Macroprudential Regulation Versus Mopping Up After the Crash*

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Abstract

How should macroprudential policy be designed when policymakers also have access to liquidity provision tools to manage crises? We show in a tractable model of systemic banking risk that there are three factors at play: First, ex-post liquidity provision mitigates financial crises, and this reduces the need for macroprudential policy. In the extreme, if liquidity provision is untargeted and costless or if it completely forestalls crises by credible out-of-equilibrium lending-of-last-resort, there is no role left for macroprudential regulation. Second, however, macroprudential policy needs to consider the ex-ante incentive effects of targeted liquidity provision. Third, if shadow banking reduces the effectiveness of macroprudential instruments, it is optimal to commit to less generous liquidity provision as a second-best substitute for macroprudential policy. Moreover, a liquidity fund that is pre-financed with resources that are raised exclusively from the banking sector is not effective in countering aggregate systemic risk.

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1 Introduction

The 2008-09 global financial crisis has significantly changed our views on the appropriateness of policy interventions to respond to financial booms and busts. The dominant view before the crisis was that the best time to intervene was ex post, at the time of crisis, rather than ex ante, when fragilities build up in the financial system. This so-called "Greenspan doctrine" held that it was preferable to "mop up" via liquidity injections after a financial crisis had materialized, since ex-ante interventions tended to be too blunt, unpredictable in their effects or too costly.[1] By contrast, there is now wide agreement that policymakers should try to contain the buildup in risks ex ante through macroprudential interventions. Ex-post crisis interventions have been criticized for being counter-productive in various ways, in particular for creating moral hazard and inducing excessive risk-taking ex ante. This shift in the policy debate is reflected in the financial reforms that were implemented in response to the crisis. For example, the Dodd-Frank reform gives the US Federal Reserve new powers in designing prudential capital and liquidity requirements at the same time as it curtails the Fed's ability to support individual institutions in a crisis.[2] The pendulum has swung away from ex-post interventions towards ex-ante interventions, and some argue that it went too far (Bernanke, Geithner and Paulson, 2019).

This policy debate has been accompanied, on the theoretical side, by a new strand of literature that analyzes the desirability of ex-ante macroprudential interventions.[3] Another

[1] See Greenspan (2002, 2011). Some economists, especially at the Bank for International Settlements (BIS), were early defenders of the view that policymakers should intervene ex ante (Borio, 2003; Bordo and Jeanne, 2002).

[2] Before Dodd-Frank the Federal Reserve was allowed to lend to a wide range of entities "in unusual and exigent circumstances" by Section 13(3) of the Federal Reserve Act. This disposition was limited in numerous ways by Dodd-Frank, including the fact that Fed loans can no longer be targeted to individual firms. This would have made many of the Fed's interventions in the 2008-09 crisis impossible.

[3] See for example Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003), Lorenzoni (2008), Jeanne and Korinek (2010, 2019), Stein (2012), He and Kondor (2016), Dávila and Korinek (2018) and Korinek (2018) for papers that motivate macroprudential intervention on the basis of pecuniary externalities, or Farhi and Werning (2016, 2017) and Korinek and Simsek (2016) for a motivation on the basis of aggregate demand externalities. There is also a related quantitative literature – see Korinek and Mendoza (2014) for

line of literature has focused on ex-post interventions.[4] However there is little work that systematically studies how to design ex-ante macroprudential regulation when policymakers also have tools to respond to financial crises ex-post.[5] The objective of our paper is to fill this gap.

We provide a tractable model of ex-ante and ex-post crisis interventions that allows us to obtain powerful analytic results. Our model features a simple collateral constraint that depends on asset prices which, following the logic of fire-sale models, may lead to financial amplification and systemic risk ex post and to overborrowing ex ante (Shleifer and Vishny, 1992; Lorenzoni, 2008; Dávila and Korinek, 2018). Ex-ante policy restricts the leverage of banks through capital requirements or a macroprudential tax on bank lending. Ex-post policy provides liquidity to banks through various channels but comes at a social cost. We show that a key feature of ex-post liquidity provision is whether they are targeted to the institutions that need liquidity the most (for example, through the disco;unt window) or injected in a way that is not tailored to individual institutions' needs (for example, through open-market operations.[6] Targeted and untargeted liquidity provision are equivalent from an ex-post perspective, but have different implications for ex-ante incentives, and macroprudential policy.

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an overview.

[4]Acharya and Yorulmazer (2008), Diamond and Rajan (2011), Philippon and Schnabl (2012), and Shleifer and Vishny (2010) compare the efficiency of different types of ex-post policy measures. Bocola and Lorenzoni (2017) and Schmitte-Grohe and Uribe (2018) study the implications of policy in the presence of multiple equilibria.

[5]Some papers focus on specific aspects of the interaction. Acharya et al (2011) study the effects of different crisis resolution regimes on banks' incentives to hold liquidity. Benigno et al. (2012, 2016) show that ex-post interventions reduce the excessive borrowing that arises from pecuniary externalities. Caballero and Lorenzoni (2014) compare ex-ante and ex-post intervention in a setting in which persistent appreciations drain the net worth of the traded sector in an emerging economy, and Bornstein and Lorenzoni (2018) show that ex-post measures to manage aggregate demand do not necessarily generate ex-ante moral hazard. Dogra (2015) designs an optimal mechanism for macroprudential policy and debt bailouts in the presence of private information.

[6]A similar distinction is drawn, in social safety nets, between targeted benefits that are distributed to the neediest individuals and universal benefits that are distributed equally. Our distinction between targeted and untargeted interventions is also similar to Goodfriend and King's (1988) distinction between "lending-in-last-resort as an input in banking policy" and "lending-in-last-resort as an input in monetary policy."

We use the model to characterize the optimal balance between ex-ante macroprudential policy and ex-post liquidity provision. The interactions between ex-ante and ex-post policies go both ways. Liquidity provision policies have an impact on the optimal design of ex-ante macroprudential policy and conversely, ex-ante macroprudential policy affects the optimal design of ex-post policies.

Our first set of results is about how ex-post liquidity provision affects the need for ex-ante macroprudential policy. Our model qualifies the standard intuition that more ex-post liquidity provision calls for tighter macroprudential regulation: to the contrary, more generous liquidity provision, no matter if targeted or untargeted, calls for a relaxation of bank capital requirements and leverage ratios – because it mitigates systemic risk and therefore makes it efficient for bankers to take on greater leverage. However, the effects of more generous liquidity provision on the optimal macroprudential tax is ambiguous since liquidity provision increases banks' willingness to borrow at the same time as it reduces their vulnerability to crises for any given level of leverage. The optimal macroprudential tax is more likely to increase with targeted liquidity interventions since these create moral hazard and inefficient incentives for bankers to take on extra leverage.

In general, ex-post liquidity provision does not obviate the need for macroprudential policy because, being socially costly, it is not used to the point of completely alleviating systemic risk. All instruments in the policy mix should be used. There are nevertheless conditions under which the Greenspan doctrine is valid, i.e. the social planner should go for the corner solution of using only ex-post liquidity intervention. The Greenspan doctrine is valid if and only if ex-post liquidity interventions are untargeted and socially costless. A special case where macroprudential policy is not necessary is if crises are self-fulfilling and can be forestalled by out-of-equilibrium lending-in-last-resort interventions. This is true even if these interventions are targeted or socially costly provided that they are not implemented in equilibrium.

We also show that quantity regulations (capital requirements or leverage ratios) are preferable to macroprudential taxes when there is parameter uncertainty about the type of liquidity provision policies that will be used by the social planner in the event of a crisis. This is because the optimal macroprudential tax depends on the extent to which liquidity interventions are targeted or not, whereas the optimal quantity regulations do not.

Our second set of results is about how macroprudential policy affects the design of liquidity provision policies. An important theme in the banking literature is that there is a benefit from committing to restricted financial safety nets in order to mitigate moral hazard. We show that this intuition does not necessarily apply when macroprudential policy is added to the policy mix. If macroprudential regulation applies to all banks and can be set optimally, it resolves any time consistency problems that may arise from the use of ex-post liquidity provision – no matter whether it is targeted or not. Optimal ex-ante interventions ensure that the ex-ante borrowing incentives of private agents are corrected given the anticipated ex-post liquidity provision. As a result, there is no welfare gain from commitment, and it is optimal to exercise complete discretion in the use of ex-post interventions. Put differently, one benefit of macroprudential policy is to allow more discretion in the use of liquidity provision.

Things are more complicated when macroprudential policy is constrained. We study a variant of the model in which a fraction of the banks (termed shadow banks) are not subject to macroprudential regulation. As a result macroprudential regulation "leaks"—a macroprudential restriction shifts financial intermediation from the regulated to the unregulated sector. We show that in such an environment it is optimal to exclude the shadow banks from targeted liquidity provision. Furthermore, commitment comes back into play: it is optimal for the social planner to commit to restricted liquidity interventions in order to limit the leverage of shadow banks.

Our last set of results is about ways in which macroprudential policy can be used to generate liquidity. We analyze the common policy proposal of using the proceeds of macroprudential taxes to finance liquidity interventions in the event of a systemic crisis. We find this proposal undesirable – it is preferable to inject fresh resources coming from outside of the banking sector in the event of a crisis, even if raising these resources is distortionary. In both cases, the reason is a form of Ricardian equivalence: banks respond to the respective policy by increasing their leverage in good times. In addition, a pre-financed liquidity fund may inefficiently limit the size of liquidity interventions in the event of a severe crisis.

The remainder of this paper is structured as follows. In the following section, we introduce the baseline model, characterize the first best and introduce the financial constraint that lies at the heart of our analysis. Section 3 introduces the ex-ante and ex-post policy instruments at the disposal of the social planner. Section 4 analyzes the optimal policy mix, and section 5 concludes.

2 Model

This section presents the assumptions of the model without social planner. The social planner's policy interventions are introduced in the next section.

2.1 Assumptions

We consider an economy with three time periods $t = 0, 1, 2$, and one homogeneous good. In the main body of the paper, we assume that there are two classes of atomistic agents in the economy: bankers and depositors. Bankers invest in productive capital on behalf of depositors.[7] For simplicity we assume that there is a mass 1 of each type of agent. Periods 0 and 1 are the periods when bankers invest in capital assets, and period 2 is when the final payoffs are realized.

[7] Appendix B presents a "double-decker" model in which we introduce firms that borrow from banks. We show that our baseline model is isomorphic to the double-decker setup in which the financial constraints on firms are slack.

The period-t utility of the representative banker or depositor is given by,

$$U_t = E_t(c), \tag{1}$$

where c is the agent's level of consumption in the final period. There is a zero-return storage technology, implying that the riskless interest rate is equal to zero.

In the initial period, bankers are endowed with equity e and raise deposits d, which they employ in an increasing and concave production function $f(\cdot)$ to produce k units of assets that deliver a riskless payoff in the final period. Bank deposits have a maturity of one period. In period 1, bankers receive an exogenous stochastic income ρ, which is the only source of uncertainty in our model. Furthermore, bankers issue new deposits d' and sell Δk units of the asset at market price p. They use these funds to invest i in the creation of more assets with the same production function $f(\cdot)$ as in the initial period and to repay the deposits d. In the final period, bankers hold a quantity of asset $k - \Delta k + f(i)$ and receive one unit of good per unit of asset. After repaying the deposits d', they consume the remainder of their income. There is no default risk on bank deposits. Based on these assumptions, the bankers' budget constraints are

$$\begin{aligned} k &= f(e+d), && (2) \\ i + d &= \rho + d' + p\Delta k, && (3) \\ c^b + d' &= k - \Delta k + f(i), && (4) \end{aligned}$$

where c^b is the final consumption of the representative banker.

Depositors are endowed with y units of consumption good in periods $t = 0, 1$. They can lend these to the bankers or save them in the zero-return storage technology. We assume that depositors cannot make productive use of the asset, i.e., the asset must be held by bankers to yield a payoff in period 2 lest it loses all its value. As a result, bankers may trade assets at a fire-sale discount, giving rise to what the literature has called "systemic risk." This captures in a simple manner that financial intermediaries are more productive in operating financial assets – an assumption that underlies much of the literature on fire sales following Shleifer and Vishny (1992) and Kiyotaki and Moore (1997). Since the bankers are identical, it must be that $\Delta k = 0$ in the laissez-faire equilibrium, so that the representative banker holds $f(e + d) + f(i)$ units of asset in the final period.

Remark 1 (Deposit Contracts) In line with the literature we model bank deposits as one-period debt contracts. Our results are unchanged if bankers can issue two-period debt in period 0 as long as this gives rise to the same incentives to renege as one-period debt (see, for example, our earlier working paper version, Jeanne and Korinek, 2013). Furthermore, our baseline analysis considers the case of uncontingent debt for simplicity of exposition. This is not an important restriction since our findings on the interaction of ex-ante and ex-post policy instruments hold even for the case of perfect foresight. For completeness, we show in Appendix D.1 that our main results are unchanged if a full set of Arrow securities is available. The financial imperfection that matters for our analysis is introduced in the next subsection.

