Macro-Hedging for Commodity Exporters*

Eduardo Borensztein
Inter-American Development Bank

Olivier Jeanne
Johns Hopkins University

Damiano Sandri
International Monetary Fund

January 31, 2012

Abstract

This paper uses a dynamic optimization model to quantify the potential welfare gains of hedging against commodity price risk for commodity-exporting countries. We show that hedging enhances domestic welfare through two channels: first, by reducing export income volatility; and second, by reducing the country's need to hold precautionary reserves and improving the country's ability to borrow against future export income. Under plausible calibrations of the model, the second channel may lead to much larger welfare gains, amounting to several percentage points of annual consumption.

JEL No. C61, E21, F30, F40, G13

Keywords: hedging, commodity exports, precautionary savings, international reserves, futures, options, default

*We would like to thank seminar participants at a NBER workshop, the Bank of Canada, the International Monetary Fund, the SED 2010 annual meeting, the Econometric Society 2010 World Congress, and the OxCarre 2010 conference as well as two referees for useful comments. The views expressed in the paper are those of the authors and do not necessarily represent those of the Inter-American Development Bank or the International Monetary Fund.

1 Introduction

Commodity exporters can insure against fluctuations in commodity prices by accumulating assets in commodity stabilization funds, or by hedging with derivative instruments. The first strategy has costs, but reliance on markets for hedging instruments remains limited in spite of the development of those markets in the last two decades. What is the trade-off between precautionary savings and hedging? How is this trade-off affected by the development of markets for hedging instruments? What is the potential welfare gains from further developing those markets?

This paper addresses these questions using a dynamic stochastic optimization model. We consider a small open economy with a representative agent that is exposed to risk in the price of the country's commodity exports. The country can insure against this risk by accumulating foreign assets (precautionary savings) or by using hedging instruments. We calibrate the model by reference to data on oil and other key commodities and characterize the optimal accumulation of foreign assets in the absence of hedging. We then quantify the welfare gains that can be seized by selling production forward at various horizons or insuring against price reductions with put options.

Hedging enhances domestic welfare through two conceptually different channels. The first channel, of course, involves income smoothing: reduced income volatility leads to lower consumption volatility and higher welfare if the representative consumer is averse to risk. The welfare gain from this channel is of the same order of magnitude as the welfare cost of the business cycle, i.e., it is not very large. For the typical commodity exporter the welfare gain from full insurance coming through this channel amounts to a permanent increase in consumption of 0.4 percent.

The second channel involves the "external balance sheet" of the economy, i.e., how the country changes its external assets and liabilities when hedging instruments are available. Hedging reduces the need to hold foreign assets as precautionary savings against risk in export income. In addition, the model unveils a novel mechanism which can be particularly welfare-enhancing for commodity exporters. By reducing the downside risk in export income, hedging improves the country's ability to borrow abroad by allowing it to issue more default-free external debt. We find that this balance sheet channel may yield much larger welfare gains than the income smoothing channel—possibly amounting to several percentage points of annual consumption.

We explore the robustness of our results with sensitivity analysis and extensions of the model. A key parameter is the country's "impatience" or willingness to borrow abroad (which is determined by domestic preferences, the growth rate, and the world interest rate). The welfare gains from hedging are large for countries that are impatient and those gains come mainly from the relaxation of the external borrowing constraint. For "patient" countries, the welfare cost of maintaining large amounts of precautionary foreign assets is smaller, and so is the welfare gain from hedging. For the impatient countries most of the welfare gains occur in the first decade following the introduction of new hedging instruments. In the long run domestic consumption is reduced by the service of the debt accumulated in the first decade. Finally, making external debt defaultable contingent on a low commodity price is an imperfect insurance mechanism that yields only a fraction of the gains from hedging. These caveats complement but do not detract from one basic message of this paper: the welfare gains from hedging come not only from income smoothing, but also from improving the country's ability to manage its external balance sheet and issue default-free debt.

This paper belongs in the literature that applies a precautionary savings framework to analyze a small open economy's optimal accumulation of foreign assets (Ghosh and Ostry (1997); Durdu, Mendoza and Terrones (2009)). More specifically, it is related to the part of this literature that focuses on risk coming from shocks in the output or price of a commodity export. For example, Arrau and Claessens (1992) apply Deaton's (1991) model of precautionary savings to derive the optimal rules for a commodity stabilization fund. Bems and de Carvalho Filho (2011) calibrate a small open economy model to quantify the role of precautionary savings in economies with exhaustible resources.[1]

The main innovation of this paper is to consider precautionary savings jointly with hedging instruments. This allows us to estimate the welfare gains from hedging when precautionary savings provides a realistic alternative. This paper is the first, to our knowledge, to provide such estimates in a stochastic dynamic optimization model of a commodity-exporting country. A similar study has been conducted by Caballero and Panageas (2007, 2008), but those authors focus on sudden stop risk rather than commodity price fluctuations. They find that using hedging strategies is preferable to relying on reserves for self-insurance, but that the gains from hedging are fairly moderate. For example, hedging in Caballero and Panageas (2007) leads to a welfare gain equivalent to a 0.3 percent permanent increase in consumption, which is significantly lower than our estimates. This

[1] See also Engel and Valdés (2000) and our companion paper Jeanne and Sandri (2012).

is because our estimates include the large welfare gains from hedging that come from the relaxation of the external borrowing constraint.

The paper is structured as follows. Section 2 presents some stylized facts about commodity exports as a source of volatility and about reliance on hedging instruments. Section 3 describes the model. Section 4 presents the calibration as well as our estimates of the welfare gains from hedging. We discuss two extensions of the basic framework in section 5 and conclude in section 6.

2 Stylized Facts

A substantial number of developing countries heavily depend on the export of one single commodity. This is especially true for Petroleum, which is by far the most internationally traded commodity, but also for Copper, Gold and Natural Gas. Table 1 lists the countries with an average ratio of commodity net export (X) to non-commodity GDP (Y) between 2002 and 2007 of at least 10 percent.[2] We identify nine commodities and more than 40 commodity-exporting countries that satisfy this criterion, with ratios that in the case of Petroleum often exceed 30 percent and reach a maximum of 81.5 percent for Saudi Arabia.

This heavy dependence on commodity exports can cause substantial volatility in the economy. Table 2 shows that the standard deviation of commodity export income, $\sigma(\tilde{X})$, tends to be more than twice as large as that of non-commodity income, $\sigma(\tilde{Y})$.[3]

The governments and private agents of commodity-exporting countries have developed various ways of insuring themselves against commodity price risk. First, many governments have accumulated a buffer stock of assets in commodity-stabilization funds. To some extent, this asset accumulation is intended to deal with predictable exhaustion of the commodity resources, but it is also meant as precautionary savings against uncertainty in future export income. There are, however, potential drawbacks to this strategy. For example, those funds may be misused because of weak governance leading to diversion for purposes other than insurance, or because the authorities

[2] Commodity export data are from UN COMTRADE and retrieved though World Integrated Trade Solution (WITS). We use the IMF Commodity Unit product aggregates based on the SITC3 classification. GDP data are from the World Bank World Development Indicators database. We consider countries with at least 3 data points over 2002-2007.

[3] Standard deviations are computed with data from the countries listed in Table 1, starting from the first year at which $X/Y > 5$ percent. The table reports the standard deviations of the log of commodity export and non-commodity income detrended with a time trend. The standard deviations by commodities are obtained as the simple average of country volatilities.

Table 1: Countries with 2002-2007 average of commodity net export share of non-commodity-GDP above 10 percent

Commodity	Country	$X/Y(\%)$	Commodity	Country	$X/Y(\%)$
Aluminum	Mozambique	18.3	Petroleum	Saudi Arabia	81.5
	Guinea	15.6		Kuwait	73.2
	Jamaica	11.2		Gabon	63.8
Cocoa	Cote d'Ivoire	16.6		Oman	63.2
	Ghana	10.7		Brunei	55.8
Copper	Zambia	23.4		United Arab Emirates	54.1
	Mongolia	19.1		Nigeria	48.2
	Chile	19.0		Qatar	40.9
	Papua New Guinea	11.4		Venezuela	37.9
Fish	Iceland	10.4		Azerbaijan	36.6
	Maldives	10.3		Bahrain	34.8
Gold	Mali	17.7		Algeria	33.4
	Papua New Guinea	14.5		Kazakhstan	33.3
	Guyana	14.0		Yemen	30.6
	Kyrgyz Republic	11.0		Iran, Islamic Rep.	25.4
	Mongolia	10.7		Norway	17.6
Iron	Mauritania	22.2		Russian Federation	14.7
Natural Gas	Brunei	29.2		Syrian Arab Republic	14.3
	Qatar	26.5		Ecuador	13.0
	Trinidad and Tobago	20.1		Trinidad and Tobago	12.7
	Algeria	17.5		Sudan	12.1
	Bolivia	13.5	Sugar	Guyana	18.2

try to offset a permanent shock. Furthermore, the accumulation of precautionary reserves comes at the cost of reducing consumption and welfare.

