# Economics of Collusion
# ENCORE

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July 6-7, 2004

## 1 Introduction

1. What is collusion?

   (a) Practical definition: Firms collude when they coordinate their prices and quantities. This is typically done with the intent of raising price and earning higher profit.

   (b) Technical definition: Firms collude when they achieve an equilibrium for a multi-period game in which payoffs exceed equilibrium payoffs for the one-period game.

2. Antitrust and Competition Law

   (a) United States

      i. Section 1 of the Sherman Act (1890): "Every contract, combination in the form of trust or otherwise, or conspiracy, in restraint of trade or commerce among the several States, or with foreign nations, is declared to be illegal."

      ii. In the 1911 American Tobacco case, the Supreme Court stated that "the words 'restraint of trade' . . . only embraced acts or contracts or agreements or combinations . . . which, either because of their inherent nature or effect or because of the evident purpose of the acts, etc., injuriously restrained trade."

      iii. *Per se* rule - When a practice can have no beneficial effects but only harmful effects, the "inherent nature" of the practice is injuriously restraining trade and is thereby *per se* illegal. Price fixing by a cartel fits this description and is thus illegal by virtue of the behavior regardless of its intent or effect. There is no allowable defense.

   (b) European Union

i. Article 65 of the Treaty of Paris (1951) prohibited agreements among firms which tend to reduce competition within the Common Market.

ii. Article 81 of the Treaty of the European Communities (1999) - "The following shall be prohibited as incompatible with the common market: all agreements between undertakings, decisions by associations of undertakings and concerted practices which may affect trade between Member States and which have as their object or effect the prevention, restriction or distortion of competition within the common market, and in particular those which:

A. directly or indirectly fix purchase or selling prices or any other trading conditions

B. limit or control production, markets, technical development, or investment

C. share markets or sources of supply

D. apply dissimilar conditions to equivalent transactions with other trading parties, thereby placing them at a competitive disadvantage

E. make the conclusion of contracts subject to acceptance by the other parties of supplementary obligations which, by their nature or according to commercial usage, have no connection with the subject of such contracts."

3. A few facts on recent price-fixing activity

(a) Annual reports of the competition/antitrust authorities

i. *European Community Competition Policy, XXXth Report on Competition Policy* (2000): "International cartels are estimated to represent a drain of hundreds of millions of euros on the European economy. ... Since 1998, the number of cartel cases investigated by the Commission has increased dramatically."

ii. *Annual Report, Antitrust Division, United States Department of Justice* (1999): "Since the beginning of [fiscal year] 1997, the Antitrust Division has prosecuted international cartels affecting over $10 billion in U.S. commerce [and] has obtained over $1.5 billion dollars in criminal fines ... The dramatic increase in fines reflects the fact that major international cartels prosecuted over the past few years have been bigger, in terms of the volume of affected commerce and the amount of harm caused to American businesses and consumers, than any conspiracies previously encountered by the Antitrust Division."

(b) U.S. antitrust activity in the 1990s

i. U.S. Department of Justice antitrust fines grew dramatically in the 1990s due to higher penalties as a result of revised sentencing guidelines and the discovery of some large cartels.

ii. Summary of DoJ price-fixing cases that led to a guilty verdict or a *nolo contendere* plea.

A. Cartels lasted around 6-7 years before discovery.

B. More price-fixing cases on an annual basis in more recent time period.

Frequency and Length of Cartels in U.S. Dept of Justice Cases

| Length of Cartel (years) | Frequency (1961-88)* | Frequency (1988-96)** |
|---|---|---|
| 0-2 | 12 | 10 |
| 2-4 | 42 | 22 |
| 4-6 | 42 | 30 |
| 6-8 | 25 | 19 |
| 8-10 | 21 | 8 |
| 10-12 | 12 | 4 |
| 12-14 | 12 | 1 |
| 14-16 | 5 | 1 |
| 16-18 | 9 | 1 |
| 18-20 | 0 | 2 |
| $> 20$ | 4 | 2 |
| Sample Size | 184 | 100 |
| Mean | 7.27 | 6.02 |
| Median | 5.80 | 5.23 |
| Minimum | 0.33 | 0.50 |
| Maximum | 40.8 | 26.0 |

*Bryant and Eckard, *Review of Economics and Statistics*, 1991

*Collected by Joseph Harrington with the assistance of Wei Xiao.

4. What is the distinction between explicit collusion and tacit collusion?

(a) Explicit collusion occurs when firms (or, more specifically, their employees) directly communicate about price, market allocation, sales quotas, and other information pertinent to coordinating their prices and quantities.

(b) Tacit collusion (also known as conscious parallelism) is when a less competitive outcome is achieved through non-direct communication such as a mutual understanding among firms, price leadership, or signalling using market instruments such as price. Judge Richard Posner describes tacit collusion as the behavior of "competing sellers ... [who] coordinate their pricing without conspiring in the usual sense of the term - that is, without any overt or detectable acts of communication." [*Antitrust Law*, 2nd Edition, 2001.]

(c) Legal distinction: Tacit collusion is, for all intents and purposes, legal while explicit collusion is illegal.

(d) Game-theoretic distinction: Existing theory does not distinguish between explicit and tacit collusion. Rather, theory characterizes when certain collusive price or quantity-setting rules are stable. It doesn't speak to the issue of how firms are able to agree to those rules.

5. What are the challenges faced by a cartel?
   (a) Sustaining a collusive outcome
   (b) Selecting a collusive outcome
   (c) Eluding detection by customers and the authorities
   (d) Example: international steel cartel agreement (1926)

6. What are the challenges faced by the competition/antitrust authorities?
   (a) Discovering cartels
   (b) Prosecuting cartels
   (c) Deterring cartels through prosecution and penalties

7. Overview of lectures
   (a) Sustaining a collusive outcome
      i. Canonical mechanism
      ii. Punishments
      iii. Monitoring and compliance
      iv. Multi-market collusion
   (b) Selection of a collusive outcome
      i. Bargaining-theoretic approach
      ii. Heuristics
   (c) Detection
      i. Eluding detection
      ii. Distinguishing between collusion and competition

8. Discussion questions
   (a) Should we seek to prosecute tacit collusion as suggested by Judge Richard Posner? If so, how would we implement such a policy? What are its advantages and disadvantages?

      Though tacit collusion remains largely immune to prosecution, some scholars and judges are not content for it to remain that way. Judge Posner argues that tacit collusion can, in some instances, be thought as a contractual arrangement and, on those terms, warrants prosecution: "... one seller communicates his "offer" by restricting output, and the offer is "accepted" by the actions of his rivals in restricting their outputs as well. It may therefore be appropriate in some cases to instruct a jury to find an agreement to fix prices if it is satisfied that there was a tacit meeting of the minds of the defendants on maintaining a noncompetitive pricing policy. ... What is being proposed is less the alteration of the substantive contours of the law than a change in evidentiary requirements to permit illegal price fixing to be found in circumstances in which an actual meeting of the minds on a noncompetitive price can be inferred even though explicit collusion cannot be proved." [Richard A. Posner, *Antitrust Law*, 2nd Ed., 2001; pp. 94-5, 98.]

(b) A distinction between the U.S. and the E.U. is that the E.U. only allows government fines while the U.S. allows damages and prison sentences in addition to government fines. Which is the better policy?

(c) What types of behavior and outcomes are signals of collusion? How do we distinguish between collusion and competition? Is parallel behavior evidence of collusion? Stability of market shares?

(d) What industry conditions are conducive to explicit collusion? (Note: Keep in mind that if firms have formed a cartel and thereby exposed themselves to possible prosecution, it is probably because they either were unable to tacitly collude or were not as effective by tacitly colluding.)

# 2 Sustaining Collusion

1. Modelling objectives

   (a) Construct an oligopoly model in which collusion can be generated as an outcome.

   (b) The resulting theory should be able to explain

      i. collusive outcomes in terms of the price level and the allocation of market share and identify their determinants

      ii. when a cartel would form and identify its determinants

2. Basic approach - enrich the standard static oligopoly model.

   (a) Assume firms interact repeatedly (either forever or indefinitely) and this is common knowledge among them.

   (b) Firms care about future profits.

   (c) Some part of the history - for example, past prices - is commonly observed so that firm behavior can condition on that history.

## 2.1 Some Game Theory

### 2.1.1 Subgame Perfect Equilibrium

1. Game

(a) $n$ players choose actions in each of $T$ periods where $T$ could be finite or infinite

(b) A strategy assigns a current action conditional on the past history known by that player

2. Subgame perfect equilibrium

   (a) A strategy profile is a *subgame perfect equilibrium* if and only if (iff), at the initial decision node for every subgame, a player's strategy prescribes an action that maximizes his payoff given

      i. all other players act according to their strategies and

      ii. this player acts according to his strategy at all decision nodes farther down the tree.

   (b) Only one-shot deviations need be considered because optimality is checked for every subgame.

3. Markov perfect equilibrium

   (a) A subgame perfect equilibrium is a *Markov perfect equilibrium* if continuation strategies depend only on the payoff-relevant portion of the history.

   (b) In some games, this implies that continuation strategies are sensitive only to elements of the history which influence the strategic form of the subgame.

   (c) Eric Maskin and Jean Tirole, "Markov Perfect Equilibrium, I: Observable Actions," *Journal of Economic Theory*, 100 (2001), 191-219.