2.2 Financial Imperfections

The quantity of deposits issued by bankers in period 1 is constrained by a financial friction. Specifically, we assume that at most the first ϕ units of the bankers' capital assets are seizable if bankers renege on their deposits, giving rise to the financial constraint

$$d' \leq \phi p, \tag{5}$$

where p is the period-1 price of the asset. Constraints of this type have been used in the recent literature on systemic risk and can be microfounded by limited commitment as follows.

Assume that a banker can make a take-it-or-leave-it offer to reduce the value of his deposits at any time. If depositors reject this offer, they can seize ϕ units of the banker's assets which they can then sell at price p, the competitive price that other bankers are ready to pay for the asset. Depositors, thus, will accept the banker's offer as long as the offered repayment is at least ϕp, the amount that she would obtain by foreclosing on the capital.[8] Without loss of generality we assume that deposits are default-free, i.e., they are not renegotiated in equilibrium. At the end of period 1, the threat of renegotiation implies that the deposits outstanding must be lower or equal to the value of the seizable collateral.

We assume that deposits could be renegotiated right after they are issued in period 1. As a result the collateral constraint involves the current price of the asset, as in the literature on fire sales. Deposits could also be renegotiated at the time of repayment (period 2). However, if constraint (5) is satisfied in period 1, the banker will not renegotiate in period 2 since the price of the asset never decreases between period 1 and period 2. As we will see, the period-1 price satisfies $p \leq 1$ whereas the period-2 price of the asset is equal to 1. There could also be a renegotiation over deposits issued in period 0, d, but we assume that the resulting constraints are not binding to focus our analysis on the interesting case with excessive borrowing/leverage. (The formal condition for this is given in Appendix D.2, which also describes the case where the period 0 constraint may be binding.)

2.3 First-Best Allocation

This section characterizes the first-best allocations without financial imperfections as a benchmark for the ensuing analysis.

[8]One implication of our setup is that bankers do not relax the collateral constraint when they buy additional assets. As a result, the asset price will not include what Fostel and Geanakoplos (2008) term a "collateral premium." This setup simplifies our analysis without affecting our qualitative results.

An alternative assumption, used in some of the literature, is that depositors can seize a *fraction* ϕ of the banker's assets, implying $d^{j\prime} \leq \phi k^j p$. We show in Appendix C that this gives rise to very similar dynamics albeit at the cost of additional analytic complexity.

We define a first-best allocation as a set of allocations (k,d) and functions $(i(\rho), c_2^b(\rho), c_2^d(\rho))$, with the latter depending on the realization of the productivity shock ρ, that maximize aggregate surplus $U_0 = U_0^b + U_0^d$ and satisfy the resource constraints of the economy. It is easy to see that all first-best allocations satisfy $f'(k) = f'(i) = 1$. We denote by k^{FB} the capital level satisfying this condition. The total welfare of the representative banker and depositor in a first-best allocation is then given by[9]

$$U_0^{FB} = 2\left[y + f\left(k^{FB}\right) - k^{FB}\right] + e + E\left(\rho\right).$$

Laissez-faire leads to the first-best allocation if the credit constraint (5) is slack in all states of nature. We assume $y \geq k^{FB}$, so that depositors have enough funds to finance the first-best level of investment in both periods.

3 Policies

A systemic financial crisis is an equilibrium in which a low realization of the liquidity shock ρ leads to a binding financial constraint. This section introduces the policy instruments that a social planner can use to mitigate the welfare cost of a systemic financial crisis. The main distinction that we focus on in this paper is between ex-ante interventions and ex-post interventions. Broadly speaking, the purpose of ex-ante interventions is to mitigate over-borrowing in period 0 whereas ex-post interventions mitigate financial amplification if there is a crisis in period 1. We discuss in the following how these interventions can be interpreted in terms of macroprudential policy, monetary policy, fiscal policy or financial safety nets.

[9] Any allocation of the available resources between consumption of bankers and consumption of depositors is first-best since both value consumption equally.

3.1 Ex-ante Interventions

The first category of policy instruments target the decision variables of banks in period 0, before binding financial constraints materialize. In our simple framework, there is just a single decision margin for bankers in period 0, which is how much to borrow and invest. Policy can affect this decision variable using quantity-based or price-based intervention. Most real-word macroprudential policy interventions involve quantity restrictions, such as a leverage or capital adequacy ratio, or maximum loan-to-value ratios for bank lending. These policies can be captured in our model by assuming that a bank's deposits must not exceed a certain multiple of its equity,

$$d \leq \bar{d} \equiv \mu e,$$

where the multiplier μ is set by regulation. In our setting, this policy is equivalent to setting a cap $\bar{d}$ on deposits.

Alternatively, the planner could use a macroprudential tax on period-0 borrowing. In this case the social planner makes each banker j pay a tax τ for every unit of deposit issued in period 0 and leaves him the net proceeds $(1-\tau)d^j$, with the tax revenue rebated to all bankers. In our framework, a given amount of deposits d can be equivalently be implemented using ex-ante price and quantity interventions. One reason why we may be interested in optimal price interventions is that they reflect the wedge introduced in the optimality condition of bankers, which is a good indicator for the regulatory burden and for the incentive to circumvent regulation.

3.2 Ex-post Interventions

During a financial crisis, real-world policymakers have a range of policies at their disposal, from liquidity provision and monetary relaxation to debt relief and bailouts. For our analysis, the characteristic of these policies that matters is whether they are untargeted or targeted to specific bankers. In our baseline analysis the social planner provides liquidity to bankers through two channels. The first channel is untargeted liquidity provision, which takes the form of open-market operations in which the social planner purchases long-term asset against liquid funds. The second channel is targeted liquidity provision, which takes the form of crisis loans that are contingent on the recipient's size or leverage. The relevant difference between targeted and untargeted intervention is that the former depends on the leverage undertaken by recipient banks in period 0 whereas the latter does not. We show in Appendix A that other forms of interventions are equivalent in reduced form to these two policies: additional examples of untargeted interventions include uniform lump sum transfers or interest rate cuts (modeled as taxes or subsidies on savings or borrowing as in Farhi and Tirole, 2012); examples of targeted interventions include discount window lending, debt relief or bank recapitalization operations.

If there is a crisis in period 1, we assume that the social planner raises a quantity of liquid funds ℓ from depositors. The social planner uses the liquid funds to conduct open market operations or provide liquidity to individual banks. Specifically, the social planner injects κp units of untargeted liquidity into the market by buying a quantity κ of long-term assets at the prevailing market price p; and the social planner provides targeted liquidity σd_j to each banker j in the form of crisis loans to be repaid in period 2. The crisis loans are not subject to the collateral constraint (5). In a symmetric allocation, the total amount of liquidity injected by the social planner is

$$\ell = p\kappa + \sigma d.$$

In period 2, the planner repays depositors ℓ using the earnings from the asset purchases and bankers' repayments on the liquidity provision. Furthermore, the planner rebates any surplus earned from asset purchases back to bankers. However, since depositors and bankers are both unconstrained in period 2 and value consumption linearly, our measure of joint social welfare is independent of the distribution of surplus in period 2 —welfare would be unchanged if depositors were not fully repaid and the liquidity provision contained an element of transfers, as is frequently the case in the real world (see e.g. Lucas, 2019).

We assume furthermore that the planner's liquidity operation incurs a social cost $g(\ell)$ that is (without loss of generality) imposed on depositors, where $g(\cdot)$ is increasing and convex. This cost may capture a number of economic distortions and phenomena. First, the deadweight cost from liquidity interventions may come from the fact that the central bank is weakly inferior at extracting value from its asset holdings. Because of lower levels of flexibility, information, or information processing capabilities, the central bank may be inferior in monitoring, in making continuation decisions, and in enforcing repayment. The social cost $g(\ell)$ can then be interpreted as the loss in the payoff of assets that are held by the government. Second, if the liquidity intervention requires that funds are raised via taxation, the social cost $g(\ell)$ may result from the distortions generated by taxation. Third, if ex-post interventions takes the form of a monetary stimulus that sets the interest rate below the natural interest rate of the economy, as in Farhi and Tirole (2012), this may generate distortions arising from inefficient investment. Finally, if the intervention involves a transfer, the deadweight cost function $g(\ell)$ may capture the social planner's concerns about the distributive effects between depositors and bankers. In the baseline analysis we simply assume a general function $g(\cdot)$ that is increasing and convex—the reader can find the different microfoundations to endogenize the function $g(\cdot)$ in Appendix A.

3.3 Equilibrium

Assuming that macroprudential policy is a tax on borrowing, the budget constraints of banker j are

$$\begin{aligned} k^j &= f\left(e + (1-\tau)d^j + \tau d\right), && (6) \\ i^j + d^j &= \rho + \sigma d^j + d'^j + p\Delta k^j, && (7) \\ c^j + \sigma d^j + d'^j &= k^j - \Delta k^j + f\left(i^j\right) + \kappa\left(1-p\right), && (8) \end{aligned}$$

where τd in the first equation is the rebate from the tax on debt, and $\kappa(1-p)$ in the third equation is the rebate from the social planner's profit on open-market operations. Using these equations it is easy to see that in a symmetric equilibrium where $d^j = d$ and $\Delta k^j = \kappa$, the welfare of the representative banker is given by

$$U_t^b = E_t\left(c_t^b\right) = f\left(e+d\right) - d + E_t\left[\rho + f\left(i\right) - i\right], \tag{9}$$

for $t = 0, 1$, whereas the welfare of the depositors is simply equal to their endowments minus the cost of the liquidity interventions,

$$U_t^d = 2y - E_t g\left(\ell\right). \tag{10}$$

In our baseline analysis, we consider subgame perfect equilibria in which the social planner maximizes welfare in a time-consistent way. As we will see the optimal ex-post policies depend only on the liquid net worth of the representative banker m where

$$m = \rho - d.$$

The social planner's period-1 policies are contingent on the state m and consist of two functions $\sigma(m)$ and $\kappa(m)$. The private sector decisions depend in turn on total liquidity $\ell + m$. A competitive equilibrium consists of

(i) a set of real allocations $(k, i(\ell+m), c^b(\ell+m), c^d(\ell+m))$;

(ii) financial allocations d and $d'(\ell+m)$ and an asset price $p(\ell+m)$;

(iii) ex-ante policies τ (or $\bar{d}$) and ex-post policies $\sigma(m)$ and $\kappa(m)$;

such that in both periods $t = 0, 1$ bankers maximize their utility subject to their budget and financial constraints and the social planner maximizes welfare.

4 Optimal Policy Mix

The core question of this paper is to characterize the optimal mix of ex-ante versus ex-post policy interventions that would be chosen by a benevolent social planner to maximize welfare in the economy. We will first focus on the optimal policy problem of a discretionary planner, since excessive discretion in the use of ex-post policy interventions is frequently cited as a reason to engage in macroprudential policies. Then we will compare the optimal policy mix under discretion with the solution under commitment.

4.1 Ex post policy

We start with an analysis of the equilibrium in period 1 after all uncertainty has been resolved.

Period-1 Investment An individual banker j enters period 1 with a deposit level d^j from the previous period and obtains the endowment income ρ that depends on the shock realization, resulting in liquid net worth $m^j = \rho - d^j$. The banker's period-1 investment cannot exceed the level that can be afforded by his private liquid net worth, the liquidity he receives from the social planner and the open market, and his private borrowing capacity

$$i^j \leq m^j + \sigma d^j + p\Delta k^j + \phi p. \tag{11}$$

The banker determines his investment i^j and asset sale Δk^j so as to maximize his welfare,

$$U_1^j = k^j + m^j + \max_{i^j,\ \Delta k^j} \left\{ f\left(i^j\right) - i^j + \left(\kappa - \Delta k^j\right)(1-p) + \lambda^j \left[m^j + \sigma d^j + p\Delta k^j + \phi p - i^j \right] \right\} \tag{12}$$

where λ^j is the shadow cost of constraint (11).[10] The first-order conditions of this problem imply $pf'(i^j) = 1$. In equilibrium, bankers invests until this yields the same return as buying the asset in the open market.[11] It follows that all bankers make the same investment in equilibrium, $i^j = i$, $\forall j$, and that the asset price is an increasing function of aggregate investment i,

$$p(i) \equiv \frac{1}{f'(i)}\ . \tag{13}$$

The economy can be in two regimes. In the unconstrained regime, the collateral constraint (11) is lax for all bankers and investment is at its first-best level, $i = k^{FB}$. In the constrained regime, the collateral constraint (11) is binding for all bankers. Aggregating the binding constraint over bankers gives $i = m + \sigma d + p\kappa + \phi p$ where $m = \int m^j dj$ and $d = \int d^j dj$ are respectively aggregate liquidity and aggregate debt, and $\kappa = \int \Delta k^j dj$ is the social planner's open market purchase of assets. The following expression for investment subsumes the two regimes

$$i = \min\left[k^{FB}, \ell + m + \phi p\left(i\right)\right], \tag{14}$$

[10]This expression is obtained by using the budget constraints (7) and (8) to substitute out c^j and d'^j in $U_1^j = E_1 c^j$. We assume that the constraint $\Delta k^j \leq k^j$ is not binding, which is true in a symmetric equilibrium as long as $\kappa < k$.

