“Whatever It Takes”: Government Default Versus Financial Repression*

Olivier Jeanne**
Johns Hopkins University

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Abstract

There are many historical examples of governments reducing their debts and avoiding default through financial repression. This paper presents a theory of optimal financial repression in a model of government debt and default. Financial repression can prevent a default when a desirable fiscal adjustment is prevented by fiscal deadlock, but should preserve the incentives to implement fiscal adjustments. A calibrated version of the model shows that optimal financial repression yields substantial welfare gains. Financial repression is a policy of last resort that should be rarely used in equilibrium, but ruling out financial repression entirely leads to an equilibrium with frequent defaults. The fiscal incentives are more difficult to preserve in a monetary union than in countries that have their own currency.

Work in Progress.

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**Olivier Jeanne, Johns Hopkins University, ojeanne@jhu.edu

1 Introduction

There are many historical examples of government debt being reduced by financial repression (Reinhart and Sbrancia, 2015; Acalin and Ball, 2023). In these episodes banks are induced through moral suasion and regulation to hold government debt and to accept a return that is lower than the market rate. In many of these episodes, furthermore, the real interest rate on government debt is reduced by inflation. Since the global financial crisis there have been concerns that the large-scale purchase of government debt by central banks could be a prelude to monetary financing and financial repression (IMF, 2022).

One question though is whether financial repression may be justified when the alternative is a government default. The purpose of this paper is to propose a framework to study the scope for optimal financial repression.

The main premises of the theory are as follows. First, we assume that financial repression is more costly for welfare than a regular fiscal adjustment, although it may be less costly than a formal default. Second, we assume that even a desirable fiscal adjustment takes some time to implement because of political frictions that may lead to a fiscal deadlock (a possibility illustrated by the current situation in many advanced economies including the U.S.).

We show that under these conditions, it is beneficial to have financial repression as a last resort policy to backstop government debt and prevent a default. The problem with not having this backstop (as would be the case for example if the central bank single-mindedly followed a strict inflation targeting mandate) is what this implies for the dynamics of government debt and the frequency of default. Ruling out financial repression leaves default as the only alternative to a fiscal adjustment. Once government default is at play, the government must pay a default risk premium that destabilizes the debt dynamics. This most often does not leave enough time to resolve fiscal deadlocks before a default and the government defaults quite frequently.

We quantify this theory by calibrating the model to the behavior of government debt in a sample of advanced economies. The model explains the long debt-increase and -decrease episodes that we observe in the data, and we use the low-frequency moments associated with these episodes to calibrate the model. The calibrated model implies that optimal financial repression significantly increases welfare. If financial repression is ruled out defaults occur every 20 years on average, whereas with optimal financial repression there is no default and financial repression is very rare—the economy spends less than 1% of the time financially repressed on average. The welfare gain from optimal financial repression is equivalent to about a one percent permanent increase in consumption.

One issue with financial repression is that it may weaken the incentives to do fiscal adjustments. The optimal financial repression policy is designed to preserve these incentives, and the high cost of financial repression is key to make that possible. It is precisely because financial repression is costly that the government is willing to adopt a fiscal adjustment in order to exit financial repression whenever possible. At the same time, financial repression should not be so costly as to make a default preferable.

The fiscal incentives are more difficult to maintain in a monetary union than in a country that has its own currency. If there is perfect capital mobility between the member countries, the cost of financial repression must be shared across the countries and cannot be allocated to the country that needs the quasi-fiscal resources from financial repression. This limits the cross-country redistribution that can be implemented in the union, a problem that can be mitigated by ringfencing national banking systems with capital controls. The optimal financial repression policies are less beneficial and more difficult to implement in a monetary union than in countries that have their own currencies.

We show that the properties of the model dramatically change if $r < g$. Then government debt converges to a finite level even if there is no fiscal adjustment and a default-free equilibrium does not need to be backstopped by financial repression. This implies that government debt tends to be very high and makes the government vulnerable to a default if the sign of the interest-growth differential changes.

Literature. The building blocks of the model are familiar in the literature on the interaction between fiscal and monetary policies and the literature on government debt default. We draw on the distinction between active and passive fiscal policy made by Leeper (1991). Government may default to avoid the distortionary cost of domestic taxation as in Pouzo and Presno (2022).

This paper is also related to the literature on the central bank backstop of government debt inspired by the 2010 euro debt crisis. An important theme in that literature is that the central bank can play a role in removing self-fulfilling government debt crises. This argument has been developed in models with self-fullfilling rollover crises a la Cole and Kehoe (Aguiar et al., 2015), and in models a la Calvo (1988) where the crisis mechanism involves the interest rate. Corsetti and Dedola (2016) show that the central bank can select the good equilibrium by purchasing government debt in a two-period model a la Calvo. Lorenzoni and Werning (2019) study the existence of self-fulfilling government debt crises a la Calvo in a more dynamic and micro-founded environment. Bacchetta, Perazzi and van Wincoop (2018) show how the central bank can remove self-fulfilling crises in a New Keynesian version of the Lorenzoni and Werning's model.

The analysis in this paper does not rely on the existence of multiple equilibria. Although financial repression can be viewed as a form of "taxation of last resort" in our model, it does not offer a free lunch and must sometimes be used in equilibrium. Thus, this paper does not support the view that central banks can costlessly remove the default risk on government debt with open market operations—what Reis (2013) calls the "mystique" surrounding the central bank balance sheet. Financial repression works in our model because it produces quasi-fiscal revenue for the government.[1] One issue with the self-fulfilling view is that in the real world we see countries where government debt seems to be on an unsustainable path even though there is no significant default risk premium in the interest rate. These debt dynamics are due to a fundamental fiscal imbalance and cannot be explained as a bad equilibrium in a model a la Calvo. For this situation to be an equilibrium the public must believe that the government will either implement a fiscal adjustment or be rescued from default in some other way that produces new revenue for the government. This is the situation that our model attempts to capture.

The paper belong to the literature on financial repression. There is a large literature on how unsustainable debt dynamics have been corrected in the past (Mauro et al., 2015) and some authors have more specifically studied the role of financial repression—see Reinhart and Sbrancia (2015) and Acalin and Ball (2023). On the theoretical side, Chari, Dovis and Kehoe (2020) make the point that it may be optimal to force banks to buy government debt so as to dissuade the government from defaulting.

2 Model

We consider a continuous-time economy with households, a banking sector and a government. As represented in Figure 1, the government issues debt

[1] Similar views were developed earlier by Jeanne (2012) and Zhang (2021).

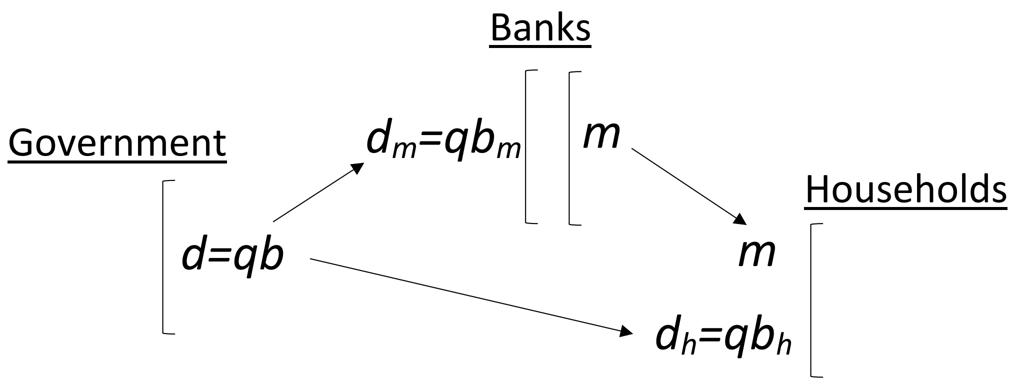


Figure 1: Balance sheets of the government, banks and households

d that is held by households and by the banking sector. The banking sector issues deposits m to the household sector. Government debt is composed of bonds with an exponentially decaying coupon, and the price of bonds q may be lowered by default risk.