#### 2.1.2 Folk Theorem

1. Preliminaries

   (a) Stage game

      i. $A_i$ is player $i's$ (pure) action set for the stage game

      ii. $v_i : A_1 \times \cdots \times A_n \to \Re$ is player $i's$ payoff function for the stage game

   (b) Infinitely repeated game

      i. A strategy is $\{f_i^t\}_{t=1}^{\infty}$ where $f_i^t : \Pi_{j=1}^n A_j^{t-1} \to A_i$

      ii. Payoff is the sum of discounted single-period utilities where $\delta \in (0, 1)$ is player $i's$ discount factor

   (c) Minimax

      i. $M_{-i}$ are strategies of the other players that minimize player $i's$ maximum payoff

      ii. $M_{-i} \in \arg\min_{a_{-i}} \max_{a_i} v_i(a_i, a_{-i})$

      iii. $v_i^* \equiv \max_{a_i} v_i(a_i, M_{-i})$

(d) Set of individually rational payoffs

   i. $(v_1, ..., v_n)$ is individually rational iff $v_i \geq v_i^* \forall i$
   ii. $U \equiv \{(v_1, ..., v_n) : \exists (a_1, ..., a_n) \in A_1 \times \cdots \times A_n \text{ with } v_i(a_i, a_{-i}) = v_i \forall i\}$
   iii. $V \equiv$ convex hull of $U$ (smallest convex set containing $U$)
   iv. $V^* \equiv \{(v_1, ..., v_n) \in V | v_i > v_i^* \forall i\}$

2. Folk Theorem

   (a) For any $(v_1, ..., v_n) \in V^*$, if $\delta$ is sufficiently close to one then there exists a Nash equilibrium such that the average payoff is $v_i \forall i$.
   (b) Proof
      i. Let $\bar{v}_i = \max_{(a_i, a_{-i})} \ v_i(a_i, a_{-i})$
      ii. Suppose a deviation is responded to by the other players using $M_{-i}$
      iii. Player $i$ prefers not to deviate if:

$$\frac{v_i}{1-\delta} \geq \bar{v}_i + \frac{\delta v_i^*}{1-\delta} \Leftrightarrow v_i \geq (1-\delta)\bar{v}_i + \delta v_i^*.$$

      iv. If $v_i > v_i^*$ then this condition holds for $\delta$ sufficiently close to 1
   (c) Subgame perfect equilibrium
      i. There is an issue as to whether other players minimaxing player $i$ is itself part of a Nash equilibrium.
      ii. The exact same theorem is true when NE is replaced with SPE
         A. when $n = 2$
         B. when $n \geq 3$, the set of stage games is subject to a mild restriction

## 2.2 Canonical Mechanism - Grim Trigger Strategy Equilibria

### 2.2.1 Static Quantity Game (Cournot)

1. Structure

   (a) $n \geq 2$ firms have homogeneous products.
   (b) Firms make simultaneous quantity decisions.
   (c) Price is set in the market so as to equate supply and demand.

2. Assumptions on the inverse market demand function

   **A1** $P(\cdot) : \Re_+ \rightarrow \Re_+$ is continuous and bounded.
   **A2** $\exists$ finite $\bar{Q} > 0$ such that $P(Q) = 0$ iff $Q \geq \bar{Q}$.
   **A3** $P(\cdot)$ is twice differentiable and $P'(Q) < 0 \forall Q \in (0, \bar{Q})$.

3. Assumptions on a firm's cost function

**A4** $C_i(\cdot) : \Re_+ \to \Re_+$ is continuous.

**A5** $C_i(\cdot)$ is twice differentiable and $C_i'(q) \geq 0 \forall q > 0$.

4. Assumptions on a firm's profit function

$$\pi_i(q_i, Q_{-i}) \equiv P(q_i + Q_{-i})q_i - C_i(q_i)$$

   (a) $\pi_i(\cdot)$ is continuous and bounded from above.

   (b) $\pi_i(\cdot)$ is twice differentiable in $q_i$ and $Q_{-i} \forall (q_i, Q_{-i}) \in \{(q_i, Q_{-i}) | q_i + Q_{-i} \in (0, \bar{Q})\}$.

**A6** $\pi_i(q_i, Q_{-i})$ is strictly quasi-concave in $q_i$ $\forall (q_i, Q_{-i}) \in \{(q_i, Q_{-i}) | q_i + Q_{-i} \in (0, \bar{Q})\}$.

5. Existence: By A1-A6, a Nash equilibrium exists.

6. Symmetric Equilibrium: If $C_i(q) = C(q) \forall i$ then, by A1-A6, a symmetric Nash equilibrium exists.

7. Interior Symmetric Equilibrium: If $C_i(q) = C(q) \forall i$ and $P(0) > C_i'(0)$ then, by A1-A6, a symmetric Nash equilibrium exists in which the quantity is positive.

8. Common additional assumptions

**A7** $P''(Q) \leq 0 \forall Q \in (0, \bar{Q})$.

**A8** $C_i''(q) \geq 0 \forall q > 0$.

9. Uniqueness: By A1-A8, a unique Nash equilibrium exists.

### 2.2.2 Infinitely Repeated Oligopoly Game

1. Stage game is symmetric Cournot game under the standard assumptions so that

   (a) a firm's best reply function, $\psi(Q_{-i})$, exists and is continuous,

$$\psi(Q_{-i}) \in \arg\max \pi(q, Q_{-i})$$

   (b) a symmetric interior Nash equilibrium exists,

$$\widehat{q} \in \arg\max \pi\left(q, (n-1)\,\widehat{q}\right)$$

   (c) joint profit-maximizing quantity exists:

$$q^m \in \arg\max \pi(q, (n-1)q)$$

2. Strategy

(a) What is a strategy?

i. A strategy maps from the space of information sets to the space of actions

ii. The form of a strategy depends on what we assume about what firms know

(b) Information sets

i. Firms know the entire history; that is, all firms' past quantities. A strategy is of the form $\{f_i^t\}_{t=1}^{\infty}$ where $f_i^t : \Pi_{j=1}^n A_j^{t-1} \to A_i$; where $A_i$ is the space of feasible quantities that firm $i$ can produce. $f_i^t$ prescribes a quantity for firm $i$ depending on the past quantities of all firms

ii. Firms know past prices and their own quantities. A strategy is of the form $\{f_i^t\}_{t=1}^{\infty}$ where $f_i^t : A_i^{t-1} \times \Re_+^{t-1} \to A_i$

(c) What is a Markov strategy?

i. A firm conditions its quantity only on those elements of the history that affect the strategic form of the game

ii. A strategy for firm $i$ is an element from $A_i$ because there is no part of history that influences either the strategy set or payoff functions

3. Payoff is the sum of discounted single-period profits where firm $i's$ discount factor is $\delta_i \in (0, 1)$. Payoff associated with some outcome path is

$$\sum_{t=1}^{\infty} \delta^{t-1} \pi_i \left(q_i^t, Q_{-i}^t\right)$$

### 2.2.3 Initial analysis

1. The set of Markov perfect equilibria is the infinite repetition of the static Nash equilibrium. If this model is to generate a richer theory, it lies in non-Markov equilibria

2. Application of folk theorem to infinitely repeated quantity game

(a) What is the set of individually rational payoff vectors?

i. Minimax payoff is zero as other firms can produce so that price equals zero and thus a firm's profit is zero and its payoff is zero

ii. Any feasible payoff vector that gives each firm a positive payoff

(b) If firms' discount factors are sufficiently close to 1 then for each feasible positive payoff vector there exists a SPE which generates that payoff vector

### 2.2.4 SPE in grim trigger strategies

1. Let us see what strategy profile can work to support collusive outcomes

2. Trigger strategy

(a) Description

$$q_i^1 = q^o$$
$$q_i^t = \begin{cases} q^o \text{ if } q_j^\tau = q^o \forall \tau \leq t-1 \forall j \\ \\ \widehat{q} \text{ otherwise} \end{cases}$$
$$t = 2, 3, \ldots, \; i = 1, \ldots, n$$

where $q^o \in (q^m, \widehat{q}]$.

(b) Deriving necessary and sufficient conditions for this strategy profile to be a SPE

i. Need to show that for any subgame (any period and any history), the strategies form a NE

ii. For each subgame, show that each player's prescribed action for that period is optimal given

A. the other players act according to their strategies in the current period and all players act according to their strategies in future periods

B. Following the logic of dynamic programming, one need only consider one-time deviations from the proposed strategy profile

(c) Consider period 1 or a period $t$ history such that $q_j^\tau = q^o \forall \tau \leq t-1 \forall j$

i. Definitions

$$\pi(q) \equiv P(nq)q - C(q)$$
$$\pi^*(q) \equiv P(\psi((n-1)q) + (n-1)q)\psi((n-1)q) - C(\psi((n-1)q))$$

ii. SPE requires that:

$$\frac{\pi(q^o)}{1-\delta_i} \geq \pi(q, (n-1)q^o) + \delta_i \left( \frac{\pi(\widehat{q})}{1-\delta_i} \right) \forall q \Leftrightarrow$$
$$\frac{\pi(q^o)}{1-\delta_i} \geq \pi^*(q^o) + \delta_i \left( \frac{\pi(\widehat{q})}{1-\delta_i} \right) \Leftrightarrow$$
$$\delta_i \geq \frac{\pi^*(q^o) - \pi(q^o)}{\pi^*(q^o) - \pi(\widehat{q})}$$

iii. A deviation trades off higher profits today for lower profits tomorrow. Such a deviation is undesirable if a firm sufficiently values future profits.

(d) Consider a period $t$ history such that $q_j^\tau \neq q^o$ for some $\tau \leq t-1$, for some $j$

i. SPE requires:

$$\frac{\pi(\widehat{q})}{1-\delta_i} \geq \pi(q, (n-1)\widehat{q}) + \delta_i \left( \frac{\pi(\widehat{q})}{1-\delta_i} \right) \forall q$$

ii. This holds trivially

(e) This strategy profile is a SPE iff:

$$\delta_i \geq \frac{\pi^*(q^o) - \pi(q^o)}{\pi^*(q^o) - \pi(\widehat{q})} \forall i \Leftrightarrow \min\{\delta_1, ..., \delta_n\} \geq \frac{\pi^*(q^o) - \pi(q^o)}{\pi^*(q^o) - \pi(\widehat{q})}.$$

3. Suppose the strategy had firm $i$ producing $\hat{q}$ only if another firm deviated

(a) Strategy is then

$$\begin{aligned} q_i^1 &= q^o \\ q_i^t &= \begin{cases} q^o \text{ if } q_j^\tau = q^o \forall \tau \leq t-1 \forall j \neq i \\ \\ \widehat{q} \text{ otherwise} \end{cases} \\ t &= 2, 3, \ldots, \ i = 1, \ldots, n \end{aligned}$$

(b) This is not a SPE.

i. Consider the following period $t$ history:

$$q_j^\tau = q^o \forall \tau \leq t-2 \forall j, \ q_j^{t-1} = q^o \forall j \neq i, \ q_i^{t-1} \neq q^o$$

ii. Firm $i'$s strategy calls for $q^o$ when $\widehat{q}$ is preferred

(c) It is a NE if $\delta$ is sufficiently high. The problem with it being a SPE is with the non-equilibrium event of player $i$ deviating from $q^o$.

