}{\alpha_H}\left(1-\frac{1}{\epsilon_x}\right)S^x.$$

Since $C_F/C_H = \alpha_F/\alpha_H S^m$ and $S^x/S^m = (1+\tau^m)(1+\tau^x)$ this implies (20).

Proof of Proposition 4. Symmetry. A Nash equilibrium is necessarily symmetric, i.e., all countries have the same allocation. To see this, observe first that as all countries have the same discount rates, they have the same value functions, $V_{jt}(\cdot) = V_{kt}(\cdot)$. The Euler condition for the national policymaker's problem, $V_t'(B_{jt}) = \beta_t R_t V_{t+1}'(B_{jt+1})$, together with the assumption that all countries have the same initial level of foreign assets $B_{j1} = 0$, implies that all countries have the same marginal utility of wealth, $V_t'(B_{jt}) = V_t'(B_{kt})$ for all j k and t. This implies $B_{jt} = B_{kt}$ and by (21) that all countries have zero foreign assets in equilibrium. Since B_{jt} is the only country-level state variable, all national policymakers face the same optimization problem in any given period and choose the same allocation in all periods.

Allocation. In a Nash equilibrium in which the national policymakers can use all the policy instruments and the ZLB constraint never binds, it follows from Proposition 3 that there is full employment in all countries and periods. As shown in the proof of Proposition 3 we have

$$\frac{C_{Ft}}{C_{Ht}} = \frac{\alpha_F}{\alpha_H}\left(1-\frac{1}{\epsilon_x}\right)S_{jt}^x.$$

It follows from (23) that $S^x_{jt} = 1$. Equation (11) and $C_{Ft} = C^W_{Ft}$ imply $C_{Ht} + C_{Ft} = \overline{L}$. This equation and $\frac{C_{Ft}}{C_Ht} = \frac{\alpha_F}{\alpha_H}(1 - 1/\epsilon_x)$ imply

$$C_H = \frac{\alpha_H}{1 - \alpha_F/\epsilon_x}, \quad C_F = \frac{\alpha_F (1 - 1/\epsilon_x)}{1 - \alpha_F/\epsilon_x}.$$

The allocation is unique. Using that C and $S^m = 1 - 1/\epsilon_x$ are constant, it follows from (15) that $1 + i_t = (1 + \pi^*)/\beta_t$. Hence condition (26) is necessary for the existence of a Nash equilibrium in which the ZLB constraint never binds. Conversely, condition (26) ensures that such an equilibrium exists.

The fact that C and C_F are constant and the same for all countries implies that all countries have the same constant marginal utility of wealth $V'(0) = u'(C)\frac{\partial C}{\partial C_F}$. Then the national policymaker's Euler equation $V'(0) = \beta_t R_t V'(0)$ implies

$$\beta_t R_t = 1.$$

Instruments. Using the fact that C and S^m are constant and that inflation is equal to the target, equations (15) and (47) and $\beta_t R_t = 1$ imply that i_{jt} and τ^b_{jt} satisfy (27). Any set of policies that satisfies (20) and (27) satisfies the conditions of a Nash equilibrium.

By Lerner symmetry it is clear that the same Nash allocation obtains if the national policymakers can use only one of the two trade taxes. To see this, consider for example the case where policymakers use the tariff on imports and not the tax on exports (the opposite case can be analyzed in the same way). Then by (20) the tariff rate is constant and given by $\tau^m = 1/(\epsilon_x - 1)$. It then follows from (27) that the tax on capital flows is equal to zero for all countries and in all periods. This shows that if countries use only one trade tax they do not need to use the tax on capital flows to implement the Nash allocation. The tax on capital flows is useful only if there is time variation in the tariff rate, which cannot be the case if countries use only one trade tax. Hence the Nash allocation remains the same if countries uses only one trade tax in addition to the nominal interest rate, as stated in the Proposition.