Households. The economy is populated by a mass 1 of identical infinitely-lived households. The utility of the representative household is given by

$$U(0) = E_0 \left\{ \int_0^{+\infty} [c(t) + u(m(t))] e^{-rt} dt \right\}, \tag{1}$$

where $c(t)$ is consumption at time t and $u(m(t))$ is the utility of real money balances (bank deposits). Given the linearity of the utility of consumption the real return on the market value of government debt must be equal to r in equilibrium. The budget constraint of the representative household is

$$c(t) + \tau(t) + m'(t) + d_h'(t) = y(t) + r d_h(t) + r_m(t) m(t), \tag{2}$$

where $d_h(t) = q(t) b_h(t)$, is the value of the households' holdings of government debt, $\tau(t)$ is a tax paid to the government, $y(t)$ is household gross income and $r_m(t)$ is the real return on bank deposits. The real return on bank deposit is equal to the difference between the nominal return on deposits and inflation, $r_m(t) = i_m(t) - \pi(t)$.

Banks. The banking sector consolidates the central bank with commercial banks and is regulated by the government. The market value of government debt held by banks is equal to the value of the deposits issued to households,

$$d_m(t) = q(t)\, b_m(t) = m(t), \tag{3}$$

(i.e., banks have zero equity).

Banks receive a real return r on their holdings of government debt and make their net interest income from the spread between their assets and their liabilities. Banks' interest income is used to pay a fixed operating cost κ and a transfer to the government θ,

$$[r - r_m(t)]\, m(t) = \kappa + \theta(t). \tag{4}$$

The transfer $\theta(t)$ is the quasi-fiscal revenue that the government extracts from financial repression.

Government. The government finances a constant stream of expenditures g by issuing debt and raising fiscal and quasi-fiscal revenue. Conditional on no default the budget constraint of the government is given by

$$g + rd(t) = \tau(t) + \theta(t) + d'(t). \tag{5}$$

Government debt is composed of real bonds with an exponentially decaying coupon like in Hatchondo and Martinez (2009). A bond issued in period 0 yields a flow of payment $(r + \alpha)\, e^{-\alpha t}$ in time t. This implies that the equilibrium price of debt $q(t)$ is equal to 1 if there is no default risk.

The government budget constraint can also be written in terms of the outstanding bonds,

$$g + (r + \alpha)\, b(t) = \tau(t) + \theta(t) + q(t)\, [b'(t) + \alpha b(t)]. \tag{6}$$

The term $(r + \alpha)\, b(t)$ on the left-hand side is the flow of payment on the outstanding bonds. The term $q(t)\, [b'(t) + \alpha b(t)]$ on the right-hand side is the net issuance of debt taking into account that debt "melts" at rate α. Equations (5) and (6) are equivalent because the price of debt satisfies the valuation equation $(r + \alpha)\, q(t) = r + \alpha + q'(t)$.

Fiscal policy. We assume that fiscal policy is in a passive regime or in an active regime as defined by Leeper (1991). In the *passive* regime the tax rate is set so as to ensure the convergence of government debt towards a long-term level $\underline{b}$. We assume that debt dynamics obey

$$b'(t) = -\sigma \left[b(t) - \underline{b} \right] \tag{7}$$

in the passive regime.

In the *active* regime the tax rate is set at a constant level $\tau_t = \tau_a$ that is too low to keep government debt on a sustainable path. We assume that the primary deficit is positive for $b = \underline{b}$,

$$\delta \equiv g + r\underline{b} - \tau_a \geq 0. \tag{8}$$

A transition from the passive regime to the active regime is a bad fiscal shock, and a transition from the active regime to the passive regime is a fiscal adjustment that puts government debt on a sustainable path.

The transition between regimes is not symmetric. While bad fiscal shocks are exogenous, fiscal adjustments involve a government choice. The government has sometimes the opportunity to implement a fiscal adjustment and switch from the active to the passive regime but it is free to do the adjustment or not. We denote by η the dummy variable capturing this decision (equal to 1 if the government adjusts, and to 0 if it does not). We assume that bad fiscal shocks and fiscal adjustment opportunities arrive with constant flow probabilities respectively denoted by μ and ϕ.

Taxation vs. financial repression. The difference between fiscal revenue and quasi-fiscal revenue can be interpreted as follows. Fiscal revenue is decided in the context of conventional fiscal policy as voted by parliament. This form of taxation may be difficult to change quickly because of the reasons that have been invoked in the political economy literature to explain fiscal deadlock and more generally inefficient delays in fiscal adjustment. This has been explained for example by wars of attrition between different political parties (Alesina and Drazen, 1991). By contrast, we assume that the quasi-fiscal revenue from financial repression $\theta(t)$ can be changed at any time.

Financial repression and taxation entail different distortions. Consider financial repression first. We assume in the following that the utility from real money balances is a power function,

$$u(m) = \mu_m \frac{m^{1-\nu}}{1-\nu}, \tag{9}$$

with $\nu > 1$. It then follows from the first-order condition for money demand $u'(m(t)) = r - r_m(t)$ and the budget constraint of banks (4) that the real return on bank deposits is given by,

$$r_m(t) = i_m(t) - \pi(t) = r - \left[\frac{\kappa + \theta(t)}{\mu_m^{1/\nu}}\right]^{\frac{\nu}{\nu-1}}. \tag{10}$$

An increase in financial repression θ reduces the resources left for banks to pay a return on their deposits.[2] Observe that if there is a zero-lower-bound constraint on $i_m(t)$ because currency in circulation (banknotes and coins) provides the same services as bank deposits, the real return on bank deposits can be negative only if there is a positive rate of inflation. Financial repression, thus, must be accompanied with inflation to produce quasi-fiscal revenue above a certain threshold.

Using again the first-order condition for money demand and (4), one can see that the utility of real money balances decreases linearly with the level of revenue from financial repression,

$$u(m(t)) = -\frac{\kappa + \theta(t)}{\nu - 1}. \tag{11}$$

Financial repression decreases the level and so the utility of real money holdings. The welfare cost of financial repression depends only on the level of quasi-fiscal revenue that it produces. We will define the financial repression policy by the path of revenue $(\theta_t)_{t\geq 0}$.

To capture the idea that taxation is distortionary too, we assume that output decreases linearly with the tax revenue levied by the government,

$$y(t) = \overline{y} - \gamma_\tau \tau(t). \tag{12}$$

As shown in Appendix A, this equation can result from the linearization of a model where the government taxes output produced with labor.

The utility cost of taxation and financial repression can then be put together as follows. Consolidating the budget constraints (2), (4) and (5),

[2] Alternatively, and in line with the evidence described by Reinhart and Sbrancia (2015), one could define financial repression as forcing the banks to accept an interest rate on government debt that is lower than the market rate. The banks then pass on the lower return on their assets by paying a lower return on their deposits. This leads to the exactly the same implications as our baseline assumption.

household consumption can be written as output net of government expenditures and the operating cost of banks,

$$c(t) = y(t) - g - \kappa. \tag{13}$$

Combining equations (11), (12) and (13) then gives the following expression for the households' flow utility,

$$c(t) + u(m(t)) = \bar{u} - [\gamma_\tau \tau(t) + \gamma_\theta \theta(t)],$$

where $\bar{u}{=}\bar{y} - g - \nu\kappa/(\nu - 1)$ and $\gamma_\theta \equiv 1/(\nu - 1)$. The two forms of government revenues have different linear disutility costs captured by parameters γ_τ and γ_θ.