**Example 1: Linear Example**

1. Assumptions

(a) Linear inverse market demand curve:

$$P(Q) = \max\{a - bQ, 0\}$$

where $a, b > 0$.

(b) Linear firm cost function,

$$C_i(q) = cq$$

where $0 \leq c < a$.

(c) Firm profit function

$$\pi(q_i, Q_{-i}) = [a - b(q_i + Q_{-i}) - c] q_i$$

where $Q_{-i} \equiv \sum_{j \neq i} q_j$.

2. Results for the stage game

(a) Best reply function

$$\psi(Q_{-i}) = \frac{a-c}{2b} - \frac{Q_{-i}}{2}.$$

(b) Static Nash Equilibrium

$$\widehat{q} = \psi((n-1)\widehat{q}) \Leftrightarrow \widehat{q} = \frac{a-c}{2b} + \frac{(n-1)\widehat{q}}{2} \Leftrightarrow \widehat{q} = \frac{a-c}{b(n+1)}$$

$$\widehat{\pi} \equiv \frac{(a-c)^2}{b(n+1)^2}$$

3. Equilibrium condition for grim trigger strategy

$$\frac{\pi(q^o)}{1-\delta} \geq \pi^*(q^o) + \frac{\delta\widehat{\pi}}{1-\delta} \Leftrightarrow \pi(q^o) \geq (1-\delta)\pi^*(q^o) + \delta\widehat{\pi} \tag{1}$$

where

$$\begin{aligned} \pi(q) &\equiv (a - bnq^o - c)\,q \\ \pi^*(q) &\equiv [a - b(\psi((n-1)q) + (n-1)q) - c]\,\psi((n-1)q) = \frac{(a-c-b(n-1)q)^2}{4b}. \end{aligned}$$

(1) is then

$$\begin{aligned} (a - bnq^o - c)\,q^o &\geq \frac{(1-\delta)(a-c-b(n-1)q^o)^2}{4b} + \frac{\delta(a-c)^2}{b(n+1)^2} \\ (a - bnq^o - c)\,q^o 4b(n+1)^2 &\geq (1-\delta)(a-c-b(n-1)q^o)^2(n+1)^2 + \delta 4(a-c)^2 \end{aligned}$$

4. Collusive solutions

(a) Define

$$\phi(q^o) \equiv (a - bnq^o - c)\,q^o 4b(n+1)^2 - (1-\delta)(a-c-b(n-1)q^o)^2(n+1)^2 - \delta 4(a-c)^2$$

so the equilibrium condition is

$$\phi(q^o) \geq 0.$$

(b) A solution always exists as it holds when $q^o$ is the static Nash equilibrium quantity:

$$\phi(\widehat{q}) = 0.$$

(c) A non-trivial solution exists by showing that $\phi'(\widehat{q}) < 0$ which implies there exists $q^o < \widehat{q}$ such that $\phi(q^o) > 0$.

**Example 2: Linear Example with Stochastic Shocks**

1. Model

   (a) Standard infinitely repeated quantity game except that demand and cost functions are subject to observable shocks

   $$P_t(Q) = a_0 + a_1 x_t + a_2 Q$$

   $$C_t(q) = (c_0 + c_1 w_t) q$$

   $(x_t, w_t)$ are *iid* demand and cost shifters that are observable in period $t$ prior to firms choosing quantity.

   (b) $\pi(q; x_t, w_t) \equiv P(nq; x_t) q - C(q; w_t)$

   (c) $\pi^*(q; x_t, w_t) = \max_{q_i} P(q_i + (n-1) q; x_t) q_i - C(q_i; w_t)$

   (d) $\widehat{\pi}(x_t, w_t)$ is the static Nash equilibrium profit

2. Optimal collusive quantity: collusive quantity is chosen to maximize profit subject to the constraint that there is no incentive to deviate.

   (a) Characterization

   $$q^*(x_t, w_t) \in \arg\max \pi(q; x_t, w_t) \text{ subject to}$$

   $$\pi(q; x_t, w_t) + \sum_{\tau=1}^{\infty} \delta^\tau E_t\left[\pi\left(q^*(x_{t+\tau}, w_{t+\tau}); x_{t+\tau}, w_{t+\tau}\right)\right] \geq$$

   $$\pi^*(q; x_t, w_t) + \sum_{\tau=1}^{\infty} \delta^\tau E_t\left[\widehat{\pi}(x_{t+\tau}, w_{t+\tau})\right] \forall (x_t, w_t)$$

   (b) Suppose $\delta$ is sufficiently low so that the constraint is binding $\forall (x_t, w_t)$. Then the optimal collusive quantity is the lowest value that satisfies the constraint. Solving

   $$\begin{aligned} & \pi(q; x_t, w_t) + \sum_{\tau=1}^{\infty} \delta^\tau E_t\left[\pi\left(q^*(x_{t+\tau}, w_{t+\tau}); x_{t+\tau}, w_{t+\tau}\right)\right] \\ = \; & \pi^*(q; x_t, w_t) + \sum_{\tau=1}^{\infty} \delta^\tau E_t\left[\widehat{\pi}(x_{t+\tau}, w_{t+\tau})\right] \forall (x_t, w_t) \end{aligned}$$

   for the appropriate root with respect to $q$ yields:

   $$q^*(x_t, w_t) = \widehat{q}(x_t, w_t) - \omega$$

   where $\widehat{q}(x_t, w_t)$ is the static NE quantity,

   $$\widehat{q}(x_t, w_t) = \frac{a_0 + a_1 x_t - c_0 - c_1 w_t}{-(N+1) a_2},$$

and

$$\omega \equiv \frac{2\sqrt{-a_2 L}}{-(N+1)\,a_2}$$

where

$$L \equiv \sum_{\tau=1}^{\infty} \delta^{\tau} E_t \left[\pi\left(q^*\left(x_{t+\tau}, w_{t+\tau}\right); x_{t+\tau}, w_{t+\tau}\right) - \widehat{\pi}\left(x_{t+\tau}, w_{t+\tau}\right)\right]$$

is the loss due to deviation.

## 2.3 Punishments in Theory and Practice

1. Perspective of a game theorist

   (a) The rationale for considering grim trigger strategies is to sustain a higher equilibrium payoff.

   (b) But then why not identify the highest equilibrium payoff that can be achieved? Consider all sorts of punishment strategies and find the one that is most severe but is still an equilibrium. Use that to support the best collusive outcome.

   (c) What do those equilibrium strategies look like? Are they complicated? Simple? Are punishments asymmetric? long?

   (d) The focus of game theory has been on the set of sustainable payoffs.

2. Perspective of an industrial organization economist

   (a) What types of punishments do we think firms actually use?

   (b) Is the severity of the punishment the only criterion for selecting among them? What about simplicity? What about norms having developed for punishments?

   (c) The focus of industrial organization economists is on firm behavior.

### 2.3.1 Theory of Optimal Punishments

**Simple strategy profiles**

1. Introduction (Abreu, *Econometrica*, 1988)

   (a) As a selection device, let us assume firms achieve, for any discount factor, the highest payoff supportable by a SPE

   (b) What is the maximal degree of collusion? What is the most severe credible punishment that can be imposed upon a deviator?

   (c) What can we say about the form of the strategy profile that supports a maximal SPE payoff? Must it make the punishment tailored to the extent of the deviation or the past record of deviations?

2. Preliminaries

   (a) Define $Q^j$ to be an outcome path for the game: $Q^j \in (A_1 \times \cdots \times A_n)^\infty$ where $A_i$ is the stage game action set for player $i$.
   (b) $\Omega^o$ is the set of SPE outcome paths
   (c) $\Omega$ is the set of all outcome paths

3. Definition: $\sigma(Q^0, Q^1, ..., Q^n)$ is a *simple strategy profile* if

   (a) players play according to $Q^0$ until some player deviates from that outcome path
   (b) for any $j \in \{1, ..., n\}$, players play according to $Q^j$ (starting with the first element) when player $j$ deviates from the current path
   (c) if two or more players simultaneously deviate then players play according to the current outcome path

4. Example of a simple strategy profile for the infinitely repeated quantity game

   (a) i. Trigger strategy with $T$ period reversion to static NE
       ii. $Q^0 = \{(q^o, ..., q^o), ...\}$
       iii. $Q^i = \{\underbrace{(\hat{q}, ..., \hat{q}), ..., (\hat{q}, ..., \hat{q})}, (q^o, ..., q^o), ...\}$

5. Main Result

   (a) **Theorem**: $Q^0 \in \Omega^o$ iff $\exists Q^i \in \Omega \forall i$ such that $\sigma(Q^0, Q^1, ..., Q^n)$ is a SPE.
   (b) Remarks
       i. If some outcome path is induced by a SPE then it is induced by a SPE in simple strategy profiles
       ii. In characterizing the set of SPE outcomes, we can then limit our attention to simple strategy profiles
       iii. This greatly simplifies matters as it means that the future path (punishment) depends only on who deviated and no other feature like how much he deviated or how often he deviated

**Maximal degree of collusion - simple example**

1. Introduction

   (a) We want to characterize strategies which achieve the maximal degree of collusion; that is, the highest payoffs for firms
   (b) This entails characterizing the most severe punishment strategy equilibria
       i. The more detrimental is the punishment, the better the outcome to which you can hold players

ii. This may require players participating in their own punishment

2. Example of using a punishment more severe than infinite reversion to a static NE

   (a) Game

| | $\alpha_2$ | $\beta_2$ | $\gamma_2$ |
|---|---|---|---|
| $\alpha_1$ | 5,5 | -2,10 | -8,1 |
| $\beta_1$ | 10,-2 | 0,0 | -10,-1 |
| $\gamma_1$ | 1,-8 | -1,-10 | -9,-9 |