Insurance against commodity price risk can also be achieved by hedging. Although markets for hedging instruments have developed significantly in recent years, economists have for some time wondered why they are not used to a greater extent (see, e.g., Caballero and Cowan (2007), Becker et al. (2007) and Powell (1989)). Figure 1 shows the average open interest position on oil futures in the New York Mercantile Exchange (NYMEX) market between 2003 and 2009. We observe that most of the hedging is limited to maturities of less than three months.[4] Furthermore, the total open

[4] The limited availability of long-term contracts does not preclude long-term hedging, which can be partially achieved by rolling forward short-maturity contracts (dynamic hedging).

Table 2: Standard deviation of the detrended log of commodity exports ($\tilde{X}$) and non-commodity GDP ($\tilde{Y}$)

Commodity	$\sigma(\tilde{X})$	$\sigma(\tilde{Y})$	$\sigma(\tilde{X})/\sigma(\tilde{Y})$
Aluminum	0.18	0.09	2.0
Cocoa	0.30	0.18	1.6
Copper	0.38	0.17	2.3
Fish	0.20	0.11	1.8
Gold	0.32	0.11	3.0
Iron	0.16	0.08	2.0
Natural Gas	0.21	0.12	1.7
Petroleum	0.34	0.16	2.2
Sugar	0.18	0.09	1.9

interest position in futures on the two largest markets for oil, the NYMEX and the Intercontinental Exchange (ICE), is currently around 2.3 billions of barrels which is less than thirty days of world production and less than six weeks of world export.[5] The limited extent of hedging is even more striking if compared with the size of proven oil reserves: the combined NYMEX and ICE open interest position is less than 0.2 percent of known oil reserves.[6]

The main purpose of this paper is to quantify the welfare gains that could be achieved by developing the use of hedging instruments to insure against commodity price risk. We believe that a correct assessment of the potential welfare gains from hedging is an important first step in understanding why hedging is currently little used. If the welfare gains are found to be small, the fact that hedging is not more extensively used should not be viewed as a puzzle. If instead hedging has a large impact on welfare, it becomes important to investigate the factors that currently prevent a larger reliance on hedging instruments and think about the policies that may relax those impediments.

[5] Similar sizes of open interest position in futures relative to world production are also reported in the IMF World Economic Outlook (2007, April) for natural gas, copper, corn, and soybeans.

[6] We underestimate the extent of hedging by leaving aside over-the-counter transactions (Campbell, Orskaug and Williams (2006)). However, open interest positions overestimate hedging by including contracts underwritten by speculators without a direct exposure to oil prices. The U.S. Commodity Futures Trading Commission reports that around 30 percent of the open interest positions on the NYMEX are held by speculators, and this share is likely to be higher on the ICE market which is subject to less stringent regulation.

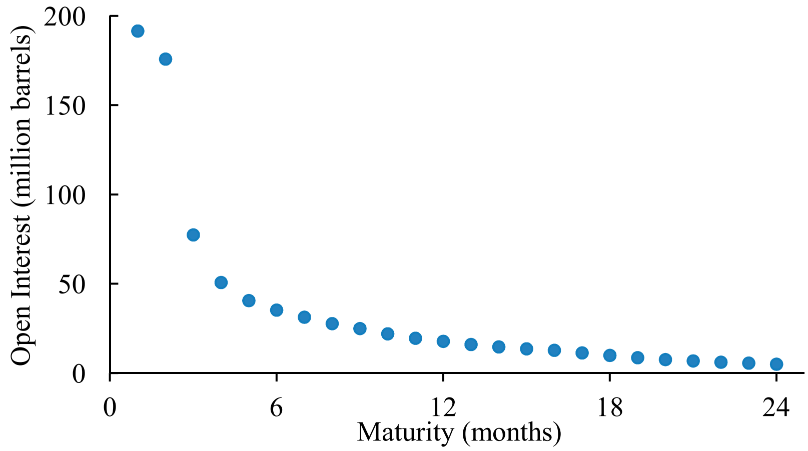

Figure 1: Average open interest and risk premium (NYMEX July 03 - May 09)

3 The model

We consider a small open economy that is exposed to shocks in the price of the commodity that it exports. We first present the assumptions of the model with precautionary savings but no hedging (section 3.1) and then introduce the possibility of selling forward (section 3.2).

3.1 No hedging

We consider a small open economy producing an exportable good (the commodity) and populated by a representative infinitely-lived consumer. The consumer derives utility from consuming another good that we will call the consumption good. The consumption good will be used as numeraire. The representative consumer maximizes his utility

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \frac{C_{t+i}^{1-\gamma}}{1-\gamma}, \tag{1}$$

subject to the budget constraint

$$B_{t+1} = (1+r)(Y_t + X_t + B_t - C_t), \tag{2}$$

where B_t is the consumer's holding of foreign assets at the beginning of period t, r is the risk-free world interest factor, Y_t is the domestic output of consumption good (non-commodity GDP), and X_t is the level of export income expressed in terms of consumption good.

In order to better focus on the consequences of uncertain export income, we assume that the domestic output of consumption good is deterministic, and grows by a factor G in every period. Thus, the budget constraint can be rewritten in units of non-exportable income as

$$b_{t+1} = \overbrace{\left(\frac{1+r}{G}\right)}^{R}(1 + x_t + b_t - c_t) \tag{3}$$

where lower-case variables are normalized by Y_t, for example $b_t = B_t/Y_t$.

The stochastic dynamics of the economy are driven by the normalized export income, x_t. Since we are interested primarily in the insurance of the price risk, we abstract from the quantity risk and assume that the country exports a constant normalized volume q of commodity.[7] Without hedging, thus, one has

$$x_t = qp_t,$$

where p_t is the spot price of the commodity. We assume that the spot price follows a multiplicative error process:

$$p_{t+1} = (p + \rho(p_t - p))\varepsilon_{t+1}, \tag{4}$$

where p without time subscript is the average price, $\rho \leq 1$ is the persistence of the price process, and ε_t is an i.i.d. non-negative random variable of mean one defined over the support $[\underline{\varepsilon}, \infty)$.[8] When estimating the price process and solving the optimization problem we will assume that ε_t is log-normally distributed so that $\underline{\varepsilon} = 0$, but for the sake of generality we now describe the model allowing also for a positive $\underline{\varepsilon}$.

We assume that the domestic consumer can borrow only against future export income and we rule out the possibility of default.[9] Therefore, to ensure solvency the debt level can never be larger than the minimum present discounted value of future export income. More formally the country is

[7] Bems and de Carvalho Filho (2011) find that price variations account for $85 - 90$ percent of 1-year changes in oil and gas revenues.

[8] We do not use an AR1 in levels to rule out negative prices. We also prefer to avoid working with an AR1 in logs, which complicates the computation of conditional expectations and the pricing of futures contracts.