[11]As a result, the equilibrium price of assets declines when bankers are constrained and invest less, even if they do not sell assets to other agents in the economy. An alternative mechanism to obtain price declines when agents are constrained would be to require them to sell assets to other agents who have lower productivity and who thus earn a lower marginal product on assets, leading to fire-sale prices (see e.g. Lorenzoni, 2008).

where $\ell = \sigma d + \kappa p$ is the aggregate liquidity injection from the social planner.

Equation (14) implicitly defines investment as an increasing function of total (private plus public) liquidity $\ell + m$. If total liquidity is above the threshold $\ell + m \geq \hat{m} \equiv k^{FB} - \phi$, then investment is at its first-best level. For $\ell + m < \hat{m}$, equilibrium investment is constrained and the asset price declines, leading to financial amplification.

One issue with equation (14) is that in general, it could determine more than one level of i, leading to multiplicity of equilibrium. In our baseline model, we rule this out by making the following assumption:

$$\phi p'(i) < 1, \quad \forall i < k^{FB}.$$

This condition ensures that the slope of the right-hand side of (14) is always less than the slope of the left-hand side (which is one), guaranteeing at most a single intersection and thus a unique equilibrium. Period-1 investment is then a well-defined function of total liquidity denoted by $i(\ell + m)$. The case when the condition is violated, and multiple equilibria may arise, will be analyzed in section 4.5.

The financial amplification mechanism is well-known from the literature. However, what is important to emphasize is that liquidity provision to constrained bankers leads to a virtuous circle: Suppose that net liquidity $\ell + m < \hat{m}$ so the financial constraint is binding, and assume that the policymaker provides a unit of extra liquidity. The impact on investment can be obtained from implicitly differentiating (14),

$$i'(\ell + m) = \frac{1}{1 - \phi p'(i)} > 1. \tag{15}$$

Intuitively, the amplification arises because bankers push up the price of collateral when they have more liquidity, which relaxes the financial constraint and allows them to obtain further liquidity from borrowing. The value of $i'(\ell + m)$ can be viewed as the sum of the geometric series $1 + \phi p'(i) + [\phi p'(i)]^2 + ...$ that captures the initial liquidity injection plus round after round of relaxation of the financial constraint.

Period-1 Problem of Policymaker For given private liquid net worth m, the social planner chooses liquidity policies σ and κ to maximize aggregate welfare in the economy. From the perspective of period 1, both measures enter the expressions for welfare of the two agents through the sum $\ell = \sigma d + \kappa p$, and it is sufficient to determine the planner's optimal choice of ℓ. The planner maximizes total welfare $U_1 = U_1^b + U_1^d$ where U_1^b and U_1^d are respectively given by (9) and (10). Therefore, the period-1 optimization problem of the planner (in which we drop constant or predetermined terms) can be expressed as

$$W(m) = \max_{\ell} f(i(\ell + m)) - i(\ell + m) - g(\ell). \tag{16}$$

When the financial constraint is binding $(m < \hat{m})$, the planner's objective function strictly increases with ℓ for $\ell = 0$ and strictly decreases with ℓ for $\ell \geq \hat{m} - m$. Thus we know that it is maximized for an interior solution $0 < \ell(m) < \hat{m} - m$ which satisfies the first-order condition,

$$[f'(i) - 1]\, i'(\ell + m) = g'(\ell). \tag{17}$$

Intuitively, the planner equates the marginal benefit of liquidity for bankers to the social marginal cost of liquidity. The marginal benefit of liquidity is to increase their period-1 investment by $i'(\ell + m) = 1/(1 - \phi p'(i))$, which earns the net marginal product of investment, $f'(i) - 1$. The optimality condition defines an optimal amount of liquidity $\ell(m)$ that is positive for $m < \hat{m}$ and is zero for $m \geq \hat{m}$.

We summarize our results on the optimal ex-post intervention in the following proposition.

Proposition 1 (Ex-Post Interventions) *Assume that ex-post interventions are distor-*

tionary ($g(\cdot) > 0$). Then:

(i) The planner provides liquidity $\ell > 0$ to bankers if and only if their liquid net worth m is strictly below the threshold $\hat{m}$ at which the credit constraint becomes binding.

(ii) The planner mitigates the credit constraint only partially.

(iii) It does not matter for period-1 allocations and period-1 welfare whether liquidity provision is targeted or not.

Proof. For point (i), the result follows immediately from equation (17): the equilibrium ℓ is strictly positive if and only if the l.h.s. is strictly larger than the r.h.s. for $\ell = 0$, that is, if and only if m is strictly lower than $\hat{m}$. To prove point (ii), observe that if the liquidity provision were to completely relax the credit constraint when $m < \hat{m}$, then the l.h.s of equation (17) would be equal to zero whereas the r.h.s. would be strictly positive, a contradiction. To prove point (iii), observe that the two measures σ and κ enter condition (17) only via ℓ. Ceteris paribus, any combination of σ, κ that satisfies $\ell = \sigma d + \kappa p$ therefore implements the same allocation from the perspective of period 1. ■

4.2 Ex ante policy

We start with the optimal period-0 policy problem when the planner's instrument is a macroprudential tax τ on borrowing; then we focus on how to implement the same allocation using a cap on deposits $\bar{d}$.

Macroprudential tax In period 0 each individual banker j chooses d^j to maximize his expected utility $U_0^j = E_0 U_1^j$. Using (12) the banker solves

$$U_0^j = \max_{d^j} f(e + (1 - \tau)\, d^j + \tau d) + E\left[\rho - d^j + V\left(m^j\right)\ \right], \tag{18}$$

where $V(\cdot)$ is the surplus from investing in period 1

$$V\left(m^j\right) = \max_{i^j,\ \Delta k^j} \left\{ f\left(i^j\right) - i^j + \left(\kappa - \Delta k^j\right)(1-p) + \lambda^j \left[\sigma\rho + (1-\sigma)\, m^j + p\Delta k^j + \phi p - i^j\right] \right\},$$

Using the envelope condition we have

$$V'\left(m^j\right) = (1-\sigma)\,\lambda(i),$$

where $\lambda(i) \equiv 1/p(i) - 1$ is the excess return between periods 1 and 2. Reducing debt by one unit raises total liquidity by $1-\sigma$ units because of the banker's reduced access to targeted liquidity. In a symmetric equilibrium with $d^j = d$ the first-order condition from (18) is

$$(1-\tau)\, f'(e+d) = 1 + E\left[(1-\sigma)\,\lambda(i)\right]. \tag{19}$$

By contrast, the social planner sets d to maximize social welfare in period 0,

$$\max_d f(e+d) + E\left[\rho - d + W(\rho - d)\ \right],$$

where $W(\cdot)$ is the surplus generated by period-1 investment, as defined in (16). The difference between $W(\cdot)$ and the bankers' utility $V(\cdot)$ is that the social planner takes into account the general equilibrium impact of aggregate borrowing on the asset price and by extension on investment. The social planner's optimality condition

$$\begin{aligned} f'(e+d) &= 1 + E\left[W'(m)\right] \\ &= 1 + E\left[\frac{\lambda(i)}{1-\phi p'(i)}\right] = 1 + E\left[g'(\ell)\right], \end{aligned} \tag{20}$$

equates the marginal benefit of investment in period 0 to the marginal social benefit of funds in period 1, which consists of one plus the shadow cost of the financial constraint $\lambda(i)$ times the amplification effects, captured by the derivative $i' = 1/(1 - \phi p'(i))$. The last equality is obtained by substituting equation (17) and shows that in equilibrium, the marginal social benefit and cost of investment must also be equal to the marginal social cost of liquidity provision to the banking sector.

Comparing the first-order conditions (19) and (20) shows that there is overborrowing under laissez-faire, in the sense that individual bankers perceive a smaller cost than the true social cost of borrowing. Specifically, in the absence of macroprudential intervention, equation (19) shows that bankers overborrow because they may benefit from targeted liquidity provision and because they do not internalize the social costs of financial amplification,

$$1 + E[(1-\sigma)\lambda(i)] \leq 1 + E[\lambda(i)] \leq 1 + E\left[\frac{\lambda(i)}{1 - \phi p'(i)}\right]. \tag{21}$$

Combining the private Euler equation (19) and the planner's Euler equation (20), we obtain the optimal macroprudential tax rate,

$$\tau = \frac{E\{[\sigma + (i' - 1)] \cdot \lambda(i)\}}{f'(e+d)}. \tag{22}$$

The two additive terms in the numerator reflect the two causes of overborrowing in this model and serve as an empirical guide for the optimal level of the macroprudential tax. The first term reflects the overborrowing induced by the expectation of targeted liquidity provision σ. Empirically, σ captures the fraction of short-term debt that is rolled over with targeted crisis lending.

The second additive term reflects the overborrowing due to the private agents' failure to internalize their contribution to financial amplification. Specifically, $i'(\ell + m) - 1$ captures the extent of amplification, i.e., how much an additional dollar provided to bankers stimulates aggregate investment, net of that additional dollar. Empirically, this term can be estimated as a credit multiplier, i.e., the credit extension that an additional dollar of liquidity injected into the banking system generates. Using (15), we can alternatively express the factor

$$i'(\ell+m)-1=\phi p'(i)\cdot i'(\ell+m). \tag{23}$$

This formulation reflects that the uninternalized benefit of an additional dollar of liquidity is to increase the asset price and thus relax the constraint by $\phi p'(i)$.

Both terms in the numerator of (22) are multiplied by $\lambda(i)$, which captures the marginal benefit (shadow price) of relaxing the financial constraint. This term can be quantified by observing the spread that constrained bankers are willing to pay to obtain credit.

Cap on deposits The planner can equivalently implement the optimal allocation by imposing a cap $\overline{d}$ on deposits that prevents bankers from issuing more than the optimal level of deposits. The first-order condition for the optimal cap on deposits is equation (20). Observe that the equation only depends on ℓ, not on the components (σ,κ), so that the cap is independent of whether liquidity is provided in a targeted or untargeted way ex post. It results from equation (19) that the debt cap $\overline{d}$ is increased by ex-post liquidity interventions.

We summarize our findings in the following proposition:

Proposition 2 (Ex-Ante Interventions) *The planner implements the optimal policy mix by following the optimal ex-post policy described in Proposition 1 and imposing*

(i) either a quantity regulation $d^j\le\bar{d}=\mu e$ as defined by equation (20), which is independent of the composition (σ,κ) of the amount of ex-post liquidity that is provided and more relaxed than in the absence of ex-post interventions,

(ii) or a macroprudential tax on deposits given by (22). The optimal macroprudential tax is higher the more of the ex-post intervention is provided in the form of targeted liquidity provision σ rather than open market operations κ.

Proof. See discussion above. ■

This proposition points to a practical benefit of quantity regulation that arises if private agents face uncertainty about whether liquidity will be provided in targeted form σ or in untargeted form κ. Weitzman (1974) observed that whether price or quantity regulation is more desirable depends on which one is more robust to changes in model parameters, aggregate shocks, and heterogeneity. In our specific application, uncertainty about how liquidity will be provided introduces parameter uncertainty into the economy. The optimal quantity regulation (whether a capital adequacy ratio or a cap on deposits) does not depend on whether liquidity is provided in the form of σ or κ, but the optimal price regulation does.[12] Quantity regulation is thus superior to macroprudential taxes in the face of this particular form of parameter uncertainty.[13]

One of the motivations of this paper was to evaluate the conditions under which the "Greenspan doctrine" (according to which policymakers should intervene only ex post and not ex ante) holds. The Greenspan doctrine is not true in general in our model, but it is interesting to delineate the assumptions that are necessary and sufficient to make it true.

Proposition 3 (Greenspan Doctrine) *Macroprudential regulation is superfluous if and only if both of the following two conditions are satisfied:*

(i) the ex-post intervention does not involve targeted liquidity provision (so $E\left[\sigma\lambda\left(i\right)\right]=0$) and

(ii) either there is no financial amplification or the ex-post intervention has no distortionary cost (so $E[(i'-1)\lambda]=E\left[\phi p'\left(i\right)g'\left(\ell\right)\right]=0$).

Proof. Using equation (22) it is easy to see that $\tau=0$ if both (i) and (ii) are true. Conversely

[12] To provide a formal description, assume that there is a parameter $x\in[0,1]$ that determines what fraction of the optimal amount of liquidity ℓ is spent on σ versus κ so that $\sigma=x\ell/d$ and $\kappa=(1-x)\,\ell/p$. In the language of Weitzman, any uncertainty about the parameter x makes it desirable to use quantity regulation instead of price-based regulation.