   (b) Punishment: infinite reversion to static NE

      i. Trigger strategy

         A. $Q^0 = \{(\alpha_1, \alpha_2), ...\}$

         B. $Q^1 = Q^2 = \{(\beta_1, \beta_2), ...\}$

      ii. SPE iff

$$\frac{5}{1-\delta} \geq 10 + \delta \left( \frac{0}{1-\delta} \right) \Leftrightarrow \delta \geq 1/2$$

   (c) More severe punishment

      i. Strategy profile

         A. $Q^0 = \{(\alpha_1, \alpha_2), ...\}$

         B. $Q^1 = \{(\beta_1, \gamma_2), (\alpha_1, \alpha_2), ...\}$

         C. $Q^2 = \{(\gamma_1, \beta_2), (\alpha_1, \alpha_2), ...\}$

      ii. SPE Conditions

         A. $i$ defects from $Q^0$ : $5 + \delta 5 \geq 10 - \delta 10 \Leftrightarrow \delta \geq 1/3$ [Ensures it is optimal to participate in the cooperative outcome]

         B. $i$ defects from $Q^i$ : $-10 + \delta 5 \geq -8 - \delta 10 \Leftrightarrow \delta \geq 2/15$ [Ensures it is optimal to participate in one's own punishment]

         C. $i$ defects from $Q^j$ : $-1 + \delta 5 \geq 0 - \delta 10 \Leftrightarrow \delta \geq 1/15$ [Ensures it is optimal to participate in the other's punishment]

         D. SPE iff $\delta \geq 1/3$

   (d) Comparison of punishments

      i. Infinite reversion to static NE results in a condition of $\delta \geq 1/2$

      ii. This punishment, being more severe, can support the same outcome when $\delta \geq 1/3$

**Maximal degree of collusion - quantity game**

- Abreu (*Journal of Economic Theory*, 1986)

1. Assumptions

(a) Stage game is two-firm quantity game with homogeneous goods and constant marginal cost, $c$

(b) $P(\cdot) : \Re_+ \to \Re_+$ is strictly monotonic and continuous

(c) $P(0) > c > 0$ ($c > 0$ is important; try to find where)

(d) $\pi(q) \equiv P(nq)q - cq$ is strictly quasi-concave in $q$ with a maximum of $q^m$

(e) The stage game has a symmetric pure-strategy NE

(f) Comments

i. Rather than place sufficient structure on the profit function directly, it is assumed that a NE exists. Is this more or less general?

ii. Is $P(Q) = \max\{a - bQ, 0\}$ a special case?

2. Statement of problem

(a) Preliminaries

i. $\Gamma$ is the set of strongly symmetric SPE; that is, for every history, the outcome path is symmetric

A. This is a proper subset of the set of SPE in which firms use symmetric strategies

B. Even for asymmetric histories, the prescribed behavior is the same for all firms

ii. $v(\gamma)$ is the payoff to a (symmetric) player from symmetric strategy profile $\gamma$

(b) Problem

i. Find $\gamma^* \in \Gamma$ such that $v(\gamma^*) \geq v(\gamma) \, \forall \gamma \in \Gamma$

ii. Objective is to characterize $\gamma^*$

3. Results

(a) Introduction

i. Interested in deriving strategies that yield the best SPE outcome

ii. Limiting attention to symmetric outcome paths. This is more restrictive than symmetric strategies (e.g., requires punishments to be symmetric)

(b) Optimal symmetric punishment path (OSP)

i. An OSP is a solution to the problem of minimizing firm $i'$s payoff subject to the outcome path being a symmetric SPE outcome path - Find $\widehat{\gamma} \in \Gamma$ such that $v(\widehat{\gamma}) \leq v(\gamma) \, \forall \gamma \in \Gamma$

ii. **Theorem**: $\exists(\bar{q}, q^o)$ such that $\{(\bar{q}, \bar{q}), (q^o, q^o), ...\}$ is an OSP

iii. Interpretation

A. The punishment is producing $\bar{q}$ which may be rather high

B. The reward to going through with the punishment is $q^o$ which may be rather low; this is required since we are looking at punishment paths which are induced by SPE

(c) Stick-and-carrot strategy

i. Simple strategy profile

A. $Q^0 = \{(q^o, q^o), ...\}$

B. $Q^1 = Q^2 = \{(\bar{q}, \bar{q}), (q^o, q^o), ...\}$ [OSP]

ii. SPE conditions

A. Define: $\pi^*(q) \equiv P(\psi(q) + q)\psi(q) - c\psi(q)$

B. Collusive stage

$$\frac{\pi(q^o)}{1-\delta} \geq \pi^*(q^o) + \delta\pi(\bar{q}) + \delta^2 \left[\frac{\pi(q^o)}{1-\delta}\right] \Leftrightarrow$$

$$\delta \geq \frac{\pi^*(q^o) - \pi(q^o)}{\pi(q^o) - \pi(\bar{q})}$$

C. Punishment stage

$$\pi(\bar{q}) + \delta \left[\frac{\pi(q^o)}{1-\delta}\right] \geq \pi^*(\bar{q}) + \delta\pi(\bar{q}) + \delta^2 \left[\frac{\pi(q^o)}{1-\delta}\right] \Leftrightarrow$$

$$\delta \geq \frac{\pi^*(\bar{q}) - \pi(\bar{q})}{\pi(q^o) - \pi(\bar{q})}$$

iii. Recall that the dynamic programming argument is that a firm need only consider a one-time deviation under the assumption that it acts optimally in the future

A. Thus, if both of these conditions hold then there is no better strategy

B. E.g., it is not preferable for a firm to deviate from the punishment path forever by always producing $\psi(\bar{q})$

C. It is preferable to go along with the punishment iff

$$\pi(\bar{q}) + \delta \left[\frac{\pi(q^o)}{1-\delta}\right] \geq \frac{\pi^*(\bar{q})}{1-\delta} \Leftrightarrow$$

$$\delta \geq \frac{\pi^*(\bar{q}) - \pi(\bar{q})}{\pi(q^o) - \pi(\bar{q})}$$

D. This is the same condition as above

(d) Optimal stick-and-carrot strategy

i. It is a strategy profile that maximizes each player's payoff subject to the strategy profile being a SPE and having a symmetric outcome path for all histories

ii. $(\bar{q}, q^o)$ satisfies

$$\delta[\pi(q^o) - \pi(\bar{q})] = \pi^*(\bar{q}) - \pi(\bar{q})$$

$$\begin{aligned} \delta[\pi(q^o) - \pi(\bar{q})] &= \pi^*(q^o) - \pi(q^o) \text{ if } q^o \neq q^m \\ \delta[\pi(q^o) - \pi(\bar{q})] &\geq \pi^*(q^o) - \pi(q^o) \text{ if } q^o = q^m \end{aligned}$$

iii. Interpretation

A. The first condition is to ensure that the punishment is credible. One wants it to be binding so that the worst punishment is inflicted.

B. The second condition generates the best collusive outcome.

C. These conditions feed into one another in that the higher is $\bar{q}$, the lower $q^o$ can be. The lower is $q^o$, the higher $\bar{q}$ can be.

(e) If $\pi(\cdot)$ and $\pi^*(\cdot)$ are continuously differentiable then $\bar{q} > \hat{q} > q^o$.

i. Some collusion is sustainable regardless of $\delta$ (this also holds for infinite reversion to the static NE)

ii. Punishment

A. The punishment involves each firm producing above that which maximizes current profit

B. The punishment is worse than infinite reversion to the static NE:

$$\pi(\bar{q}) + \delta \left[ \frac{\pi(q^o)}{1-\delta} \right] < \frac{\pi(\hat{q})}{1-\delta}$$

C. This allows a higher degree of collusion

(f) Restriction to symmetric SPE outcome paths

i. This strategy profile yields the maximum degree of collusion subject to the restriction that the strategy profile is a SPE and yields a symmetric outcome path for all histories

ii. An asymmetric outcome path may be natural where the deviator is punished differently from everyone else

iii. When do we know that this is the best collusive outcome for all SPE?

A. When the punishment payoff is zero as there can be no more severe punishment as this is each firm's minimax payoff

B. The punishment payoff is zero when $\delta$ is sufficiently close to 1 so it is the globally OSP

4. Discussion

(a) Why does Abreu assume $c > 0$ rather than $c \geq 0$?

(b) In the homogeneous products price game, what is an example of a most severe punishment strategy equilibrium? Does it make a difference if price is required to be non-negative?

**Renegotiation-Proofness**

1. Introduction

   (a) The criterion for a credible punishment is that it is a SPE.

      i. If all other firms enact the punishment then it is optimal for an individual firm to do so.

      ii. For example, if all other firms choose to produce the static NE quantity in all future periods then an individual firm's optimal response is to do the same.

   (b) A criticism is that it may be in the *collective* interest of firms to renegotiate.

      i. Firms agree to replace the punishment equilibrium with an equilibrium that is Pareto superior.

      ii. For example, replace the punishment by re-starting the equilibrium with the initial collusive outcome.

      iii. If this is anticipated - that the punishment will not occur because of renegotiation - then the original strategy profile is no longer an equilibrium and, in particular, firms will choose to cheat rather than collude.

2. Solution concepts immune to renegotiation

   (a) Some theorists have argued that any symmetric punishment equilibrium is incredible (Farrell and Maskin, *Games and Economic Behavior*, 1989)

      i. With such a punishment, there exists a Pareto superior equilibrium to which all firms can agree to choose instead.

      ii. This argues for asymmetric punishment equilibria for which there are no SPE that are Pareto superior. For example, an equilibrium in which the deviator's payoff is low and any other equilibrium would lower the payoff of non-deviators.

   (b) Some theorists argue that symmetric punishment equilibria can be credible (Pearce, Yale University, 1987)

      i. Argument against the previous definition.

         A. A response to the above argument is that there would be no point in trying to go to a Pareto superior equilibrium which served to undermine the credibility of the punishment as then firms actually don't receive that higher payoff. There is an internal inconsistency to the argument.