Proof of Proposition 5. Consider a symmetric allocation with unemployment ($L_{j1} < \overline{L}$). Then Table D2 implies

$$\frac{1 + i_1}{u'(C^W_1) C^W_1} \frac{\partial U_{j1}}{\partial i_{j1}} = -\alpha_H \epsilon_i - \gamma\eta < 0,$$

where the subscript W stands for the representative country.

Hence a national policymakers raises home welfare by lowering the interest rate as long as there is unemployment. The rest of the proof is stated in the text.

Proof of Proposition 6. The inflation target appears in equations (57) and (58), in both cases through the factor $(1 + i_{j1}) / (1 + \pi_j^*)$. Hence increasing π_j^* given i_{j1} is equivalent to reducing i_{j1} given π_j^*. As shown in the proof of Proposition 5, reducing i_{j1} raises home welfare if there is unemployment. Increasing π_j^* has the same effect. In a Nash equilibrium, all national policymakers raise their inflation targets π_j^* until there is full employment, i.e., all policymakers set an inflation target such that $\beta_1 \leq 1 + \pi_j^*$. The inflation target of country j is irrelevant for welfare and so indeterminate as long as it satisfies this condition.

Proof of Proposition 7. We first write the first-order condition for the national policymaker's problem in the general case (this expression will be useful for the proofs of the following propositions). We can write the Lagrangian for the policymaker of country j

$$\mathscr{L}_{j1} = u(C_{j1}) + \beta_1 V_2(R_1 X_{j1}) + u'(C_1^W)\left[\lambda\left(\overline{L} - L_{j1}\right) + \frac{C_1^W}{1 + i_1}\mu i_{j1}\right]$$

where λ and μ are the costate variables for the labor constraint and the ZLB constraint respectively. The costate variables have been scaled to simplify the first-order condition.

The first-order condition for policy instrument $n_{j1} = i_{j1}$, τ_{j1}^m, τ_{j1}^x, τ_{j1}^b is

$$u'(C_{j1})\frac{\partial C_{j1}}{\partial n_{j1}} + \beta_1 R_1 V_2'(0)\frac{\partial X_{j1}}{\partial n_{j1}} + u'(C_1^W)\left[-\lambda\frac{\partial L_{j1}}{\partial n_{j1}} + \frac{C_1^W}{1 + i_1}\mu \mathbb{1}_{n=i}\right] = 0,$$

where $\mathbb{1}_{n=i}$ is the indicator variable equal to 1 iff the instrument is the nominal interest rate. Assuming a symmetric allocation, we then use (56) to substitute out $V_2'(0)$; equation (47) with [] and $C_{F1} = \alpha_F (S_1^m)^{\alpha_H} C_1$ to substitute out $u'(C_2)$; and the definitions of the elasticities, to obtain

$$e(C, n) + \alpha_F \frac{1 + \tau_2^m}{1 + \tau_1^m}\frac{1}{1 + \tau_1^b} e(X, n) - \lambda e(L, n) + \mu \mathbb{1}_{n=i} = 0. \quad (68)$$

In Proposition 7. we consider the case where the instruments are the nominal interest rate i_{j1} and the tariff τ_{j1}^m, and $\lambda = 0$ because there is unemployment. Writing (68) for $n = \tau^m$, $\tau_1^b = 0$, $\mathbb{1}_{n=i} = 0$ and the elasticities from Table D1 gives equation (38).

Proof of Proposition 8. The fact that the equilibrium described in (i) exists if and only if $\beta_1 \leq 1+\pi^*$ is an implication of Proposition 4. As noted in that Proposition, when policymakers can use i and τ^m there is a Nash equilibrium with a non-binding ZLB constraint if and only if $\beta_t \leq 1+\pi^*$.

By Proposition 7, the equilibrium described in (ii) exists if and only if there is unemployment when the tariffs are set to (37) and (38) and the ZLB constraint is binding. Using (34) this is true iff

$$\beta_1 > \beta^*$$

where the discount rate threshold is defined by

$$\beta^* \equiv (1+\pi^*)\left(1+\alpha_H^2 \frac{1/\epsilon_i - 1}{1-\alpha_F/\epsilon_x}\right)^{1/\epsilon_i}\left(\frac{\alpha_H}{\epsilon_i}+\alpha_F\right)^{-(\alpha_H/\epsilon_i+\alpha_F)}.$$

This proves part (ii) of the proposition.

