Managing credit booms and busts: A Pigouvian taxation approach

Olivier Jeanne[a], Anton Korinek[b,1,*]

[a] *Johns Hopkins University, 3400 North Charles St., Baltimore, MD 21218, USA*
[b] *Department of Economics and Darden GSB, University of Virginia, 248 McCormick Rd, Charlottesville, VA 22904, USA*

A R T I C L E   I N F O

A B S T R A C T

The interaction between debt accumulation and asset prices magnifies credit booms and busts. Borrowers do not internalize these feedback effects and therefore suffer from excessively large booms and busts in both credit flows and asset prices. We show in a dynamic model that a time-consistent policymaker finds it optimal to internalize these externalities by imposing a Pigouvian tax on borrowing that is the product of three simple sufficient statistics. A numerical illustration shows that the optimal tax is countercyclical: it rises during booms but can be set to zero in busts when the financial constraint is binding. The optimal macroprudential tax is a non-trivial function of the environment.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction

The interaction between debt accumulation and asset prices contributes to magnify the impact of booms and busts. Increases in borrowing and in collateral prices feed each other during booms. In busts, the feedback turns negative, with credit constraints leading to asset price declines and further tightening of credit. This type of mechanism has received a lot of attention following the US financial crisis, and it has been suggested that prudential policies could be used to mitigate the build-up in systemic vulnerability during the boom (Lorenzoni, 2008; Shleifer and Vishny, 2011).

This paper formalizes such policies in a dynamic optimizing model of consumption-based asset pricing and collateralized borrowing. Our model is stripped down to the essence of the mechanism that we want to study. We consider a group of borrowers who enjoy a comparative advantage in holding an asset and who can use this asset as collateral. Their borrowing capacity is therefore increasing in the price of the asset. The asset price, in turn, is driven by their aggregate borrowing

---

capacity. This introduces a mutual feedback loop between asset prices and credit flows, so that small shocks may be amplified and lead to large simultaneous booms or busts in asset prices and credit flows. When borrowing and asset prices respond strongly to each other, we also show that the feedback loop between the two can give rise to multiple equilibria. We illustrate this in a simple example that can be characterized analytically, and we show how to impose assumptions that ensure uniqueness of equilibrium.

The model captures a number of economic settings in which the systemic interaction between credit and asset prices may be important in a stylized way. The borrowers can be interpreted as a group of entrepreneurs who have more expertise than outsiders to operate a productive asset, or as households putting a premium on owning durable consumer assets such as their homes. Alternatively, the borrowers could represent a group of investors who enjoy an advantage in dealing with a certain class of financial assets, for example because of superior information or superior risk management skills. The borrowers could also be interpreted as the residents of a country who borrow from foreign investors. One advantage of studying these situations in a common framework is to bring out the commonality of the problems and of the required policy responses.

The free market equilibrium is constrained inefficient in our setting. The asset-debt loop entails a pecuniary externality that leads borrowers to undervalue the benefits of conserving liquidity as a precaution against busts. A borrower who has one more dollar of liquid net worth when the economy experiences a bust relaxes not only his private borrowing constraint but also the borrowing constraints of all other borrowers. Not internalizing this spillover effect, borrowers take on too much debt during good times. As a result, a time-consistent policymaker finds it optimal to impose a cyclical tax on debt to prevent borrowers from taking on socially excessive levels of debt—in the spirit of macroprudential policy (see Borio, 2003). The optimal tax rate can be expressed as the product of three sufficient statistics.

We illustrate the quantitative implications of the model in a parsimonious calibration to the US small and medium-sized enterprise (SME) sector in the 2008–09 crisis. We find that the optimal tax converges to 0.6% of the amount of debt outstanding over the course of a boom; by contrast, when a bust occurs, the tax can be set to zero. Borrowing by the US household sector is subject to externalities of similar magnitude. The optimal tax rate is not very large – it reduces borrowing by only half a percent of GDP compared to the laissez-faire allocation. Since busts are infrequent events, it is not desirable for the policymaker to lean too heavily against the credit boom. However, the tax rate is sufficient to reduce the financial amplification dynamics in a meaningful way: conditional on a bust occurring, the decline in consumption is reduced by about one percent, and the fall in the asset price is reduced by about two percent on average.

Even though our framework is stylized, we find that the optimal macroprudential tax on borrowing is a non-trivial function of the environment. The optimal macroprudential tax on borrowing may respond to changes in parameter values in surprising ways. For example, an increase in the probability of a bust may call for *lower* macroprudential taxation. This is because a riskier environment may increase private self-insurance sufficiently that there is less need for public intervention. Furthermore, when the impatience of borrowers exceeds a certain threshold, their optimal borrowing is a corner solution and macroprudential taxation is irrelevant – the tax would have to be higher than what is desirable to affect real allocations and relax the collateral constraint in the future.

We study three extensions of the basic model and find that its essential properties are preserved. First, we change the nature of the shock by assuming that it affects the availability of credit rather than the income of borrowers. Then we look at the case where borrowers can issue long-term debt or equity. All three of these extensions change some features of the boom-bust cycle equilibrium, but it remains true that the constrained optimum can be achieved by a cyclical tax on debt, and this tax is of the same order of magnitude as in the benchmark model.

The structure of the remainder of the paper is as follows. Section 2 presents the assumptions of the model. Section 3 compares the laissez-faire equilibrium with a social planner. Section 4 presents a calibration of our model and explores its quantitative implications. Section 5 lays out extensions of the benchmark model. Section 6 discusses the related literature and concludes.

2. The model

We consider a group of identical atomistic individuals, indexed by $i \in [0, 1]$, in infinite discrete time $t = 0, 1, 2, \ldots$. The utility of individual i at time t is given by,

$$U_{i,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{i,s}),$$ (1)

where $\beta < 1$ is a discount factor and the utility function has constant relative risk aversion,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$ (2)

We will consider equilibria in which individuals are impatient and borrow—thus we call them "borrowers." They receive two kinds of income, the payoff of an asset that can serve as collateral, and an endowment income. Borrower i maximizes his utility under the budget constraint

$$c_{i,t} + a_{i,t+1} p_t + \frac{w_{i,t+1}}{R} = (1-\alpha)y_t + a_{i,t}(p_t + \alpha y_t) + w_{i,t},$$ (3)

where $a_{i,t}$ is the borrower's holdings of the collateral asset at the beginning of period t and p_t is its price; $w_{i,t}$ is his financial wealth in the form of bond holdings at the start of period t; y_t is total income in period t and is the same for all borrowers, and αy_t is the share of that income that comes from the asset (which, in a richer model, could be interpreted as capital income). Since he is impatient, the representative borrower's wealth w will be negative in equilibrium and we can call $-w$ his debt. The representative borrower's debt is held by outside lenders who have an indefinite demand/supply for risk-free bonds at the safe interest rate $r = R - 1$.

Total income y_t follows a stochastic process which, for the sake of simplicity, we assume to be independent and identically distributed, although it would be straightforward to extend the analysis to the case where it is a more general Markov process. Assuming an i.i.d. process for y_t is not too restrictive given that, in the calibration, we will consider shocks that represent relatively rare crises events rather than business cycle fluctuations.

The collateral asset is not reproducible and the available stock of the asset is normalized to 1. The asset can be exchanged between borrowers in a perfectly competitive market, but we do not allow them to sell the asset to outside lenders and rent it back because borrowers derive important benefits from the control rights that ownership provides. For simplicity we assume that the asset would become worthless if it was sold to outside lenders. Therefore $a_{i,t}$ must be equal to 1 in a symmetric equilibrium. This assumption can be relaxed to some extent (see Section 4.3), but we need some restriction on asset sales so that borrowers issue collateralized debt in equilibrium.

Furthermore, we assume that the only financial instrument that can be traded between borrowers and lenders is uncontingent one-period debt. The assumption that debt is uncontingent can be justified e.g. on the basis that shocks to borrowers are not verifiable and cannot be used to condition payments. The feature that debt is short-term provides borrowers with adequate incentives (we present an extension to long-term debt in Section 4.2). Both assumptions correspond to common practice across a wide range of financial relationships.

The amount of debt that the borrowers can roll over is limited by a collateral constraint. The microfoundation for the collateral constraint is similar to that in Kiyotaki and Moore (1997). We assume that after rolling over his debt in period t, a borrower can renegotiate the level of his debt in the same period. If the negotiation fails, the creditors can seize an amount ϕ of collateral asset. The borrower has all the bargaining power, and so negotiates his debt down to the amount that the creditors can recover in a default. In order to discourage renegotiation, the level of debt must satisfy the collateral constraint,

$$\frac{w_{i,t+1}}{R} + \phi p_t \geq 0, \tag{4}$$

In general, the parameter ϕ will depend on legal rights of lenders as well as the borrower's ability to hide or abscond with his assets.

We could specify the constraint in alternative ways. For example, one could assume that creditors can seize a *fraction* ϕ instead of an *amount* ϕ of the collateral asset so that $w_{i,t+1}/R + \phi a_{i,t+1} p_t \geq 0$. We employ the version of the constraint given by (4) to simplify the analytical derivations in the paper, but we present the equilibrium conditions resulting from this alternative approach in Online Appendix A2 and discuss why the qualitative and quantitative implications of the two specifications are almost identical.