         B. However, suppose firms could move to another equilibrium for which, for all histories, the payoff is higher than what firms currently are receiving. That move is preferred and it seems credible in the sense that there is no internal inconsistency.

ii. Defining a new concept

A. An alternative definition is that an equilibrium is not renegotiation-proof if there exists another equilibrium for which its minimal payoff exceeds the minimal payoff for the original equilibrium.

B. For example, reversion to static NE for $T'$ periods is not r-proof by this definition if the same cooperative outcomes can be supported by reversion to static NE for $T' - 1$ periods.

### 2.3.2 Punishments in Practice

1. Cases

(a) Citric acid (1991-95) (Connor, 2001)

i. Citric acid is an organic chemical used as an additive in foods and detergents. It is a fairly homogeneous product purchased by industrial buyers.

ii. In the event that realized market shares did not match targeted shares, a buy-back system was put in place whereby, at the end of the year, firms whose market share exceeded their allocation would compensate those whose shares were below target. Inter-firm sales provided a means by which to make such side payments.

(b) Sugar (1927-36) (Genesove and Mullin, 2001)

i. The Sugar Institute was formed as a trade association. It was comprised of 14 firms which had nearly all of the cane sugar refining capacity in the U.S.

ii. Punishments

A. Individual violations were never met with reversion to competition, much less subcompetitive prices.

B. Deviations were either ignored or matched. One time deviations were ignored if the practice was stopped. Continual cheating was responded to by matching the practice.

C. Large scale cheating was responded to with intense competition.

iii. Purpose of meetings

A. Forum for accusation and rebuttal in connection with suspected cheating. The Sugar Institute served as a court and thereby reduced the noise in the signals.

B. Revised the rules to close loopholes which allowed firms to cheat.

C. Gave firms an opportunity to provide prior notification for some future action.

(c) Graphite electrodes (1992-97) (Levenstein, Suslow, and Oswald, 2004)

i. A graphite electrode is an input in the manufacturing of steel; its function being to conduct high levels of electricity in an electric arc furnace in order to melt scrap steel.

   ii. Side payments were made in response to unexpected demand fluctuations in different regional markets.

(d) Sodium gluconate (1993-95)

   i. Industrial metal and glass cleaner used for bottle washing, food service and utensil cleaning, food process equipment cleaning, and paint removal.

   ii. Had a compensation scheme whereby if a firm oversold its quota then its quota the following year was reduced. (Related to Athey and Bagwell, 2001.)

(e) International steel agreement (1926)

2. Two-tier description of punishments

   (a) Penalties for overproduction

      i. These were explicit in describing how a firm which produced above its quota was to compensate those firms below their quotas.

      ii. These penalties were minor and may entail simply buying output so that sales matched the target.

      iii. This may not have been to punish cheating - though it could serve to deter cheating - but more to take account of the fact that firms could only imperfectly control their sales so that they matched their target.

   (b) Threat of collapse of collusion

      i. Implicit is that if there is continual cheating or firms do not honor the compensation scheme that collusion may collapse.

      ii. This could result in a reversion to the non-collusive outcome or something more severe.

## 2.4 Compliance and Imperfect Monitoring

1. A key feature to the collusive mechanism is that deviation by a firm is observed and punished. This credible punishment serves to deter firms from cheating by pricing high or producing too much.

2. In many cartels, monitoring of firms' behavior is difficult.

   (a) Firms' supply is not easily observed though the market price may be.

   (b) When customers are large (e.g., firms), price is often negotiated in which case firms' prices are not common knowledge.

3. Objectives

   (a) The modelling objective is to characterize collusion when monitoring is imperfect and thus compliance is problematic.

(b) This will not only enrich the model in a direction of descriptive reality but will also expand the set of behavior that we can generate.

   i. Without imperfect monitoring, we can explain collusion but not price wars.

   ii. As will be seen, imperfect monitoring can explain price wars.

### 2.4.1 Quantity-Setting Game

**Theory**

1. Introduction

   (a) Assume firms' quantities are private information but price is still common knowledge. Does this invalidate our previous analysis?

   (b) Now assume $P_t = P(Q_t) + \epsilon_t$ where $\epsilon_t$ is an unobserved random component

2. Model (Porter, *Journal of Economic Theory*, 1983)

   (a) Demand: $P^t = \theta^t P\left(Q^t\right) = \theta^t(a - bQ^t)$

      i. $\theta$ is an iid $r.v.$ with cdf $F(\cdot)$

      ii. $F(0) = 0$, $F(\theta^o) = 1$, $\theta^o < \infty$

      iii. $F(\cdot)$ is continuously differentiable and convex

   (b) Cost: $C(q) = c_o + c_1 q$

   (c) Informational structure

      i. In period $t$, a firm knows all past prices and all of its past quantities

      ii. Only past prices are common knowledge

      iii. A strategy is an infinite sequence of functions in which the period $t$ function maps from $\Re_+^{2(t-1)}$ into $\Re_+$

3. Trigger strategies with imperfect monitoring

   (a) Description

      i. If in the *cooperative phase* in period $t-1$ and

         A. $P^{t-1} \geq \widetilde{P}$ then $q_i^t = q^o$ and remain in the cooperative phase

         B. $P^{t-1} < \widetilde{P}$ then $q_i^t = \hat{q}$ and go to the punishment phase

      ii. If in the $\tau^{th}$ period of the *punishment phase* in period $t-1$ and

         A. $\tau < T-1$ then $q_i^t = \hat{q}$ and remain in the punishment phase

         B. $\tau \geq T-1$ then $q_i^t = q^o$ and go to the cooperative phase

   (b) Variables

      i. $\widetilde{P}$ is the trigger price

      ii. $T-1$ is the length of the punishment or $T$ is the time between when a punishment starts and when firms return to the cooperative outcome

4. SPE conditions

   (a) Notation

      i. $\pi(q_i, Q_{-i}) \equiv \int [\theta P(q_i + Q_{-i}) - c_o - c_1 q_i] F'(\theta) \, d\theta$
      ii. $\pi(q) \equiv \int [\theta P(nq) - c_o - c_1 q] F'(\theta) \, d\theta$
      iii. $\hat{q} \in \arg\max \pi(q, (n-1)\hat{q})$ (static Nash equilibrium quantity)
      iv. $q^o$ is a generic collusive quantity
      v. $q^*$ is equilibrium collusive quantity

   (b) $V(q^o)$ is a firm's payoff when firms are in the cooperative phase and the collusive quantity is $q^o$

      i. It is defined recursively by

$$
\begin{aligned}
V(q^o) &= \pi(q^o) + \Pr\left(\theta P(nq^o) \geq \widetilde{P}\right) \delta V(q^o) \\
&\quad + \Pr\left(\theta P(nq^o) < \widetilde{P}\right) \left[\sum_{\tau=1}^{T-1} \delta^{\tau} \pi(\hat{q}) + \delta^T V(q^o)\right]
\end{aligned} \tag{2}
$$

      ii. $\Pr\left(\theta P(nq^o) < \widetilde{P}\right) = F\left(\frac{\widetilde{P}}{P(nq^o)}\right)$ is the probability of triggering a punishment given that all firms are cooperating
      iii. Solve (2) for $V(q^o)$,

$$
V(q^o) = \frac{\pi(\hat{q})}{1-\delta} + \frac{\pi(q^o) - \pi(\hat{q})}{1 - \delta + \left(\delta - \delta^T\right) F\left(\frac{\widetilde{P}}{P(nq^o)}\right)}
$$

      iv. The first term is "normal" profit discounted
      v. The second term is the additional profit when in the cooperative phase which is discounted by how much of the time one spends in the cooperative phase which is decreasing in $T$ and decreasing in $\widetilde{P}$

   (c) Condition defining the equilibrium collusive quantity

      i. $q^*$ is the value of $q^o$ such that if all other firms produce $q^o$ in the cooperative phase, a firm also wants to produce $q^o$.
      ii. Set of conditions
         A. There are two classes of states: cooperative and punishment
         B. By dynamic programming, we need only consider one-time deviations from all (both) states and then acting according to the candidate strategy (which, if all such conditions are satisfied, means acting optimally thereafter)
      iii. A firm clearly wants to go along with the punishment phase
         A. Producing a different quantity lowers current profit
         B. Does not affect future payoff

iv. We then need only focus on the optimality of producing $q^o$ when in the cooperative phase

(d) First-order condition on $q^*$

$$\begin{aligned} q^* \in \; & \arg\max \pi\left(q,(n-1)q^*\right) + \\ & \left[1 - F\left(\frac{\widetilde{P}}{P\left(q+(n-1q^*)\right)}\right)\right]\delta V\left(q^*\right) + \\ & F\left(\frac{\widetilde{P}}{P\left(q+(n-1q^*)\right)}\right)\left[\left(\frac{\delta-\delta^T}{1-\delta}\right)\pi\left(\widehat{q}\right)+\delta^T V\left(q^*\right)\right] \end{aligned}$$

$$\begin{aligned} \frac{\partial\cdot}{\partial q_i} = \; & \frac{\partial\pi\left(q^*,(n-1)q^*\right)}{\partial q_i} + \\ & F'\left(\frac{\widetilde{P}}{P\left(q^*+(n-1q^*)\right)}\right)\left[\frac{\widetilde{P}P'\left(nq^*\right)}{P\left(nq^*\right)^2}\right] \times \\ & \left\{\delta V\left(q^*\right) - \left(\frac{\delta-\delta^T}{1-\delta}\right)\pi\left(\widehat{q}\right) - \delta^T V\left(q^*\right)\right\} = 0 \end{aligned}$$

Substitute for $V(q^*)$ from (2) and the first-order condition is

$$\begin{aligned} \frac{\partial\cdot}{\partial q_i} = \; & \frac{\partial\pi\left(q^*,(n-1)q^*\right)}{\partial q_i} + \\ & F'\left(\frac{\widetilde{P}}{P\left(q^*+(n-1q^*)\right)}\right)\left[\frac{\widetilde{P}P'\left(nq^*\right)}{P\left(nq^*\right)^2}\right] \times \\ & \left\{\left(\delta-\delta^T\right)\left[\frac{\pi(\hat{q})}{1-\delta} + \frac{\pi(q^*)-\pi(\hat{q})}{1-\delta+\left(\delta-\delta^T\right)F\left(\frac{\widetilde{P}}{P(nq^*)}\right)}\right] - \left(\frac{\delta-\delta^T}{1-\delta}\right)\pi\left(\widehat{q}\right)\right\} = 0 \end{aligned}$$

$$\begin{aligned} 0 = \; & \frac{\partial\pi\left(q^*,(n-1)q^*\right)}{\partial q_i} + \\ & \left(\delta-\delta^T\right)F'\left(\frac{\widetilde{P}}{P\left(q^*+(n-1q^*)\right)}\right) \times \\ & \left[\frac{\widetilde{P}P'\left(nq^*\right)}{P\left(nq^*\right)^2}\right]\left[\frac{\pi\left(q^*\right)-\pi\left(\widehat{q}\right)}{1-\delta+\left(\delta-\delta^T\right)F\left(\frac{\widetilde{P}}{P(q^*+(n-1q^*))}\right)}\right] \end{aligned}$$

(e) Note that a firm's quantity does not affect $V(q^*)$ but only current profit and the probability of staying in the cooperative phase.