[13] Other forms of parameter uncertainty will naturally lead to different degrees of desirability for price versus quantity regulations. For example, Perotti and Suarez (2011) show that when banks differ in their investment opportunities, Pigouvian taxes are superior to quantity interventions since they encourage banks with greater opportunities to invest more. This result can also be replicated in our setup. See Sublet (2018) for additional examples on price versus quantity regulations when banks are heterogeneous.

if $\tau = 0$ it must be that the two terms in the numerator are equal to zero, which requires (i) and (ii).[14] ■

A necessary condition for the Greenspan doctrine to hold and for macroprudential regulation to be superfluous, captured by part (i) of the proposition, is that liquidity is provided exclusively through open market operations—otherwise the expectation of targeted liquidity provision is sufficient to generate overborrowing which must be offset by ex-ante macroprudential regulation.

Conditional on this, part (ii) of the proposition states that there are two different scenarios under which the Greenspan doctrine holds. One scenario is that liquidity provision is costless, $g'(\ell) = 0$, allowing the planner to rely entirely on ex-post interventions to relax any binding constraints. As a result there is no systemic risk–and therefore no need to impose costly macroprudential regulation. This argument has been developed in greater detail in Benigno et al. (2012). The second scenario is an economy in which government revenue may be costly but there is no financial amplification so $p'(i) = 0$. The planner then finds it optimal to distribute resources to constrained bankers in period 1 until their marginal valuation of wealth equals the resource cost plus the deadweight cost of transferring resources. However, this transfer is constrained efficient: since there is no financial amplification, bankers correctly internalize the social cost of borrowing, and there is no reason for macroprudential intervention in period 0.

Let us summarize our findings so far: except in the knife-edge case of Proposition 3, the social planner uses both ex-ante and ex-post interventions because neither type of intervention fully alleviates the financial friction. Ex-ante intervention reduces the risk and severity of financial crises, but crises still occur. When they do, it is optimal for the social planner to resort to ex-post interventions. This result is consistent with the theory of second-best taxation. Both macroprudential regulation and the liquidity provision introduce a second-

[14] This also encompasses the trivial case that the probability for the constraint to bind is zero.

order distortion into the economy but achieve a first-order benefit from mitigating binding constraints through two alternative channels.

Dependence on cost of liquidity provision An important question is how ex-ante and ex-post interventions respond to the cost of ex-post interventions as captured by the cost function $g(\ell)$. In particular, how is the need for ex-ante macroprudential policy affected if we assume that the social planner is allowed to use a wider range of ex-post liquidity-provision instruments, thus reducing the distortionary cost of her interventions?

To answer this question, let us introduce a scale factor $\gamma > 0$ into the cost function so that providing ℓ units of liquidity imposes a social cost of $\gamma g(\ell)$. Our baseline case is captured by $\gamma = 1$. The optimal level of liquidity is then given by a variant of equation (20),

$$[f'(i(\ell + m)) - 1] \cdot i'(\ell + m) = \gamma g'(\ell). \tag{24}$$

Applying the implicit function theorem, we can see that for a given level of m, the response of liquidity provision to the scale factor γ is

$$\frac{d\ell}{d\gamma} = \frac{g'(\ell)}{f''(i)(i')^2 - \gamma g'' + [f'(i) - 1] i''}.$$

The numerator of this expression is always non-negative. The denominator reflects the planner's second-order condition for liquidity provision and needs to be negative for an interior optimum. The first two term are always negative; a sufficient condition for the third term to be negative is that $sign\{i''\} = sign\{p''\} < 0$, which is the case for conventional specifications of $f(\cdot)$. As a result, for any interior optimum, making liquidity provision more costly strictly reduces the quantity of liquidity that the planner provides.

The impact of more costly liquidity provision on ex-ante policy depends on whether it is specified in terms of a capital adequacy requirement or a macroprudential tax. For given m, lower liquidity provision ℓ implies that the marginal benefit of additional net worth $W'(m)$ rises. Returning to the first equality in equation (20) shows that it is then desirable to reduce deposits d and preserve more net worth, which the planner can implement by tightening the deposit cap $\bar{d}$ or the capital adequacy requirement μ. The effects on the optimal macroprudential tax, on the other hand, are in general ambiguous. With reduced liquidity provision, increased amplification effects call for higher macroprudential taxation, but this is counteracted by the fact that bankers also find it privately optimal to invest less.

Our results are summarized in the following proposition.

Proposition 4 *Assume that the asset price is a weakly concave function of investment, $p''(i) \leq 0$. Then more costly liquidity provision (higher γ) reduces the equilibrium amount of liquidity provided ex post, and calls for stricter capital adequacy requirement μ or deposit cap $\bar{d}$ ex ante. The impact on the optimal macroprudential tax τ is ambiguous in general.*

Proof. See discussion above. ■

The results of the Proposition run counter to the popular intuition that greater ease of liquidity provision ex post increases the need for macroprudential policy to offset moral hazard ex ante. What this intuition misses is that more borrowing in response to greater availability of ex-post liquidity is constrained efficient – because the liquidity reduces the social cost of systemic crises ex post.[15] Conversely, the more difficult it is to provide liquidity, the less the planner mitigates the financial constraints and the more macroprudential regulation is indicated. Observe, however, that the popular intuition holds if (i) the provision of liquidity is targeted; and (ii) the instrument of macroprudential regulation is a Pigouvian tax on bank leverage.

[15]In a similar vein, Jeanne and Zettelmeyer (2005) emphasize that an increase in borrowing in response to better financial safety nets does not in general reflect moral hazard. Stavrakeva (2015) finds that better financial safety nets – as enabled by greater fiscal capacity – reduce the need for macroprudential regulation.

4.3 Commitment vs. discretion

One question that arises when studying the optimal policy mix is that of commitment—whether or not the social planner should commit in period 0 to her future policy interventions. An important theme in the literature on financial crises is that policymakers tend to be excessively interventionist ex post because they ignore the implications of their policies for ex-ante private risk taking. A related theme is that it is important to set ex-ante limits and constraints on the use of ex post interventions.

In our initial anaysis, we stacked the deck against ex-post interventions by assuming that the social planner cannot commit. We now compare the optimal policy mix under discretion to the one under commitment.

Proposition 5 (Commitment Vs. Discretion) *The optimal allocation obtained under discretion coincides with the optimal allocation under commitment.*

Proof. The behavior of private agents is described by their period-0 Euler equation (19) and the investment rule (14). Given this and omitting constant terms, a planner under commitment in period 0 chooses a deposit level d and state-contingent liquidity provision ℓ to solve

$$\max_{d,\ell} f(e+d) - d + E\left[f\left(i\left(\rho - d + \ell\right)\right) - i\left(\rho - d + \ell\right) - g\left(\ell\right)\ \right].$$

The optimality conditions are identical to equations (17) and (20) of the problem under discretion. As a result, the planner chooses the same allocation under commitment as under discretion. ■

It turns out that commitment does not allow the planner to improve on the allocation obtained under discretion. By implication, the optimal policy mix of Proposition 2 implements the constrained efficient allocation of the economy. Intuitively, the benefit of committing to a lower level of bailouts in models of financial constraints is that it induces bankers to borrow less. In our framework, macroprudential policy already reduces borrowing directly without ancillary distortions. This enables the planner to provide the socially efficient level of liquidity when necessary ex post. In other words, macroprudential policy enables the planner not to worry about "moral hazard" from ex-post policy interventions such as liquidity provision. In particular, Proposition 5 holds even if the period-1 intervention occurs in the form of targeted liquidity provision $\sigma > 0$ – in that case, the optimal macroprudential tax (22) rises, but the real allocation in the economy is unchanged and remains constrained optimal. As a result, the planner is indifferent about which ex-post policy instrument is used.[16]

In the next section, we show that it is crucial for the result of Proposition 5 that macroprudential policy is at its optimal level. We do this by considering the case in which macroprudential policy is suboptimal because it does not apply to part of the financial sector.

4.4 Shadow banking

In practice, macroprudential policy may not be able to implement the optimal allocation described above. The reasons include that policymakers have only recently started to explicitly consider macroprudential motives in setting financial regulation, and that many financial regulators even lack a macroprudential mandate. Furthermore, financial regulation in general gives rise to circumvention by the private sector. We now introduce a shadow banking sector in the form of a subset of bankers that are not regulated or can circumvent regulation. There is no market segmentation between the two sectors in the sense that all bankers, whether regulated or not, exchange the asset in the same market in period 1. All bankers are subject to the same financial constraint (5) in period 1. We denote by n^r the number of bankers

[16]This result reflects a more general insight about time consistency in optimal policy problems: time consistency problems result from a lack of policy instruments and can be solved if a planner has sufficient (unrestricted) instruments available. Time inconsistency arises when the expectation of a planner's optimal actions affects the behavior of private agents in earlier periods in an undesirable way. In our setup, if the planner can control borrowing in period 0 directly via a macroprudential policy instrument τ, then there is no more reason to deviate from the ex-post optimal level of debt, and the time consistency problem disappears.

that are subject to regulation and by n^u the number of unregulated bankers ($n^r + n^u = 1$).

Irrespective of their type, bankers face the same choice between investing i and buying or selling the asset at price p in period 1. Hence, provided that the constraint $\Delta k^j \leq k^j$ is not binding, all the bankers invest until $pf'(i) = 1$, implying that bankers invest the same i irrespective of their type. This implies that in equilibrium the bankers are either all constrained ($i < k^{FB}$) or all unconstrained ($i = k^{FB}$). Differences in period-0 borrowing and investment across bankers manifest themselves only in whether bankers buy or sell assets in period 1.

Equation (14) applies, with $\ell = n^r \ell^r + n^u \ell^u$ and $m = n^r m^r + n^u m^u$ respectively denoting the aggregate liquidity provision and aggregate liquid net worth of bankers.[17] Thus the period-1 equilibrium is characterized by the same two equations (13) and (14) as in the absence of shadow banking. In particular, ceteris paribus, the period-1 equilibrium does not depend on how the liquidity ℓ is distributed between the regulated and shadow banking sectors. Shifting liquidity provision between the two sectors implies that the sector that obtains more liquidity buys more of the asset, but does not change the asset price or the aggregate level of investment.

The allocation of a given amount of liquidity between the two sectors affects the welfare of individual bankers but not aggregate welfare because of the linearity in bankers' utility function. Therefore, the social planner is indifferent ex post about which type of banker receives liquidity. As we will see, however, the social planner may want to target liquidity provision towards the regulated banking sector because of ex-ante incentives that this gives to shadow bankers.

Only the regulated sector is subject to the macroprudential tax τ ex ante, and shadow banks may not receive the same level of targeted liquidity as regular banks. As a result the first-order conditions for period-0 investment are not the same in the two sectors, and are

[17] Expression (14) holds whether or not bankers are identical in period 1.

given by the analog of equation (19) for each sector

$$(1-\tau) f'(e+d^r) = 1 + E[(1-\sigma^r)\lambda(i)], \tag{25}$$

$$f'(e+d^u) = 1 + E[(1-\sigma^u)\lambda(i)], \tag{26}$$

where like before $\lambda(i) = f'(i) - 1$. Observe that the social planner can mitigate overborrowing in the shadow banking sector by providing no targeted liquidity to that sector, $\sigma^u = 0$. As noted above this policy is time consistent since how liquidity is distributed is irrelevant for welfare ex post. Even with $\sigma^u = 0$, however, there may be excessive borrowing in the shadow banking sector because shadow banks do not internalize their contribution to systemic risk.

The lack of macroprudential policy in the shadow banking sector creates a role for commitment that is absent when macroprudential policy is optimal as in Proposition 5. We study the difference between commitment and discretion by introducing the following notations. Let us denote by $\ell_c(m)$ the liquidity provided under commitment, when the social planner can set her period-1 interventions in period 0, and by $\ell_d(m)$ the liquidity provided under discretion, the case already analyzed in Proposition 1.