Note that the borrower can renegotiate his debt right after it is issued but is committed to repay in the following period. This captures the notion that the liquidity of the asset in period t matters. We could allow for renegotiation at the time of repayment too, which would introduce an additional constraint involving p_{t+1} rather than p_t on the right-hand side of (4). The financial amplification dynamics, however, come from the feedback loop between p_t and c_t and would not be significantly altered.

2.1. Equilibrium conditions under laissez-faire

We derive in the online appendix the first-order conditions for the optimization problem of a borrower i. We then use the fact that in a symmetric equilibrium, all individuals are identical and hold one unit of collateral asset ($\forall i, t \ a_{i,t} = 1$). Variables without the subscript i refer to the *representative* borrower (or equivalently, to aggregate levels, since the mass of borrowers is normalized to 1). This gives the following two conditions

$$u'(c_t) = \lambda_t + \beta R E_t \left[u'(c_{t+1}) \right], \tag{5}$$

$$p_t = \frac{\beta E_t \left[u'(c_{t+1})(\alpha y_{t+1} + p_{t+1}) \right]}{u'(c_t)}, \tag{6}$$

where λ_t is the costate variable for the borrowing constraint (4). The first equation is the Euler condition and the second one is the standard asset pricing equation.

The equilibrium is characterized by a set of functions mapping the state of the economy into the endogenous variables. Given that y_t is i.i.d. and $a_t = 1$, we can summarize the state by one variable, the beginning-of-period liquid net wealth (excluding the value of the collateral asset),

$$m_t \equiv y_t + w_t. \tag{7}$$

In a symmetric equilibrium the budget constraint (3) simplifies to

$$c_t + \frac{w_{t+1}}{R} = m_t, \tag{8}$$

and the collateral constraint (4) can be written, in aggregate form,

$$c_t \leq m_t + \phi p_t. \tag{9}$$

A recursive laissez-faire equilibrium is then characterized by three functions, $c(\cdot)$, $p(\cdot)$ and $\lambda(\cdot)$ that satisfy the first-order optimality conditions,

$$c(m)^{-\gamma} = \lambda(m) + \beta RE\big[c(m')^{-\gamma}\big], \tag{10}$$

$$\lambda(m) = \big[(m + \phi p(m))^{-\gamma} - \beta RE\big(c(m')^{-\gamma}\big)\big]^{+}, \tag{11}$$

$$p(m) = \beta E\big[c(m')^{-\gamma}(\alpha y' + p(m'))\big]c(m)^{\gamma}, \tag{12}$$

and the law of motion for net wealth

$$m' = y' + R(m - c(m)). \tag{13}$$

2.2. Multiple equilibria

Models with endogenous collateral constraints such as ours may exhibit multiplicity of equilibrium. This section analyzes the mechanism underlying multiplicity in a special case and gives a heuristic account of how to ensure uniqueness more generally. The multiplicity comes from the self-reinforcing loop that links consumption to the price of the collateral. In the constrained regime, financial amplification arises because a fall in the price of the collateral asset decreases the borrowers' level of consumption, which in turn tends to depress the price of the asset. If the effect of this loop is strong enough, it may lead to self-fulfilling crashes in the price of the asset.

More formally, using (6) the credit constraint (9) can be written

$$c_t \leq m_t + \phi \beta \frac{E_t\big[u'(c_{t+1})(\alpha y_{t+1} + p_{t+1})\big]}{u'(c_t)}. \tag{14}$$

The right-hand side of (14) is increasing in c_t because the credit constraint on each individual is relaxed by a higher level of aggregate consumption, which raises the price of the asset (by lowering u' in the denominator). Multiplicity may arise if the left-hand side and the right-hand side of (14) intersect for more than one level of c_t.

Example of multiplicity. We consider a special case of the model that can be solved analytically – the case where y is constant and $\beta R = 1$. The model then has steady state equilibria in which consumption and the asset price satisfy $c^{SS}(m) = \beta y + (1 - \beta)m$ and $p^{SS} = \alpha y / r$. The collateral constraint is loose in these equilibria if and only if

$$m \geq \bar{m} \equiv y - R\phi p^{SS}, \tag{15}$$

where $\bar{m}$ is the threshold value of m for which the steady state equilibrium is marginally unconstrained.

We restrict our attention to multiplicity in period 1 and assume that the economy is in a steady state from period 2 onwards. For a given initial level of wealth m_1 the equilibrium in period 1 satisfies

$$c_1 = \min\big\{c^{SS}(m_2), m_1 + \phi p_1\big\}, \tag{16}$$

$$p_1 = \beta \left(\frac{c_1}{c_2}\right)^{\gamma}(\alpha y + p^{SS}) = p^{SS}\left(\frac{c_1}{c_2}\right)^{\gamma}, \tag{17}$$

where $m_2 = y + R(m_1 - c_1)$. Observe that as $c_1 \leq m_1 + \phi p_1 \leq m_1 + \phi p^{SS}$, one has $m_2 \geq y - R\phi p_1 \geq \bar{m}$ so that it is indeed possible for the economy to settle in the steady state equilibrium in period 2.

As captured by the first equation, the equilibrium in period 1 is either unconstrained or constrained. An unconstrained period-1 equilibrium is feasible if $m_1 \geq \bar{m}$. In that case, the economy may settle immediately in the steady state described by $c^{SS}(m_1)$ and p^{SS}.

Conversely, a constrained equilibrium is defined by the binding constraint $c_1 = m_1 + \phi p_1$. Substituting c_1 and $c_2 = y + r(m_1 - c_1)$ from the expression for p_1, the first-period consumption level can be obtained as a solution to the implicit equation

$$c_1 = m_1 + \phi p^{SS} \cdot r^{-\gamma}\left(\frac{y + rm_1}{y + r(m_1 - c_1)} - 1\right)^{\gamma}. \tag{18}$$

Fig. 1 shows how the two sides of this equation vary with c_1 for two different levels of ϕ. As can be seen in the figure, for $\gamma \geq 1$, equilibrium multiplicity is possible only if the unconstrained steady state equilibrium exists, which requires $m_1 \geq \bar{m}$. In addition, equilibrium multiplicity requires the slope of the r.h.s. of (18) to be larger than 1.
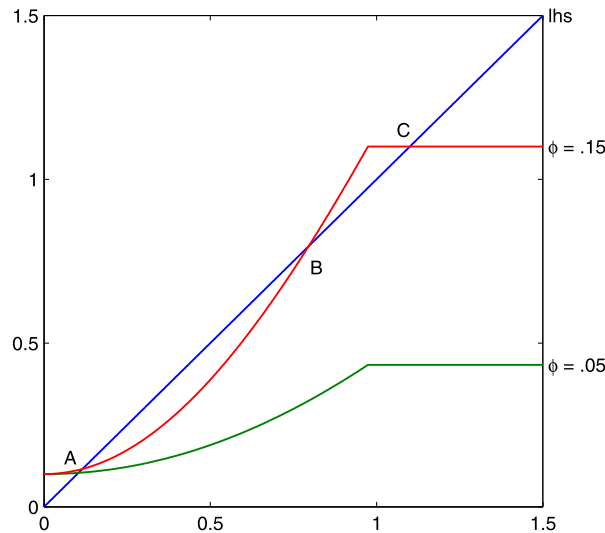
Fig. 1. L.h.s. and r.h.s. of Eq. (18) for $\phi = 0.05$ and $\phi = 0.15$. *Notes:* The remaining parameter values for the figure are $y = 1$, $R = 1.03$, $\alpha = 0.2$, $\beta = 1/R$, $\gamma = 2$ and $m_1 = 0.1$.

Sufficient conditions for unique equilibrium. The equilibrium must be unique if the slope of the r.h.s. of (18) is smaller than 1. In the case $\gamma \geq 1$, the r.h.s. is a convex function of c_1 in the constrained region, as shown in Fig. 1. Hence the slope of the r.h.s. is at its maximum when the economy enters the unconstrained region. Using that $c_1 = c_2$ when the economy is marginally constrained, the maximum slope is given by,

$$\left. \frac{\partial rhs}{\partial c_1} \right|_{c_1 = c^{SS}(\bar{m})} = \phi \left(1 + \frac{1}{r} \right) \frac{\gamma \alpha y}{c^{SS}(\bar{m})}. \tag{19}$$

To avoid multiplicity, it is sufficient that the slope (19) be smaller than 1. In the given example, we can substitute for $c^{SS}(\bar{m}) = y(1 - \alpha\phi)$ to express this as a condition on ϕ in terms of fundamental parameters,

$$\phi \leq \hat{\phi} := \frac{1}{\alpha \left[\left(1 + \frac{1}{r} \right) \gamma + 1 \right]}. \tag{20}$$

If this condition is satisfied, then the slope of the r.h.s. of (14) is lower than 1 everywhere and the equilibrium is unique. Conversely, if this condition is not satisfied, then there is multiplicity for m_1 slightly above $\bar{m}$.