We assume that $\gamma_\theta > \gamma_\tau$, i.e., financial repression has a larger utility cost than taxation. The value of γ_θ can be calibrated from the literature estimating the interest elasticity of money demand and we show below that the assumption $\gamma_\theta > \gamma_\tau$ holds for plausible calibrations. In addition γ_θ could be further increased by costs that have not been taken into account in the model, such as the crowding out of bank credit to the private sector by government debt, or by the reputational loss of increasing inflation for the central bank.

The linearity of the distortionary costs makes it clear that our results are not driven by the traditional second-best argument that the marginal cost of government revenue should be equated across sources of revenue. The only reason that a welfare-maximizing government might want to use financial repression in our model is that a tax adjustment is not possible because of policy deadlock.

Default. The government may default at any time. The trade-off involved in a default is that it reduces the output cost of raising government revenue but involves an output cost γ_d. Taking default into account the equation for the level of output (12) becomes

$$y(t) = \overline{y} - \gamma_\tau \tau(t) - \delta_T(t)\gamma_d,$$

where $\delta_T(t)$ is the Dirac delta function for a default at time T, and γ_d is the present discounted value of the output loss caused by a default.[3]

[3] The integral of the Dirac delta function in an infinitesimal interval around time T is equal to 1, that is, $\int_{T-\varepsilon}^{T+\varepsilon} \delta_T(t)\,dt = 1$ for an arbitrarily small ε.

We assume that the debt of a defaulting government is reduced to $\underline{b}$ (the same level to which debt converges in the passive fiscal regime). In a default, creditors receive $\underline{b}/b$ unit of new debt per unit of old debt where b is the face value of debt at the time of default. We assume that after a default banks are recapitalized by a lump-sum transfer of government debt from households so as to always maintain zero equity in the banking sector. A defaulting government implements a fiscal adjustment.

The government may default of its own volition, or it may be forced to default because creditors refuse to roll over the government's debt. We call the two types of default *opportunistic default* and *rollover default* respectively. We assume that the government can roll over its debt as long as the price is larger than a threshold

$$q(t) \geq \underline{q}, \tag{14}$$

and that a rollover default occurs when the price of debt falls to $\underline{q}$. This assumption sets a limit on the government's ability to dilute its creditors.

Equilibrium. Let us denote by $\omega(t) = a, p$ the state of fiscal policy at time t (a for active and p for passive respectively). We consider Markov equilibria in which exogenous and endogenous variables are functions of the state, which is summarized by ω and b. Financial repression policy is given by an exogenous function $\theta = \Theta_\omega(b)$. Given the financial repression policy, an equilibrium is composed of policy functions for fiscal policy and default as well as a function for the bond price $Q_\omega(b)$. The policy functions for fiscal policy and default maximize welfare. The bond price function $Q_\omega(b)$ clears the market for government debt. We denote by $V_\omega(b)$ the equilibrium value functions for welfare.

The following two properties of the equilibrium are of special interest:

1. the equilibrium is *incentive-compatible* if the government implements the fiscal adjustment whenever it has the opportunity, that is

$$V_p(b) \geq V_a(b), \tag{15}$$

   for all $b \in B$, where B is the set of possible equilibrium values for bonds;

2. the equilibrium is *default-free* if the government never defaults, that is

$$Q_\omega(b) = 1, \text{ and } V_\omega(b) \geq V_d, \tag{16}$$

for all ω and $b \in B$, where $V_d = V_p\left(\underline{b}\right) - \gamma_d$ is the value associated with default.

3 A tractable specification

This section focuses on a specification of the model in which the equilibrium can be characterized in closed form: the case where fiscal policy is initially active and fiscal adjustment is irreversible ($\omega(0) = a$ and $\mu = 0$). The general case can be solved only numerically. Looking at a special case with closed-form solutions is useful to derive analytical results, to guess the form of the equilibrium in the general case, and to develop intuition for the numerical results.

The analysis proceeds in two steps. First, we derive the equilibrium when the government is committed not to resort to financial repression (for example because monetary policy and banking regulation are delegated to a central bank that targets inflation). We then characterize the financial repression policies that maximize welfare.

3.1 Equilibrium without financial repression ($\theta = 0$)

Our first result is that an equilibrium without financial repression cannot be default-free. To see this, consider the decision to default or not under a passive fiscal policy. In the passive regime welfare is given by the present discounted value of potential output minus the distortionary cost of the taxation required to repay the debt,

$$V_p\left(b\right) = \frac{\overline{u} - \gamma_\tau g}{r} - \gamma_\tau b. \tag{17}$$

Welfare under default involves instead the distortionary cost of repaying the post-default debt $\underline{b}$ as well as the default cost,

$$V_d = \frac{\overline{u} - \gamma_\tau g}{r} - \gamma_\tau \underline{b} - \gamma_d. \tag{18}$$

There is no default in the passive regime as long as $V_p\left(b\right) \geq V_d$, that is, if and only if debt is lower than $b_p^d = \underline{b} + \gamma_d/\gamma_\tau$. In this debt range the price of debt is equal to 1 in the passive regime and a default cannot be triggered by a rollover crisis.

There is no default-free equilibrium because b exceeds b_p^d with positive probability in the active regime. Hence the set B includes values of b that do not satisfy condition (16). Although the government will sooner or later have the opportunity of doing a fiscal adjustment, this may happen too late for the government to choose the adjustment over a default.

Once the equilibrium involves the possibility of default the price of government bonds is below 1. The price of government bonds is governed by the valuation equation

$$(r+\alpha)\, q_a(t) = r + \alpha + \phi\eta(t)\left[1 - q_a(t)\right] + q_a'(t). \tag{19}$$

where $q_a\left(t\right)$ is the price of a bond conditional on staying in the active regime. The left-hand side is the return on government debt demanded by households given that it automatically depreciates at rate α. The right-hand side is the flow of payment on debt plus the valuation gain, including the valuation gain that occurs if the government implements the fiscal adjustment and the price of debt jumps up to 1 (which occurs with flow probability $\phi\eta(t)$, the flow probability of a an adjustment opportunity times the probability that it translates into an actual adjustment).

As long as fiscal policy stays in the ative regime the price of government debt decreases over time and reaches $\underline{q}$ in finite time, at which point there is a rollover crisis and a default. By arbitrage the market value of debt cannot jump at the time of default, implying that default occurs when debt is at the threshold

$$b_a^d \equiv \frac{\underline{b}}{\underline{q}}. \tag{20}$$

We assume that the government is better off adjusting than defaulting when it is close to default, that is$V_p\left(b_a^d\right) \geq V_d$. Using equations (17) and (18) this implies a lower bound on the bond price that triggers a rollover crisis

$$\underline{q} \geq \left(1 + \frac{\gamma_d}{\gamma_\tau \underline{b}}\right)^{-1}. \tag{21}$$

Under this condition the government implements the fiscal adjustment whenever it has the opportunity, i.e., the equilibrium is incentive compatible. Figure 2 shows the variations of welfare with b under the two fiscal egimes and under default. Welfare in the active regime converges to V_d when b converges to the default threshold b_a^d. It is possible to show that $V_a\left(b\right)$ stays between V_d and $V_p\left(b\right)$ for lower values of b as shown on the figure (see the proof of