(f) The second-order condition is

$$\begin{aligned} \frac{\partial^2\cdot}{\partial q_i^2} = \; & \frac{\partial^2\pi(q^*,(n-1)q^*)}{\partial q_i^2} - F''\left(\frac{\widetilde{P}}{P(q^*+(n-1q^*))}\right)\left[\frac{\widetilde{P}P'(nq^*)}{P(nq^*)^2}\right]^2\Delta \\ & + F'\left(\frac{\widetilde{P}}{P(q^*+(n-1q^*))}\right)\widetilde{P}\left[\frac{P''(nq^*)P(nq^*)^2-2\left[P'(nq^*)\right]^2}{P(nq^*)^4}\right]\Delta < 0 \end{aligned}$$

where $\Delta \equiv \left\{\delta V(q^*) - \left(\frac{\delta-\delta^T}{1-\delta}\right)\pi(\widehat{q}) - \delta^T V(q^*)\right\} > 0$. The second derivative is negative because

i. The first term is negative by strict concavity of the profit function.
ii. Since $F$ is convex then $F'' > 0$ so the second term is negative.
iii. The third term is negative as $F' > 0$, $P'' = 0$, and $-2\left[P'(nq^*)\right]^2 < 0$.

5. Equilibrium quantities

(a) $q^* = \hat{q}$ is one value that satisfies the first-order condition and thus is a collusive equilibrium quantity

(b) If $q^* \in [q^m, \hat{q})$, why is $q^* < \psi((n-1)q^*)$? If firm $i$ marginally lowers its quantity below $\psi((n-1)q^*)$, there is no first-order effect on current profit as $\frac{\partial \pi_i}{\partial q_i} = 0$. There is a first-order effect on future profit as it reduces the probability of moving into the punishment phase since

$$F'\left(\frac{\widetilde{P}}{P(q_i + (n-1q^*))}\right)\left[\frac{\widetilde{P}P'(nq^*)}{P(nq^*)^2}\right] < 0$$

$\forall q_i$ including $q_i = \psi((n-1)q^*)$

6. Discussion

(a) Why is the collusive quantity defined by a first-order condition in the model with imperfect monitoring but not in the model with perfect monitoring?
(b) Why are there price wars when no one deviates in equilibrium?
(c) How does the price path differ from when firms do not collude? How could we distinguish collusion and competition?
(d) Is it possible that price wars are not an equilibrium phenomenon? Why might they emerge?

7. Most severe punishment strategy (Abreu, Pearce, and Stachetti, *Journal of Economic Theory*, 1986)

(a) Strategy profile

i. If in the *cooperative phase* in period $t-1$ and
   A. $P^{t-1} \geq \bar{P}$ then $q_i^t = q^o$ and remain in the cooperative phase
   B. $P^{t-1} < \bar{P}$ then $q_i^t = \overline{q}$ and go to the punishment phase

ii. If in the *punishment phase* in period $t-1$ and
   A. $P^{t-1} \leq \underline{P}$ then $q_i^t = q^o$ and go to the cooperative phase
   B. $P^{t-1} > \underline{P}$ then $q_i^t = \overline{q}$ and remain in the punishment phase

(b) Price path

i. First-order Markov process
ii. Regime switching with endogenous duration

**Railroad Cartel (1880s)**

- Ellison, Glenn, "Theories of Cartel Stability and the Joint Executive Committee," *RAND Journal of Economics*, 25 (1994), 37-57.
- Porter, Robert H., "A Study of Cartel Stability: The Joint Executive Committee, 1880-1886," *Bell Journal of Economics*, 14 (1983), 301-314.

1. As cartels were legal in the U. S. prior to 1890, the railroads entered a cartel agreement in 1879 in order to stabilize price. This agreement created the Joint Executive Committee (JEC). The role of the JEC was to set the rail rate of eastbound freight shipments from Chicago to the Atlantic coast.
2. Testing for collusion
   (a) By the collusive theory of imperfect monitoring, a test for collusion is to see whether there are regime switches in the intensity of competition. That is, changes in price-cost margins that cannot be explained by cost and demand shocks.
   (b) Porter (1983) assumed an empirical model in which the regime switches are *iid*.
   (c) He did not test whether switches to intensified competition were triggered by downward demand shocks.
3. Regime switching between collusion and competition
   (a) Figure 5.8, Cartel Pricing of Rail Rates for Shipment of Grain from Chicago to the Atlantic Seaboard, 1880–1886
   (b) Rail rates were relatively high in weeks 0–80 and 120–220. There appeared to have been price wars in weeks 80–120 and periodically over 220–360.
   (c) The dark line below the grain rate indicates periods in which Porter concluded that collusion broke down
4. Ellison (1994)
   (a) Assumed an empirical model in which the regime switchers are Markovian.
   (b) Found strong persistence in collusion
      i. Probability of colluding in $t$ given colluding in $t-1$ is estimated to be .975.
      ii. Probability of colluding in $t$ given price war in $t-1$ is .067.

### 2.4.2 Price-Setting Game

- Harrington and Skrzypacz, 2004

1. Introduction
   (a) Many price-fixing cartels, such as the lysine cartel, involve firms selling to industrial buyers. As price can be settled through private negotiation, it is not typically observable.
   (b) In such cases, compliance with the collusive agreement is based on firms' sales.
   (c) The objective is to explore collusion when monitoring is in terms of sales rather than price.

2. Stage game
   (a) Bertrand price game with discrete demand.
      i. Firms have homogeneous products and constant marginal cost, $c \geq 0$.
      ii. Firms simultaneously choose price.
   (b) Demand
      i. Fixed at $m \in \{1, 2, \ldots\}$ discrete units.
      ii. $\phi : \{0, 1, \ldots, m\} \times \Re^2 \rightarrow [0, 1]$ is the probability function on firm 1's demand.
         A. $\phi(b; p_1, p_2)$ is the probability that firm 1 sells $b \in \{0, 1, \ldots, m\}$ units given its price is $p_1$ and its rival's price is $p_2$.
         B. As total demand is fixed at $m$ units, $\phi(m - b; p_1, p_2)$ is the probability that firm 2's demand is $b$.
      iii. Assumptions
         **A1** $\phi$ is continuously differentiable with respect to $p_1$ and $p_2$.
         **A2** $\phi(b; p', p'') = \phi(m - b; p'', p') \; \forall b \in \{0, 1, \ldots, m\}, \forall (p', p'') \in \Re^2_+$.
         **A3** $\frac{\partial \phi(b;p,p)}{\partial p_1} + \frac{\partial \phi(b;p,p)}{\partial p_2} = 0 \; \forall b \in \{0, 1, \ldots, m\}, \forall p \in \Re_+$.
      iv. A1 is standard.
      v. A2 imposes symmetry.
      vi. A3 is the key restriction and is satisfied in many models.
         A. Sufficient condition is that $\phi$ depends only on the price difference, $p_1 - p_2$, or the price ratio, $p_1/p_2$.
         B. For example, suppose $\exists \xi : \{0, 1, \ldots, m\} \times \Re \rightarrow [0, 1]$ such that

$$\phi(b; p_1, p_2) = \xi(b; \Delta) \, \forall b \in \{0, 1, \ldots, m\}, \forall (p_1, p_2) \in \Re^2_+,$$

where $\Delta \equiv p_1 - p_2$. Then

$$\frac{\partial \phi(b; p, p)}{\partial p_1} + \frac{\partial \phi(b; p, p)}{\partial p_2} = \frac{\partial \xi(b; 0)}{\partial \Delta} - \frac{\partial \xi(b; 0)}{\partial \Delta} = 0,$$

so that A3 holds.

vii. An example from the literature that conforms to this specification is an $m$-buyer generalization of Cabral and Riordan (*Econometrica*, 1994).

A. The probability that firm 1 sells to a particular buyer is $F(p_2 - p_1)$ where $F : \Re \to [0,1]$ is continuously differentiable and non-decreasing. Assume $F'$ is symmetric around zero.

B. Buyers' decisions as to whom to buy from are *iid* which implies that a firm's demand is binomially distributed,

$$\phi(b; p_1, p_2) = \left(\frac{m!}{b!\,(m-b)!}\right) F(p_2 - p_1)^b \left(1 - F(p_2 - p_1)\right)^{m-b},$$

and thus depends only on the price difference.