Ruling out multiplicity. Multiplicity of equilibrium raises a number of issues, in particular about equilibrium selection and discontinuous policy functions, that we prefer to leave aside in order to focus on pecuniary externalities. For the remainder of the paper, we will thus assume that the equilibrium is unique by employing sufficiently low values of $\phi < \hat{\phi}$, which contain the strength of the amplification effects below the level where it leads to multiple equilibria. Although Eq. (20) was derived under very specific assumptions, the equation also provides guidance for the magnitude of $\hat{\phi}$ in the more general case with $\beta R < 1$ and stochastic y. In our baseline parameterization (see Table 1 in Section 3.1), we numerically find the threshold to be $\hat{\phi} = 0.072$, which is also the result of the analytic condition (20). Excluding multiplicity requires relatively low values of ϕ. This means that multiplicity is likely to arise for sectors with significant leverage, for example the highly-leveraged financial sector, as discussed in Section 3.1. However, if only a fraction of debt needs to be rolled over every period, higher leverage is consistent with a unique equilibrium, as we show in Section 4.2 on long-term debt.

2.3. Social planner

We introduce a time-consistent constrained social planner into the economy who determines borrowing, but does not directly intervene in asset markets, i.e., is constrained to take the asset price as determined by the marginal rate of substitution between assets and consumption goods. Formally, Eq. (6) serves as a constraint on the planner's problem. This corresponds to a setup in which the planner can choose allocations in a time-consistent manner subject to the same constraints as borrowers and with no additional instruments, in the spirit of the constrained planner setup of Stiglitz (1982).[2]

---

[2] In our time-consistent setting, the planner can only change the future by affecting the endogenous state variables, i.e. by affecting how much borrowing is carried into the next period. If the planner had the power to commit, she could lower risk premia and raise asset prices by promising agents a smoother consumption profile further in the future, thus relaxing the borrowing constraint. We do not pursue this case in the current paper.
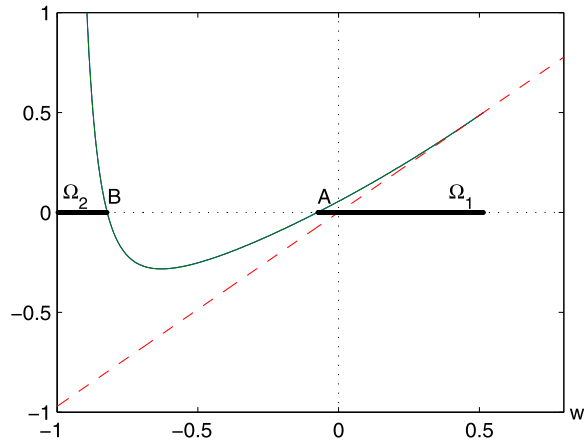
Fig. 2. L.h.s. of Eq. (24) depicted as solid curved line and feasible set $\Omega(m)$ depicted by bold bars. *Notes:* The left-hand side of inequality (24) as a function of w' is evaluated at $m = 0.5$ using the planner's policy functions for the parameter values $R = 1.04$, $\beta = 0.96$, $\gamma = 2$, $\alpha = 0.2$, $\phi = 0.046$ and a binomial output process $y \in \{0.96, 1\}$ where the lower value occurs i.i.d. with probability $\pi = 0.05$. The dashed line indicates w'/R.

We could alternatively describe the planner's optimal policy as a Markov perfect policy with a tax on borrowing. This is because time-consistent behavior implies that the planner's policy is only a function of the current state variables, and since the planner takes the asset pricing decisions of private agents as given and private agents take the planner's tax on borrowing as given, the two are part of a Markov perfect equilibrium.

Given the level of liquid net wealth m the planner makes the consumption/savings decision of the representative borrower and determines c and w'. Let us denote by $\tilde{c}(m)$ and $\tilde{p}(m)$ the policy functions for consumption and the collateral price in the constrained social planner's allocation (with tildes to differentiate them from the laissez-faire equilibrium). An allocation c, w', p is *feasible* given m if it satisfies the conditions,

$$c + \frac{w'}{R} = m, \tag{21}$$

$$\frac{w'}{R} + \phi p \geq 0, \tag{22}$$

$$p = \frac{\beta E\left\{u'\left(\tilde{c}(w' + y')\right)\left(\tilde{p}(w' + y') + y'\right)\right\}}{u'(c)}. \tag{23}$$

The first and second equations are the budget and credit constraints respectively, whereas the third equation captures that the equilibrium price of the asset is determined by the optimality condition for asset holdings of private agents. In a recursive equilibrium, the time-consistent constrained social planner chooses the optimal allocation rationally anticipating that she will apply the same policy functions in the next period.

Using (21) and (23) to substitute out p and c from (22) gives a condition involving w' and m,

$$\frac{w'}{R} + \phi \cdot \frac{\beta E\left\{u'\left(\tilde{c}(w' + y')\right)\left(\tilde{p}(w' + y') + y'\right)\right\}}{u'(m - w'/R)} \geq 0. \tag{24}$$

This condition defines the set of feasible w', which we denote by $\Omega(m)$. It is easy to see that the set $\Omega(m)$ expands with m since the l.h.s. of (24) increases with m. The maximum possible value of w' for which this inequality is defined is given by $w'^{\max} = Rm$, ensuring that consumption in the current period is non-negative, as captured by the denominator. The minimum possible w' ensures that consumption in the following period is non-negative, as captured by $\tilde{c}(w' + y')$ in the numerator. The social planner's problem can then be written in a recursive manner,

$$\tilde{V}(m) = \max_{w' \in \Omega(m)} u\left(m - w'/R\right) + \beta E\tilde{V}(y' + w'). \tag{25}$$

One issue in solving this problem is that the set $\Omega(m)$ is not in general a simple interval. The sign of variation of the l.h.s. of (24) with respect to w' is ambiguous: An increase in w' raises the first term but also raises the denominator in the second term. This implies that the set $\Omega(m)$ may not be connected, as illustrated by Fig. 2, which shows a typical situation encountered in our numerical simulations. The set $\Omega(m)$ is composed of two disconnected intervals denoted by bold black lines labeled Ω_1 and Ω_2. Intuitively, the two intervals can be interpreted as two different strategies for the planner to ensure that the constraint (24) is satisfied: Within the set Ω_1, the planner keeps borrowing sufficiently low (w'/R sufficiently high) so that the constraint is satisfied for levels of the asset price that are typical of what is encountered in the decentralized equilibrium. Within Ω_2, by contrast, the planner inflates the asset price by increasing borrowing (lowering w'/R) to the

point where next-period consumption c' is at a very low level. In our simulations, we found that for values of ϕ that are low enough to rule out multiple equilibria in the laissez-faire case, this second strategy leads to significantly lower welfare than the first strategy since private agents dislike the steep drop in next-period consumption. In the remainder of our analysis, we therefore focus on the case where ϕ is low enough that the social planner chooses w' in Ω_1, i.e., the largest connected subset of $\Omega(m)$ containing the non-negative values of w'.

This subset is an interval $\Omega_1 = [\bar{w}(m), Rm]$ where $\bar{w}(m)$ corresponds to point A in Fig. 2. The lower bound $\bar{w}(m)$ is negative and decreases with aggregate liquid net wealth m. Restricting the social planner's choice set to this interval is a reasonable assumption if ϕ is small. As ϕ decreases the l.h.s. of (24) converges to the first term and the set $\Omega(m)$ converges to the interval $[\bar{w}(m), Rm]$. In the limit $\phi = 0$ the set $\Omega(m)$ converges to the interval $[0, Rm]$.

Observe that we need to assume low values of ϕ to obtain a well-behaved equilibrium both under laissez-faire and with the social planner. The reason, in both cases, is that high values of ϕ lead to powerful financial amplification that make the problem of solving for the equilibrium highly non-linear. The implied difficulty, however, is not the same under laissez-faire and with the social planner. Low values of ϕ are required to rule out multiplicity in the decentralized equilibrium. With the social planner, the difficulty comes from a disconnected choice set rather than multiplicity since the social planner picks the unique policy that maximizes welfare. When ϕ is low enough, the planner can also implement the optimal policy with linear taxes without giving rise to multiplicity in the resulting competitive equilibrium under taxes.

If the social planner chooses w' in the interval $[\bar{w}(m), Rm]$, the collateral constraint becomes very easy to handle since it takes the form of a simple inequality

$$w' \geq \bar{w}(m). \tag{26}$$

The collateral constraint sets an upper bound on the level of borrowing, which is an increasing function of aggregate liquidity. When the planner chooses an allocation for which the inequality holds strictly, the asset price is given by

$$\bar{p}(m) = -\bar{w}(m)/\phi R, \tag{27}$$

which increases with m.

Conditional on the assumptions made so far we have the following result.

Proposition 1 (Constrained Social Planner). *The optimum of the time-consistent constrained social planner is characterized by three functions, $\tilde{c}(\cdot)$, $\tilde{p}(\cdot)$ and $\tilde{\lambda}(\cdot)$ that are given by Eqs. (11), (12) and*

$$\tilde{c}(m)^{-\gamma} = \tilde{\lambda}(m) + \beta RE\big[\tilde{c}(m')^{-\gamma} + \phi\tilde{\lambda}(m')\tilde{p}'(m')\big], \tag{28}$$

where $\tilde{p}'(m')$ is the first derivative of the next-period asset price with respect to next-period aggregate liquid net wealth and is strictly positive.