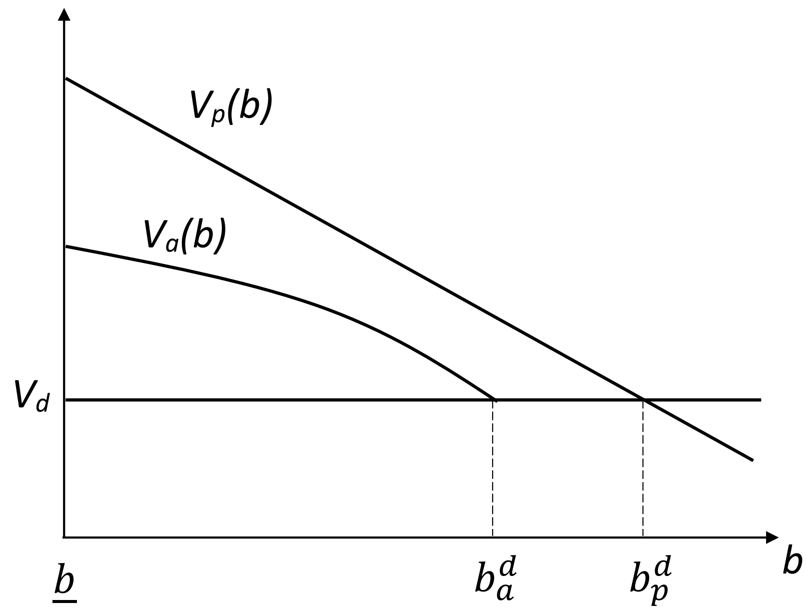


Figure 2: Value functions

Proposition 1). Hence the government does not default opportunistically and implements the fiscal adjustment whenever possible when debt has not yet reached the default threshold. A default leads not only to the payment of the default cost but also to an increase in the level of the tax because of (8). Thus it is always optimal for the government to postpone a default and pay these costs later. The government rolls over its debt by diluting outstanding creditors until the price of bonds alls to $\underline{q}$.

Our results are summarized in the following Proposition.

Proposition 1 *Consider the tractable case ($\omega(0) = a$ and $\mu = 0$) and assume condition (21) is satisfied. Then there is a unique equilibrium in which: (i) the debt level b increases and the price of debt q decreases over time as long as the government does not implement the fiscal adjustment; (ii) the government implements the fiscal adjustment whenever it has the opportunity; and (iii) the government defaults in finite time.*

Proof. See Appendix B ■

Conditional on no fiscal adjustment the dynamics of debt is deterministic and the government defaults at a time that is known ex ante. Two equations will be useful to derive the equilibrium dynamics of debt and of the debt price. Let us denote by $T_a(b)$ the time to default conditional on b, i.e., the time that it takes for the government to default conditional on no no fiscal adjustment. Using the fact that the price of debt at the time of default is $\underline{q}$, equation (19) can be integrated with $\eta(t) = 1$ to give a relationship between the price of debt before default and the time to default,

$$Q_a(b) = 1 - \left(1 - \underline{q}\right) e^{-(r+\alpha+\phi)T_a(b)}. \tag{22}$$

The second equation involves the market value of debt $d(t) = q(t)b(t)$. As long as there is no fiscal adjustment, equation (5) implies $g + rd(t) = \tau_a + d'(t)$. Using the fact that the market value of debt qb is equal to $\underline{b}$ at the time of default, this equation can be integrated into

$$Q_a(b)\, b = \underline{b} - \delta \frac{1 - e^{-rT_a(b)}}{r}. \tag{23}$$

Equations (22) and (23) uniquely define the policy functions $Q_a(b)$ and $T_a(b)$ in the active regime. These equations can be used for comparative statics. The price of debt and the time to default are both decreasing in b. Given b, an increase in α (i.e. a shortening of the maturity of debt) raises the price of debt but reduces the time to default. The time to default converges to zero when α goes to infinity—default is immediate with zero-maturity debt because it cannot be diluted. An increase in the probability of fiscal adjustment ϕ has the same impact as a shortening of debt maturity. A higher flow probability of adjustment raises the price of debt but also the rate at which this price decreases over time if the expectation of adjustment does not materialize.

3.2 Optimal financial repression

We now consider the government's optimal financial repression policy. We restrict our attention to equilibria in which financial repression is used only before a default or a fiscal adjustment and preserves the incentives not to default and to implement the fiscal adjustment. These requirements are natural as it is not optimal to use financial repression if a less distortive source of revenue is available.

For the purpose of deriving the equilibrium it is easier to assume that the government can commit to a path $(\theta(t))_{t\geq 0}$ and then show that the solution is time-consistent. This path is chosen to maximize period-0 welfare, taking into account how it affects the government's incentives to fiscally adjust or to default. The value function of the government in the active regime satisfies

$$rU_a(t) = \overline{u} - [\gamma_\tau \tau_a + \gamma_\theta \theta(t)] + \phi [V_p(b(t)) - U_a(t)] + U_a'(t). \tag{24}$$

The term factored by ϕ is the probability of fiscal adjustment times the change in the value function if there is a fiscal adjustment, where $V^p(b_t)$ is given by (17). The equilibrium is default-free and incentive compatible if the no-default and fiscal adjustment constraints, $U_a(t) \geq V_d$ and $U_a(t) \leq V_p(b(t))$, are satisfied. Integrating this equation and using the transversality conditions gives an expression for initial welfare

$$U_a(0) = \int_0^{+\infty} [\overline{u} - \gamma_\tau \tau_a - \gamma_\theta \theta(t) + \phi V_p(b(t))] e^{-(r+\phi)t} dt.$$

The problem is to maximize $U_a(0)$ subject to the dynamic equation for the accumulation of debt, $b'(t) + \tau_a + \theta(t) = g + rb(t)$; the no-default constraint, $U_a(t) \geq V_d$; the fiscal adjustment incentive constraint, $U_a(t) \leq V_p(b(t))$; and the non-negativity constraint on non-fiscal revenue, $\theta(t) \geq 0$. The welfare maximizing policy is characterized in the following Proposition.

Proposition 2 *The welfare-maximizing financial repression policy is to use financial repression to prevent debt from exceeding a threshold b_a^m and to use it only once debt has reached this threshold. The optimal threshold is given by:*

$$b_a^m = \underline{b} + \frac{(r+\phi)\gamma_d - (\gamma_\theta - \gamma_\tau)\delta}{r\gamma_\theta + \phi\gamma_\tau}. \tag{25}$$

This policy is time consistent.

Proof. See Appendix B. ■

Proposition 2 states that it is optimal to use financial repression as a *last resort*. It is not optimal to use financial repression to slow down the accumulation of debt early on because it is socially more costly than regular taxation. Financial repression must be used only when it is unavoidable to prevent a default.

There is a set of possible values for b above $\underline{b}$ if and only if $b_a^m \geq \underline{b}$. Equation (25) shows that this is possible iff

$$\gamma_\theta \leq \gamma_\tau + \frac{(r+\phi)\,\gamma_d}{\delta}. \tag{26}$$

If the cost of financial repression is too high default is always preferable to financial repression.

4 Quantitative analysis

We discuss in this section the quantitative properties of the calibrated model. We explain how the model is calibrated in section 4.1. Section 4.2 then discusses the benefits from financial repression.

4.1 Calibration

Our baseline calibration is reported in Table 1. Part of the calibration is based on the literature, but the parameters related to fiscal policy are calibrated by reference to debt-increase and decrease episodes observed in the data.

Potential output is normalized to $\overline{y} = 1$. The real interest rate r is set to 2 percent.[4] The debt repayment parameter α is set to 0.15, which implies a government debt duration of about 6 years, close to the average maturity of government debt in OECD countries. Parameter g is set to 0.4 in order to match the average share of government spending in GDP in OECD countries. The target level of debt under the active fiscal rule is set to 0.6 by reference to the Maastricht Treaty. The value of $\underline{q}$ is justified by the fact that private holders of Greek debt accepted a haircut of 50% in the 2011 Greek default.