We then have the following result:

Proposition 6 *Consider an economy in which a fraction $n^u > 0$ of the bankers (the shadow banking sector) are not subject to macroprudential regulation. Then the social planner does not provide targeted liquidity to shadow banks ($\sigma^u = 0$), which is time consistent. Furthermore, the social planner provides less liquidity under commitment than under discretion, i.e., $\forall m$, $\ell_c(m) < \ell_d(m)$.*

Proof. The planner chooses deposit levels d^r and d^u and the state-contingent liquidity policies ℓ_c and ℓ_c^r, ℓ_c^u to maximize the Lagrangian

$$n^r f(e+d^r) + n^u f(e+d^u) + E\left[m + f(i(\ell_c+m)) - i(\ell_c+m) - g(\ell_c) + \zeta^r\sigma_c^r + \zeta^u\sigma_c^u\right]$$
$$-\chi\left[f'(e+d^u) - E(1-\sigma_c^u)\lambda(i(\ell_c+m)) - 1\right],$$

where $m = \rho - n^r d^r - n^u d^u$, ζ^θ is the shadow price on the non-negativity constraint on σ_c^θ and χ is the shadow cost on the implementability constraint (26). The planner will subsequently set κ_c so that $\ell_c = n^r\sigma_c^r d^r + n^u\sigma_c^u d^u + \kappa_c p$ is satisfied, and the tax rate τ such that (25) is satisfied. Using $\lambda'(i) = f'(i)$, the optimality conditions are

$$\begin{aligned}
FOC(d^r) &: \quad f'(e+d^r) = 1 + E\{[f'(i)-1]i'\} + \chi E f''(i)(1-\sigma_c^u)i', \\
FOC(d^u) &: \quad f'(e+d^u) = 1 + E\{[f'(i)-1]i'\} + \chi\left[f''(e+d^u)/n^u + Ef''(i)(1-\sigma_c^u)i'\right] \\
FOC(\ell_c) &: \quad g'(\ell_c) = [f'(i)-1]i' + \chi f''(i)(1-\sigma_c^u)i', \\
FOC(\sigma_c^u) &: \quad \chi\lambda(i) = \zeta^u.
\end{aligned}$$

According to the second condition, the shadow price χ satisfies

$$\chi = \frac{f'(e+d^u) - 1 - E\{[f'(i)-1]i'\}}{f''(e+d^u)/n^u + Ef''(i)(1-\sigma_c^u)i'} = \frac{E[(1-\sigma_c^u - i')\lambda(i)]}{f''(e+d^u)/n^u + Ef''(i)(1-\sigma_c^u)i'},$$

where the second equality results from (26). Because $i' > 1$ the shadow price is positive, $\chi > 0$, if there is overborrowing. The shadow price ζ^u is positive by the fourth optimality condition, proving that the constraint $\sigma_c^u \geq 0$ is binding. The third optimality condition then reveals that the planner reduces the liquidity ℓ_c provided compared to the optimal policy mix described in Proposition 1 and equation (17), proving that the social planner provides less liquidity under commitment.

■

Even when $\sigma^u = 0$ so that shadow banks do not obtain targeted liquidity, liquidity provision is excessive under discretion because a discretionary policymaker does not take into account her impact on the shadow banking sector's incentives to borrow ex ante. At the margin, a small reduction in the size of liquidity provision to regulated banks has a second-order welfare cost ex post but a first-order welfare gain by reducing leverage in the shadow banking sector. Thus, it is not enough to exclude the shadow banking sector from targeted liquidity provision. Such a policy takes care of excessive borrowing generated by moral hazard but does not address excessive borrowing due to uninternalized systemic risk.[18]

Ruling Out Ex-Post Interventions? In the presence of commitment problems, it is at times desirable for policymakers to commit to simple rules. This brings up the question of whether it is advantageous to ban ex-post liquidity provision, i.e., commit to $\ell_c = 0$, in order to provide better incentives than under the discretionary liquidity policy ℓ_d of the policymaker. Formally, the question boils down to comparing welfare in the two scenarios,

$$n^u f\left(e + \hat{d}^u\right) + n^r f\left(e + \hat{d}^r\right) + W\left(\rho - \hat{d}\right) \gtrless n^u f\left(e + d^u\right) + n^r f\left(e + d^r\right) + W\left(\rho - d + \ell\right)$$

where we denote by hats all variables under a commitment to no liquidity provision. For example,. $\hat{d} = n^u \hat{d}^u + n^r \hat{d}^r$ denotes the aggregate debt level carried into period 1 under no liquidity provision. In general, the comparison depends on all the parameters of the model, but we can make specific predictions for simple limit cases. On one extreme, if the shadow banking system (parameter n^u) is sufficiently small, maintaining discretion in liquidity provision is optimal. On the other extreme, if the cost of liquidity provision $g(\ell)$ is sufficiently high, committing to no liquidity provision is optimal. More generally, if the policymaker can commit to a maximum level of liquidity provision, we can show that the constrained optimal maximum level is generally greater than zero, i.e. it is not generally

[18]Another avenue to capture the potential effects of regulatory arbitrage is that it limits the maximum size of the macroprudential tax, $\tau \leq \overline{\tau}$. The results are similar to what we described above. (See our earlier working paper, Jeanne and Korinek, 2013.)

optimal to commit to no liquidity provision whatsoever.

4.5 Self-fulfilling crises

Equation (14) may in general admit multiple equilibria. This is illustrated in Figure 1 in a case where $p(i)$ is convex.[19] There are two stable equilibria (points A and B) and one unstable equilibrium. Point B corresponds to a self-fulfilling fire sale in which banks want to sell the asset because they are financially constrained and they are financially constrained because the price of the asset is low. We assume that if there are multiple equilibria, the equilibrium is selected by an exogenous sunspot variable $s \in \{L, H\}$. Specifically, if $s = H$ and the parameters of the economy are consistent with multiple equilibria, then bankers coordinate on the high equilibrium, i.e., they rationally expect the high equilibrium to occur, act accordingly, and thereby make the good equilibrium materialize. Similarly, if $s = L$, they coordinate on the low equilibrium if it exists. If the economy admits only one equilibrium, the sunspot variable is irrelevant.

For simplicity we assume that ρ is constant so that the sunspot is the only source of crisis, and that the economy is not constrained in the good equilibrium, as exemplified by point A in the figure. Let us denote the equilibrium investment function by $i(\ell + m, s)$. As can be seen for the figure, increasing the total level of liquidity to which banks have access, $\ell + m$, shifts the curve upwards and raises the level of investment in the bad equilibrium, $i(\ell + m, L)$. For a sufficiently high level of liquidity, the bad equilibrium disappears. Let us denote by μ the level of liquidity above which there is no bad equilibrium, i.e., the lowest level of $\ell + m$ such that $i(\ell + m, L) = i(\ell + m, H)$. The threshold μ depends only on the exogenous parameters of the model. Function $i(\cdot, L)$ is discontinuous in μ since the equilibrium jumps from B to A when $\ell + m$ crosses μ from below.

Let us consider the social planner equilibrium under discretion. In line with the analysis

[19] Function $p(i)$ is convex for example if $f(\cdot)$ is quadratic: $f(k) = a - \frac{b}{2}k^2$ implies $p(i) = 1/(a - bi)$, which is a strictly increasing convex function of i for $i < a/b$. In general, $p(\cdot)$ could be locally convex or concave.

conducted in section 4, we assume that the social planner chooses the level of liquidity ℓ in period 1, taking the state m, s as well as the reaction function of the banking sector $i(\ell + m, s)$ as given. The social planner policy is characterized by a function $\ell(m, s)$. To fix ideas we assume that liquidity is provide through the open market, $\ell = p\kappa$.

If the state is high, the social planner does not need to intervene. If the state is low, the social planner compares the maximum period-1 surplus that can be achieved if liquidity is not high enough to remove the bad equilibrium,

$$W_L^B(m) = \max_{\ell + m < \mu} \left[f(i(\ell + m, L)) - i(\ell + m, L) - g(\ell) \right], \tag{27}$$

with the surplus achieved by setting ℓ just high enough to remove the bad equilibrium,

$$W_L^A(m) = f\left(i^{FB}\right) - i^{FB} - g(\mu - m), \tag{28}$$

(setting liquidity above $\mu - m$ would only reduce welfare).

Observe that in general, $W_L^A(m)$ could be larger or lower than $W_L^B(m)$. The optimal policies are characterized in the following proposition.

Proposition 7 (Multiplicity) *Assume that the economy is vulnerable to a self-fulfilling fire sale under laissez-faire. If the social planner makes its liquidity provision a function of the state (m, s), then the low equilibrium can be ruled out if and only if*

$$W_L^B(m) < W_L^A(m), \tag{29}$$

where $W_L^B(m)$ and $W_L^A(m)$ are given by (27) and (28). If ex-post interventions are not costless, the social planner uses macroprudential regulation in period 0 *irrespective of whether self-fulfilling fire sales occur or not.*

Proof. If the inequality (29) is satisfied, the planner would find it optimal to incur the cost

$g(\mu - m)$ to implement the first-best level of investment with total welfare $W_L^A(m)$ in the low state. If the inequality is violated, a discretionary planner will find it optimal to remain in the low equilibrium and achieve welfare $W_L^B(m)$.

In both cases the social planner intervenes in equilibrium, $\ell > 0$, in the low sunspot state. If the bad equilibrium is removed and there were no macroprudential regulation, bankers would invest k^{FB} and borrow $d = k^{FB} - e$ in period 0. Then there would be a first-order gain (in terms of lower liquidity provision in the low state) and a second-order loss from reducing k below k^{FB}. If the bad equilibrium is not removed, the case for macroprudential intervention relies on the fact that like before, private agents do not internalize the financial multiplier. ■

In order to remove the bad equilibrium, the social planner must buy a sufficient amount of asset in the market that bankers are left with too few assets to drive their price to the fire-sale level. Merely promising to intervene is not sufficient to remove the bad equilibrium—there is none of the free lunch sometimes associated with lending in last resort. Because interventions are costly, the social planner may prefer to let self-fulfilling fire sales occur and intervene only to mitigate their costs. Macroprudential policy remains necessary because ex-post interventions are costly.

The assumption of discretion is not crucial per se for Proposition 7. The same result would hold if we allowed the social planner to commit in period 0 to a policy $\ell_c(m, s)$, for the same reason as in Proposition 5 (macroprudential policy obviates the need for commitment). The social planner would have to deliver on a commitment to provide liquidity in the low state when that state is realized. However, the results are different if we change the nature of the equilibrium by allowing liquidity provision to be a function of the bankers' investment i rather than a function of the state m, s. To see this, assume that the planner commits to the following ex-post intervention

$$\ell = i^{FB} - i.$$

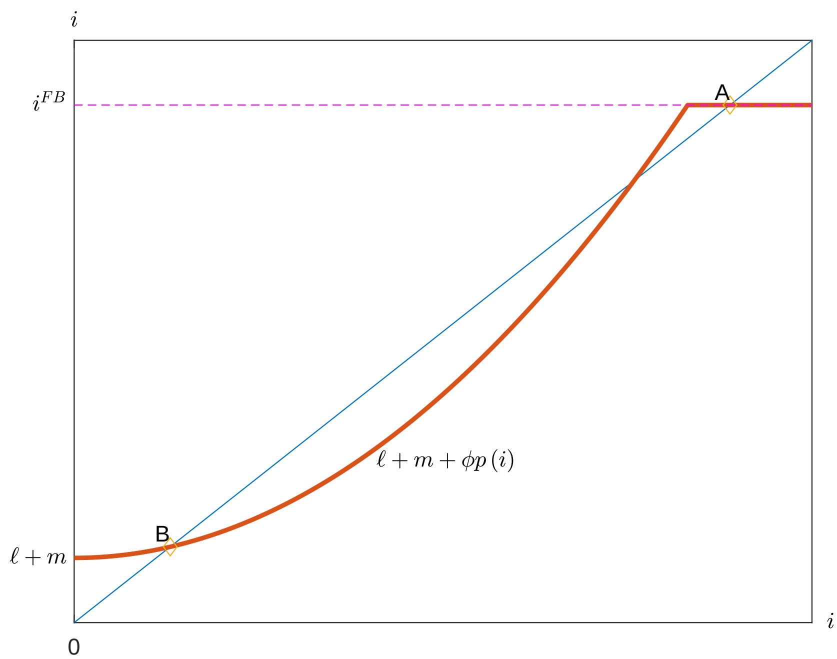

Figure 1: Multiple equilibria

This policy ensures that each banker has sufficient liquidity to implement the first-best level of investment, independently of the sunspot variable and of any concerns about coordination. Then the only solution to the equilibrium condition $i = i\,(\ell, s)$ is $i = i^{FB}$ so the bad equilibrium is removed and surplus from period-1 investment is at the first-best level $W^{FB} = f\left(i^{FB}\right) - i^{FB}$. No liquidity needs to be provided in equilibrium (there is now a free lunch). Propositions 2 and 3 imply that no macroprudential regulation is required. The following proposition summarizes our results.

Proposition 8 (Ruling Out Multiplicity) *The planner can always implement the high equilibrium by committing to provide sufficient liquidity to finance the first-best level of investment if $s = L$. In this case, no macroprudential regulation is required.*

Proof. See discussion above. ∎

This is an example where liquidity provision ex post reduces the need for macroprudential policy ex ante. Our results in Proposition 3 on the "Greenspan doctrine" still apply because the planner's promise to provide liquidity will not be carried out in equilibrium so that the two conditions of the Proposition, $E\left[\sigma f'\left(i\right)\right] = 0$ and $E[\phi p'(i) g'(\ell)] = 0$, are met in equilibrium.

4.6 Liquidity Fund

Since it is optimal to impose macroprudential restrictions in period 0 and to provide liquidity in period 1, one might be tempted to combine the two policy measures and use the proceeds of a macroprudential tax to provide liquidity in the event of a crisis. This can be done by accumulating the macroprudential tax proceeds in a fund that is lent in the future if bankers experience binding financial constraints, similar to the common practice in deposit insurance systems. It would seem preferable to finance liquidity provision with resources accumulated by a corrective macroprudential tax rather than through a tax that creates distortions outside of the banking sector.