Since it is impossible to have unemployment with a non-binding ZLB constraint, the only other possible type of equilibrium must involve full employment and a binding ZLB constraint, as described in (iii). To find the conditions under which such an equilibrium exists we write the first-order condition (68) for $n=i$ and $n=\tau^m$

$$\lambda \frac{C_{H1}}{C_1}\left(\alpha_H\epsilon_i+\alpha_F+\epsilon_x\frac{C_{F1}}{C_{H1}}\right)+\mu = \alpha_H\epsilon_i+\alpha_F\left(\epsilon_x-\alpha_H\epsilon_i-\alpha_F\right)\frac{1+\tau_2^m}{1+\tau_1^m}, \quad (69)$$

$$\lambda\frac{C_{H1}}{C_1} = \frac{\epsilon_i}{1-\epsilon_i}\left[\frac{1+\tau_2^m}{1+\tau_1^m}\left(\frac{\alpha_H}{\epsilon_i}+\alpha_F\right)-1\right]. \quad (70)$$

As shown by the second equation, if $\epsilon_i < 1$ there is full employment ($\lambda > 0$) if and only if $\frac{1+\tau_2^m}{1+\tau_1^m} > \left(\frac{\alpha_H}{\epsilon_i}+\alpha_F\right)^{-1}$.

Using the second equation to substitute out λ from the first one and noting that $\frac{C_F}{C_H} = \frac{\alpha_F}{\alpha_H}S_1^m = \frac{\alpha_F}{\alpha_H}\frac{\epsilon_x}{\epsilon_x-1}\frac{S_1^m}{S_2^m}$ gives

$$\frac{\epsilon_i}{1-\epsilon_i}\left[\frac{1+\tau_2^m}{1+\tau_1^m}\left(\frac{\alpha_H}{\epsilon_i}+\alpha_F\right)-1\right]\left[\alpha_H\epsilon_i+\alpha_F+\frac{\alpha_F}{\alpha_H}(\epsilon_x-1)\frac{1+\tau_2^m}{1+\tau_1^m}\right]+\mu$$
$$= \alpha_H\epsilon_i+\alpha_F\left(\epsilon_x-\alpha_H\epsilon_i-\alpha_F\right)\frac{1+\tau_2^m}{1+\tau_1^m}. \quad (71)$$

Figure 7 shows the r.h.s. and l.h.s. of this equation as functions of $S_1^m/S_2^m = (1+\tau_2^m)/(1+\tau_1^m)$ for $\mu = 0$. (The figure was constructed with the parameter values in Table 1 but is representative of the case $\epsilon_i < 1$.) An equilibrium where the ZLB constraint is not binding ($\mu = 0$) corresponds to the intersection of the two curves (point B), which implies $S_1^m/S_2^m = 1$, i.e., $\tau_1^m = \tau_2^m$. There is full employment because $S_1^m/S_2^m > \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)^{-1}$ so that $\lambda > 0$. The associated nominal interest rate satisfies $1 + i_i = (1 + \pi^*)/\beta_1$, so that the ZLB constraint is not binding if and only if $\beta_1 \leq 1 + \pi^*$. This is the equilibrium described in part (i) of the Proposition.