[4] A maintained assumption of the model is that r is positive. This assumption is not obvious, given the papers that have found a negative interest-growth differential in advanced economies (Blanchard, 2019). However, Barrett (2018) points out that confidence intervals for the estimates of the long-run interest-growth are large and that one cannot exclude values of 1% or 2% at conventional levels of statistical significance in countries where the point estimate is negative. We discuss the case $r < 0$ in section 5.

The calibration of ϕ, μ, δ and σ is based on empirical moments related to the long swings in the debt-to-GDP ratio in the data. We identify debt-increase and debt-decrease episodes in the same way as Zhang (2021). A debt-increase episode starts in a year where the government debt to GDP ratio increases by more than 1%, and ends when the debt to GDP ratio either falls by more 1% for three years in a row, or falls by more than 4% in one year. A debt-decrease episode starts when the debt-to-GDP ratio falls and ends when the ratio increases for two years in a row or by more than 6% in one year. The episodes must last for more than 10 years to be included. Figure 3 shows the results of this identification method for selected economies.

This methodology identifies 20 debt episodes in 17 advanced economies over the period 1981-2017.[5] We calibrate the model with financial repression to match some features of the data. We exclude euro area members after 2008 from the data because the financial repression backstop was arguably in doubt at least for some euro area countries at that time.

We assume that the probability of switching from the active fiscal regime to the passive regime is the same as the probability from switching from the latter to the former ($\phi = \mu$). We then set ϕ, σ and δ so as to match three moments: the fraction of the time that the economy spends in debt-increase or -decrease episodes (55% in our sample), the unconditional volatility in the annual change in the debt to GDP ratio (4.9%), and the average level of debt-to-GDP ratio observed in 2017.

The calibration of γ_τ is based on the model reported in Appendix A. In that model fiscal revenue comes from the taxation of output which is produced with labor. The value of γ_τ comes from the linearization of the model and assuming conventional values for the Frisch elasticity of labor supply.

The parameter for the welfare cost of financial repression is given by $\gamma_\theta \equiv -1/(\nu - 1)$ where ν is the inverse of the semi-elasticity of money demand with respect to the nominal interest rate. The literature on the elasticity of money demand gives a range of estimates that include $1/\nu = 0.2$ (see e.g. Teles and Zhou, 2005). This implies $\gamma_\theta = 0.25$ as reported in Table 1. We need to calibrate parameter μ_m in the utility from real money holdings (9) to estimate the level of inflation required by financial repression. We should

[5]The country sample includes: Australia, Austria, Belgium, Canada, Switzerland, Spain, Finland, France, Great Britain, Greece, Iceland, Italy, Japan, Norway, New Zealand, Portugal and the United States.

Parameter	Value	Source
$\bar{y}$	1	Normalization
r	2	Barrett (2018)
α	0.15	Average maturity of debt 6 years
g	0.4	Average share of government spending in GDP (OECD)
$\underline{b}$	0.6	Maastricht Treaty debt target
$\underline{q}$	0.5	Haircut in 2011 Greek default
σ	0.05	Targets debt-episode moments described in text
δ	0.019	Targets debt-episode moments described in text
ϕ	0.15	Targets debt-episode moments described in text
μ	0.15	Targets debt-episode moments described in text
γ_τ	0.10	Appendix A
γ_θ	0.25	Appendix A
γ_d	0.4	

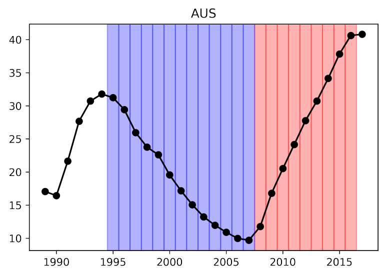
Table 1: Baseline calibration

think of money as a broad money aggregate such as M2. We use the first-order condition $u'(m(t)) = i(t) - i_m(t)$ and the observation that in 2000-05 M2 amounted to 50% of US GDP on average while the opportunity cost of holding M2 was about 2% on average (Judson, Schlusche and Wong, 2014). This gives $\mu_m = 1/1600$. We set the banking operating cost κ to zero.

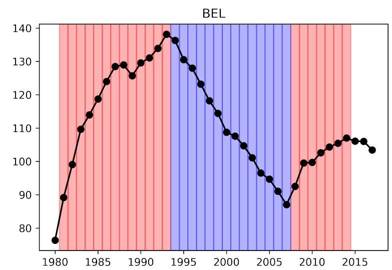
4.2 The benefits of financial repression

We first consider the quantitative implications of the tractable model and then move on to the numerical simulations in the general case.

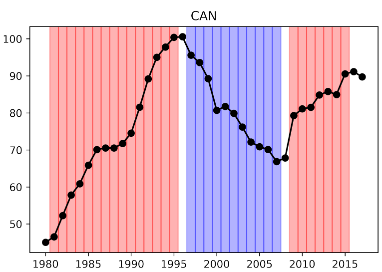
Tractable model. The analysis of the tractable model relied on several assumptions that are satisfied by our calibration. First, the calibration implies that the utility cost of financial repression is larger than the utility cost of taxation ($\gamma_\theta > \gamma_\tau$). This ensures that governments implement a fiscal adjustment to exit financial repression when they have the opportunity. Second, condition (26) is satisfied by a wide margin (the right-hand side is equal to 3.68) so that financial repression is preferable to default over a wide range of debt. Under our calibration the financial repression is optimally triggered when debt reaches $b_a^m = 3.86$, i.e. 386% of potential GDP. This happens very infrequently because the government generally has the time to fiscally adjust



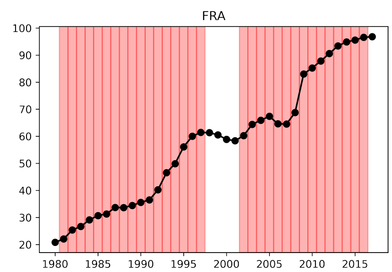
(a) Australian: 1989-2017



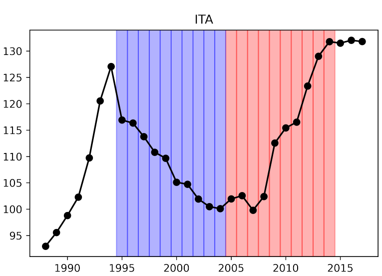
(b) Belgium: 1981-2017



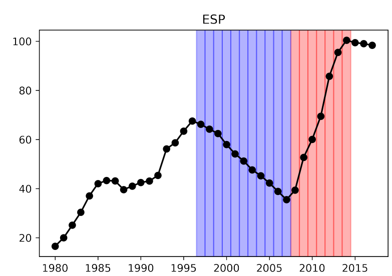
(c) Canada: 1981-2017



(d) France: 1981-2017



(e) Italy: 1989-2017



(f) Spain: 1981-2017

Figure 3: Debt-to-GDP ratios in selected advanced economies. Debt increase episodes are highligted in red and debt decrease episodes are highlighted in blue (Source: Zhang, 2021).

before debt reaches that threshold.

Financial repression must produce a quasi-fiscal revenue $\theta = \delta + r\left(b_a^m - \underline{b}\right) = 0.084$, i.e., 8.4% of potential GDP. The implied inflation rate is obtained by setting $i_m(t) = 0$ in equation (10), which gives an annual inflation rate of 26.6%. This is high but well below the inflation levels that would be expected to be associated with massive government debt monetization. The quasi-fiscal revenue associated with financial repression is conceptually equivalent to seigniorage derived from the printing of broad money, which is much higher than the seigniorage from printing base money for any given level of inflation.

In the absence of financial repression a rollover crisis is triggered as soon as the face value of debt reaches $b_a^d = \underline{b}/\underline{q} = 1.2$, i.e., 120% of potential GDP.[6] Debt reaches that level relatively fast if there is a default risk premium in the passive regime.