[9] We allow for default in section 5.2 in order to assess its potential as an alternative insurance mechanism.

subject to the no-default constraint:

$$\frac{b_{t+1}}{R} \geq -\min_{t} \left(\sum_{i=1}^{+\infty} R^{-i} x_{t+i} \right) = -\Phi_t, \tag{5}$$

where the minimum is taken at time t over all the possible paths $(x_{t+i})_{i \geq 1}$. Let us define the operator $\underline{E}_t p_{t+i}$ which given the price level at t returns the lowest possible price at $t+i$. This can be computed from equation (4) by assuming that the shock ε is equal to its worst realization $\underline{\varepsilon}$ in every period between t and $t+i$

$$\underline{E}_t p_{t+i} = \underline{p} + (\underline{\varepsilon}\rho)^i (p_t - \underline{p}), \tag{6}$$

where $\underline{p} = [1 - (1-\underline{\varepsilon})/(1-\rho\underline{\varepsilon})]\, p$ is the lower bound to which p_t converges if the price shock is always equal to $\underline{\varepsilon}$. In the absence of hedging, the maximum borrowing ability of the country is therefore given by :

$$\Phi_t = q \sum_{i=1}^{+\infty} R^{-i} \underline{E}_t p_{t+i}, \tag{7}$$

which, using (6), can be written as a linear function of the current price p_t,

$$\Phi(p_t) = q \left[\frac{\underline{p}}{R-1} + \frac{\underline{\varepsilon}\rho}{R - \underline{\varepsilon}\rho} (p_t - \underline{p}) \right]. \tag{8}$$

It will be convenient to use as state variable in the consumer's optimization problem the country's "total current wealth", defined as the sum of the country's external wealth and current income

$$w_t \equiv 1 + x_t + b_t.$$

As can be seen using (3) and (5), period-t consumption must be lower than the country's total current wealth plus its pledgeable future export income,

$$c_t \leq w_t + \Phi(p_t). \tag{9}$$

The country's consumption/saving problem can thus be expressed in Bellman form as

$$v(w_t, p_t) = \max_{c_t} \left\{ u(c_t) + \beta G^{1-\gamma} E_t v(w_{t+1}, p_{t+1}) \right\}, \tag{10}$$

subject to (4), (9) and the transition equation for wealth,

$$w_{t+1} = 1 + R(w_t - c_t) + q p_{t+1}. \tag{11}$$

We assume the impatience condition

$$\frac{\beta(1+r)}{G^{\gamma}} < 1, \tag{12}$$

which, as discussed in Carroll (2008), prevents degenerate solutions with infinite accumulation of foreign assets.[10] We derive the policy function for consumption, $c(w_t, p_t)$, numerically by policy function iteration as explained in appendix B. The dynamics of w_t are then obtained from the intertemporal budget constraint (11) with $c_t = c(w_t, p_t)$. The dynamics of the net foreign asset position b_t are derived by netting out current income $1 + x_t$. As discussed in detail by Jeanne and Sandri (2012), the model features a target net foreign asset position, $\hat{b}$, towards which the economy tends to converge over time.[11] The target could be positive or negative. It is increasing with the variance in export income and with the domestic consumer's patience.

3.2 Selling forward

In the absence of hedging, the country has to sell its commodity production at the spot price and is therefore fully exposed to price fluctuations. In this section we solve the country's optimization problem if the exports can be sold forward. More specifically, we allow the country's representative consumer to sell exports θ periods ahead at the conditional expected price given by the operator,[12]

$$F(p_t, \theta) = E_t(p_{t+\theta} | p_t) = p + \rho^{\theta}(p_t - p). \tag{13}$$

[10] This condition is sufficient if $\rho < 1$. A stronger restriction is required if $\rho = 1$.

[11] Formally the target is defined as the level of b_t at which $E_t b_{t+1} = b_t$. For a formal proof of the existence of this target see Carroll (2008).

[12] We should emphasize that θ represents the hedging horizon and not the maturity of the underlying hedging instruments. Hedging at horizon θ can be achieved using futures with shorter maturities by rolling the hedge forward.

Note that there is no risk premium. Introducing a risk premium makes the optimization problem intractable if we allow the country to sell a different fraction of its export income in each period.[13] Since commodity importers are also interested in reducing commodity price volatility and could act as counterparts to these contracts, a zero risk premium seems a sensible benchmark to use. We will report in section 4.3 the welfare gains from hedging when the country fully hedges and hedging involves a cost that is proportional to the export income.

Selling forward improves the welfare of the representative consumer through several channels. The first channel is that hedging reduces the volatility in export income. Hedging allows the country to sell its commodity exports at the conditional expected price, $(p + \rho^{\theta}(p_{t-\theta} - p))$, rather than at the spot price, p_t. Income volatility, thus, is reduced by the factor ρ^{θ}. The associated benefits are conceptually similar to the welfare gains from eliminating the business cycle, as measured by Lucas (1987).

Hedging brings other welfare gains through changes in the country's external assets and liabilities (the "balance-sheet channel"). First, decreasing the volatility of income allows the country to decrease its stock of precautionary savings, and increase the present discounted utility value of domestic consumption without making any sacrifice in terms of consumption volatility. Second, hedging relaxes the external borrowing constraint. This effect is theoretically more subtle than the other ones but – as we will see – has a larger impact on domestic welfare.

The optimization problem with hedging can be solved without complicating the definition of the state in the following way. First, we redefine total current wealth w_t to include the present discounted value of the export income that has been sold forward (in the current period or in the past),

$$w_t = 1 + b_t + \sum_{i=0}^{\theta} R^{-i} x_{t+i}. \tag{14}$$

Since $x_{t+i} = qF(p_{t+i-\theta}, \theta)$ for $i \geq \theta$, the transition equation for wealth becomes

$$w_{t+1} = 1 + R(w_t - c_t) + qR^{-\theta} F(p_{t+1}, \theta). \tag{15}$$

[13] With a positive risk premium, we would have to solve for the share of production that the country wants to sell forward at each available maturity, thus enlarging the choice set. The state space would become much larger since we would need to keep track of the shares of production that have been sold forward in each past period.

In each year total current wealth increases by the present discounted value of the commodity export that is sold θ periods ahead, rather than by current export valued at the spot price.

The external credit constraint, then, is still given by equation (9) if Φ_t is redefined as the minimum present discounted value of the export income that will be sold forward in the future,

$$\Phi_t = \min_t \left(\sum_{i=\theta+1}^{+\infty} R^{-i} x_{t+i} \right). \tag{16}$$

Like before, the current price p_t contains all the information that is needed to compute Φ_t. The time-$(t+i)$ exports are sold forward at time $t+i-\theta$ and at price $F(p_{t+i-\theta}, \theta)$. The minimum possible value for this price is $F(\underline{E}_t p_{t+i-\theta}, \theta)$. Using (6) and (13) to substitute out $F(\underline{E}_t p_{t+i-\theta}, \theta)$, we have,

$$\begin{aligned} \Phi_t &= q \sum_{i=\theta+1}^{\infty} R^{-i} F(\underline{E}_t p_{t+i-\theta}, \theta), \\ &= \frac{q}{R^\theta} \left[\frac{(1-\rho^\theta)p + \rho^\theta \underline{p}}{R-1} + \frac{\varepsilon \rho^{\theta+1}}{R - \varepsilon\rho} (p_t - \underline{p}) \right], \end{aligned}$$

which defines function $\Phi(\cdot)$. Note that the new definitions for w_t and Φ_t are consistent with and generalize those used in the case without hedging, which is obtained for $\theta = 0$.

The country's consumption/saving problem can be expressed in Bellman form as (10), subject to the constraints (4), (9), and (15). The equilibrium can then be solved for numerically in the same way as in the case without hedging.

In order to see how hedging relaxes the country's external borrowing constraint, consider the situation of a country that unexpectedly starts to hedge at time t. Then the country's maximum level of consumption, $w_t + \Phi_t$, increases by,

$$\sum_{i=1}^{\theta} R^{-i} \left[F(p_t, i) - \underline{E}_t p_{t+i} \right] + \sum_{i=\theta+1}^{+\infty} R^{-i} \left[F(\underline{E}_t p_{t+i-\theta}, \theta) - \underline{E}_t p_{t+i} \right] \geq 0. \tag{17}$$

This expression is positive because every term in the sum is positive as can be readily seen from (6) and (13). Hedging allows the country to secure a higher minimum level of export income, and so to issue more default-free debt. The minimum level of export income is increased by hedging because the exports that are sold forward are valued at the expected price rather than at the lowest possible spot price that can be later obtained.

4 The welfare gains from hedging

In this section we calibrate the model and quantify the welfare gains from hedging at various horizons. We then explore the sensitivity of our results to the key parameters and report the welfare gains from hedging for specific commodities.