Proof. The constrained social planner's problem is to maximize the utility of the representative borrower subject to the budget constraint $w'/R + c = m$ and, using the function $\bar{p}(m)$ introduced above, to the credit constraint

$$\frac{w'}{R} + \phi\bar{p}(m) \geq 0, \tag{29}$$

It is easy to see that the first-order condition for consumption and saving is

$$u'(c) = \lambda + \beta RE\big[u'(c') + \phi\lambda'\bar{p}'(m')\big]. \tag{30}$$

To finalize the proof of the proposition, observe that $\bar{p}(m) = \tilde{p}(m)$ by definition whenever the collateral constraint is binding in equilibrium. This implies that the last terms in the Euler Eqs. (28) and (30) coincide. Conversely, when the collateral constraint is slack next period, $\lambda' = \tilde{\lambda}(m') = 0$ and so the last terms in the two Euler equations vanish. We conclude that the two equations coincide both when the constraint is binding and when the constraint is slack. We have $\tilde{p}' = \bar{p}' > 0$ because $\bar{p}(m) = -\bar{w}(m)/(R\phi)$ is increasing in m. $\square$

By comparing the Euler Eqs. (10) and (28), one can see that the social planner raises saving above the laissez-faire level, strictly so if there is a risk that the collateral constraint will bind in the next period. The planner's wedge is proportional to the expected product of the shadow cost of the credit constraint times the derivative of the debt ceiling with respect to wealth. The planner internalizes that increasing aggregate savings today raises tomorrow's asset price and relaxes tomorrow's credit constraint.

The social planner augments precautionary savings by a macroprudential component. This does not come from the fact that the planner estimates risks better than individuals. Decentralized agents are aware of the risk of credit crunch and maintain a certain amount of precautionary saving (they issue less debt than if this risk were absent). But they do not internalize the contribution of their precautionary savings to reducing the *systemic* risk coming from the debt-asset deflation spiral.

Eq. (28) illustrates that the planner's motive is purely prudential and forward-looking. When the constraint in the current period is slack, there is no benefit from relaxing the constraint. Conversely, when the credit constraint in the current period is binding, the planner does not take any action – even though she fully internalizes how her period-t savings and consumption choices affect the period-t asset price. The reason is that it is impossible for the planner to borrow more when

the economy is constrained without violating the collateral constraint, and that it is not desirable to save more when the binding constraint reflects that the planner would rather like to borrow more. This is the intuition behind our observation in the proof that $\bar{p}(m) = \tilde{p}(m)$ in constrained states – the planner does not have any free decision margins when she is constrained and just borrows the maximum amount possible, which is given by the function $\bar{p}(m)$. Note that as indicated by Eq. (23), the asset price function $\tilde{p}(m)$ in the planner's allocation—both in constrained states and in unconstrained states—differs from the asset price function $p(m)$ under laissez faire, since for any given m, the planner will chose different consumption allocations in the future so in general $c(m) \neq \tilde{c}(m)$.

The planner's Euler equation provides guidance for how the constrained optimal equilibrium can be implemented via taxes on borrowing. Decentralized borrowers undervalue the social cost of debt by the term $\phi E[\tilde{\lambda}(m')\tilde{p}'(m')]$ on the right-hand side of the planner's Euler Eq. (28). The planner's equilibrium can be implemented by a Pigouvian tax $\tau = \tau(m)$ on borrowing that is rebated as a lump sum transfer $T = -\tau w'/R$, implying the budget constraint

$$c = y + w - \frac{w'}{R}(1 - \tau) + T. \tag{31}$$

The Pigouvian tax that implements the social planner equilibrium is characterized in the following result.

Corollary 2 (Implementation). *The optimal time-consistent policy can be implemented by imposing a Pigouvian tax on borrowing of*

$$\tau(m) = E\left[\phi \cdot \frac{\beta R \tilde{\lambda}(m')}{u'(\tilde{c}(m))} \cdot \tilde{p}'(m')\right] \geq 0. \tag{32}$$

Proof. The tax introduces a wedge in the borrowers' Euler equation,

$$(1 - \tau)u'(c) = \lambda + \beta R E\left[u'(c')\right]. \tag{33}$$

By substituting into (33), one can see that the implementation replicates the social planner's optimum described in Proposition 1. □

The optimal tax rate $\tau(m)$ is the expected product of three sufficient statistics: the collateralizability parameter, ϕ, the discounted shadow cost of the binding constraint, $\beta R \tilde{\lambda}'/u'(c')$, and the response of the asset price to additional net worth next period, $\tilde{p}'(m')$, all of which are non-negative. We call these terms sufficient statistics in the spirit of Chetty (2009) because they are all that is required to determine the level of the tax rate.

The function $\tau(m)$ pins down a unique level of the tax when the financial constraint is slack for a given m, i.e. when $\tilde{\lambda}(m) = 0$. However, for all m for which the financial constraint is binding in the current period, there is a continuum of tax rates that all implement the same allocation. When borrowing is determined by the constraint, the Euler Eq. (33) pins down the level of the shadow price λ for a given real allocation and tax rate. A small change in τ simply induces an adjustment in λ but leaves the consumption allocation unaffected. By imposing $\lambda \geq 0$ in Eq. (33), we can see that the same real allocation is implemented for any tax rate that satisfies

$$\tau \leq 1 - \frac{\beta R E\left[u'\left(\tilde{c}'(m')\right)\right]}{u'(\tilde{c}(m))}. \tag{34}$$

In our application below, we will set the tax rate to zero whenever borrowing is determined by binding constraints, capturing that there is no need for policy intervention in those periods.

Naturally, the optimal policy can also be implemented by imposing a quantitative limit on borrowing, corresponding to

$$w' \geq \tilde{w}(m) = R[m - \tilde{c}(m)]. \tag{35}$$

3. Numerical illustration

We now turn our attention to assessing the magnitude of the externalities stemming from pecuniary externalities in the asset price and to analyzing how they evolve over booms and busts, how they depend on the parameters of the model and how they respond to policy intervention. We think of our model as applicable to individual sectors of an economy that use the same types of assets as collateral and thereby impose fire-sale effects on each other, e.g. households borrowing against real estate, rather than as capturing an aggregate economy as a whole. This translates easily into the way macroprudential policy is conducted in practice, which involves targeting individual sectors of the economy that experience overheating (see e.g. Bank of England, 2011). In our numerical illustration, we choose the parameter values of our model economy so as to capture selected quantitative features of the SME sector in the wake of the US crisis of 2008. We do not attempt to match moments of an aggregate economy at the business cycle frequency.

In order to generate a persistent motive for borrowing, we assume that borrowers are impatient relative to outsiders, i.e.,

$$\beta R < 1. \tag{36}$$

For convenience we denote by $d = -w'/R$ the amount of debt issued in the current period. Furthermore, we assume that in addition to the collateral value ϕp of their asset, borrowers can also borrow an exogenous amount ψ. This may capture fixed

Table 1
Benchmark calibration.

R	1.03	
β	0.96	
γ	2	
π	0.05	1 crisis every 20 years
α	0.2	p/y, US SMEs 2008Q2
ϕ	0.046	$\Delta d/\Delta p$, US SMEs 2008Q2–2009Q2
ψ	1.97	d/p, US SMES 2008Q2
y_H	1	normalization
y_L	0.96	$\Delta p/p$, US SMEs 2008Q2–2009Q2

default costs or debts that do not need to be rolled over period-by-period. As a result, the collateral constraint (4) becomes

$$d \leq \psi + \phi p. \tag{37}$$

3.1. Parameter values

Although our model is very stylized, we attempt to calibrate it meaningfully by picking parameter values that reproduce the decline in borrowing and in asset prices observed in certain sectors of the US economy between 2008 and 2009. Our benchmark calibration is reported in Table 1. A period is a year. The top three variables are calibrated by reference to the literature. The riskless real interest rate is set to 3% (one period corresponds to one year). The discount factor is set to 0.96, a value that is low enough to induce the borrowers to borrow and expose themselves to the risk of a credit crunch. The risk aversion parameter is equal to 2, a standard value in the literature.

We assume that the process for income is binomial: total income is high (equal to y_H) with probability $1 - \pi$, or low (equal to y_L) with probability π. The high state is the normal state that prevails most of the time, whereas the realization of the low state is associated with a bust in the asset price and in credit, which occurs infrequently. Thus, we calibrate our model by reference to rare and large events rather than real business cycle fluctuations. We assume that a bust occurs once every twenty years on average.

The other parameters are calibrated to the experience of the US small and medium enterprises (SMEs) in 2008–09.[3] Calibrating the model to the household sector gives very similar results (see our working paper Jeanne and Korinek, 2010b for further details). By contrast, we exclude the financial sector from our analysis since ϕ is much higher there and would place the sector in the region with multiple equilibria. Furthermore, we exclude the non-financial corporate business sector since that sector was able to increase its borrowing during the 2008 financial crisis, suggesting that it was not subject to binding constraints.

The share of the asset in income, α, was inferred from the ratio of the asset price to value added in the SME sector. Abstracting from the risk of bust, the price of the asset converges to $p = \beta/(1 - \beta) \cdot \alpha y$ in the high state so that

$$\alpha = \frac{1 - \beta}{\beta} \frac{p}{y}. \tag{38}$$

The ratio p/y was proxied by taking the ratio of the value of SME assets to their value added.[4] Note that at 20%, our estimate of the share of capital in the value added of SMEs is smaller than the share of capital income in total GDP, which is closer to 30%. This may reflect that SMEs are less capital intensive than large corporations, or that a larger share of labor income goes to self-employed entrepreneurs.