Numerical simulations. In the general case we numerically solve for the equilibrium policy functions and simulate the economy with and without financial repression over a period of 10,000 years. We then count the number of times that defaults or financial repression episodes occur in the simulated economy. This exercise confirms the insights from the tractable model.

Under the optimal policy, financial repression is used once every one thousand years on average and the economy spends 0.8% of the time (10 months per century) under financial repression. The occurrence of financial repression episodes increases the average inflation rate by 0.2%.

By contrast, if financial repression is excluded the government defaults every 20 years on average. This reduces welfare because of the fixed output cost of default. The welfare loss relative to the equilibrium with optimal financial repression is equivalent to a 0.9% permanent decrease in consumption.

The difference between the two equilibria is illustrated by Figures 4 and 5, which show the path for the face value of debt over 200 years of simulation starting from the same initial conditions. Figure 4 shows the path with optimal financial repression. The debt path exhibits the long swings that we observe in the data. During this 200-year period the government always implements a fiscal adjustment before debt reaches the threshold that would

[6]This threshold could be increased by assuming a lower value for the bond price that triggers a rollover crisis. Condition (14) sets a lower bound of 0.13 on $\underline{q}$ so there is room to do that.

trigger financial repression, so that there is neither financial repression nor default. Figure 5 shows the equilibrium when financial repression is ruled out. Debt increases much faster because of the risk premium coming from the expectation of a default, which leaves little time for the government to do the adjustment and indeed frequently leads to a default.

Remark. Our calibration assumes some symmetry between debt-increase and debt-decrease episodes, in particular by assuming that these episodes have the same average duration. This assumption is not satisfied in the data: debt-increase episodes last 3 years more than debt-decrease episodes on average in our sample. Furthermore the debt ratio tends to increase faster during debt-increase episodes than it decreases during debt-decrease episodes. To some extent this may be due to our definition of episodes but this also reflects the fact that there is an upward drift in the debt-to-GDP ratio in the data. If we calibrate the model to reflect this drift in the debt-to-GDP ratio, the economy ends up spending most of its time with high debt and financial repression in the long run. We have instead calibrated the model based on the assumption that the government debt to GDP ratio does not drift and stay at the same average level as in 2017.

5 Extensions

This paper being in part inspired by the experience of the euro area, we look how our analysis of optimal financial repression changes in a monetary union. We then discuss the case where $r < g$ and its implications for optimal financial repression.

5.1 Monetary union

Consider a monetary union with n countries, where each country is like the closed economy described in section 2. If there is free trade all the countries have the same inflation rate because of the law of one price. If there is perfect capital mobility bank deposits must pay the same real return across countries, and equation (10) implies that all countries must extract the same quasi-fiscal revenue θ from financial repression.

The optimal financial repression policy depends on whether international transfers are allowed and on the degree of international capital mobility inside the union. We consider two polar cases in this section, the case with

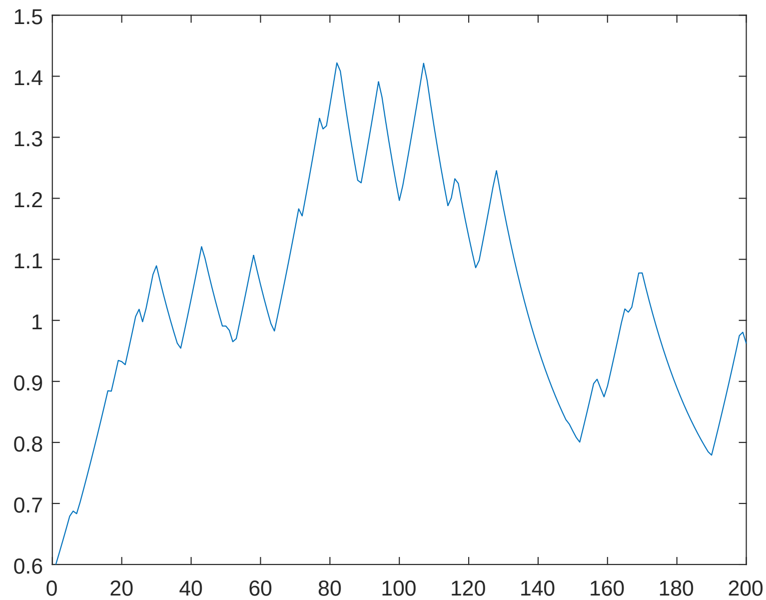


Figure 4: Debt path with optimal financial repression

free capital mobility and transfers, and the case without transfers and with financial market segmentation.

First, consider the case with transfers and free capital mobility. We assume that countries are identical ex-ante and agree to the contingent transfers that maximize the ex-ante welfare of the representative country. Countries have the same parameter values and start from the same level of debt. We further consider the tractable specification of section 3.

Financial repression is introduced at the level of the union when the debt of at least one country reaches a threshold b_a^m. We denote by θ the quasi-fiscal revenue from financial repression per country. A country that has reaches the debt threshold receives θ plus a transfer z from the rest of the union to stabilize its debt at the threshold. The transfer z should be low enough that the receiving country remains incentivated to do the fiscal adjustment. Simple manipulations of the incentive condition $V_a\left(b_a^m\right) \leq V_p\left(b_a^m\right)$ and the budget constraint $g + rb_a^m = \tau_a + \theta + z$ show that incentives are preserved as long as the transfer does not exceed a fraction of the per-country revenue of

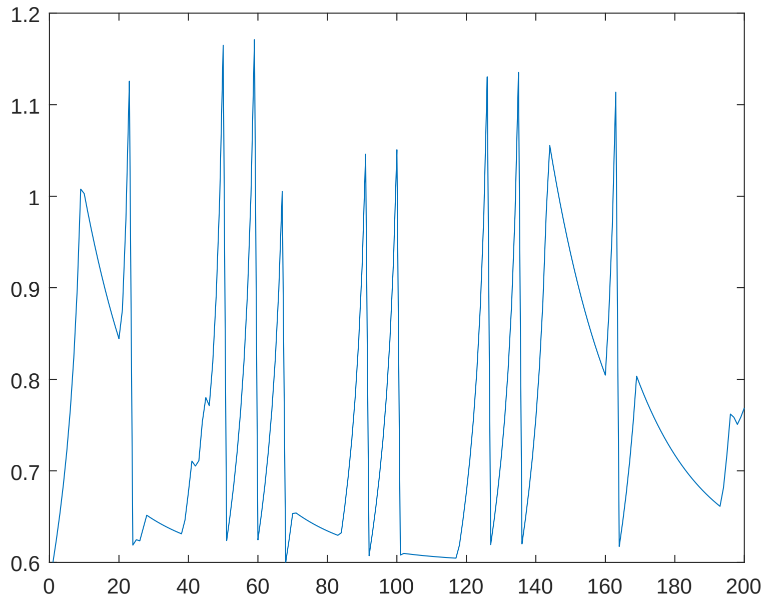


Figure 5: Debt path without optimal financial repression

financial repression,

$$z \leq \left(\frac{\gamma_\theta}{\gamma_\tau} - 1\right)\theta.$$

If the number of countries at the threshold is $n' \leq n$, the transfer could reach up to $(n/n' - 1)\,\theta$ per receiving country. Thus, the incentive constraint is binding as soon as $n' < \frac{\gamma_\tau}{\gamma_\theta} n$, that is, the fraction of countries whose government debt is backstopped by financial repression is lower than $\gamma_\tau/\gamma_\theta$. For our baseline calibration this fraction is $1/4$. When the incentive constraint is binding, financial repression is excessive in the sense that financial repression produces revenue that is not entirely distributed to the countries that need it.