The ZLB constraint binds ($\mu > 0$) if and only if the l.h.s. of (71) is below the r.h.s.. Figure 7 shows that this is the case iff $S_1^m/S_2^m < 1$, i.e., $\tau_1^m > \tau_2^m$. Hence an equilibrium with full employment and a binding ZLB constraint ($\lambda > 0$ and $\mu > 0$) exists if and only if $\left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)^{-1} < S_1^m/S_2^m < 1$. By equation (34), full employment and a binding ZLB constraint imply

$$\frac{\ell\left(S_1^m\right)}{\ell\left(S_2^m\right)}\left(\frac{S_1^m}{S_2^m}\right)^{\alpha_F \epsilon_i}\left(\frac{\beta_1}{1+\pi^*}\right)^{-\epsilon_i} = 1.$$

This condition determines S_1^m given β_1. It determines $S_1^m = S_2^m$ for $\beta_1 = 1+\pi^*$ and $S_1^m = S_2^m\left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)^{-1}$ for $\beta_1 = \beta^*$. Because of condition (39), $\ell\left(S_1^m\right)\left(S_1^m\right)^{\alpha_F \epsilon_i}$ strictly decreases with S_1^m when S_1^m varies between $S_2^m\left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)^{-1}$ and S_2^m. Hence, for any value of β_1 such that $\beta^* < \beta_1 < 1 + \pi^*$, there exists one unique S_1^m between $S_2^m\left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)^{-1}$ and S_2^m that satisfies the equilibrium conditions of an equilibrium with full employment and a binding ZLB constraint. This proves part (iii) of the proposition.

Proof of Proposition 10. As shown in Table D1, the policymaker of country j can increase net exports X_{j1} without changing consumption C_{j1}, and thus increase home welfare, by lowering τ_{j1}^x in a symmetric allocation with unemployment. Hence there cannot be unemployment in a symmetric Nash equilibrium.

Note that by (23) we have $S_t = 1/\left(1 + \tau_t^x\right)$. If $\beta_1 \leq 1 + \pi^*$, we know from Proposition 4 that there is an equilibrium in which the ZLB constraint is not binding in period 1 and $\tau_1^x = \tau_2^x = 1/\left(\epsilon_x - 1\right)$. To show that the equilibrium is unique, we write the first-order conditions (68) for $n = i, \tau^x$ with $\tau_1^m = \tau_2^m = \tau_1^b = 0$. The first-order condition for i_1 is (69) and the first-order condition for τ_1^x is

$$\lambda \frac{C_{F1}}{C_1} = \alpha_F\left(1 - \frac{1}{\epsilon_x}\right)\Bigg(\tag{72}$$

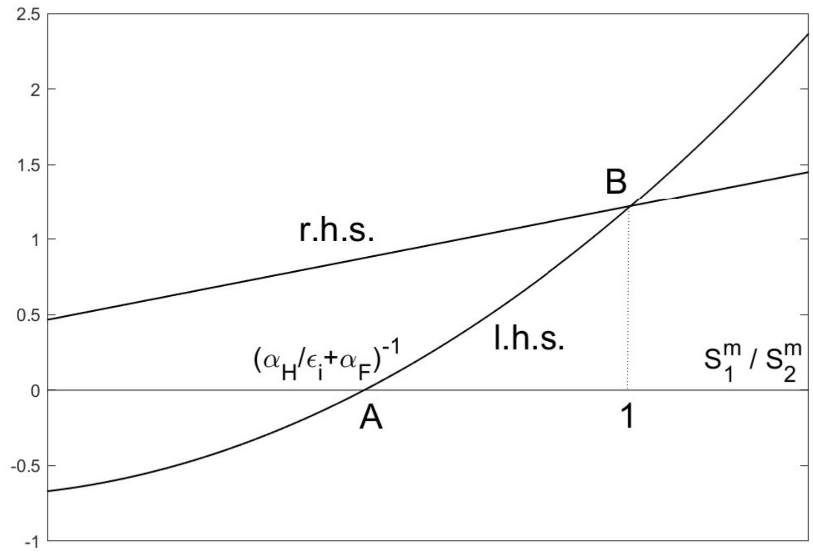

Figure 7: Equation (71)

Eliminating λ between the two equations and using $\frac{C_{H1}}{C_{F1}} = \frac{\alpha_H}{\alpha_f}\frac{1}{S_1} = \frac{\alpha_H}{\alpha_F}\left(1+\tau_1^x\right)$ we obtain,

$$\frac{\epsilon_x - 1}{\epsilon_x}\alpha_H\left(\alpha_H\epsilon_i + \alpha_F\right)\left(1+\tau_1^x\right) + \mu = \alpha_H\left(\alpha_H\epsilon_i + \alpha_F\right). \tag{73}$$