The two parameters in the collateral constraint, ψ and ϕ, were calibrated using information about SME assets and liabilities during a one-year time window centered on the peak of the crisis (the fall of 2008). The value of ϕ was estimated by dividing the fall in debt by the fall in asset value between the second quarter of 2008 and the second quarter of 2009. Analytically, this can be seen by differencing the borrowing constraint (37) to obtain

$$\phi = \frac{\Delta d}{\Delta p}. \tag{39}$$

The resulting value for ϕ is lower than suggested by the microeconomic evidence on the maximum amount of collateral asset that creditors can seize in a default. The measure of ϕ that is relevant for the purpose of calibrating our model, however, is the responsiveness of the debt constraint to a fall in the price of collateral inside a given period. This could be lower than the share of the asset that serves as collateral, for example because lenders accept temporary deviations from

---

[3] The source is the Federal Reserve's Flow of Funds database in 2010. The data for SMEs come from Table B.103 (Nonfarm Noncorporate Business) lines 1 and 23. The nonfarm noncorporate business sector comprises partnerships and limited liability companies, sole proprietorships and individuals who receive rental income. This sector is often thought to be composed of small firms, although some of the partnerships included in the sector are large companies.

[4] The data for national income and the value added of the noncorporate business sector are taken from the Bureau of Economic Analysis' NIPA statistics, Table 1.13 (annual data for 2008).
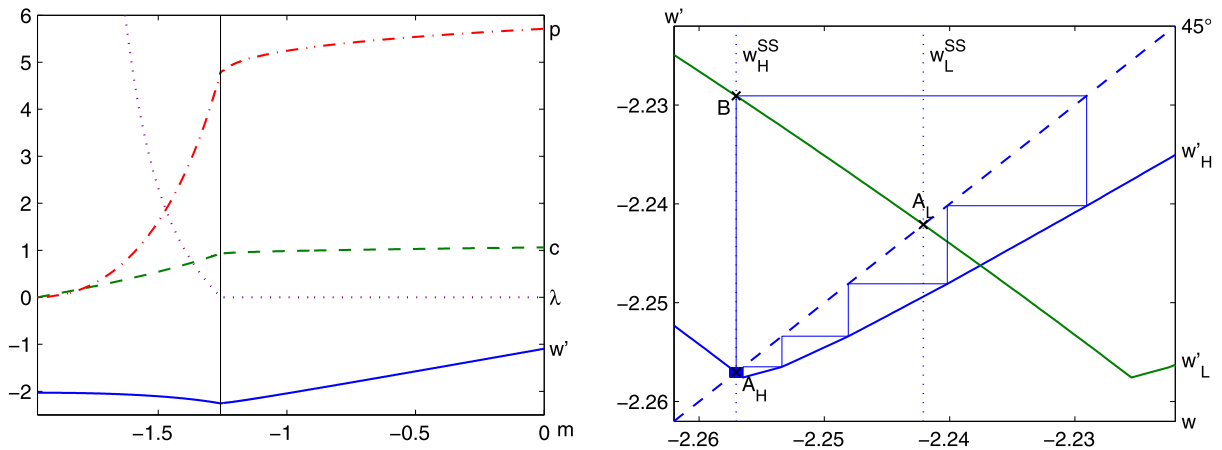
Fig. 3. Left panel: policy functions $c(m)$, $p(m)$ and $\lambda(m)$; right panel: wealth dynamics around the constraint.

the collateral constraint or because debt has a maturity longer than one period and does not respond instantaneously to a fall in the price of collateral. We investigate an extension of the model with long-term debt in Section 4.2.

Abstracting from the risk of a bust, the ratio of debt to asset value converges to $\psi/p + \phi = \frac{\psi(1-\beta)}{\alpha\beta y} + \phi$, so that the default penalty ψ can be calibrated as

$$\psi = \frac{\alpha\beta y}{1-\beta}\left(\frac{d}{p} - \phi\right), \tag{40}$$

where d/p is the ratio of debt to asset value. We proxied d/p by taking the ratio of debt to total assets for US SMEs in the second quarter of 2008. We then applied formula (40) using the values of α and ϕ derived before and $y = 1$.

Finally, income was normalized to 1 in the high state and y_L was calibrated so as to reproduce the fall in asset value observed in the 2008-09 bust.

3.2. Results

We numerically solve the model using a variant of Carroll's (2006) endogenous grid point method. (The details can be found in Online Appendix B. See also Hintermaier and Koeniger, 2010, and Carroll, 2011, for similar approaches.) The left-hand panel of Fig. 3 shows the policy functions $c(m)$, $p(m)$ and $\lambda(m)$ in the laissez-faire equilibrium for the benchmark calibration in Table 1. The equilibrium is unconstrained if and only if wealth is larger than $\bar{m} = -1.26$. In the unconstrained region, consumption, saving and the price of the asset are all increasing with wealth. Higher wealth raises current consumption relative to future consumption, which bids up the price of the asset.

The levels of consumption and of the asset price vary more steeply with wealth in the constrained region than in the unconstrained region, reflecting the collateral multiplier. Both consumption and the asset price fall to zero when wealth is equal to $-\psi = -1.97$. By contrast, saving w' *decreases* with wealth in the constrained region. Higher wealth is associated with an increase in the price of collateral, which relaxes the borrowing constraint on borrowers and allows them to roll over larger debts.

The right-hand panel of Fig. 3 zooms in to the wealth level where the constraint becomes binding and indicates how saving depends on the level of wealth, $w'(w)$, for the two states $y = y_L$, y_H. One can obtain the curve for the low state by shifting the curve for the high state to the right by $\Delta y = y_H - y_L$. The curves intersect the 45º line in two points, A_H and A_L, which determine the steady state levels of wealth conditional on remaining in each state, respectively denoted by w_H^{SS} and w_L^{SS}. We observe that both A_H and A_L are on the downward-sloping branches of each curve, which means that borrowers borrow to the point where they are financially constrained in both states. Furthermore, borrowers tend to borrow more in the high steady state than in the low steady state ($w_H^{SS} < w_L^{SS}$), which they can do because the price of the collateral asset is higher.

The figure also shows the dynamics of the economy when the steady state is disturbed by a one-period fall in y. At the time of the shock, the economy jumps up from point A_H to point B, as borrowers are forced to reduce their debts by the fall in the price of collateral.[5] The dynamics are then determined by the saving function in the high state (since we have assumed that the low state lasts only one period). The economy converges back to point A_H. As it approaches A_H,

---

[5] The price of the collateral asset falls by 12.3%, from 4.81 to 4.22. Thus the borrowing ceiling falls by $\phi \cdot 0.59 \approx 0.03$, which is the distance between A_H and B.
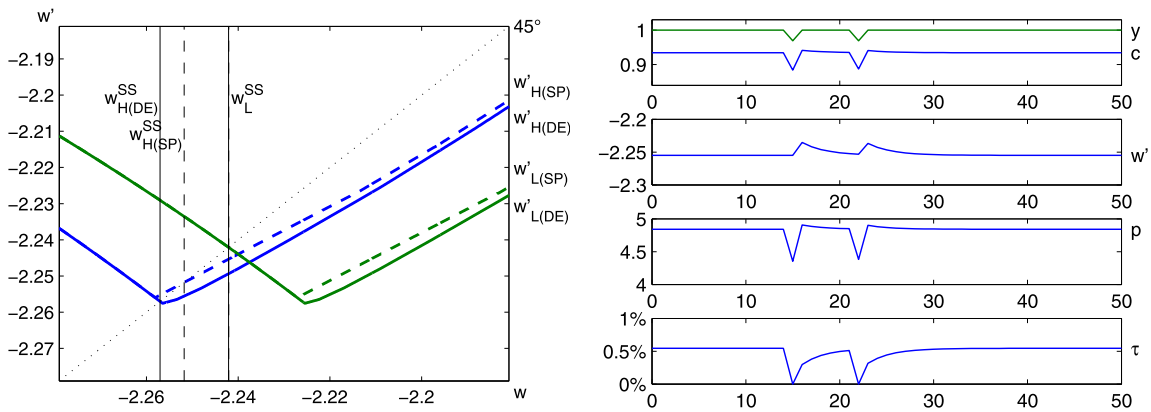
Fig. 4. Left panel: policy functions of decentralized agents and planner; right panel: sample path of y, c, w', p and τ under planner for 50 periods.

wealth follows oscillations of decreasing amplitude (creating what looks like a small black rectangle in the figure). There are oscillations because saving is decreasing with wealth in the constrained regime. There is convergence because the slope of the saving curve is larger than -1 in point A_H for our benchmark calibration. This is not true for any calibration and the equilibrium can exhibit cyclic or chaotic dynamics if ϕ is larger.

The left-hand panel of Fig. 4 shows how the social planner (dashed line) increases saving relative to laissez-faire (solid line).[6] The social planner saves more, implying that the economy has a higher level of wealth and it is no longer financially constrained in the high steady state. The $\tilde{w}'(w)$-line is closer to the 45° line with the social planner, implying that following a one-period fall in y, the economy reaccumulates debt at a lower pace than under laissez-faire. In the constrained branch, the policy functions in the laissez-faire equilibrium and under the planner differ but are so close to each other that the difference is not visible to the naked eye. This is because asset prices in the two allocations are very close to each other for given w.

The right-hand panel of Fig. 4 illustrates the dynamics of the main variables of interest in the social planner equilibrium with a stochastic simulation. The top panel shows how consumption falls at the same time as output when there is a negative shock. Even with the social planner, consumption falls by more than income because of the fall in the price of collateral. Consumption increases above its long-run level in the period after the shock, when the economy is unconstrained and borrowers inherit low debt from the credit crunch. The same pattern is observed for the price of the collateral.