We analyze this policy proposal by assuming that the planner stores the proceeds of the macroprudential tax $\tau \int d^j dj = \tau d$ in a liquidity fund. The fund is used to provide liquidity to constrained bankers in period 1. Using the fund does not entail any deadweight cost ex post, so that the liquidity is provided to constrained bankers until either the constraint is completely alleviated or the fund is exhausted. The liquidity that is left after bankers are no longer financially constrained, if any, could be rebated lump-sum to either bankers or depositors (it does not matter which for total welfare given that both have linear utility). We assume, without loss of generality, that the liquidity is distributed via open market operations, which do not distort borrowing in period 0, and that the payoffs from the assets that the policymaker accumulates as well as any remaining resources in the fund are distributed to bankers in lump sum fashion so that the welfare of depositors is unaffected.

The welfare properties of a liquidity insurance fund are described in the following.

Proposition 9 (Irrelevance of Pre-financed Liquidity Fund) *A liquidity fund that accumulates the revenue from a macroprudential tax does not affect real allocations as long as the bankers subject to the tax are unconstrained.*

Proof. We denote all variables when the liquidity fund is accumulated by bars, e.g. $\bar{d}$ for period 0 borrowing. We prove the result under the assumption that liquidity provision is limited to the resources in the liquidity insurance fund, $\bar{\ell} = \tau\bar{d}$, that is accumulated by the macroprudential tax τ. The proof in the case when we admit additional liquidity provision ℓ at cost $g(\ell)$ is equivalent. The ex-ante welfare of a banker j who is unconstrained in period 0 and obtains liquidity $\bar{\ell} = \tau\bar{d}$ in period 1 is given by

$$f(e + (1-\tau)\bar{d}^j) + E\left\{\max_{\bar{\imath}^j}\left[f(\bar{\imath}^j) + \rho - \bar{d}^j - \bar{\imath}^j + \tau\bar{d} + \lambda^j\left(\rho - \bar{d}^j + \tau\bar{d} + \phi p - \bar{\imath}^j\right)\right]\right\}.$$

where λ^j is the shadow price on the financial constraint in period 1. The first-order condition is,

$$(1-\tau)f'(e + (1-\tau)\bar{d}^j) = E\left[f'\left(\bar{\imath}^j\right)\right] \quad \text{where} \quad \bar{\imath}^j = \min\left\{i^{FB}, \rho - \bar{d}^j + \tau\bar{d} + \phi p\right\}.$$

Without a liquidity fund – if the macroprudential tax proceeds τd are rebated in period 0 or if a cap on deposits is used instead, the first-order condition is,

$$(1-\tau)f'(e + (1-\tau)d^j + \tau d) = E\left[f'\left(i^j\right)\right] \quad \text{where} \quad i^j = \min\left\{i^{FB}, \rho - d^j + \phi p\right\}.$$

In both cases the first-order condition can be written in terms of equilibrium investment levels k in period 0 and i in period 1,

$$(1-\tau)f'(k) = E\left[f'(i)\right] \quad \text{where} \quad i = \min\left\{i^{FB}, \rho + e - k + \phi p(i)\right\}.$$

This problem is solved by a unique allocation $(k, i(\rho))$. Thus the allocation is the same whether there is a liquidity fund or not. The real allocation is unchanged because bankers who are unconstrained in period 0 borrow more to offset the revenue invested by the social planner in the liquidity fund so that $\bar{d} = d/(1-\tau)$. Hence welfare is the same as without such a fund. ■

As a result, accumulating a liquidity fund while bankers are unconstrained does not generate any welfare gains compared to the allocation under which the optimal macroprudential tax is rebated or under which macroprudential regulation is enacted via quantity regulation. Conversely, when bankers are financially constrained in period 0 so that the accumulation of the bailout fund reduces the resource that they have available for investment, then investment in period 0 is reduced, similar to the effects of a binding macroprudential capital requirement. However, assuming that the macroprudential tax is set at its optimum level, this reduction in investment actually reduces welfare.

Corollary 10 *Restricting liquidity provision to the resources in a liquidity fund unambiguously reduces welfare compared to the optimal ex-post liquidity provision policy.*

Proof. Proposition 9 demonstrated that there are no welfare gains from introducing a liquidity insurance fund compared to the case when no liquidity is provided. Proposition 1 tells us that it is desirable to provide a positive amount of ex-post liquidity. Taken together, this implies the statement of the corollary. ■

Intuitively, these results capture that introducing a liquidity insurance fund does not yield any risk-sharing benefits — if our concern is aggregate risk, then it is equivalent for the planner to hold precautionary savings in a liquidity insurance fund or for individual bankers to hold an identical amount of precautionary savings on their balance sheets. The planner has no comparative advantage in holding precautionary savings. The tax that is used to finance the liquidity insurance fund achieves its intended macroprudential benefit in reducing borrowing. The fund, however, does not achieve the gains of other ex-post policy interventions that channel resources from outside the banking sector towards banks because private bankers simply finance their contribution to the fund by issuing more deposits, leaving the net amount of liquid resources on bank balance sheets unchanged. Put differently, aggregate risk cannot be reduced or diversified if bankers and the policymaker only have access to uncontingent financial instruments.

Under the described conditions, macroprudential regulation can do anything that could be accomplished by liquidity insurance funds. In practice, we note three factors for why such funds may have benefits that go beyond our framework. First, whereas our paper is focused on addressing aggregate and systemic risk, one benefit of such funds that is that they allow for the sharing of uninsurable idiosyncratic risk. If a planner can pool the idiosyncratic risks of *heterogeneous* bankers in a common fund, then she can reduce the total amount of liquidity held and thereby improve efficiency. However, as our findings highlight, it is important to remember that this benefit does not apply to the aggregate systemic risk that is at the core of our analysis. Second, liquidity insurance funds may be easier to implement and monitor than instructing banks to accumulate greater precautionary savings. If bank balance sheets are opaque and banks engage in moral hazard vis-a-vis their regulators, then the actual precautionary reserves of banks may be significantly less than what they report on their balance sheets. Third, such funds may also carry political benefits to the extent that voters do not appreciate our equivalence result.

5 Conclusions

This paper develops a simple framework to analyze optimal policies in an environment where collateral-dependent borrowing constraints lead to financial amplification. Except in knife-edge cases, all policies fall into the category of second-best interventions, i.e., they achieve first-order welfare gains by mitigating binding borrowing constraints in the economy, but at the expense of introducing second-order distortions. In accordance with the theory of the second-best (see Lipsey and Lancaster, 1956), it is optimal to use all second-best instruments available in such a setting. In particular, we show that it is optimal to both restrict borrowing ex-ante via macroprudential regulation and to relax borrowing constraints ex-post by providing liquidity. This implies that policymakers should both "lean against the wind" and "mop up after the crash."

The two policies are substitutes for each other since they address the same goal from different angles, implying that in general, macroprudential policy is used more when liquidity policies are restricted and vice versa. We also show that there is no time consistency problem if the optimal mix of ex-ante macroprudential regulation and ex-post liquidity provision is implemented. However, if macroprudential policy is restricted, e.g. by the presence of unregulated shadow banking, then committing to less liquidity provision and to providing it exclusively in untargeted form serves as a second-best device for reducing excessive borrowing.

Our setup is designed to focus on the financial implications of different ex-ante and ex-post policies to reduce crisis risk so as to deliver sharp results on how these policies interact with fire sales and balance sheet constraints. From this perspective, low interest rates, for example, feed into the economy by raising asset prices and relaxing binding constraints on bankers. Our focus contrasts with a second important aspect of crisis management policies, their aggregate demand effects, which can be studied in New Keynesian model frameworks. We refer the interested reader e.g. to Aoki et al. (2016), Farhi and Werning (2016), and Korinek and Simsek (2016).

There are a number of important questions that we leave for future analysis. First, financial policies such as macroprudential regulation and liquidity provision have distributive implications. Although we noted that the cost of liquidity provision $g(\ell)$ can be interpreted as the planner's penalty for redistributing funds, we have not paid attention to the political economy aspects of the choice between macroprudential regulation and mopping up after the crash. It is clear in our setup that bankers will dislike the former and greatly value the latter. This creates an important role for special interests and lobbying.

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APPENDICES

A Alternative Ex-Post Interventions and Their Deadweight Losses

This appendix lays out microfoundations of six alternative ex-post policy measures that provide liquidity to constrained bankers and show how they all map into the framework we described in Section 3. The ex-post measures that we describe can be categorized along two main dimensions:

The first dimension is whether a given ex-post policy measure is untargeted or targeted. This is the main distinction relevant for our optimal policy analysis, since targeted ex-post policy measures affect ex-ante incentives and may therefore require the planner to adjust her ex-ante policy measures.

The second dimension is whether a given ex-post policy measure redistributes wealth from depositors to bankers or whether it only provides liquidity, i.e. it is fully repaid. This distinction is not relevant for efficiency in our baseline setup, since both sets of agents have linear preferences over final consumption. However, it is highly relevant for the distribution of surplus between bankers and depositors and thus carries important political implications. One point worth emphasizing is that liquidity provision to constrained agents always generates a surplus (by allowing for productive investment opportunities to be taken) that frequently accrues to bankers and that is separate from any potential redistribution of net worth between depositors and bankers.

For each of the described ex-post policy measures, this appendix also spells out microfoundations for why ex-post measures may lead to deadweight losses, giving rise to a deadweight loss function $g(\ell)$ that is increasing and convex, as we assume throughout our analysis in the main text. Although we model these channels separately for clarity, they are frequently complementary, and it is likely that all of them (plus perhaps several more) play a role in practice when policymakers provide liquidity to combat a financial crisis. Analytically, this can be captured by viewing the deadweight loss function $g(\ell)$ faced by actual policymakers as the overall social cost of providing liquidity ℓ simultaneously through different channels in a way that minimizes the overall social cost.

The table below summarizes the properties of all the discussed policy measures along these dimensions. The second column captures whether a given ex-post policy is untargeted or targeted. The third column lists whether the most natural form of implementation of a given policy involves a redistribution of net worth from depositors to bankers or not. The fourth column lists potential sources of deadweight loss that typically arise for each policy measure.

Policy measure	Targeted?	Redistribution?	Sources of deadweight loss $g(\ell)$
open market purchases	untargeted	liquidity only	Inferior screening/monitoring capacity
lump-sum transfers	untargeted	redistribution	Distortions from taxation; redistribution
interest rate cuts	untargeted	redistribution	Intertemporal distortion; redistribution
targeted liquidity provision	targeted	liquidity only	Inferior screening/monitoring capacity
debt relief	targeted	redistribution	Distortions from taxation; redistribution
recapitalization	targeted	liquidity only	Inferior screening/monitoring capacity

Table A.1: Typical properties of alternative ex-post policy measures

A.1 Open Market Purchases

Assume that the planner borrows ℓ units of liquid funds from depositors in period 1 and uses them for open market purchases by buying $\kappa = \Delta k = \ell/p$ units of capital assets at the prevailing market price p from bankers.[20] We assume κ satisfies $\kappa + \phi \leq k + f(i)$ so that bankers keep enough collateral for borrowing from private lenders.

If the planner is equally productive in managing the assets, she would earn κ units of consumption good in period 2, generating a net profit on the asset purchases of $\kappa - \ell \geq 0$ after returning ℓ units of funds to depositors. This net profit reflects part of the surplus generated by the policy intervention. W.l.o.g. we assume that the planner rebates her net profit to the bankers. In period 2, bankers thus earn $k - \kappa + f(i)$ from their capital holdings and receive the transfer $\kappa - \ell$. The budget constraints of bankers in periods 1 and 2 can then be summarized as

[20] The analysis would be little changed if the policymaker bought assets at a subsidized value, e.g. at the full first-best face value of 1, or at any other arbitrary value $q \geq p$ at which bankers are willing to part with their assets. The only difference is that this would reduce the payoff of the policymaker from holding the capital assets, $h(\kappa) - q\kappa$, which would require the central bank to commensurately reduce its period 2 transfer to bankers or potentially raise taxes on bankers or depositors in period 2.

$$i = \ell + m + d', \tag{30}$$

$$c^b + d' = k + f(i) - \ell. \tag{31}$$

Deadweight Losses from Inferior Capacity to Manage Capital Assets More generally, we assume that the social planner is weakly inferior at monitoring, managing, and extracting value from her asset holdings, captured by a production function $h(\kappa)$ that satisfies $h(0) = 0$, $h'(0) = 1$ and $h'' \leq 0$ so that $h(\kappa) \leq \kappa$. This may reflect e.g. that the private sector has greater flexibility, an information advantage, or greater information processing capabilities that make it superior in monitoring, in making continuation decisions, and in enforcing repayment. It may also reflect the risk of rent extraction that arises when productive assets are held by government entities.