4.1 Calibration

Our benchmark calibration is reported in Table 3. The values of r, γ, and β are standard in the literature. We choose a deterministic growth rate of 1 percent and will discuss the sensitivity to this parameter in the next section. The impatience condition (12) is satisfied because of the positive growth in domestic income, against which the domestic consumer would like to borrow.

Table 3: Benchmark calibration

r	γ	β	G	ρ	σ	qp
0.04	2	1.04^{-1}	1.01	0.9	0.2	0.2

The persistency and volatility of the price shocks are calibrated by looking at the time series properties of commodity prices. We estimate the AR1 process with multiplicative error term in equation (4) for the 9 commodities that were listed in Table 1.[14] We assume that the error term is lognormally distributed, and perform the estimation using Maximum Likelihood as described in Appendix C. The results are reported in Table 4 together with the average ratios of commodity exports to non-export GDP, qp.

As previously reported in the literature (Cashin, Liang and McDermott (2000)), we find that shocks to commodity prices are very persistent, with autocorrelation coefficients usually above 0.85. Our benchmark value for the persistence parameter, 0.9, is somewhat higher than the average level of ρ across commodities (0.87), but lower than the estimated persistence for oil. We find that the year-to-year volatility in the price varies from 11 to 34 percent depending on the commodity: in

[14] We used annual commodity price data taken from the International Financial Statistics and deflated by the export unit value for industrial countries.

Table 4: Calibration by commodity

Commodity	ρ	σ	$qp(\%)$
Aluminum	0.72	0.14	15.0
Cocoa	0.87	0.23	13.6
Copper	0.89	0.20	18.2
Fish	0.87	0.11	10.3
Gold	0.89	0.15	13.6
Iron	0.98	0.14	22.2
Natural Gas	0.89	0.28	21.4
Petroleum	0.94	0.23	38.0
Sugar	0.74	0.34	18.2

the calibration we choose $\sigma = 0.2$, close to the cross-commodity average as well as the volatility of oil prices. Finally, we set the ratio of commodity export to non-commodity GDP to 20 percent, close to the average across commodities.

4.2 Benchmark results

We measure the welfare gains from hedging by comparing the utility of a consumer who does not use forward contracts with the utility of a consumer who sells domestic exports forward at horizon $\theta > 0$. We assume that in both cases the consumer starts with the target level of net foreign assets conditional on no hedging, $b_1 = \hat{b}$ and that the commodity price is initially at the average level $p_1 = p$).[15] As common in the literature, we express the welfare gains from hedging as the permanent percentage increase in annual consumption that yields the same welfare gain in the economy without hedging. More formally, the welfare gain from hedging is the α that solves:

$$(1+\alpha)^{1-\gamma} v(w(\hat{b}, p), p) = v^f(w^f(\hat{b}, p), p), \tag{18}$$

where total current wealth is denoted by $w^f(\hat{b}, p)$ and $w(\hat{b}, p)$ in the equilibria with and without hedging respectively. We use the superscript "f" to differentiate the equilibrium with forward

[15] The welfare gains are slightly smaller for countries with net foreign assets above target since they can better smooth price fluctuations in the absence of hedging. We can also obtain an unconditional measure of the welfare gain by taking the average of the gain over a large number of realizations of the states. We have verified that these two alternative measures give almost identical values for the gains from full hedging.

hedging.

Figure 2 shows how the welfare gains from hedging varies with the horizon θ. Selling the country's commodity export one-year forward yields welfare gains equivalent to a permanent increase in consumption of more than one percent. Extending the hedging horizon to five years multiplies these gains by more than three. The gains from hedging are marginally decreasing with the horizon, and selling forward at ten years provides most of the benefits.

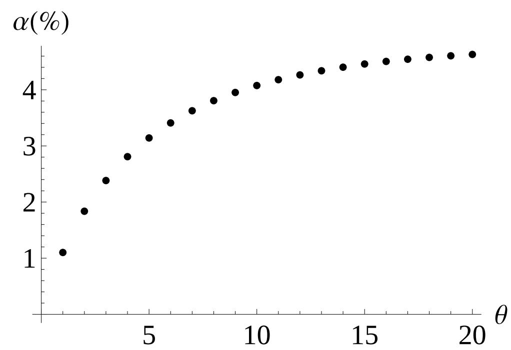

Figure 2: Full welfare gains (allowing for the optimal reduction in b_t)

As already discussed in section 3.2, hedging improves welfare through two conceptually different channels: first, by reducing the volatility of income; and second, by allowing the country to reduce its net foreign asset position. In order to measure the welfare gain from the first channel, we use the smoother income process allowed by hedging but keep the same consumption function (and so the same target level of assets) as in the absence of hedging. We are therefore exclusively assessing the gains from reducing income variance without allowing for the optimal depletion of the net foreign asset position. We use an upper bar to denote variables related to this exercise.

Figure 3 shows the welfare gains from income smoothing as a function of the hedging horizon. We observe that the gains are about ten times smaller than in Figure 2. Selling the export income one-year forward yields relatively small welfare gains, equivalent to a permanent increase in annual consumption of less than one-tenth of a percent, similar to the benefits from eliminating the business cycle as quantified by Lucas (1987).[16] Even extending the hedging horizon to twenty years provides the country with limited gains, less than one half of a percent.

The reason that the welfare gains from hedging are much larger with endogenous net foreign

[16] Pallage and Robe (2003) have found business cycle fluctuations to be more costly in developing countries than in advanced economies because their economies are more volatile.

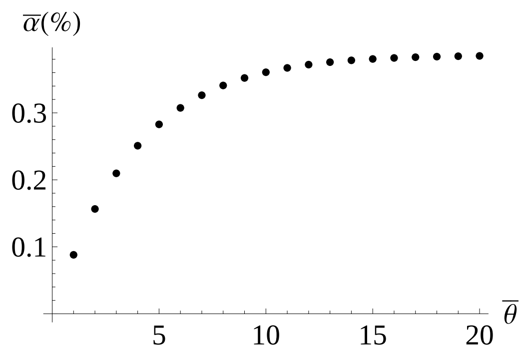

Figure 3: Welfare gains from consumption smoothing only (holding constant the target level $\hat{b}$)

assets is that securing the future export price allows the country to increase consumption by issuing debt. This can be seen from the left panel of Figure 4, which shows how the consumption function varies with hedging.[17] The curve labeled with $\theta = 0$ corresponds to the case without hedging, whereas the other curves correspond to the cases in which the country can sell its exports forward at horizons of 1, 3 and 10 years and infinity respectively. We observe that the ability to sell forward increases consumption, which is financed by depleting the net foreign asset position. The right-hand side panel shows how the target for net foreign assets decreases with the hedging horizon, from zero when there is no hedging to a debt of about 80 percent of income for one-year hedging, and amounting to five years of income when selling twenty-year forward.[18]

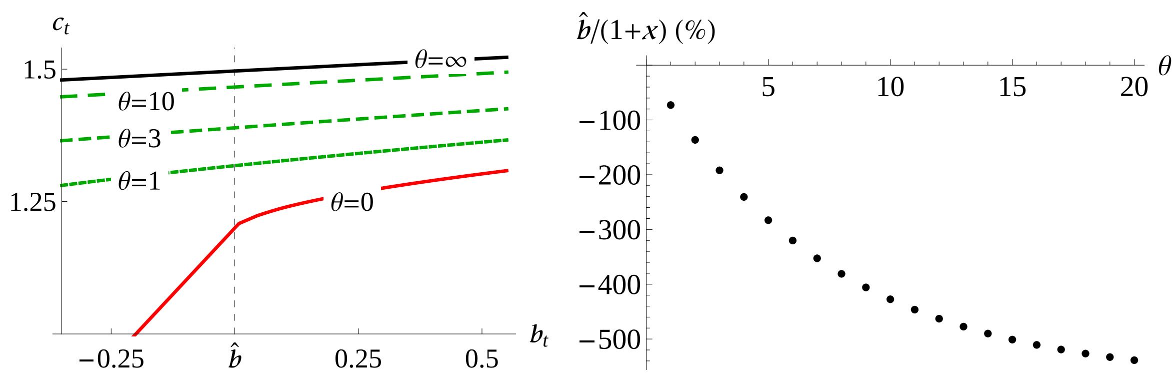

Figure 4: Consumption functions and target net foreign asset position

The fact that the introduction of hedging leads to a decrease in foreign assets raises interesting questions about the distribution of the welfare gains over time. Figure 5 shows, on the left-hand

[17] Figure 4 shows the consumption function assuming that the commodity price is equal to its average, p.