The bottom-right panel shows that the optimal Pigouvian tax rate is positive in the high state and zero in the low state. The optimal tax rate in the high steady state is $\tau_H^{SS} = 0.56\%$. Following Eq. (32), it is obtained as the probability of crisis $\pi = 0.05$ times three factors, or "sufficient statistics:" (i) the collateralizability parameter $\phi = 0.046$, (ii) the normalized shadow price of the binding constraint in the low state $\beta R \tilde{\lambda} \left(m_L^{SS\prime} \right) / u'(c) = 0.134$ and (iii) the asset price response to extra net worth $\tilde{p}' \left(m_L^{SS\prime} \right) = 18$, where $m_L^{SS\prime}$ is the borrower's net wealth in the event that he was in the high steady state in one period and is hit by a low output shock (bust) in the following period. When borrowing is constrained, we have set the tax to zero. Any value $\tau \leq \lambda$, including any subsidy to debt $\tau < 0$, would result in the same allocation in that case since the equilibrium is not determined by the Euler equation but by the binding constraint.

This optimal tax rate is not very large: it reduces borrowing by only 0.5% of GDP compared to the laissez-faire allocation. Since busts are infrequent events, it is not desirable for a policymaker to reduce debt by too much—the planner does not attempt to completely avoid binding financial constraints when a bust occurs. Instead, he reduces the decline in consumption when the sector experiences a bust after a number of boom periods from -6.2% to -5.2%, and the fall in the asset price during a bust from -12.3% to -10.3%.

Another way of interpreting the magnitude of the tax is to use a back-of-the-envelope calculation to translate it into an equivalent capital requirement: if we approximate the cost of capital of financial institutions by their return on equity, which was around 15% in the decade before the financial crisis,[7] and assume that this additional capital must be held in safe assets at a return of 3%, then our Pigouvian tax of 0.56% is equivalent to an additional capital requirement of $0.56/(0.15-0.03) = 4.67\%$.

Note also the cyclical pattern of the tax rate in the figure: In a bust, the tax is zero, although, as we observed earlier, the tax rate is not pinned down uniquely when the constraint is binding. However, after the bust, when the level of the tax is uniquely defined, the tax rate does not immediately go back to the long-run level because the economy temporarily

---

[6] We have also verified that the social planner's optimal savings choice always lies in what we defined as interval $\Omega_1(m)$ in Section 2.3 above.

[7] This data is taken from the St. Louis Fed's FRED, series USROE (Return on Average Equity for all U.S. Banks), which is accessible at https://fred.stlouisfed.org/series/USROE.
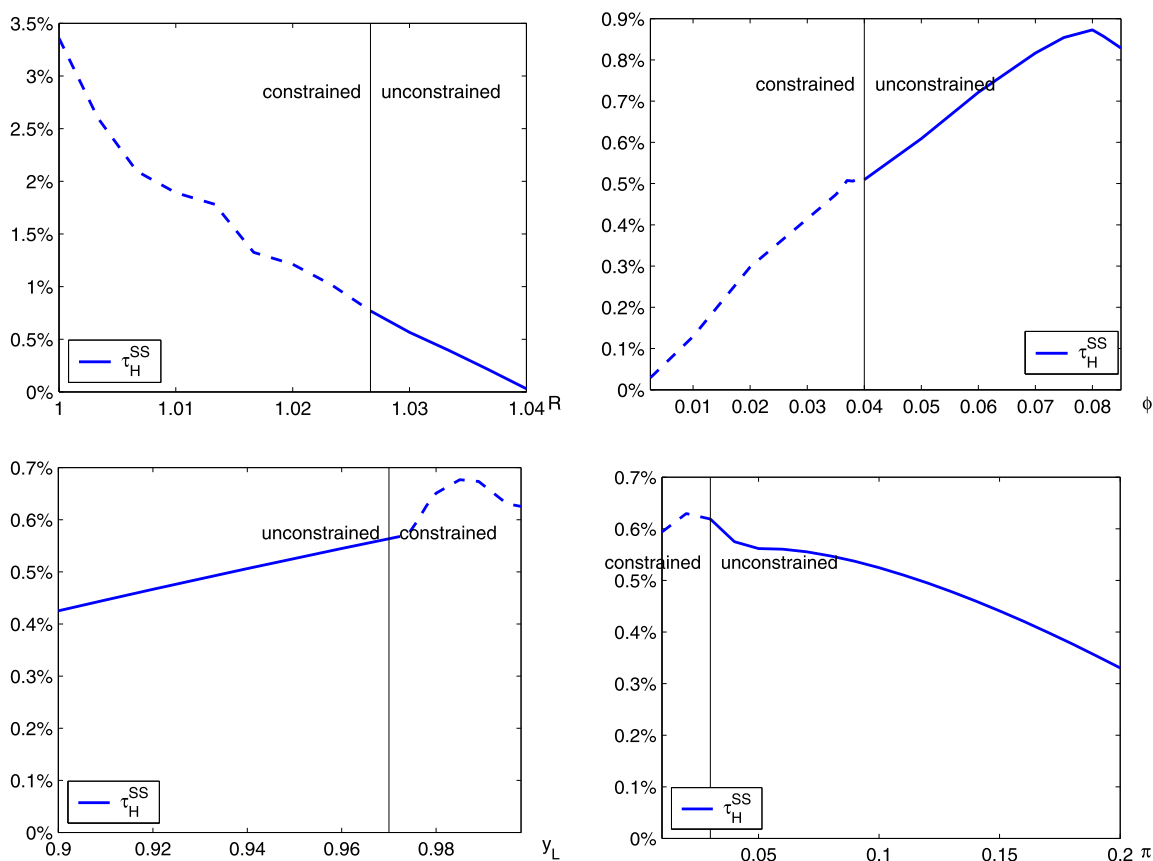
Fig. 5. Optimal macroprudential tax as a function of different parameters. Top-left panel: interest rate R; top-right panel: collateralizability parameter ϕ; bottom-left panel: magnitude of bust $1 - y_L$; bottom-right panel: probability of bust π.

has lower debt, which reduces the period-ahead risk of financial amplification. Subsequently, the tax rate increases with the sector's vulnerability to a new credit crunch.

3.3. Sensitivity analysis

We investigate how the optimal Pigouvian taxation depends on the parameters. The top-left panel of Fig. 5 shows how τ_H^{SS} (the steady state rate of tax in the high state) varies with the gross interest rate R. For $R = 1.04$, the optimal steady-state tax in the economy is close to zero since $\beta R \approx 1$ and borrowers accumulate a level of precautionary savings that is sufficient to almost entirely avoid debt deflation in case of busts.

As the interest rate declines, it becomes more attractive for borrowers to borrow and the economy becomes more vulnerable to debt deflation in busts. Lower interest rates therefore warrant higher macro-prudential taxation to offset the externalities that individual agents impose on the economy. This effect can be large: the optimal tax rate is multiplied by two when the interest rate is reduced from 2% to 1%.

For $R \leq 1.026$ (when the line is dashed in the figure), the level of debt accumulated by the social planner is high enough that the economy is constrained even in the high steady state A_H. As we discussed earlier, this means that the social planner could lower the tax rate to zero as soon as the economy becomes constrained—although he could also maintain the tax rate at the level shown by the dashed line or any level in between without changing the equilibrium. In this case, macroprudential taxation matters only in the transition: its role is to slow down the build-up of risk and financial vulnerability after a bust, and thus delay the transition to the constrained regime where the tax no longer matters.

The top-right panel of Fig. 5 depicts the response of the steady-state tax rate τ_H^{SS} to changes in parameter ϕ. An increase in ϕ means that the credit constraint becomes more sensitive to the price of collateral. We observe that the optimal tax rate increases with ϕ. A higher ϕ strengthens the potential amplification effects when the borrowing constraint becomes binding, and requires tighter macroprudential regulation.

The level of the parameter ϕ also determines whether prudential taxation is transitory or permanent. For low levels of ϕ, we find that the amplification effects are small enough that the planner chooses a constrained equilibrium in steady state and macroprudential taxation is only relevant in the transition from busts to booms.

The two bottom panels of Fig. 5 show how the optimal tax varies with the size and probability of the underlying shock. The optimal tax rate is not very sensitive to those variables: it changes by less than 0.3% when the size of the income shock varies between 0 and 10% and its probability varies between 0 and 20%. The sign of the variation is paradoxical. The bottom-left panel shows that τ_H^{SS} is increasing with y_L, i.e., the optimal rate of prudential taxation is *decreasing* with the size of the income shock. This result comes from the endogenous response of precautionary savings by private agents to increased riskiness in the economy. As the size of the shock increases, borrowers raise their own precautionary savings, which alleviates the burden on prudential taxation. The tax rate is the highest when the amplitude of the shock is the smallest, but again, these high tax rates do not bind in equilibrium if the income shock is very small (below 2 percent).

We observe a similar pattern for the variation of the optimal tax with the probability of a shock (bottom right panel of the figure). The optimal tax rate is decreasing with π because of the endogenous increase in private precautionary savings. The tax is not binding (and could be set to zero in the long run) if the probability of bust falls below 3%. Prudential taxation thus responds the most to "tail risk", i.e., a risk that is realized with a small probability, but not so small that even the social planner can ignore it in the long run. The long-run tax rate is binding and is at its maximum when the probability of shock is between 3 and 5%.