We denote the resulting deadweight loss from providing ℓ units of liquidity to bankers by $g(\ell) = \kappa - h(\kappa) = \ell/p - h(\ell/p)$, which satisfies $g(0) = g'(0) = 0$ and $g(\ell), g'(\ell) > 0$ for $\ell > 0$. In period 2, the planner's net profit on the asset purchases is then $h(\kappa) - \ell = \kappa - \ell - g(\ell)$. No matter if the deadweight loss experienced by the planner reduces the transfer to bankers or is borne by depositors via taxes, the deadweight loss function $g(\ell)$ ultimately enters the social planner's objective as captured by equation (16) in Section 4.1. In short, the described policy relaxes the financial constraint of bankers by ℓ units at the cost of introducing the deadweight loss $g(\ell)$ into the economy.

Ultra-safe Collateral A variant of this setup is that the planner may be able to provide a certain amount of liquidity, say $\bar{\ell}$, without any distortions so $g(\ell) = 0$ for $\ell \leq \bar{\ell}$, for example because bankers have a certain amount of ultra-safe collateral such as Treasuries on their balance sheet. However, once the threshold $\bar{\ell}$ is surpassed, asset purchases may give rise to distortions so $g(\ell) > 0$ for $\ell > \bar{\ell}$. An example is depicted in Figure 2.

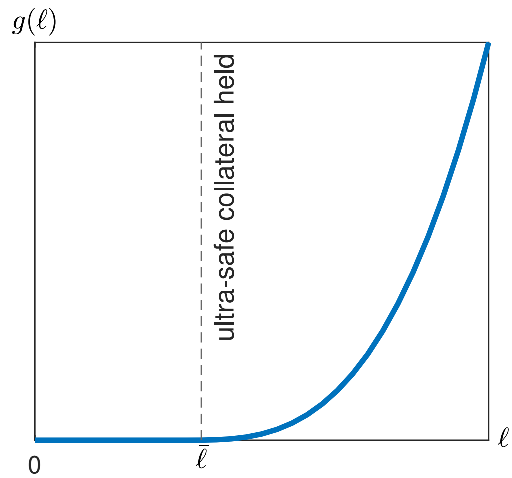

Figure 2: Example of deadweight loss function with ultra-safe collateral

A.2 Lump-Sum Transfers

Lump-sum transfers are pure transfers that are provided to economic agents regardless of their choices and needs. They are the classic example of untargeted transfers in economics and are thus a useful benchmark to analyze. In the following, we demonstrate that open market purchases and lump-sum transfers have equivalent efficiency implications in our setting.

Assume the planner in our economy provides a lump-sum transfer ℓ from households to constrained bankers in period 1. The economic effect on bankers in that period is the same as that of open market purchases: the funds available for investment in period 1 increase by ℓ as indicated by budget constraint (30), allowing for a commensurate increase in period 1 investment. However, unlike in the case of pure liquidity provision, there is no transfer back to depositors so the policy generates not only efficiency gains but also a redistribution of wealth, leaving the period 2 budget constraint of bankers unaffected at

$$c^b + d' = k + f(i) \,. \tag{32}$$

Except for the different distribution of consumption in period 2, the allocations in the economy are the same as in the case of open market purchases.

Deadweight Losses from Taxation In practice, raising the revenue required to provide transfers introduces distortions into the economy. In our example of lump-sum transfers, the deadweight loss function $g(\ell)$ captures these distortions. To provide a specific example, we assume that the planner raises the liquidity ℓ by imposing a tax on an economic activity of depositors in period 1. Specifically, we assume that depositors produce a quantity of good q at cost $C(q)$ in period 1, where $C(\cdot)$ is an increasing and convex function that satisfies $C(0) = C'(0) = 0$ and $\lim_{q \to \infty} C'(q) > 1$. (The economic activity could capture for example labor supplied by individuals, which comes at a disutility captured by the cost function $C(q)$ expressed in units of output.) We define the net income from this activity as $y(q) = q - C(q)$ and observe that the first-best level of the economic activity q that delivers the maximum level of net income y^{FB} is defined by $C'\left(q^{FB}\right) = 1$ so $y^{FB} = q^{FB} - C\left(q^{FB}\right)$. Furthermore, observe that $y'\left(q^{FB}\right) = 0$ because of the envelope theorem.

Assume that the planner imposes tax τ on the economic activity so that the after-tax net income of depositors is given by $(1-\tau) q - C(q)$. Depositors choose an optimal level of the economic activity $q(\tau)$ that is pinned down by the optimality condition $C'(q) = 1 - \tau$ and that satisfies $q(0) = q^{FB}$ and is declining in τ. The revenue obtained from the tax, $\ell(\tau) = \tau q(\tau)$, is strictly increasing in τ up to a level $\ell^{\max}$, which represents the maximum of the Laffer curve for the tax. Inverting this revenue function, $\tau(\ell)$ captures the tax rate necessary to raise revenue $\ell \in [0, \ell^{\max}]$. Then the deadweight cost of raising liquidity ℓ is given by

$$g(\ell) = y^{FB} - y\left(q\left(\tau\left(\ell\right)\right)\right).$$

It can easily be verified that $g(0) = g'(0) = 0$ and that $g(\ell)$ is strictly increasing and convex in ℓ. The after-tax period-1 net income of the representative depositor is $(1-\tau) q - C(q) = y^{FB} - \ell - g(\ell)$, and her period 2 consumption before any rebates is $2y^{FB} - \ell - g(\ell)$. The social planner maximizes the joint surplus of bankers and depositors, and her objective is again given by equation (16).

Deadweight Losses from Redistribution An alternative microfoundation for the deadweight loss function $g(\ell)$ is that the social planner cares about the distribution of resources between bankers and depositors, and that the function $g(\ell)$ captures the planner's concerns about distributive effects of such "bailout" transfers to bankers. Specifically, assume that the planner values the consumption of bankers and depositors according to a social welfare function

$$\tilde{W} = c^b + w\left(c^d\right),$$

where $w(\cdot)$ is an increasing but strictly concave function, $w' > 0 > w''$. W.l.o.g. assume that this function satisfies $w'(2y) = 1$. Let us define $g(\ell) = w(2y) - \ell - w(2y - \ell)$ as the difference between the risk-neutral valuation of a lump-sum tax ℓ on depositors and the actual valuation of such a tax, which represents the welfare losses arising from the redistribution. It can easily be verified that the cost function $g(\ell)$ satisfies $g(\ell) = g'(\ell) = 0$ and $g''(\ell) > 0$ for $\ell > 0$. The social planner's welfare function can then be expressed as

$$W = c^b + w(2y - \ell) = c^b + w(2y) - \ell - g(\ell) = w(2y) + e + \rho + f(k) - k + f(i) - i - g(\ell)$$

Dropping constants, the planner's objective is isomorphic to (16).

A.3 Interest Rate Cuts

A third example of untargeted ex-post interventions are interest rate cuts. We follow Farhi and Tirole (2012) and describe interest rate policy in a real setting as a policy that makes the equilibrium interest rate in the economy deviate from the natural rate of interest, which corresponds to the return on the safe storage technology and equals to 1. This can be captured either by taxing the return of the safe storage technology or by subsidizing all lending transactions.

Consider an economy in which the financial constraint is binding and assume that the social planner imposes a tax $\tau > 0$ on the use of the safe storage technology so that the equilibrium gross real interest rate in the economy becomes $R = 1 - \tau < 1$. Bankers then obtain d'/R units of liquid funds in period 1 in exchange for the promise to repay d' units of funds in period 2. For a given amount of borrowing as determined by the constraint, $d' = \phi p$, bankers then obtain extra liquidity $\ell = (1/R - 1)\, d' = \tau/(1 - \tau)\, \phi p > 0$, and bankers' period-1 budget constraint can be written as (30). In this example, the interest rate cut amounts not only to a transfer of liquidity but also a transfer of net worth from depositors to bankers so budget constraint (32) applies in period 2.

As we observed earlier, since both depositors and bankers value consumption linearly in period 2, joint social welfare is unchanged even if no return transfer is performed in period 2.

Deadweight Losses from Interest Rate Cuts We follow Farhi and Tirole in assuming that there is a distribution of projects that have unit cost in period 1 and pay off $r < 1$ in period 2, with a continuous cumulative distribution function $H(r)$ that satisfies $H(1) = 1$. We assume that depositors will jointly invest into any projects that seem profitable at the given market interest rate, and that the planner funds the interest subsidy by lump sum taxes on depositors. To provide liquidity ℓ, the interest rate must be lowered to $R(\ell) = d' / (\ell + d')$. Depositors will then finance all projects with $r \geq R(\ell)$, generating a social deadweight loss given by

$$g(\ell) = \int_{R(\ell)}^{1} (1 - r)\, dH(r).$$

The resulting deadweight loss function satisfies the usual properties $g(0) = g'(0) = 0$ and g is increasing and convex in ℓ.

More generally, if central banks cannot target the advantageous interest rate exclusively to constrained bankers, a distortion may arise because an interest rate below the natural rate may lead to the financing of investment projects with negative social net present value.

A.4 Targeted Liquidity Provision

Our baseline example of targeted liquidity provision is that the planner provides liquidity depending on banker j's level of period 0 borrowing, for example by providing σ units of liquidity for each unit of deposit d^j, amounting to total liquidity provision $\ell^j = \sigma d^j$. This liquidity intervention is targeted because bank j's access to liquidity is conditional on the size of the deposits issued by the bank in period 0. (One could also assume, without affecting the substance or our results, that the access to liquidity increases with deposits in a non-linear way, or that it depends on the bank's assets k^j rather than its liabilities.) A common practical example of targeted liquidity provision is discount window lending, which is targeted to each bank j's eligible collateral asset holdings.

The effects of targeted liquidity provision on the period-1 and -2 budget constraints are then given by (30) and (31) and for given state variables at the beginning of period 1, the problem is isomorphic to untargeted liquidity provision. However, as we show in Section 4, targeted liquidity provision differs in how it affects (and distorts) period-0 decisions of bankers.

Deadweight Losses from Targeted Liquidity Provision via Repos In practice, policymakers typically provide liquidity by lending against collateral, for example via repurchase agreements (repos), in which central banks purchase collateral at a given price and bankers promise to repurchase it at the original price plus interest. Repurchase agreements expose the lender to losses if bankers default on their promise to repurchase and the lender has an inferior ability to extract value from such asset holdings, as in our example of open market operations in Appendix A.1 above. To capture this formally, assume that there is an idiosyncratic shock to bankers in period 2 that determines their individual default probability, which is an increasing function of their repurchase obligation ℓ so it can be captured by a distribution function $G(\ell)$. A planner who provides a given level of liquidity ℓ will end up with unmet repurchase obligations and thus with capital asset holdings of $\kappa(\ell) = G(\ell)\,\ell$ and will experience a deadweight loss given by $g(\ell) = \kappa(\ell) - h(\kappa(\ell))$, which satisfies the same conditions as our earlier deadweight loss functions.

A.5 Debt Relief

Another example of targeted liquidity provision is debt relief. Assume that the planner provides relief on a fraction σ of each banker j's outstanding period-0 debt when a crisis occurs. This amounts to a transfer of liquid funds $\ell^j = \sigma d^j$ from depositors to bankers, as in the case of targeted liquidity provision. As in our example of interest rate cuts, debt relief provides not only liquidity but also redistributes net worth from depositors to bankers. The period-1 and -2 budget constraints thus coincide with (30) and (32). Deadweight losses may arise as described in Appendix A.2, i.e., either from the distortions of taxation or from the social cost of redistributions, generating a deadweight loss function $g(\ell)$. The resulting problem of the planner is given by equation (16).

A.6 Recapitalizations

Our final example are recapitalizations. Assume the planner injects capital into constrained bankers to assist them in paying off the deposits that they incurred in period 0, either in the form of senior debt or equity. We again denote the capital injection by $\ell^j = \sigma d^j$ where σ captured how much equity the planner injects per unit of deposits. W.l.o.g. assume that the planner requires a gross rate of return of unity on the injected capital. In that case, the period 1 and 2 budget constraints coincide with (30) and (31), and from the perspective of period 1, the planner's policy is isomorphic to the targeted liquidity provision we described earlier. If the rate of return differs from unity (e.g. because the planner provides a subsidy or charges a premium on the equity injection), then the liquidity injection also involves an element of redistribution between bankers and the planner but joint welfare is unaffected. Deadweight losses from recapitalizations may arise because the planner has inferior capacity to monitor bankers as a capital holder, or because a public capital stake in bankers may lead to rent extraction, similar to our model of deadweight losses in Section A.1.

B Double-Decker Model

This appendix explicitly describes the intermediation chain of funds from depositors via banks to the firms in the real economy that operate capital assets, following similar steps as the double-decker model of Tirole (2006). If financial frictions are binding solely between depositors and the bankers who borrow from them, then we show that this setup is isomorphic to our baseline model in which bankers and firms are represented by a single agent. In that case, binding financial constraints in the economy depend on bank capital and only affect the first step of the described intermediation chain. This observation reflects the "folk theorem" that collapsing two agents between whom financial markets are complete into a single agent does not change the allocations of other agents in the economy. Note that the same argument holds if we interpret the intermediation chain as going from depositors via banks to homebuyers, as we discuss below.