[18] Without hedging, the country cannot be a net debtor because the process (4) implies that future export income could fall to zero with a nonzero probability.

side, the dynamics of the net foreign asset position in the absence of price shocks when the country starts selling forward at one, five and ten years. On the right-hand side, we plot the change in consumption induced by hedging computed j years after the adoption of hedging.

The impact of hedging on domestic consumption is positive over the first ten years, but turns negative after that due to the service of foreign debt. Consumption is reduced by more than 5 percent forty years after the country has started to sell forward at 5 years. This suggests that in an overlapping-generations model, expanding the hedging possibilities would increase the welfare of current generations at the cost of future generations.[19]

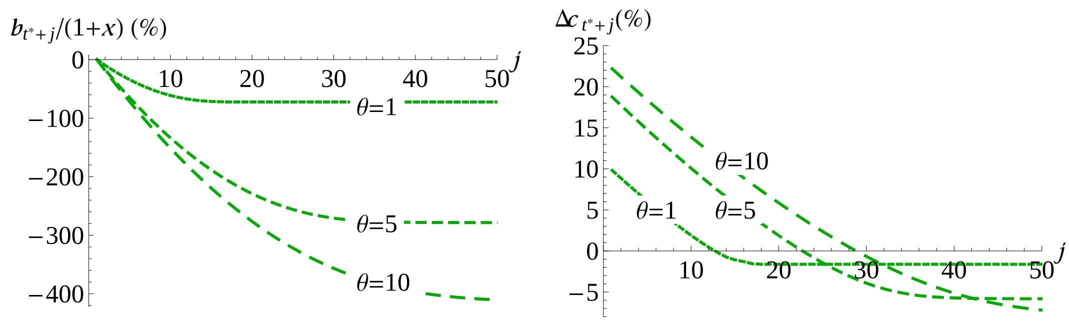

Figure 5: Dynamics of net foreign assets and consumption following the introduction of hedging

To sum up, the welfare gains from hedging export income may be much larger than the welfare costs of the business cycle. This is because hedging does more than simply smoothing domestic income: it also relaxes the country's external borrowing constraint, which may yield a large welfare gain if the domestic consumer is impatient.

4.3 Sensitivity analysis

Under the benchmark calibration, the model leads to three main results. First, the welfare gains from selling forward can be large. Second, most of the welfare gains come from the relaxation of the borrowing constraint. Third, medium-term hedging provides most of the gains. We now discuss the sensitivity of these results to alternative parameter values.

We investigate first the role of the discount factor and of the growth rate, with and without allowing for changes in foreign assets. These are important parameters since they alter the con-

[19]Indeed, our model can be interpreted as one with overlapping generations and perfectly altruistic (but impatient) parents.

sumer's level of impatience and thus his desire to borrow against future income. The top-left panel of Figure 6 shows that as we make the consumer more patient by increasing the value of β, the country targets in the absence of hedging a larger net foreign asset position as a percentage of GDP. As β approaches G^{γ}/R the impatience condition (12) becomes an equality and the foreign asset target goes to infinity.

Intuitively, a higher degree of patience should reduce the utility cost of holding precautionary assets and so should also reduce the welfare gains from substituting those assets by hedging. The top-right panel of Figure 6 confirms that this is indeed the case. The figure shows how the welfare gains from hedging at an infinite horizon vary with β under two assumptions: if the country does not adjust its foreign assets target ($\overline{\theta} = +\infty$) and if it does ($\theta = +\infty$). As β increases, the welfare gains coming from the balance sheet channel, measured by the gap between the two curves, shrinks and converges to zero as β comes close to its upper bound. This is so even though hedging reduces the level of precautionary foreign assets by a very large amount when β is large. The large reduction in foreign assets does not lead to large welfare gains because the consumer is patient and does not suffer much from postponing consumption. The main lesson from Figure 6 is that hedging yields large welfare gains if the domestic consumer is impatient and that the large welfare gains come from relaxing the consumer's external borrowing constraint.

The bottom panels of Figure 6 show the variations of the target level of foreign assets and of the welfare gains from hedging with respect to the growth rate. Increasing the growth rate has the same effect as decreasing patience since this makes the consumer more willing to borrow against future income. Thus the welfare gains from relaxing the borrowing constraint increase with the growth rate. In other words, fast growing emerging markets are the economies that benefit the most from hedging because they can better smooth consumption intertemporally by borrowing abroad.

We now consider the sensitivity of the results to the price process. The left-hand side plot of Figure 7 shows that under the benchmark calibration, changing the persistence of the price process, ρ, does not affect the target level for foreign assets in the absence of hedging, which remains equal to zero. The middle plot displays the welfare gains from moving to full insurance respectively with and without allowing the country to borrow. We observe that while the welfare gains from a smoother income are relative small, they become significant if the income shock is very persistent. This is due to the fact that the unconditional variance of the price process increases with its persistence.

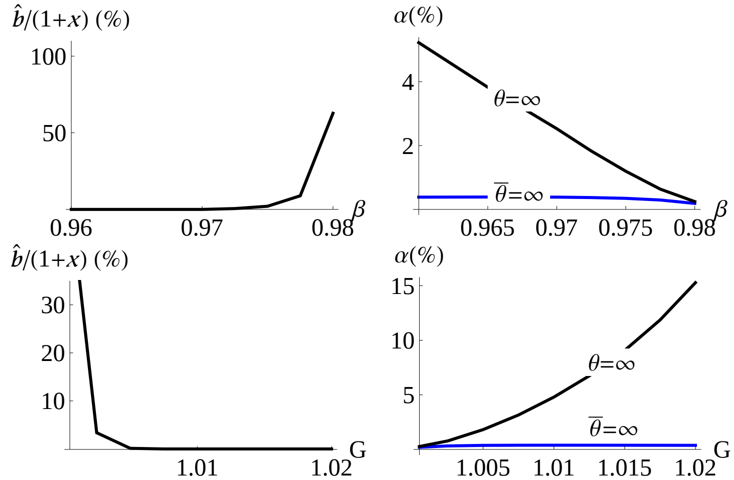

Figure 6: Welfare gains as a function of discount factor and growth rate

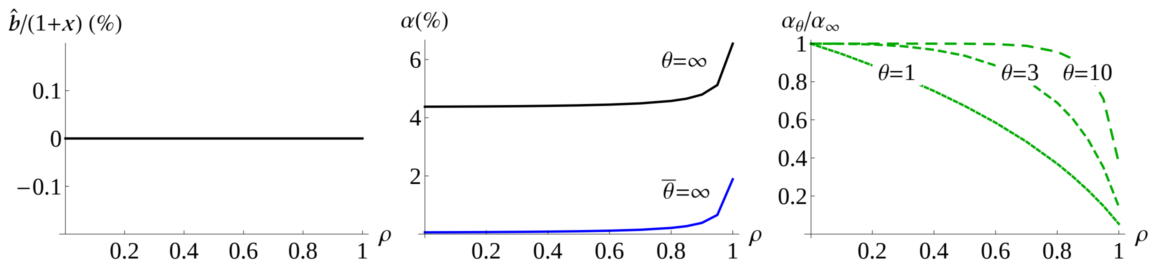

Figure 7: Welfare gains as a function of the shock persistency, ρ

Finally, the right-hand side plot shows the fraction of the welfare gains from full insurance that can be obtained with hedging at finite horizons. If the shocks are fully transitory ($\rho = 0$), complete insurance can be achieved with one-year forward contracts. As the persistence of the shock increases, short-term hedging yields a lower share of the maximum welfare gains.

The sensitivity of the welfare gains to the standard deviation of the shock in export income is shown in Figure 8. In the absence of hedging, higher volatility leads to a larger stock of precautionary savings. The welfare gains from moving to full insurance are also increasing with the volatility of the price innovation, and the gains from income smoothing become significant for high levels of income volatility. By contrast, the welfare gains from increasing the horizon of hedging are not sensitive to the level of income volatility.