If the constraint is binding the recipient countries's welfare is $V_p\left(b_a^m\right)$. Hence the no-default constraint is satisfied as long as $b_a^m \leq b_p^d$ (see Figure 2). The debt threshold that triggers financial repression is higher than in the case with one country because the recipient countries do not bear the full cost of financial repression. Other things equal, raising the debt threshold makes financial repression less likely ex ante and raises welfare.

Second, consider the case without international transfer. For example, if the union is composed of countries that have different fiscal processes or initial conditions, the countries that are more likely to be contributors than recipients may oppose any transfer or even any financial repression. But if financial repression is completely ruled out the whole union is in an equilibrium with frequent defaults. A fiscally heterogeneous monetary union is likely to be marked by instability and tensions between member countries.

This problem can be mitigated if national banking systems are segmented by capital controls. Then financial repression can be implemented in one country at least up to the level that does not require higher inflation. This lowers the debt threshold triggering financial repression below the threshold prevailing in one country with its own currency or in a union with transfers. In that scenario financial repression would be limited to the countries that need it but it would become more frequent.

5.2 Negative interest-growth differential

In this model with zero growth so a negative interest-growth differential is captured by assuming a negative real interest rate, $r < 0$. The value functions that we have manipulated so far do not have well-defined values when the discount rate is negative. Thus, we assume that the government is impatient in the sense that it discounts the future at a rate ρ that is higher than the real interest rate and is strictly positive. This assumption is often made in the sovereign debt literature to mitigate the precautionary savings motive of the government, and it can be justified by a political agency problem that reduces the horizon of the policymaker.

The negativity of the real interest rate affects the dynamics of debt. In the active regime government debt converges to a finite level,

$$\lim_{t \to +\infty} b_a(t) = \underline{b} + \frac{\delta}{(-r)},$$

instead of diverging to infinity. It does not increase welfare to default or to fiscally adjust because in this topsy-turvy world, a lower debt leads to an increase in taxation. Thus the equilibrium is default-free but incentive-incompatible in the absence of financial repression. If the interest rate permanently switches to a positive level there is an immediate default for high levels of debt. It would be interesting to study an extension of the model where r follows a Markov process taking negative and positive values but this is left for future research.

6 Conclusions

We presented a model where the government makes endogenous policy choices over fiscal policy, financial repression and default. A plausible calibration of the model implies that financial repression is more costly than fiscal policy but less costly than a default if it is used optimally. Financial repression should be used to contain the increase in government debt and avoid a default when government reaches a threshold and fiscal policy is in a deadlock. Because the expectation of financial repression stabilizes the debt dynamics the government is most often able to implement a fiscal adjustment before debt reaches the threshold so that financial repression is rarely implemented in equilibrium. However completely ruling out financial repression leads to chronic default premia and frequent defaults in equilibrium.

This has implications for the optimal governance of monetary and financial policies. If there are reasons to insulate those policies from short-run political influences and entrust them to an independent central bank, our analysis does not invalidate this conclusion most of the time. It suggests however that the objective of not letting the government default should sometimes override the normal-times objectives of monetary and financial policies. This escape clause should be used very rarely. We have not discussed at length the governance of the escape clause itself because the assumption of a benevolent government ensures that financial repression is used optimally. This question becomes more interesting if a political agency problem makes the government's objective function differ from welfare.

The analysis could be extended in several other directions. First, our model assumed that government debt is real (i.e., CPI-indexed) but in the real world most government debt is nominal. Nominal debt introduces new channels that have been left aside in the paper. Nominal debt is inflated away during episodes of financial repression but the expectation of debt debasement raises the nominal interest rate ex ante. The inflation risk premium can destabilize the debt dynamics in the same way as a default risk premium, but the magnitude of this effect could be moderate if inflation remains moderate, i.e., if debt debasement is not the main mechanism by which debt is reduced in episodes of financial repression.

The micro-foundations behind the cost of financial repression and the cost of default could be further developed. The banking sector could invest in assets other than government debt and to the extent that certain investment opportunities can be financed only by banks the cost of financial repression would include a crowding-out effect. The cost of government default could also be larger when banks hold larger amounts of government debt. Finally, our analysis was based on exogenous active and passive fiscal policy rules. It should be possible to assume shocks to spending and endogenize the policy rule.

APPENDIX A. MODEL WITH ENDOGENOUS OUTPUT

This appendix presents an extension of the baseline model in which output is produced with labor. This allows us to calibrate the parameters for the cost of fiscal revenue, γ_τ.

Assumptions. Assume individuals maximize

$$U_0 = E_0 \left\{ \int_0^{+\infty} \left[c(t) + u(m(t)) - v(\ell(t))\right] e^{-rt} dt \right\}, \tag{27}$$

where ℓ is labor and the disutility of labor $v(\ell)$ is increasing and convex.

Output is produced with labor, $y = f(\ell)$, where the production function is given by

$$f(\ell) = A\ell^{1-\alpha}.$$

Fiscal revenue is financed by a tax on production τ_y so that

$$\tau = \tau_y y.$$

Calibrating γ_τ. We calibrate γ_τ by linearizing welfare

$$\frac{d}{d\tau}\left[f(\ell) - v(\ell)\right] = -\gamma_\tau, \tag{28}$$

where the link between ℓ and τ is implicitly defined by the following two equations

$$\tau = \tau_y f(\ell), \tag{29}$$

$$(1-\tau_y) f'(\ell) = v'(\ell). \tag{30}$$

Equation (29) states that fiscal revenue is financed by the tax on production τ_y . Equation (30) is the first-order condition for firms' maximization of their profits $(1-\tau_y) f(\ell) - v(\ell)$. We linearize the model around the point where fiscal revenue covers government expenditures, $\tau = g$.

Equation (28), (30), and $f'(\ell) = (1-\alpha)\, y/\ell$ imply

$$\gamma_\tau = -\left[f'(\ell) - v'(\ell)\right] \frac{d\ell}{d\tau} = -\tau_y f'(\ell) \frac{d\ell}{d\tau} = -(1-\alpha) \frac{\tau}{\ell} \frac{d\ell}{d\tau}. \tag{31}$$

Log differentiating (30) gives

$$\frac{d\tau_y}{1-\tau_y} = -(\alpha+\psi)\frac{d\ell}{\ell}, \tag{32}$$

where $\psi = v"(\ell)\,\ell/v'(\ell)$is the inverse of the Frisch elasticity of labor supply. Log differentiating (29) gives

$$\begin{aligned}
\frac{d\tau}{\tau} &= \frac{d\tau_y}{\tau_y} + (1-\alpha)\frac{d\ell}{\ell}, \\
&= -\frac{1-\tau_y}{\tau_y}(\alpha+\psi)\frac{d\ell}{\ell} + (1-\alpha)\frac{d\ell}{\ell}, \\
&= -\left[\left(\frac{y}{g}-1\right)(\alpha+\psi)-(1-\alpha)\right]\frac{d\ell}{\ell},
\end{aligned} \tag{33}$$

where the second line is derived using (32) and the last line uses $\tau_y = g/y$. Using (33) to substitute out $\frac{\tau}{\ell}\frac{d\ell}{d\tau}$ in (31) finally gives

$$\gamma_\tau = \left[\left(\frac{y}{g}-1\right)\frac{\alpha+\psi}{1-\alpha}-1\right]^{-1}.$$

The value reported in Table 1 is derived assuming $y/g = 3$, $\alpha = 1/3$, and $\psi = 5$.

APPENDIX B. PROOFS

Proof of Proposition 1. The only part of Proposition **1** that is not proved in the main text is that condition (21) is sufficient to ensure the incentives condition $V_a(b) \leq V_p(b)$ for all $b \leq b_a^d$. This results from the following Lemma, where $U_a(t)$ denotes the level of welfare if the economy is in the active regime at time t and $U_p(t) = V_p(b(t))$ is the level of welfare if the economy switches to the passive regime at time t.