Accordingly, we introduce into our setup a third set of agents called firms, shorthand for firm owners, who have linear preferences and who employ capital assets in a linear production function in period 2. Firms are endowed with n units of capital assets and rent the additional capital assets k generated by banks to produce $y^f = A(n+k)$ units of output in period 2.[21] We assume that firms are subject to a moral hazard problem that limits the maximum amount of capital assets that banks entrust them to a multiple ψ of their endowment n. (An alternative interpretation is that the capital assets represent residential real estate and that the third set of agents are homebuyers who convert these capital assets into consumption services of housing using the linear production function $y^f = A(n+k)$ given above.)

The maximization problem of firms is[22]

$$\max_k U^f = A(n+k) - Rk \qquad \text{s.t.} \qquad k \leq \psi n$$

Denoting by μ as the shadow price on the financial constraint, the optimality condition of firms is $A = R + \mu$.

If the financial constraint on firms is slack, then $R = A$, reflecting that firms compete the return on capital assets down to the technological rate of return. It can be seen that the setup nests our baseline setup if the financial constraint on firms is slack and we parameterize the model so that $A = 1$.

On the other hand, if the firm financial constraint is binding, then $R < A$, reflecting that a scarcity in demand for capital assets pushes the market return below the return in the first best. In that case, liquidity provision or transfers to firms would be desirable, in a similar manner as the liquidity provision policies discussed in section 3.2 that are aimed at bank balance sheets are desirable when the financial constraints on banks are binding.

[21] We could equivalently provide firms with a Cobb-Douglas production function $y^f = k^\alpha \ell^{1-\alpha}$ and assume that they can rent labor ℓ at a given wage w. Given firms' maximizing choice of $\ell = [(1-\alpha)/w]^{1/\alpha} k$, this gives rise to a reduced-form production function $y^f = Ak$ where $A = [(1-\alpha)/w]^{(1-\alpha)/\alpha}$.

[22] In principle, firms could be owned by depositors, as long as we do not allow depositors to directly fund capital assets without the help of banks, i.e. as long as they need to rely on the financial intermediation services of banks. Similarly, home buyers could be a subset of depositors who, for life cycle reasons, purchase homes, as long as the other depositors cannot directly reshuffle funds to the homebuyers. Otherwise the banking sector would be irrelevant.

C Alternative Constraint Specification

We now consider an alternative version of the collateral constraint created by the threat of renegotiation in period 1. In our baseline analysis, we assumed that depositors could seize only up to ϕ units of a banker's assets, giving rise to the financial constraint $d^{j\prime} \leq \phi p$. In this appendix, we assume that in the event of renegotiation, depositors can seize a fraction ϕ of the existing assets carried into period 2, which we denote by $k^{j\prime} = k^j - \Delta k^j$ where $k^j = f\left(e^j + d^j\right)$. This gives rise to the constraint

$$d'^j \leq \phi p k'^j.$$

For simplicity, we assume that the two ex-post policy measures employed by the planner are untargeted lump-sum transfers ℓ_s as well as targeted debt relief at rate σ so that the total amount of liquidity provided by the planner is $\ell = \ell_s + \sigma d$. Results for other ex-post policy measures can be derived analogously.

Period 1 Analysis

The period-1 utility of the banker is then captured by the function

$$V\left(\ell^j + m^j, k^j\right) = k^j + \ell^j + m^j + \max_{i^j, \Delta k^j} \left[f(i^j) - i^j + p\Delta k^j + \lambda^j \left(\ell^j + m^j + \phi p \left(k^j - \Delta k^j\right) + p\Delta k^j - i^j\right)\right].$$

Unlike in our baseline setup, the capital k^j created in period 0 is an argument to the value function since it influences the tightness of the constraint and the individual stimulus received. The derivatives of the banker's value function satisfy $V_m = f'\left(i^j\right) > 0$ and $V_k = 1 + \phi p \lambda^j > 0$.

The optimality condition for fire sales Δk^j is given by $p\left[1 + \lambda^j \left(1 - \phi\right)\right] = 1$, capturing that only a fraction $(1 - \phi)$ of every unit of asset held needs to be financed with (constrained) period 1 funds, whereas a fraction ϕ can be financed by borrowing. In general equilibrium, substituting the shadow price $\lambda = f'(i) - 1$ yields the asset price equation

$$p(i) = \frac{1}{1 + (1 - \phi)\left[f'(i) - 1\right]}. \tag{33}$$

Compared to our baseline version, (33) captures that asset holdings also have collateral value.

In equilibrium, period-1 investment is given by the fixed-point equation

$$i = \min\left[i^{FB}, \ell + m + \phi p(i)k\right]$$

which defines a function $i\,(\ell + m, k)$ that now also depends on capital k created in period 0. To ensure uniqueness of equilibrium, we modify the assumption imposed in our baseline setup to

$$\phi p'(i)k < 1, \forall i, k \leq k^{FB}.$$

If this condition is not satisfied, the same considerations as in Section 4.5 on multiple equilibria apply.

When the financial constraint is slack, the analysis is unchanged from our baseline setup. When the constraint is binding, the marginals of the investment function satisfy

$$\begin{aligned} i_m\,(\ell + m, k) &= \frac{1}{1 - \phi p\,(i)\,k} > 1, \\ i_k\,(\ell + m, k) &= \phi p\,(i) > 0. \end{aligned}$$

The period-1 value function is defined analogously to (16) but also depends on k so $W\,(m, k) = k + \max_\ell m - g(\ell) + f(i(\ell + m, k)) - i(\ell + m, k)$, giving rise to an optimality condition for the optimum amount of liquidity $[f'(i) - 1]\, i_m(\ell + m, k) = g'(\ell)$ that is equivalent to (17).

Proposition 1 carries through without any changes.

Period 0 Analysis

The period 0 decision problem of bankers is

$$\max_{d^j, k^j} E[V^j\left(\rho - (1-\sigma)\,d^j + \ell_s, k^j\right)] - \eta^j\left[k^j - f\left(e + (1-\tau)d^j + \tau d\right)\right],$$

where η^j reflects the shadow value of capital created in period 0. The associated optimality conditions for d^j and k^j, where we substitute the envelope conditions in the second step, are

$$E\left[V_m^j \cdot (1-\sigma)\right] = E\left[(1-\sigma)\left(1+\lambda^j\right)\right] = \eta^j (1-\tau) f'\left(k^j\right),$$
$$E\left[V_k^j\right] = E\left[1+\phi p \lambda^j\right] = \eta^j,$$

and can be combined to

$$E\left[(1-\sigma)\left(1+\lambda^j\right)\right] = E\left[1+\phi p \lambda^j\right](1-\tau) f'\left(k^j\right).$$

The planner's optimization problem is described by $\max_d E\left[W\left(\rho - d, f\left(e+d\right)\right)\right]$, giving rise to the optimality condition

$$E\left[1+\lambda i_m\left(\cdot\right)\right] = E\left[1+\lambda i_k\left(\cdot\right)\right] f'\left(k\right). \tag{34}$$

In equilibrium, subtracting the two optimality conditions from each other delivers the optimal tax rate

$$\tau = \frac{E\left[\sigma f'\left(i\right) + \lambda\left[i_m\left(\cdot\right) - 1\right]\right]}{E\left[1+\phi p \lambda\right] f'\left(k\right)}$$

The expectations term in the denominator reflects that bankers value period-0 capital investment not only for its intrinsic marginal benefit but also for expanding the constraint.

Alternatively, if the planner employs quantity regulations, the optimal leverage ratio or deposit cap is determined by (34). Given these modifications, Propositions 2, 3, 5 and the remaining results can be replicated with minor modifications.

D Further Results [Online Appendix]

D.1 Complete Markets

This appendix replicates the analysis of the period-0 problem for the case of complete markets. The period-1 problem is unchanged from our earlier analysis in Section 4.2 and continues to be described by the same value functions $V(\cdot)$ and $W(\cdot)$. For clarity of notation, we add a superscript denoting the state of nature $\omega \in \Omega$ to all state-contingent variables. In period 0, bankers are free to issue different amounts of securities d^{ω} contingent on the state of nature ω. Given the risk-neutrality of lenders, the period 0 constraint of bankers is $k = f\left(e + E\left[d^{\omega}\right]\right)$, and the optimization problems of private bankers under laissez faire as well as the planner are

$$\max_{d^{\omega}} f\left(e + E[d^{\omega}]\right) + E\left[V\left(\rho^{\omega} - d^{\omega}\right)\right] \quad \text{and} \quad \max_{d^{\omega}} f\left(e + E[d^{\omega}]\right) + E\left[W\left(\rho^{\omega} - d^{\omega}\right)\right].$$

The optimality conditions for the security issuance in each state of nature are

$$f'\left(\cdot\right) = V'(m^{\omega}) \;\; \forall\omega \quad \text{and} \quad f'\left(\cdot\right) = W'(m^{\omega}) \;\; \forall\omega.$$

The term $f'(\cdot)$ is non-stochastic so $V'(m^{\omega})$ and $W'(m^{\omega})$ must be constant across states of nature – bankers and the planner choose to fully insure the period-1 shock so the liquid net worth $m^{\omega} = \rho^{\omega} - d^{\omega} = \bar{m}$ is constant in all states of nature. This is unsurprising since lenders are risk-neutral and offer actuarially fair insurance. If $\bar{m} \geq \hat{m}$, then bankers can insure away the binding constraints and attain a first-best allocation in which $k = k^{FB}$ and there is no role for policy intervention. Otherwise, constraints are equally binding in all states of nature. The optimal cap on security issuance in state ω is $\bar{d}^{\omega} = \bar{d} + \rho^{\omega} - E[\rho^{\omega}]$ where $\bar{d}$ is given by an optimality condition analogous to equation (20),

$$f'\left(e + \bar{d}\right) = 1 + \frac{f'\left(i\right) - 1}{1 - \phi p'\left(i\right)}.$$

Period-1 investment i is the same in all states of nature so we omitted the expectations operator. The optimal tax on securities issued against any state of nature $\omega \in \Omega$ is the same and is given by an expression analogous to equation (22),

$$\tau^{\omega} = \frac{\left(\sigma + i' - 1\right) \cdot \lambda\left(i\right)}{f'\left(e + d\right)}$$

where we omitted expectations and the superscript ω for period 1 variables since they are constant across states of nature. It is straightforward to obtain analogons to Propositions 3, 2, 5 and 6 as well as our remaining results.

In the described case, bankers fully insure their shock since depositors are risk-neutral. It would be straightforward to extend our analysis by considering risk-averse depositors. In that case, bankers would no longer insure fully and the constraint would be binding to different degrees in different states of nature, as in our baseline analysis with uncontingent debt contracts.

D.2 Collateral constraint in Period 0

This appendix analyzes the conditions under which a banker will be tempted to renegotiate the deposits issued in period 0. We assume that bankers can make a take-it-or-leave-it offer to renegotiate their deposits at any time. If depositors reject this offer, they can seize ϕ units of the banker's assets and sell them to other bankers at the prevailing market price. The incentive compatibility constraints that induce bankers to refrain from reneging at all time periods are as follows:

(C#) Period	Deposits	Collateral
(C1) end of period 0	$d \leq$	$\phi p_0 = \phi / f'(e+d)$
(C2) beginning of period 1	$d \leq$	$\phi p_1 = \phi / f'(i)$
(C3) end of period 1	$d' \leq$	$\phi p_1 = \phi / f'(i)$
(C4) beginning of period 2	$d' \leq$	ϕ

In the main text we have focused on the collateral constraints for deposits d' issued in period 1, (C3) and (C4), and we observed that (C3) is always weakly tighter than (C4) so we could focus our attention on (C3) alone. Here we focus on the collateral constraints (C1) and (C2) for the deposits d issued in period 0.

For constraint (C1), observe that the constraint is guaranteed to hold for small levels of d as long as $\phi, e > 0$. Both sides of the inequality are increasing in d. For sufficiently high levels of ϕ and e the constraint is slack even for the first-best level of investment. Specifically, this will be the case when $e + \phi \geq i^{FB}$.

Constraint (C2) requires that $d \leq \phi / f'(i)$. If the constraint (C3) is loose then $i = i^{FB}$ and (C2) can be transformed to $d \leq \phi$, which is satisfied for sufficiently high values of ϕ. Otherwise, the constraint (C2) can be expressed as

$$d \leq \phi p\left(i\left(\ell + \rho - d\right)\right).$$

This inequality is satisfied for all states of nature as long as both ϕ and the lowest shock realization $\rho^{\min}$ are sufficiently high.

If one of the constraints (C1) or (C2) was binding, then deposit issuance in period 0 would be a corner solution that is determined by the binding constraint, and there is nothing the social planner could do using ex-ante interventions. An analysis of macroprudential interventions would thus be meaningless. However, the period-1 decisions would be unchanged from our analysis in Section 4.1 and Proposition 1.