(c) Nash equilibrium - general case

i. Firm 1's payoff function

$$\sum_{b=0}^{m} \phi(b; p_1, p_2)(p_1 - c)\, b.$$

ii. Symmetric equilibrium price (first-order condition):

$$\sum_{b=0}^{m} \phi(b; \widehat{p}, \widehat{p})\, b + \sum_{b=0}^{m} \frac{\partial \phi(b; \widehat{p}, \widehat{p})}{\partial p_1} (\widehat{p} - c)\, b = 0.$$

(d) Nash equilibrium - binomially distributed demand

i. Firm 1's payoff function

$$\begin{aligned}
\pi_1(p_1, p_2) &= \sum_{b=1}^{m} \left(\frac{m!}{b!\,(m-b)!}\right) F(p_2 - p_1)^b \left(1 - F(p_2 - p_1)\right)^{m-b} (p_1 - c)\, b \\
&= \sum_{b=1}^{m} \left(\frac{m!}{b!\,(m-b)!}\right) F(p_2 - p_1)^b F(p_1 - p_2)^{m-b} (p_1 - c)\, b.
\end{aligned}$$

by the symmetry of $F'$ around zero.

ii. Symmetric equilibrium price (first-order condition):

$$(1/2)^m \sum_{b=1}^{m} \left(\frac{m!}{b!\,(m-b)!}\right) b + (1/2)^{m-1} F'(0)(p - c) \sum_{b=1}^{m} \left(\frac{m!}{b!\,(m-b)!}\right) b\,(m - 2b) = 0,$$

from which it follows that

$$\begin{aligned}
\widehat{p} &= c + \left(\frac{1}{2F'(0)}\right) \left(\frac{\sum_{b=1}^{m} \left(\frac{m!}{b!(m-b)!}\right) b}{\sum_{b=1}^{m} \left(\frac{m!}{b!(m-b)!}\right) b\,(2b - m)}\right) \\
\widehat{p} &= c + \frac{1}{2F'(0)}.
\end{aligned}$$

iii. Why is $\widehat{p}$ decreasing in $F'(0)$?

3. Infinitely repeated game

(a) There is an infinite horizon and each firm's payoff is the expected present value of its profit stream where the common discount factor is $\delta \in (0,1)$.

(b) Information

i. Firms' price decisions are private information and never revealed.

ii. Past realized sales are common knowledge. Given that it is common knowledge that market demand is fixed and each firm observes its demand then this is immediate.

iii. Wlog, a history at the start of period $t$, denoted $h^{t-1}$, is a sequence of quantities sold for firm 1.

(c) A firm's strategy is an infinite sequence of price functions, $\left\{\rho_i^t(\cdot)\right\}_{t=1}^{\infty}$, where $\rho_i^t : \{0,1,\ldots,m\}^{t-1} \rightarrow \Re_+$.

4. An Impossibility Result

(a) Consider equilibria that take full advantage of the symmetry of the model.

i. For a particular strategy profile, let

$$v_i^t(\cdot) : \{0,1,\ldots,m\}^{t-1} \rightarrow \Re$$

denote the continuation payoff starting at $t$.

ii. A *symmetric Nash equilibrium* is a Nash equilibrium in which continuation payoffs are identical when the history is symmetric.

A. Of course, if $m$ is not even then the only symmetric history is the initial null history.

B. When $m$ is even, it also includes those histories in which each firm had demand of $m/2$ in every period.

iii. An *exchangeable Nash equilibrium* is a Nash equilibrium in which

$$v_1^t\left(b^1,\ldots,b^{t-1}\right) = v_2^t\left(m-b^1,\ldots,m-b^{t-1}\right) \ \forall \left(b^1,\ldots,b^{t-1}\right) \in \{0,1,\ldots,m\}^{t-1}, \ \forall t.$$

A. If the history is permuted with respect to the firms then the payoffs are permuted as well so the identity of the firm doesn't matter.

B. In defining a permutation, a history at the start of period $t$ is the sequence of firms' quantities, $\{(q_1^\tau, q_2^\tau)\}_{\tau=1}^{t-1} = \{(b^\tau, m-b^\tau)\}_{\tau=1}^{t-1}$. Representing this history as the sequence of firm 1's quantities, $\{b^\tau\}_{\tau=1}^{t-1}$, its permutation is $\{m-b^\tau\}_{\tau=1}^{t-1}$.

iv. A *strongly symmetric Nash equilibrium* is a Nash equilibrium in which continuation payoffs are identical across firms for all histories:

$$v_1^t\left(b^1,\ldots,b^{t-1}\right) = v_2^t\left(b^1,\ldots,b^{t-1}\right) \ \forall \left(b^1,\ldots,b^{t-1}\right) \in \{0,1,\ldots,m\}^{t-1}, \ \forall t.$$

v. Let this common continuation payoff be denoted $v^t(\cdot)$.

(b) **Theorem**: The set of strongly symmetric exchangeable Nash equilibrium outcomes for the infinite horizon game coincides with the set of symmetric Nash equilibrium outcomes for the stage game.

(c) Intuition

i. In that pricing too low induces more sales for the deviator and less sales for the other firm, punishment ought to occur when market shares are sufficiently skewed. High market share for firm 1 is consistent with firm 1 having undercut the collusive price, while low market share for firm 1 is consistent with firm 2 having undercut the collusive price. Strong symmetry implies that the punishment entails identical payoffs, regardless of who had the higher market share. Exchangeability implies that the punishment depends only on how skewed is market share.

ii. This result hinges on the fact that when firm 1 sets a price marginally below the collusive price, it reduces the likelihood of having a low demand (say, $b' < m/2$) and, at the same time, raises the probability of having a high demand (say, $m - b'$). The ensuing reduction in the probability of $b'$ turns out to exactly equal the rise in the probability of $m - b'$ so that the probability of $b'$ or $m - b'$ remains constant for a marginal change in price. Since the payoff is the same for $b'$ and $m - b'$, the probability distribution over the continuation payoff is unaffected. If $p^*$ is to be an equilibrium then it must maximize expected current profit since, at the margin, the effect of price on the expected continuation payoff is zero. This implies the equilibrium price must be the same as that for a Nash equilibrium for the stage game.

5. Collusion with asymmetric punishments and side payments

(a) Demand specification

i. Assume there are two buyers: $m = 2$.

ii. The probability that firm 1 sells to a particular buyer equals $F(p_2 - p_1)$ where $F : \Re \rightarrow [0,1]$ is twice continuously differentiable and non-decreasing. Furthermore, $F'$ is symmetric around zero.

iii. Buyers' decisions as to whom to buy from are *iid* which implies that a firm's demand is binomially distributed.

(b) Symmetric (but not strongly symmetric) strategy profile.

i. If in the collusive state in period $t$ then set $p_i^t = p^*$ and

A. if $(q_1^t, q_2^t) = (1,1)$ then remain in the collusive state in period $t+1$

B. if $(q_1^t, q_2^t) = (2,0)$ then go to the type 1 punishment state in period $t+1$

C. if $\left(q_1^t, q_2^t\right) = (0, 2)$ then go to the type 2 punishment state in period $t+1$

ii. If in the type $i$ punishment state in period $t$ then firm $i$ pays $z$ to firm $j(\neq i)$.

iii. If firm $i$ pays $z$ to firm $j$ then

A. switch to the collusive state in period $t$ with probability $\gamma$

B. play the static Nash equilibrium forever with probability $1-\gamma$

iv. If firm $j$ does not pay $z$ to firm $j$ then play the static Nash equilibrium forever.

(c) Let $v$ and $w$ denote the collusive payoff and the static Nash equilibrium payoff, respectively.

(d) The Nash equilibrium price is

$$c + \frac{1}{2F'(0)}$$

and therefore

$$w = \frac{1}{2F'(0)(1-\delta)}. \tag{3}$$

(e) Equilibrium condition on the collusive price

i. When in the collusive state, the payoff of firm 1 is

$$\begin{aligned} &2F(p_2 - p_1) F(p_1 - p_2)(p_1 - c + \delta v) \\ &+ F(p_2 - p_1)^2 \left[2(p_1 - c) + \delta(-z + \gamma v + (1-\gamma) w)\right] \\ &+ F(p_1 - p_2)^2 \delta \left[z + \gamma v + (1-\gamma) w\right]. \end{aligned} \tag{4}$$

ii. Evaluating the first-order condition at a common price of $p$ and solving for $p$ yields:

$$p = c + \frac{1}{2F'(0)} + \delta z. \tag{5}$$

iii. The collusive payoff is then the expression in (4) evaluated at the price in (5) which, after some manipulation, can be shown to equal

$$v = \frac{1 + F'(0)\left[2\delta z + \delta(1-\gamma) w\right]}{F'(0)\left[2 - \delta(1+\gamma)\right]}. \tag{6}$$

(f) Equilibrium condition on making the side payment

i. The condition which ensures that firm $i$ wants to transfer $z$ in the type $i$ punishment state is

$$-z + \gamma v + (1-\gamma) w \geq w \Leftrightarrow \gamma(v - w) \geq z. \tag{7}$$

ii. Substituting (6) into (7):

$$\begin{aligned} \gamma\left(\frac{1 + F'(0)\left[2\delta z + \delta(1-\gamma) w\right]}{F'(0)\left[2 - \delta(1+\gamma)\right]} - w\right) &\geq z \Leftrightarrow \\ \gamma\left(1 - 2(1-\delta) F'(0) w\right) &\geq F'(0)\left[2 - \delta(1+3\gamma)\right] z. \end{aligned} \tag{8}$$

iii. Using (3) in (8):

$$0 \geq F'(0)\left[2 - \delta(1 + 3\gamma)\right] z. \tag{9}$$

iv. Since $z > 0$, a necessary and sufficient condition for (9) is

$$\delta \geq \frac{2}{1 + 3\gamma}.$$

v. If $\gamma > \frac{1}{3}$ then the strategy profile is a subgame perfect equilibrium with

$$p^* = c + \frac{1}{2F'(0)} + \delta z$$

as long as $\delta$ is sufficiently close to 1.

(g) In practice, the transfer $z$ could be implemented by having one firm buy the quantity of another firm.

### 2.4.3 Monitoring and Compliance in Practice

1. Citric acid (1991-95)

   (a) In order to monitor the agreement, each firm was required to submit monthly sale volumes to a representative of Hoffman-La Roche who would then compile these statistics and distribute the information among the other cartel members.