Lemma 3 *If $U_a(t^*) > U_p(t^*)$ for some time t^*, then $U_a(t) > U_p(t)$ for all $t \geq t^*$.*

Proof. Denote by $t^* + \Delta t$ the lowest time after t^* such that $U_a(t^* + \Delta t) = U_p(t^* + \Delta t)$, assuming such a time exists. The fact that $U_a(t) \geq U_p(t)$ for all t between t^* and $t^* + \Delta t$ and equation (24) with $\theta = 0$ imply that

$$U_a'(t) \geq rU_a(t) - (\overline{y} - \gamma_\tau \tau_a),$$

for all $t \in (t^*, t^* + \Delta t)$. Multiplying by e^{rt} and integrating by parts between t^* and $t^* + \Delta t$ then gives

$$\begin{aligned} U_a(t^* + \Delta t) &\geq U_a(t^*) - \left[\frac{\overline{y} - \gamma_\tau \tau_a}{r} - U_a(t^*)\right]\left(e^{r\Delta t} - 1\right), \\ &> U_a(t^*) - \gamma_\tau \left[\frac{g - \tau_a}{r} + b(t^*)\right]\left(e^{r\Delta t} - 1\right), \end{aligned} \tag{34}$$

where the second inequality was derived from $U_a(t^*) > U_p(t^*)$ and using (17) to substitute out $U_p(t^*)$.

The inequality $q_a(t) \leq 1$ implies $b'(t) \geq g + rb(t) - \tau_a$ in the active regime. Multiplying by e^{rt} and integrating by parts between t^* and $t^* + \Delta t$ then gives

$$b(t^* + \Delta t) \geq b(t^*) e^{r\Delta t} + (g - \tau_a) \frac{e^{r\Delta t} - 1}{r}.$$

Using equation (17) then gives

$$U_p(t^* + \Delta t) \leq U_p(t^*) - \gamma_\tau \left[\frac{g - \tau_a}{r} + b(t^*)\right]\left(e^{r\Delta t} - 1\right). \tag{35}$$

Equations (34), (35) and $U_a(t^*) > U_p(t^*)$ imply $U_a(t^* + \Delta t) > U_p(t^* + \Delta t)$, whence a contradiction. There cannot exist a Δt such that $U_a(t^* + \Delta t) = U_p(t^* + \Delta t)$, which proves the lemma. ■

To prove the Proposition, observe that $V_a\left(b_a^d\right) \leq V_p\left(b_a^d\right)$ implies $V_a(b) \leq V_p(b)$ for all $b \leq b_a^d$. If there were a time t^* such that $b(t^*) < b_a^d$ and $U_a(t^*) > U_p(t^*)$, functions $U_a(t)$ and $U_p(t)$ would have to intersect each other at a later time t, which would contradict the lemma.

Proof of Proposition 2. The problem is to maximize period-0 welfare in the active regime

$$U_a(0) = \int_0^{+\infty} \left[\overline{u} - \gamma_\tau \tau_a - \gamma_\theta \theta(t) + \phi V_p(b(t))\right] e^{-(r+\phi)t} dt.$$

over the policies pursued contingent on staying in the active regime, $(\theta(t))_{t\geq 0}$ and $(b(t))_{t\geq 0}$, subject to the government's budget constraint, $b'(t) + \tau_a + \theta(t) = g + rb(t)$; the no-default constraint, $U_a(t) \geq V_d$; the fiscal adjustment incentive constraint, $U_a(t) \leq V_p(b(t))$; and the non-negativity constraint on non-fiscal revenue, $\theta(t) \geq 0$.

We leave aside the fiscal adjustment incentive constraint and will show that it is satisfied in equilibrium. Leaving aside unimportant constants, the Lagrangian of the problem is

$$\mathcal{L}_0 = \int_0^{+\infty} \left\{-\gamma_\theta \theta(t) + \phi V_p(b(t)) + \lambda(t)\left[b'(t) + \theta(t) - rb(t)\right] + \mu(t) U_a(t) + \nu(t)\theta(t)\right\} e^{-(r+\phi)t} dt.$$

where $\lambda(t)$, $\mu(t)$ and $\nu(t)$ are the costate variables for, respectively, the government's budget constraint, the no-default constraint and the non-negativity constraint on quasi-fiscal revenue.

Let us define $M(t) = \int_0^t \mu(s)\, ds$. Integrating by parts and using the transversality condition we have

$$\int_0^{+\infty} \lambda(t) b'(t) e^{-(r+\phi)t} dt = -\lambda(0) - \int_0^{+\infty} \left[\lambda'(t) - (r+\phi)\lambda(t)\right] b(t) e^{-(r+\phi)t} dt,$$

and

$$\begin{aligned} \int_0^{+\infty} \mu(t) U_a(t) e^{-(r+\phi)t} dt &= -M(0) U_a(0) - \int_0^{+\infty} \left[U_a'(t) - (r+\phi) U_a(t)\right] M(t) e^{-(r+\phi)t} dt, \\ &= -M(0) U_a(0) - \int_0^{+\infty} \left[\overline{u} - \gamma_\tau \tau_a - \gamma_\theta \theta(t) + \phi V_p(b(t))\right] M(t) e^{-(r+\phi)} \end{aligned}$$

where the second line is obtained by using (24), the valuation equation for $U_a(t)$.

Using the integration-by-part formula to substitute out the corresponding terms in $\mathcal{L}_0$, differentiating with respect to $\theta(t)$ and $b(t)$and using $V_p'(b) = -\gamma_\tau$ give the first-order conditions

$$\lambda(t) + \nu(t) = \gamma_\theta [1 + M(t)], \tag{36}$$
$$\lambda'(t) = \phi \{\lambda(t) - \gamma_\tau [1 + M(t)]\}. \tag{37}$$

Using these conditions it is possible to prove the following lemma.

Lemma 4 *The government uses quasi-fiscal revenue from financial repression only if the no-default constraint is binding: at any time t, either* $U_a(t) < V_d$ *and* $\theta(t) = 0$ *or* $U_a(t) = V_d$ *and* $\theta(t) > 0$.

Proof. First, assume that $\theta(t) > 0$, that is, $\nu(t) = 0$. By (36) and (37) this implies $\lambda'(t) > 0$ and so $M'(t) = \mu(t) > 0$. Hence $U_a(t) = V_d$.

Conversely, assume $\theta(t) = 0$. Using (24) and $V_p(b(t)) - U_a(t) \geq 0$ this implies $U_a'(t) < 0$ which is consistent with the no-default constraint only if $U_a(t) > V_d$. ■

Proposition 2 follows from the lemma. Equation (36) and $M(0) = 0$ imply $\lambda(0) + \nu(0) = \gamma_\theta$. If $\nu(0) > 0$, quasi-fiscal revenue is initially equal to 0. As long as this is the case, constraint $U_a(t) \leq V_d$ is not binding and $M(t)$ stays equal to 0. By equation (37) the dynamics of $\lambda(t)$ are given by $\lambda'(t) = \phi[\lambda(t) - \gamma_\tau]$. Hence it must be that $\lambda(0) > \gamma_\tau$ and $\lambda(t)$ increase over time until reaches γ_θ, at which point $\nu(t)$ starts to be equal to 0 and $M(t)$ starts to be strictly positive and increasing over time. In other terms, one can divide time into two intervals. In the first time interval, the government sets quasi-fiscal revenue θ to zero and let its debt increase until its welfare is equal to the default level V_d. Once it reaches this level, the government starts to use quasi-fiscal revenue from financial repression to prevent debt from increasing and keep welfare at V_d. The level of debt for which this is the case satisfies $V_a(b_a^m) = V_d$. Using (24), (17), (18), the definition of δ in equation (8) and simple manipulations give (25).

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