4. Extensions

We discuss three extensions of the basic model. In the first one, we assume that the indebted agents are subject to financial shocks that abruptly restrict their borrowing constraints. The second and third subsections expand the range of liabilities by assuming that the borrowers can issue equity or long-term debt.

4.1. Fluctuations in creditworthiness

It has been suggested (see e.g. Jermann and Quadrini, 2012) that the recent global financial crisis was driven more by fluctuations in the availability of credit ("financial shocks") than by developments in the real economy ("endowment shocks"). In our framework, the availability of credit is a function of the parameters ψ and ϕ in the borrowing constraint. We now assume that the economy may be hit by shocks that reduce ψ rather than y.

As in our previous calibration, we choose our parameter values to replicate the declines in credit and asset prices observed in the SME sector during the financial crisis of 2008–09. Income is now deterministic and equal to $y = 1$. The parameter ψ_H is calibrated so as to reproduce the pre-crisis debt-to-income ratio, and ψ_L is calibrated to match the observed fall in the asset price at the time of a bust. This results in a pair of values $(\psi_L, \psi_H) = (1.94, 1.97)$. The other parameters remain the same as in Table 1.

We solve for the constrained planner's problem in the model with credit shocks and find that the behavior of the model economy is very similar to the case of output shocks. Results are reported in Online Appendix A3. A planner would impose an optimal Pigouvian tax on borrowing of $\tau_H^{SS} = 0.61$ percent if the economy has reached its steady state during a boom. In a bust, the planner lowers the tax and slowly raises it back to its high steady-state value as the economy re-accumulates debt.

The general magnitude of the externality—and by implication of optimal policy measures targeted at internalizing it—therefore seems to depend not so much on the source of shocks as on the extent of amplification when the borrowing constraint becomes binding. The optimal policy measures in the economy are similar as long as we calibrate the model in a way that reproduces similar frequencies and magnitudes of crisis as our benchmark model with endowment shocks.

4.2. Debt maturity

We have observed in the data that outstanding debt fell by substantially less than asset values in the US corporate and household sectors during the financial crisis. This implied a relatively low value of ϕ in the calibration. However, the small sensitivity of outstanding debt to collateral value could be due to the fact that a substantial fraction of the debt is medium- or long-term so that the full impact of low collateral values on outstanding debt is observed over several periods. We capture this idea in a tractable way by generalizing the collateral constraint (4) as follows,

$$\frac{w_{t+1} - (1-\delta)w_t}{R} + \psi + \delta\phi p_t \geq 0. \tag{41}$$

The parameter δ represents the fraction of the debt principal that comes due in any given period, i.e., the inverse of the duration of debt. The case of short-term debt corresponds to $\delta = 1$, which gives Eq. (4). In the general case $\delta < 1$, the collateral constraint applies to the flow of new debt issued in period t.

The dynamic behavior of the economy in case of shocks is modified: when the credit constraint binds, a unit decline in the current asset price reduces debt only by a fraction $\delta\phi$ as opposed to ϕ in our benchmark model. This mitigates the debt deflation dynamics in the economy. We present the derivation of equilibrium and the numerical solution method in Online Appendices A.4 and B.2 respectively.
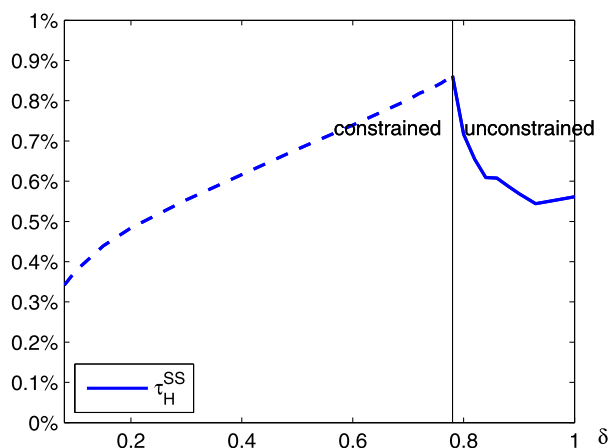
Fig. 6. Dependence of macroprudential tax on inverse debt duration δ.

Fig. 6 illustrates how the optimal steady-state tax in the high state τ_{SS}^H varies with debt duration for the parameters of our benchmark calibration as listed in Table 1. As we increase debt duration by moving leftwards in the graph from $\delta = 1$, the debt deflation effects that arise during binding constraints are mitigated. As a result, borrowers reduce their precautionary savings and the externality of a given dollar of debt at first rises. For $\delta \leq 0.78$, busts are sufficiently mild that a planner chooses not to insure against binding constraints when the steady state is reached. In this region, a planner uses macroprudential taxation only during the transition from a bust to the next boom in order to slow down the build-up of risk.

Another effect of higher debt duration (lower δ) is to make the economy more resilient in the sense of admitting higher values of ϕ without leading to multiple equilibria—long-term debt insulates borrowers against self-fulfilling panics.

4.3. Outside equity

We could assume that borrowers can sell equity rather than debt. Let us assume that the borrower can sell a claim on a share $s_t \leq \bar{s}$ of total income to outsiders. This claim will be sold to outsiders at price

$$q = \frac{E(y)}{R - 1},$$ (42)

and the budget constraint of borrowers becomes

$$c_t + a_{t+1} p_t + \frac{w_{t+1}}{R} = (1 - \alpha) y_t + a_t (p_t + \alpha y_t) + w_t - s_t y_t + (s_{t+1} - s_t) q.$$ (43)

It is easy to see that the borrowers will always sell as much equity as possible, which is a way for them to insure against their income risk (at no cost since outsiders do not require a risk premium on equity) and benefit from the greater patience of outsiders. Thus, $s_t = \bar{s}$ in every period. The introduction of equity leads to a consumption boom, but the long-run equilibrium is the same as before except that total income is reduced by the factor $(1 - \bar{s})$. Our model is homogenous of degree 1 in income y, aside from the borrowing constraint. In a sample simulation in which we set $\bar{s} = .5$, we found the optimal macroprudential tax to be $\tau_H^{SS} = 0.59\%$, which is very close to the level in our benchmark model (0.56%).

Although equity has better risk-sharing properties than debt, it is not used to reduce risk in the long run. Borrowers issue equity to increase their consumption, eventually leaving them with more liabilities and the same level of debt. Allowing borrowers to issue equity, thus, does not reduce the need for the prudential taxation of debt in the long run.

5. Related literature and final remarks

The literature about financial amplification effects has considerably expanded since this paper was first circulated (Jeanne and Korinek, 2010b) but counts a few important contributions that pre-date the global financial crisis. Bernanke and Gertler (1989) and Greenwald and Stiglitz (1993) show analytically that financial imperfections may amplify the response of an economy to fundamental shocks. Kiyotaki and Moore (1997) investigate the scope for amplification in a model of credit constraints that depend on the value of collateral assets, similar to ours. Papers that study similar constraints from a positive perspective include He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) in continuous time settings. Carlstrom and Fuerst (1997) and Bernanke et al. (1999) present an alternative mechanism of amplification based on endogenous changes in external finance premia. Gertler and Karadi (2011) describe models of financial amplification in which the financial sector is at the center stage.

On the normative side, Gromb and Vayanos (2002) have pioneered the study of externalities from financial amplification involving asset prices. This question has given rise to a considerable body of literature in recent years. Caballero and Krishnamurthy (2003), Lorenzoni (2008), Stein (2012) and Caballero and Lorenzoni (2014) have analyzed pecuniary externalities of financial amplification in stylized three-period models. These papers focus on distributive pecuniary externalities that arise because changes in asset prices change the relative terms-of-trade of buyers and financially constrained sellers of financial assets. As highlighted by Dávila and Korinek (2018), such distributive externalities can only be signed in very specific circumstances.

Our paper, by contrast, considers pecuniary externalities that arise because asset prices directly enter the financial constraints of borrowers. Our contribution is to develop and analyze the simplest possible dynamic model of consumption-based asset pricing and collateralized borrowing and characterize the pecuniary externalities that arise. We show that in such a setting, pecuniary externalities always gives rise to over-borrowing. We also show how a state-dependent Pigouvian tax on borrowing or equivalent quantity regulations may induce borrowers to internalize these externalities and increase welfare. In a related setting, Nikolov (2010) shows that, if borrowers are always sufficiently financially constrained, then a planner does not reduce debt further.

Bianchi and Mendoza (2018) also analyze pecuniary externalities in a dynamic model with price-dependent collateral constraints. As in our 2010 working paper and in this paper, they analyze a planner who takes into account that asset prices are fully determined by the optimizing decisions of private agents. Their work focuses on a quantitative description of a small open production economy with endogenous labor input and imported intermediate goods. This setup gives rise to multiple pecuniary externalities and multiple different distorted decision margins that each call for policy interventions, with potentially ambiguous signs. Our paper, by contrast, presents a model in which the optimal time-consistent policy is easy to interpret as a macroprudential tax that is shown to be non-negative and that restricts borrowing. Furthermore, we transparently address the potential for multiple decentralized equilibria and disconnected choice sets in the planner's problem in settings with price-dependent borrowing constraints. The welfare cost of the credit constraint in our framework consists of deviations from consumption smoothing – a cost that is more generic in the sense that it mirrors the costs of financial amplification in any economy with borrowers who have a concave payoff function. The optimal Pigouvian tax on debt in our numerical illustration is somewhat lower but of the same order of magnitude as theirs, suggesting that mitigating booms and busts in consumption is, in and of itself, a significant determinant of such taxes. Finally, our quantitative exercise focuses on a specific sector of the economy that engages in collateralized borrowing, reflecting the way in which macroprudential policy is targeted in practice in advanced countries, rather than on an aggregate small open economy.