The large welfare gains come in part from our assumption that hedging is costless (in particular, the country does not have to pay a risk premium on the forward contracts). Given that both producers and consumers have an interest in avoiding volatility in commodity prices, risk premia

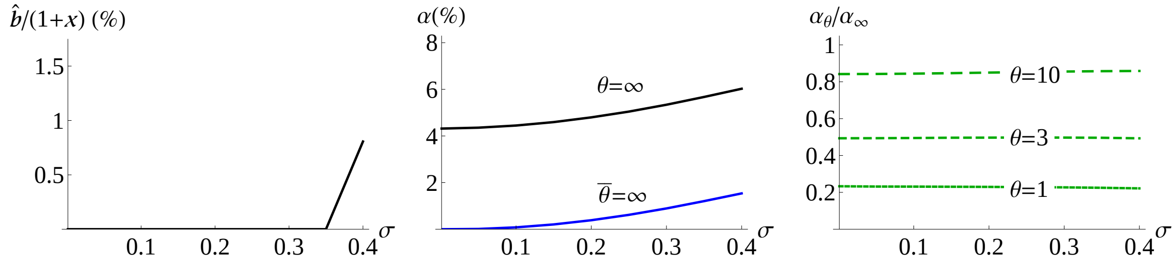

Figure 8: Welfare gains as a function of the shock variance, σ

in well developed hedging markets may indeed be fairly small. However, hedging is currently not costless in the real world. For example, the risk premium on the one-month oil futures in the New York Mercantile Exchange (NYMEX) market amounted to 1.7 percent on average between 2003 and 2009.[20] Hedging at longer horizons using dynamic hedging strategies would entail a larger cost.

In order to see how our results depend on the cost of hedging, we have solved for the welfare gains from hedging under different costs of hedging using the benchmark calibration in Table 3. In order to keep the problem tractable, we assume that the country still fully hedges its export income but must pay a fraction of the export income as a risk premium or a transaction cost. Figure 9 shows that the welfare gains from hedging may remain positive even if the cost of hedging is substantial. At the ten-year horizon, for example, the net gains from hedging become zero only if the cost of hedging exceeds 21 percent.

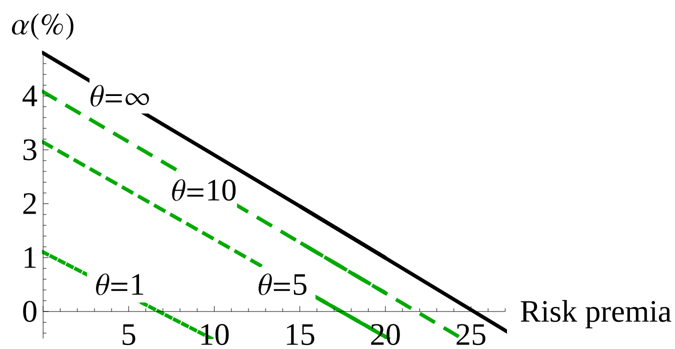

Figure 9: Variation of welfare gains from full hedging with the cost of hedging (in percent of export income)

[20]The risk premium is computed as the average ex-post forecast error over the sample period. Lower risk premia are generally found using a longer time sample (Pagano and Pisani (2009)). Over the last five years oil prices have been mostly rising and therefore part of the gap between spot and futures prices can be due to a peso effect.

4.4 Welfare gains by commodities

Countries typically export one main commodity, which makes it interesting to see how the welfare gains from hedging depend on the commodity that is exported. Table 5 reports the welfare gains from hedging for 9 different commodities based on the parameters from Table 4. The first two columns report the benefits of full insurance, respectively with and without adjustment of the target level for foreign assets. The subsequent columns report the gains reaped with hedging at various horizons (1, 3, 5, 10 and 20 years).

Table 5: Welfare gains by commodity

Commodity	α_∞	$\bar{\alpha}_\infty$	α_1	α_3	α_5	α_{10}	α_{20}
Aluminum	3.8	0.0	1.7	2.9	3.4	3.8	3.8
Cocoa	3.8	0.2	1.0	2.1	2.7	3.4	3.8
Copper	4.5	0.3	1.1	2.3	3.0	3.9	4.4
Fish	3.1	0.0	0.7	1.6	2.1	2.7	3.0
Gold	3.7	0.1	0.9	1.9	2.4	3.2	3.6
Iron	5.4	0.7	0.5	1.2	1.8	2.9	4.0
NaturalGas	5.4	0.8	1.3	2.8	3.7	4.7	5.2
Petroleum	7.8	2.1	1.4	3.2	4.4	6.1	7.3
Sugar	4.6	0.4	2.0	3.5	4.1	4.5	4.6

We observe that the welfare gains from hedging are large for petroleum, ranging from 3 percent to almost 8 percent of consumption. As discussed in section 4.3, the benefits are increasing with the persistence and variance of the price shocks and are obviously greater for countries whose commodity export income accounts for a larger share of total GDP. Countries exporting commodities with low shock persistence, such as aluminium, reap most of the gains by hedging at a relatively short horizon, while exporters of commodities subject to more persistent shocks, like petroleum, derive large welfare benefits from extending the horizon of hedging.

5 Extensions

We present two extensions of the model. In the first one, the country hedges against the *downside* risk in the commodity price by using options. In the second extension, we allow for defaultable debt and study its ability to act as an insurance mechanism when default occurs in response to large negative shock prices.

5.1 Hedging against downside risk with options

Selling forward prevents the country from benefiting from an increase in the price of the commodity. Governments may be reluctant to adopt a hedging strategy that may look ex post, in the eyes of the public, like a money-losing speculative position. A politically more viable alternative may be to insure exclusively against the downside risk, for example by purchasing put options that pay off only if the price of the commodity falls below the strike price.

Consistently with our treatment of hedging by selling forward, we assume that the country is given the possibility to insure θ periods in advance against the risk that the commodity price will fall below a threshold ζp. More specifically, the country can purchase put options at price $\xi_\theta(p_t)$, where θ is the maturity of the option and p_t the current price. We compute these prices assuming that that there is no risk premium, so that the country will always find it optimal to fully cover its export income at the maximum hedging horizon θ. When the country begins to use options, its total financial wealth falls by the amount spent to purchase the options

$$w_t = 1 + b_t + x_t - q \sum_{i=1}^{\theta} \xi_i(p_t), \tag{19}$$

while the minimum present discounted value of export income increases thanks to the floor on prices guaranteed by the options. At any future time $t+i$, the country will earn at least $q \max(\underline{E}_t p_{t+i}, \zeta p)$ and pay at most $q\xi_\theta(\underline{E}_t p_{t+i})$ to purchase new options. This implies that

$$\Phi_t = q \sum_{i=1}^{\infty} R^{-i} \left[\max(\underline{E}_t p_{t+i}, \zeta p) - \xi_\theta(\underline{E}_t p_{t+i}) \right]. \tag{20}$$

The maximization problem can still be expressed in Bellman form as (10), subject to the constraints (9) and (4) and the following new transition equation for current wealth

$$w_{t+1} = 1 + R(w_t - c_t) + q \left[\max(p_{t+1}, \zeta p) - \xi_\theta(p_{t+1}) \right]. \tag{21}$$

We compute the welfare gains from hedging with options at different maturities and strike prices using the same procedure as with forward contracts and under the benchmark calibration in Table 3. The results are shown in Figure 10. The dotted lines correspond to the case of strike prices at respectively 40, 60 and 80 percent of the average price. As shown in the left-hand side panel, options allow the country to issue more default-free debt by limiting the downside risk in export income. The welfare gains (plotted on the right-hand-side panel) are increasing with the hedging horizon and strike price.

For comparison, the continuous line reports the results obtained by selling forward. Selling forward leads to larger gains by hedging risk on both sides. However, with an hedging horizon shorter than five years, most of the welfare gains from selling forward can be captured with options, even relatively inexpensive options with low strike prices. This is no longer true in the case of long-term hedging, for which high strike prices are required to make options a good substitute to selling forward.