If the ZLB constraint is non-binding ($\mu = 0$), solving for the tax on exports gives $\tau_1^x = 1/\left(\epsilon_x - 1\right) = \tau_2^x$. In turn, equation (34) with $S_1^m = S_2^m$ and $L_1^W = \overline{L}$ shows that the ZLB constraint is non-binding in this equilibrium if and only if $\beta_1 \leq 1+\pi^*$. Thus, there exists an equilibrium with a non-binding ZLB constraint if and only if $\beta_1 \leq 1+\pi^*$ and in this equilibrium, the tax on exports is the same in the short run as in the long run ($\tau_1^x = \tau_2^x$).

Condition (73) implies that the ZLB constraint is binding if and only if $\tau_1^x < 1/\left(\epsilon_x - 1\right)$, i.e., the export tax is lower in period 1 than in the long run, so that $S_1 > S_2$. An equilibrium with a binding ZLB constraint exists if and only if there exists $S_1 > S_2 = 1 - 1/\epsilon_x$ such that $L_1 = \overline{L}$ when $i_1 = 0$. Using (34) and observing that $\ell\left(S_1\right)\left(S_1\right)^{\alpha_F\epsilon_i}$ is increasing with S_1 for $S_1 \geq S_2$ (this is an implication of condition $\epsilon_x \geq 1 + \alpha_H\left(\frac{1}{\epsilon_i} - 1\right)$), we can conclude that a Nash equilibrium with a binding ZLB constraint exists if and only if $\beta_1 > 1+\pi^*$. This achieves to prove the proposition.

Policy mix (i, τ^m, τ^x)**.** We know from Proposition 4 that if $\beta_1 \leq 1 + \pi^*$, there exists an equilibrium in which the ZLB constraint is not binding in period 1, $\tau_1^m = \tau_2^m$ and $\tau_1^x = \tau_2^x$ and $1+i_1 = (1+\pi^*)/\beta_1$. The uniqueness of this equilibrium can be shown like in the proofs of the two previous Propositions, by writing the first-order conditions for i_1, τ_1^m and τ_1^x, and showing that if the ZLB constraint is not binding one must have $\tau_1^m = \tau_2^m$ and $\tau_1^x = \tau_2^x$.

If $\beta_1 > 1 + \pi^*$ the ZLB constraint must be binding in period 1. If it were not, one would have $1 + i_1 = (1+\pi^*)/\beta_1$, as just stated, whence a contradiction.

The equilibrium in the case $\beta_1 > 1 + \pi^*$ can be derived as follows. Consider a Nash equilibrium in which the ZLB constraint is binding ($\mu > 0$). By (34) there is full employment in period 1 if and only if S_1^m satisfies

$$\ell\left(S_1^m\right)\left(S_1^{m-}\right)^{\alpha_F \epsilon_i} = \left(\frac{\beta_1}{1+\pi^*}\right)^{\epsilon_i} \ell\left(S_2^m\right)\left(S_2^m\right)^{\alpha_F \epsilon_i}. \tag{74}$$

Equation (74) gives the value of S_1^m. Then it follows from (70), (72) and $C_{F1}/C_{H1} = \alpha_F/\alpha_H S_1^m$ that

$$1 + \tau_1^m = \left(1 + \tau_2^m\right)\left[1 + \alpha_H\left(\frac{1}{\epsilon_i} - 1\right)\left(1 - \frac{S_2^m}{S_1^m}\right)\right] \tag{75}$$

This gives the value of τ_1^m, which is larger than τ_2^m. Then

$$1 + \tau_1^x = \frac{1}{\left(1 + \tau_1^m\right) S_1^m}, \tag{76}$$

gives the value of τ_1^x, which is smaller than τ_2^x. Equation (74) implies that S_1^m is increasing with β_1. Then equations (75) and (76) show that τ_1^m and τ_1^x are respectively increasing and decreasing with β_1.