Our model can be applied to the optimal taxation of international capital flows for an open economy, similar to Jeanne and Korinek (2010a), but it does not have a real exchange rate since there is only one good. Korinek (2010, 2018), Bianchi (2011) and Benigno et al. (2016) analyze models in which the collateral constraint involves the real exchange rate and characterize the optimal taxation of capital inflows for an emerging market economy.

Whereas our paper studies optimal macro-prudential regulation to reduce the cost of financial crises ex-ante, Benigno et al. (2011, 2013, 2016) and Jeanne and Korinek (2013) show that in the described models of financial amplification, there is also scope for ex-post intervention in the event of binding constraints if a planner has additional policy instruments. They show furthermore that effective ex-post interventions may allow borrowers to take on a larger quantity of debt ex-ante by reducing the necessity for precautionary savings. In the limit, if a planner has the tools required to completely undo binding financial constraints, macroprudential regulation is no longer indicated. This point has also been emphasized by Katagiri et al. (2017).

The analysis presented in this paper could be extended in several directions. First, it would be interesting to provide a numerical analysis of the case where the sensitivity of the credit constraint to the collateral price (parameter ϕ) is large enough to produce multiple equilibria and self-fulfilling asset price busts. This is the relevant case to consider if one wants to apply the model to leveraged financial institutions in systemic liquidity crises. Policies to remove the bad equilibria, such as coordinated rollovers or lending-in-last-resort, are likely to be appropriate. The optimal Pigouvian tax is likely to be higher than with the calibrations that we have considered in this paper. The optimal taxation might be implemented through the kind of countercyclical capital surcharges that are being discussed in the debates about the macroprudential regulation of banks. Schmitt-Grohé and Uribe (2016a,c) analyze models with price-dependent borrowing constraints and consider examples with multiple equilibria.

Another direction of enquiry would take into account the fact that asset price and credit busts might have a permanent negative effect on long-run output. The data suggest that output does not generally catch up with its pre-crisis trend following a financial crisis (IMF, 2009). This may be the case if the collateral constraint reduces productivity-enhancing expenditures (Ma, 2016). The welfare cost of asset price busts may be larger in this case, leading to larger welfare gains from prudential taxation in booms, and a higher optimal Pigouvian tax level.

Finally, one would like to incorporate money to the model in order to derive insights for the debate on whether and how credit and asset price busts interact with aggregate demand. In models of nominal stickiness, there is a separate objective for intervention deriving from aggregate demand externalities (see e.g. Farhi and Werning, 2016; Korinek and Simsek, 2016; Schmitt-Grohé and Uribe, 2016b). An interesting line of future research is to provide a fuller and more quantitative analysis of how these interact with the pecuniary externalities of asset price booms and busts that we study in this paper.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2018.12.005

References

Bank of England, 2011. Instruments of macroprudential policy. BoE Discussion Paper

Benigno, G., Chen, H., Otrok, C., Rebucci, A., Young, E.R., 2011. Revisiting overborrowing and its policy implications. In: Cespedes, L.F., Chang, R., Saravia, D. (Eds.), Monetary Policy under Financial Turbulence. Central Bank of Chile, pp. 145–185.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., Young, E.R., 2013. Financial crises and macro-prudential policies. J. Int. Econ. 89 (2), 453–470.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., Young, E.R., 2016. Optimal capital controls and real exchange rate policies: a pecuniary externality perspective. J. Monet. Econ. 84, 147–165.

Bernanke, B.S., Gertler, M., 1989. Agency costs, net worth and business fluctuations. Am. Econ. Rev. 79 (1), 14–31.

Bernanke, B.S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, Vol. 1C. Elsevier, Amsterdam, pp. 1341–1393.

Bianchi, J., 2011. Overborrowing and systemic crises in the business cycle. Am. Econ. Rev. 101 (7), 3400–3426.

Bianchi, J., Mendoza, E., 2018. Optimal time-consistent macroprudential policy. J. Polit. Econ. 126 (2), 588–634.

Borio, C., 2003. Towards a macroprudential framework for financial supervision and regulation? CESifo Econ. Stud. 49 (2), 181–215.

Brunnermeier, M.K., Sannikov, Y., 2014. A macroeconomic model with a financial sector. Am. Econ. Rev. 104 (2), 379–421.

Caballero, R.J., Krishnamurthy, A., 2003. Excessive dollar debt: financial development and underinsurance. J. Finance 58 (2), 867–894.

Caballero, R.J., Lorenzoni, G., 2014. Persistent appreciations and overshooting: a normative analysis. IMF Econ. Rev. 62 (1), 1–47.

Carlstrom, C., Fuerst, T.S., 1997. Agency costs, net worth, and business fluctuations: a computable general equilibrium analysis. Am. Econ. Rev. 87 (5), 893–910.

Carroll, C.D., 2006. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. Econ. Lett. 91 (3), 312–320.

Carroll, C.D., 2011. Theoretical Foundations of Buffer Stock Saving. Manuscript, Department of Economics, Johns Hopkins University.

Chetty, R., 2009. Sufficient statistics for welfare analysis: a bridge between structural and reduced-form methods. Annu. Rev. Econ. 1, 451–487.

Dávila, E., Korinek, A., 2018. Pecuniary externalities in economies with financial frictions. Rev. Econ. Stud. 85 (1), 352–395.

Farhi, E., Werning, I., 2016. A theory of macroprudential policies in the presence of nominal rigidities. Econometrica 84 (5), 1645–1704.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. J. Monet. Econ. 58 (1), 17–34.

Greenwald, B., Stiglitz, J.E., 1993. Financial market imperfections and business cycles. Q. J. Econ. 108 (1), 77–114.

Gromb, D., Vayanos, D., 2002. Equilibrium and welfare in markets with financially constrained arbitrageurs. J. Financ. Econ. 66 (2–3), 361–407.

He, Z., Krishnamurthy, A., 2013. Intermediary asset pricing. Am. Econ. Rev. 103 (2), 732–770.

Hintermaier, T., Koeniger, W., 2010. The method of endogenous gridpoints with occasionally binding constraints among endogenous variables. J. Econ. Dyn. Control 34 (10), 2074–2088.

IMF, 2009. What's the damage? Medium-term output dynamics after financial crises. In: IMF World Economic Outlook, ch.4, pp. 121–151.

Jeanne, O., Korinek, A., 2010a. Excessive volatility in capital flows: a Pigouvian taxation approach. Am. Econ. Rev. Pap. Proc. 403–407.

Jeanne, O., Korinek, A., 2010b. Managing credit booms and busts: a Pigouvian taxation approach. NBER Working Paper 16377

Jeanne, O., Korinek, A., 2013. Macroprudential regulation versus mopping up after the crash. NBER Working Paper 18675

Jermann, U., Quadrini, V., 2012. Macroeconomic effects of financial shocks. Am. Econ. Rev. 102 (1), 238–271.

Katagiri, M., Kato, R., Tsuruga, T., 2017. Prudential capital controls or bailouts? The impact of different collateral constraint assumptions. Econ. Theory 63 (4), 943–960.

Kiyotaki, N., Moore, J.H., 1997. Credit cycles. J. Polit. Econ. 105 (2), 211–248.

Korinek, A., 2010. Regulating Capital Flows to Emerging Markets: An Externality View. Manuscript, Department of Economics, University of Maryland.

Korinek, A., 2018. Regulating capital flows to emerging markets: an externality view. J. Int. Econ. 111, 61–80.

Korinek, A., Simsek, A., 2016. Liquidity trap and excessive leverage. Am. Econ. Rev. 106 (3), 699–738.

Lorenzoni, G., 2008. Inefficient credit booms. Rev. Econ. Stud. 75 (3), 809–833.

Ma, C., 2016. Financial Stability, Growth and Macroprudential Policy. Manuscript, Department of Economics, Johns Hopkins University.

Nikolov, K., 2010. Is Private Leverage Excessive? MPRA Working Paper 28407.

Schmitt-Grohé, S., Uribe, M., 2016a. Adjustment to Small, Large, and Sunspot Shocks in Open Economies With Stock Collateral Constraints. NBER Working Paper w22971

Schmitt-Grohé, S., Uribe, M., 2016b. Downward nominal wage rigidity, currency pegs, and involuntary unemployment. J. Polit. Econ. 124, 1466–1514.

Schmitt-Grohé, S., Uribe, M., 2016c. Multiple equilibria in open economy models with collateral constraints: overborrowing revisited. NBER Working Paper w22264

Shleifer, A., Vishny, R., 2011. Fire sales in finance and macroeconomics. J. Econ. Perspect. 25 (1), 29–48.

Stein, J., 2012. Monetary policy as financial-stability regulation. Q. J. Econ. 127 (1), 57–95.

Stiglitz, J.E., 1982. The inefficiency of the stock market equilibrium. Rev. Econ. Stud. 49 (2), 241–261.