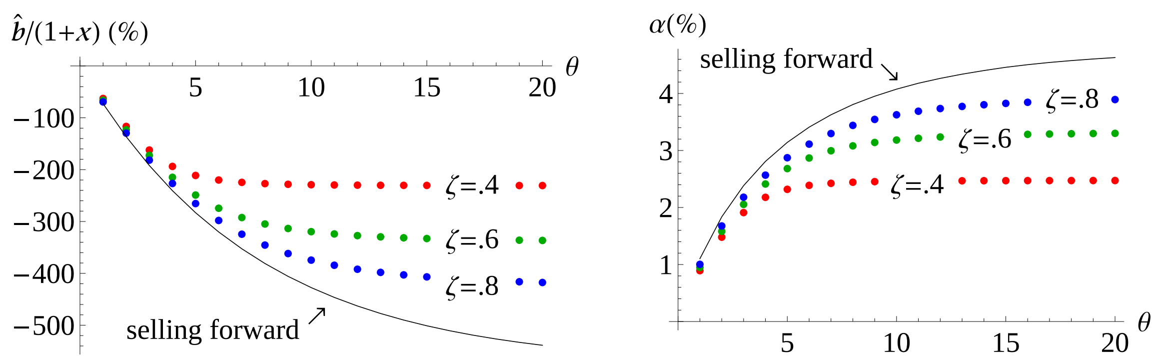

Figure 10: Net foreign assets and welfare gains if selling forward (continuous line) or hedging only against downside risk (dotted lines)

5.2 Default

Defaultable debt may reduce the welfare gains from hedging in two ways. First, default itself may be viewed as a substitute to hedging if it tends to occur when the price of the commodity export is low.[21] Second, allowing the country to default relaxes the no-default constraint and allow impatient countries to issue more debt—which was an important benefit from hedging. Since default occurs in

[21] The country could insure itself perfectly by issuing liabilities that are contingent on the commodity price. Some authors in the literature on sovereign debt have viewed default as a way of approximating the optimal state contingency (see, e.g., Grossman and Van Huyck (1988).

the real world, it seems important to see the extent to which defaultable debt provides a substitute for hedging. In this section we show that even if default does not entail any cost in terms of output or access to foreign borrowing, it generally does not provide a good substitute to formal hedging.

We assume that the country defaults entirely on its foreign debt if the export price falls below a certain percentage threshold of the equilibrium price, ζp. There is no repayment following a default (the debt is completely erased). We assume that defaulting is costless: no output is lost and the country maintains access to the international debt market.[22] The only cost for the country of having the option to default is that it must pay an interest rate premium on its debt to compensate foreign lenders for the risk of default. These assumptions obviously bias the analysis in favor of default, so that the following estimates should be viewed as an upper bound for the welfare gains from defaultable debt.

Since default is assumed to happen only if the price falls below ζp, the country is still subject to the constraint of being able to repay if the price never falls below that level. The worst possible outcome, conditional on no default, is that the export price drops permanently to exactly ζp. In order to compute the new borrowing constraint, we also have to consider that lenders will demand a higher rate of return to compensate for the risk of default. In particular, the required interest rate at $t+1$ is given by $R/(1-\pi(p_t))$, where $\pi(p_t)$ is the probability of default conditional on the current price level. The maximum borrowing capacity under defaultable debt is thus given by

$$\begin{aligned}\Phi_t &= q\left[\frac{\zeta p(1-\pi(p_t))}{R} + \frac{\zeta p(1-\pi(p_t))(1-\pi(\zeta p))}{R^2} + \frac{\zeta p(1-\pi(p_t))(1-\pi(\zeta p)^2)}{R^3} + \ldots\right], \\ &= q\zeta p \frac{1-\pi(p_t)}{R-1+\pi(\zeta p)},\end{aligned}$$

where $\pi(\zeta p)$ is the default risk from year $t+2$ onwards since we are imposing that the price falls permanently to ζp.

The introduction of defaultable debt does not affect the current level of total wealth, w_t, but

[22] In models where the country cannot commit to repay, such as Arellano (2008), the absence of default cost implies that the country cannot borrow. An interesting question (for future research) would be to study the benefits of hedging in a model with a willingness-to-repay problem.

changes the dynamic budget constraint to

$$w_{t+1} = \begin{cases} (w_t - c_t)R/(1-\pi_{t,1}) + qp_{t+1} & \text{if } (w_t - c_t) < 0 \text{ and } p_{t+1} \geq p\zeta, \\ qp_{t+1} & \text{if } (w_t - c_t) < 0 \text{ and } p_{t+1} < p\zeta, \\ (w_t - c_t)R + qp_{t+1} & \text{if } (w_t - c_t) > 0. \end{cases} \tag{22}$$

As in the case of hedging, the welfare gains from defaultable debt are closely related to the relaxation of the borrowing constraint. A higher default threshold ζ increases the level of export income that the country can borrow against, ζp, but also the probability of default and consequently the default risk premium. The impact of the default threshold on the borrowing ability of the country is therefore non monotonic as shown in Figure 11. We see that the maximum borrowing potential is achieved at a low levels of ζ, around 15 percent, and so are the maximum welfare gains, which are less than 1 percent of consumption. Somewhat unexpectedly, the possibility of default leads to the highest benefits when its occurrence is limited to very low prices. In this way, the country insures itself against the worst tail events and maximizes its borrowing capacity.[23]

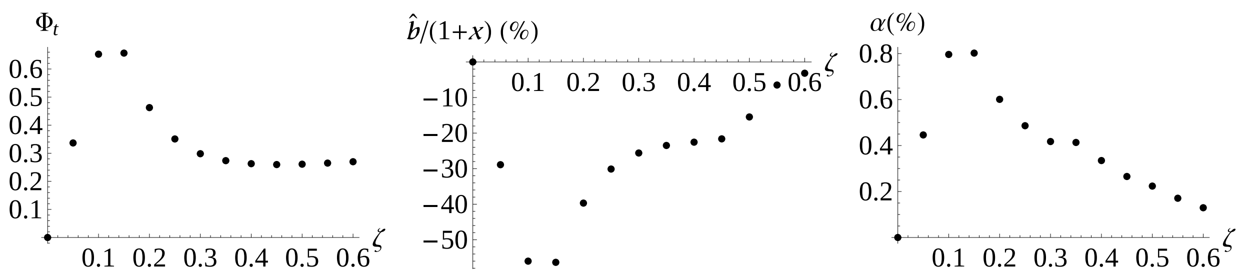

Figure 11: Borrowing capacity, equilibrium net foreign assets and welfare gains with defaultable debt

This exercise shows that defaultable debt can act as form of insurance and yield some welfare gains. However, even though we made the unrealistic assumption that default does not involve any cost in terms of output or market access, it is still dominated by a combination of non-defaultable debt and hedging (provided that the country engages in hedging at a sufficiently long horizon). Clearly a more realistic treatment of default costs would further reduce the net value of defaultable

[23] These results, however, clearly depend on the parameters of the model and in particular on the discount factor. With a lower discount factor or growth rate, the country's desire to borrow against future income decreases and the relative importance of reducing the volatility of income increases. Therefore, a higher ζ with more frequent default becomes more appealing.

debt as a form of insurance, and magnify the relative gains from formal hedging.

6 Conclusion

Many developing countries derive a substantial share of their national income from exporting a single commodity. These countries are exposed to fluctuations in commodity prices that tend to be very volatile and persistent. In this paper we developed a small open economy model in order to investigate the welfare gains that hedging against commodity price risk can provide to commodity exporters.

Under our benchmark calibration, selling the commodity export income ten-year forward can lead to very large welfare gains, equivalent to a four percent permanent increase in consumption. These gains are much larger than previously estimated in the literature, and the difference comes from the fact that hedging can relax the country's external borrowing constraint by reducing the downside risk in future export income.

We show that most of the welfare gains accrue to "impatient" countries with a high propensity to borrow against future export income. A country with a high discount factor or a relatively low growth rate does not make a large sacrifice by maintaining a large stock of precautionary savings. Hedging is much more beneficial for fast-growing emerging market economies that are willing to borrow against higher future income. Those welfare gains come from a debt-financed consumption boom that lasts about a decade and is followed by relatively lower consumption to repay the debt. Finally, hedging against the downside risk in the commodity price with options can be a valid alternative to forward contracts, while defaultable external debt is a less efficient form of insurance even if one leaves aside the default costs.