Proof of Proposition 11. We assume that the economy is in a global liquidity trap before time T and that the policymakers can use tariffs. For any period $t < T$ the representative policymaker's value function can be written,

$$V_t\left(B_t\right) = \max_{\tau_t^m, i_t \geq 0} u\left(C_t\right) + \beta_t V_{t+1}\left(R_t\left(X_t + B_t\right)\right), \tag{77}$$

where C_t and X_t are functions of the policy instruments and the state B_t defined through the equilibrium conditions (11), (13), (14) and (15) assuming that next-period variables are policy functions of the state B_{t+1}. The maximization is done under constraint (4).

The first-order condition for the policymaker's problem (77) can be written (in a symmetric equilibrium)

$$u'(C_t)\, e(C,\tau^m) + \beta_t R_t V'_{t+1}(0) \frac{C_{Ft}}{C_t} e(X,\tau^m) = 0,$$

where the elasticities are given in Table D1. Using the expressions in that table to substitute out the elasticities and $C_{Ft}/C_t = \alpha_F (S_t^m)^{\alpha_H}$ and (79) one obtains

$$u'(C_t)\,(S_t^m)^{-\alpha_H} = \beta_t R_t V'_{t+1}(0) \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right) \tag{78}$$

The envelope condition does not apply to (77) since C_t and X_t are separately defined as functions of the state B_t by the equilibrium conditions. However if $1-\beta_t$ is first order, the partial derivatives $\partial C_t/\partial B_t$ and $\partial X_t/\partial B_t$ are second-order and can be neglected to a first order of approximation. Thus we have

$$V'_t(0) = \beta_t R_t V'_{t+1}(0).$$

Iterating on this equation gives

$$V'_t(0) = \left(\prod_{s=t}^{T-1} \beta_s R_s\right) V'_T(0), \tag{79}$$

so that condition (78) can be re-written

$$u'(C_t)\,(S_t^m)^{-\alpha_H} = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\left(\prod_{s=t}^{T-1} \beta_s R_s\right) V'_T(0). \tag{80}$$

Using the Euler equation (15) with $i_t = 0$ and $R_t = 1/(1+\pi_{t+1})$ from (14) with $\tau_t^b = i_t = 0$ and $S_t = S_{t+1} = 1$ (in a symmetric allocation without export taxes) one gets

$$\begin{aligned} u'(C_t)\,(S_t^m)^{\alpha_F} &= \beta_t R_t u'(C_{t+1})\left(S_{t+1}^m\right)^{\alpha_F}, \\ &= \left(\prod_{s=t}^{T-1} \beta_s R_s\right) u'(C_T)\,(S_T^m)^{\alpha_F}. \end{aligned} \tag{81}$$

Dividing (81) by (80) and using $V'_T(0) = u'(C_T)\left(1 + \frac{\alpha_H}{\epsilon_x}\frac{1}{S_T^m}\right)\ell(S_T^m)$ (from equation (56)) gives

$$\frac{1}{S_t^m} = \left(\frac{\alpha_H}{\epsilon_i} + \alpha_F\right)\frac{1}{S_T^m}.$$

This is the same as (38), obtained in the case $T = 2$. Hence the tariff rate is the same as in Proposition 7.

Point (ii) can be proven like for Propositions 10. The policymakers can increase the trade balance without distorting consumption by reducing τ^x as long as there is unemployment, implying that there must be full employment in the Nash equilibrium. Using conditions (45) and (46) this implies

$$\ell\left(S_t\right)\left(S_t\right)^{\alpha_F \epsilon_i} = \left(\frac{\prod_{s=t}^{T-1} \beta_s}{1+\pi^*}\right)^{\epsilon_i} \ell\left(1-\frac{1}{\epsilon_x}\right)\left(1-\frac{1}{\epsilon_x}\right)^{\alpha_F \epsilon_i},$$

which generalizes (74).

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