Regarding future research, it would be interesting to endogenize investment in our model. Hedging could increase welfare by reducing the uncertainty in the return on investing in the commodity sector, which could lead to an expansion of productive capacity. It would also be interesting to extend the analysis to the case where the commodity output is stochastic and this risk cannot be insured. Another useful extension of the model would be to consider the dependence of government fiscal revenues on commodity exports. In the countries where fiscal revenues are sensitive to commodity prices, hedging may lead to additional gains that are not captured by our model.

More generally, the type of models that we use in this paper can be used to quantify the benefits of hedging against other country risks such as natural disasters or sudden stops in capital flows.

Appendix

A Commodity Price Data

Table 6: Commodity price data from International Finance Statistics

Commodity	IFS name	price data coverage
Aluminum	Aluminum: Canada	1957 – 2008
Cocoa	Cocoa beans: Ghana	1957 – 2008
Copper	Copper: United Kingdom	1957 – 2008
Fish	Fish: Norway	1980 – 2008
Gold	Gold: United Kingdom	1970 – 2008
Iron	Iron Ore: Brazil	1950 – 2008
Natural Gas	Natural Gas: Russian Federation	1985 – 2008
Petroleum	Petroleum West Texas Intermediate	1970 – 2008

B Notes on numerical solution

The Euler equation for problem (10) is

$$c_t^{-\gamma} = \max\left[\frac{\beta(1+r)}{G^\gamma}E_t\left(c_{t+1}^{-\gamma}\right), (w_t + \Phi(p_t))^{-\gamma}\right]. \tag{23}$$

We derive the policy function $c(w_t, p_t)$ that satisfies this equation and the transition equations (4) and (11) with policy function backward iteration over the state space (w_t, p_t), following the endogenous gridpoints method proposed by Carroll (2006). We discretize the end-of-period savings $(w_t - c_t)$ over the interval $[0, 350]$, and p_t over $[0, 10p]$ using respectively 50 and 20 points with a triple exponential growth. Policy functions over the entire state space are constructed with linear interpolation. To limit the interpolation error for the value function, we interpolate its transformation $((1-\gamma)v_t)^{(1/(1-\gamma))}$. The grid to test for convergence is constructed with 10 values for w_t and 10 for p_t, drawn with triple exponential growth respectively between $[0, 30]$ and $[0, 5p]$. The price shock ε_{t+1} is discretized using 7 equally likely bins with the addition of a zero realization with one out of a million probability mass. We have also experimented using Gauss-Hermite quadrature instead of equally likely bins and obtained very similar results.

The welfare gains using the consumption function solved under no hedging in Figure 3 have been computed with 10,000 Monte Carlo simulations of income and consumption over 250 years. Monte Carlo simulations have also been used to compute option prices, simulating 1,000,000 income patterns starting from 40 different values for p_t drawn from the interval $[0, 20p]$ with triple exponential growth rate.

C Maximum Likelihood Estimation

Consider the price process:

$$p_t = \overbrace{(p + \rho(p_{t-1} - p))}^{\mu_t} \varepsilon_t \tag{24}$$

where ε_t is lognormally distributed with mean 1 and variance σ^2. By taking logs (identified with a tilde), the price process can be rewritten as:

$$\begin{aligned} \tilde{p}_t &= \tilde{\mu}_t + \tilde{\varepsilon}_t && (25) \\ \tilde{\mu}_t &= \ln(p + \rho(p_{t-1} - p)) && (26) \end{aligned}$$

so that

$$\tilde{p}_t | I_{t-1} \sim N(\tilde{\mu}_t - \sigma^2/2, \sigma^2) \tag{27}$$

where I_{t-1} is the information set at time $t-1$. The likelihood can be written as:

$$\begin{aligned} L &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^T \prod_{t=1}^{T} \exp\left(-\frac{1}{2}\frac{\left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)^2}{\sigma^2}\right) && (28) \\ \ln(L) &= -\frac{T}{2}\ln(2\pi\sigma^2) + \sum_{t=1}^{T}\left(-\frac{1}{2}\frac{\left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)^2}{\sigma^2}\right) && (29) \end{aligned}$$

with first order conditions:

$$\frac{\partial \ln(L)}{\partial p} = \sum_{t=1}^{T} \left(\frac{(1-\rho)\left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)}{\mu_t \sigma^2} \right) = 0 \tag{30}$$

$$\frac{\partial \ln(L)}{\partial \rho} = \sum_{t=1}^{T} \left(\frac{(p_{t-1}-p)\left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)}{\mu_t \sigma^2} \right) = 0 \tag{31}$$

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{T}{2\sigma^2} + \sum_{t=1}^{T} \left(-\frac{1}{2} \frac{\left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)\sigma^2 - \left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)^2}{\sigma^4} \right) \tag{32}$$

$$= \left(-\frac{1}{2\sigma^2}\right) \left(T + \sum_{t=1}^{T} \left(\frac{\left(\tilde{p}_t - \tilde{\mu}_t + \sigma^2/2\right)\left(\sigma^2/2 - \tilde{p}_t + \tilde{\mu}_t\right)}{\sigma^2} \right) \right) = 0. \tag{33}$$

References

Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review*, 98(3): 690–712.

Arrau, Patricio, and Stijn Claessens. 1992. "Commodity Stabilization Funds." *World Bank Policy Research Working Paper WPS 835.*

Becker, Torbjorn, Olivier Jeanne, Paolo Mauro, and Romain Rancière. 2007. "Country Insurance: The Role of Domestic Policies." *Occasional Paper No. 254, IMF.*

Bems, Rudolfs, and Irineu de Carvalho Filho. 2011. "Current Account and Precautionary Savings for Exporters of Exhaustible Resources." *Journal of International Economics*, 84(1): 48–64.

Caballero, Ricardo J., and Kevin N. Cowan. 2007. "Financial Integration Without the Volatility." *Unpublished.*

Caballero, Ricardo J., and Stavros Panageas. 2007. "A Global Equilibrium Model of Sudden Stops and External Liquidity Management." *Working Paper.*

Caballero, Ricardo J., and Stavros Panageas. 2008. "Hedging Sudden Stops and Precautionary Contractions." *Journal of Development Economics*, 85: 28–57.

Campbell, Patrick, Bjorn-Erik Orskaug, and Richard Williams. 2006. "The Forward Market for Oil." *Bank of England Quarterly Bulletin*, 66–74.

Carroll, Christopher D. 2006. "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optmization Problems." *Economic Letters*, 312–320.

Carroll, Christopher D. 2008. "Theoretical Foundation of Buffer Stock Saving." *Unpublished.*

Cashin, Paul, Hong Liang, and C. John McDermott. 2000. "How Persistent Are Shocks to World Commodity Prices?" *IMF Staff Papers*, 47(2): 177 – 217.

Deaton, Angus. 1991. "Saving and Liquidity Constraints." *Econometrica*, 59(5): 1221 – 1248.

Durdu, Ceyhun Bora, Enrique G. Mendoza, and Marco E. Terrones. 2009. "Precautionary Demand for Foreign Assets in Sudden Stop Economies: an Assessment of the New Merchantilism." *Journal of Development Economics*, 89: 194 – 209.

Engel, Eduardo, and Rodrigo Valdés. 2000. "Optimal Fiscal Strategy for Oil Exporting Countries." *IMF Working Paper 00/118.*

Ghosh, Atish R., and Jonathan D. Ostry. 1997. "Macroeconomic Uncertainty, Precautionary Saving and the Current Account." *Journal of Monetary Economics*, 40(1): 121–139.

Grossman, Herschel, and John Van Huyck. 1988. "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation." *American Economic Review*, 78(5): 1088–97.

Jeanne, Olivier, and Damiano Sandri. 2012. "Precautionary Savings in Representative-Agent Open Economies." *Manuscript, Johns Hopkins University.*

Lucas, Robert E. 1987. *Models of Business Cycles.* Basil Blackwell, New York.

Pagano, Patrizio, and Massimiliano Pisani. 2009. "Risk-Adjusted Forecasts of Oil Prices." *European Central Bank Working Paper No.999.*

Pallage, Stephane, and Michel A. Robe. 2003. "On the Welfare Cost of Business Cycles in Developing Countries." *International Economic Review*, 44(2): 677–98.

Powell, Andrew. 1989. "The Management of Risk in Developing Country Finance." *Oxford Review of Economic Policy*, 5(4): 69–87.

World Economic Outlook. 2007, April. *International Monetary Fund.*