   (b) Meetings

      i. Full scale meetings occurred about every eight weeks.

      ii. Some of them took place during the meetings of the European Citric Acid Manufacturers' Association (ECAMA). It is quite typical to use trade association meetings as a pretense for firms' representatives to get together.

      iii. There would be a discussion of the latest cartel sales report towards monitoring the agreement.

      iv. Also discussions of trends in demand and cost which would lead up to a decision on the new cartel price.

      v. They would discuss any problems with the cartel's operation such as accusations about cheating.

2. Lysine (1992-95)

   (a) Each month, the five companies telephoned or mailed their lysine sales to an employee of Ajinomoto who prepared a spreadsheet that was handed out at the quarterly maintenance meetings.

   (b) Sales volumes were calculated for four regions: North America, Latin America, Europe/Middle East/Africa, and Asia/Oceania.

3. Graphite electrodes (1992-97)

    (a) In 1995, the cartel formalized the reporting of sales with the creation of the Central Monitoring System which called for Tokai Carbon to collect data from the other members and compare actual sales with the agreed-upon market shares.
    (b) Hierarchy of meetings
        i. "Top level" or "top guy" meetings were attended by chief executives at which price levels were set.
        ii. Meetings with lower level officers such as sales managers would organize details and exchange information.
        iii. Meetings could be global, European (excluded Japanese firms), and national or regional.

4. Sugar (1927-36)

    (a) Rule number one of the Sugar Institute was the requirement of "open prices and publicly announced terms."
    (b) They generally prohibited discriminatory terms. It was very wide ranging in terms of the contractual features it covered because almost every contractual term could mask a price cut.
    (c) The Institute was primarily a mechanism to enhance detection of secret price cuts.

5. Electrical and mechanical carbon and graphite products (1990-2000)

    (a) Over 140 cartel meetings.
    (b) Topics discussed
        i. Setting price levels - general price, prices for large customers, prices that would undercut non-cartel competitors.
        ii. Exchanged price quotations issued to customers to ensure that they did not undercut each other.

6. Sodium gluconate (1993-95) - held at least 25 meetings.

### 2.5 Factors relevant to collusion

1. Multi-market interaction

    - Bernheim, B. Douglas and Michael D. Whinston, "Multimarket Contact and Collusive Behavior," *RAND Journal of Economics*, 21 (1990), 1-26.
    - Harrington, Joseph E. Jr., "Collusion in Multiproduct Oligopoly Games under a Finite Horizon," *International Economic Review*, 28 (1987), 1-14.

- Spagnolo, Giancarlo, "On Interdependent Supergames: Multimarket Contact, Concavity, and Collusion," *Journal of Economic Theory*, 89 (1999), 127-139.

2. Demand movements

   - Bagwell, Kyle and Robert W. Staiger, "Collusion Over the Business Cycle," *RAND Journal of Economics,* 28 (1997), 82-106.
   - Haltiwanger, John and Joseph E. Harrington, Jr., "The Impact of Cyclical Demand Movements on Collusive Behavior," *RAND Journal of Economics*, 22 (1991), 89-106.
   - Kandori, Michihiro, "Correlated Demand Shocks and Price Wars During Booms," *Review of Economic Studies*, 58 (1991), 171-180.
   - Rotemberg, Julio J. and Garth Saloner, "A Supergame-Theoretic Model of Price Wars During Booms," *American Economic Review*, 76 (1986), 390-407.

3. Private information

   - Athey, Susan and Kyle Bagwell, "Optimal Collusion with Private Information," *RAND Journal of Economics*, 32 (2001), 428-465.
   - Athey, Susan, Kyle Bagwell, and Chris Sanchirico, "Collusion and Price Rigidity," Columbia University, pdf version, March 2000.

### 2.5.1 Multi-market collusive pricing

**Theory**

- Bernheim and Whinston (*RAND Journal of Economics*, 1990)
- Evans and Kessides, (*Quarterly Journal of Economics*, 1994)

1. Summary

   (a) Firms interact in multiple markets with perfect monitoring

   (b) Results

      i. If markets are identical then there is no gain from linking a punishment in a market with a deviation in another market.

      ii. If markets are different then a greater degree of collusion can be sustained in a particular market by having any deviation result in a punishment that goes across markets.

         A. In one market, maximal collusive prices may be sustainable and there is punishment to spare. This excess punishment can be used to support higher collusive prices in another market where maximal collusion cannot be sustained.

B. The greater the degree of multimarket contact between firms, the easier it is collude and the higher PCMs should be.

2. Model - Infinitely repeated Bertrand price game

   (a) Two markets: A and B
   (b) Market A
      i. Set of firms is $\{1, \ldots, m\}$
      ii. Homogeneous products
      iii. $D_A(\cdot) : \Re_+ \rightarrow \Re_+$ is continuous, nonincreasing, and bounded.
      iv. Common constant marginal cost, $c_A$, and $D_A(c_A) > 0$.
      v. $P_A^m \in \arg\max\ (P - c_A)D_A(P)$
      vi. $\pi_A^m \equiv (P_A^m - c_A)D_A(P_A^m)$
   (c) Market B
      i. Set of firms $\{1, \ldots, n\}$ where $n \geq m$
      ii. $D_B(\cdot) : \Re_+ \rightarrow \Re_+$ is continuous, nonincreasing, and bounded.
      iii. Common constant marginal cost, $c_B$, and $D_B(c_B) > 0$.
      iv. $P_B^m \in \arg\max\ (P - c_B)D_B(P)$
      v. $\pi_B^m \equiv (P_B^m - c_B)D_B(P_B^m)$
   (d) Assume firms $1, ..., m$ supply products in both markets while firms $m + 1, ..., n$ supply only market B.
      i. If $n = m$ then all firms serve both markets.
      ii. If $n > m$ then some firms serve both markets and some serve only market B.

3. Equilibrium conditions for collusion in separate markets

   (a) Consider trigger strategy equilibria in each market.
      i. Collusive price in market $I$ is $P_I^c \in (c_I, P_I^m]$.
      ii. Deviation is punished by infinite reversion to the static Nash equilibrium which involves pricing at $c_I$. Punishment payoff is zero.
      iii. Optimal deviation is to arbitrarily undercut the collusive price and take the entire demand.
   (b) Market A
      i. SPE condition

$$\left(\frac{1}{1-\delta}\right)(P_A^c - c_A)D_A(P_A^c)\left(\frac{1}{m}\right) \geq (P_A^c - c_A)D_A(P_A^c) \Leftrightarrow$$

$$1 \geq m(1-\delta).$$

ii. Since the condition is independent of the collusive price, if it holds then the firms should set $P_A^c = P_A^m$.

(c) Market B

i. SPE condition

$$\left(\frac{1}{1-\delta}\right)(P_B^c - c_B)D_B(P_B^c)\left(\frac{1}{n}\right) \geq (P_B^c - c_B)D_B(P_B^c) \Leftrightarrow$$

$$1 \geq n(1-\delta).$$

ii. Since the condition is independent of the collusive price, if it holds then the firms should set $P_B^c = P_B^m$.

4. Irrelevance result when $n = m$.

(a) Single-market collusion

i. When $1 \geq n(1-\delta)$ or, equivalently, $\delta \geq 1/(n-1)$, the maximal collusive price is the monopoly price in both markets.

ii. When $\delta < 1/(n-1)$, the maximal collusive price is marginal cost in both markets.

(b) Multi-market collusion

i. The most severe punishment is to go to the static Nash equilibrium in both markets in response to a deviation in either or both markets.

ii. The optimal deviation is then to cheat in both markets.

iii. Cheating is not optimal iff

$$\begin{aligned} &\left(\frac{1}{1-\delta}\right)(P_A^c - c_A)D_A(P_A^c)\left(\frac{1}{n}\right) + \left(\frac{1}{1-\delta}\right)(P_B^c - c_B)D_B(P_B^c)\left(\frac{1}{n}\right) \\ \geq\ & (P_A^c - c_A)D_A(P_A^c) + (P_B^c - c_B)D_B(P_B^c) \Leftrightarrow \end{aligned}$$

$$\begin{aligned} (P_A^c - c_A)D_A(P_A^c)\left[1-(1-\delta)n\right] + (P_B^c - c_B)D_B(P_B^c)\left[1-n(1-\delta)\right] &\geq 0 \Leftrightarrow \\ \left[(P_A^c - c_A)D_A(P_A^c) + (P_B^c - c_B)D_B(P_B^c)\right]\left[1-n(1-\delta)\right] &\geq 0 \Leftrightarrow \\ 1-n(1-\delta) &\geq 0. \end{aligned}$$

iv. This is the same condition as with single-market collusion.

(c) Result: If all firms are in the same set of markets and have identical products, costs, and discount factors then the maximal degree of collusion with multi-market contact is the same as with single-market contact.

5. Multi-market collusion when $n > m$

(a) Assumptions

i. There are $m$ firms serving both markets A and B and $n - m$ firms serving only market B.

ii. Single-market collusion is feasible in market A but not in market B:

$$n(1-\delta) > 1 > m(1-\delta).$$

(b) Strategy for firms $1,\ldots,m$

i. Price at the monopoly price in both markets in period 1.

ii. In period $t$, price at the monopoly price in market A if

A. firms $1,\ldots,m$ priced at the monopoly price in market A $\forall\tau < t$ and

B. firms $1,\ldots,,m$ priced at the monopoly price in market B $\forall\tau < t$ such that firms $m{+}1,\ldots,n$ priced at the monopoly price in market B in $\tau$.

iii. In period $t$, price at the monopoly price in market B if

A. firms $1,\ldots,m$ priced at the monopoly price in market A $\forall\tau < t$ and

B. firms $1,\ldots,n$ priced at the monopoly price in market B $\forall\tau < t$.

(c) Strategy for firms $m+1,\ldots,n$

i. Price at the monopoly price in market B in period 1.

ii. In period $t$, price at the monopoly price in market B if firms $1,\ldots,n$ priced at the monopoly price in market B $\forall\tau < t$.

(d) Market shares

i. In market A, firms $1,\ldots,m$ equally share demand

ii. In market B

A. $\lambda$ is the market share for firm $i$, $i \in \{1,...,m\}$, where $\lambda \in \left[0,\frac{1}{n}\right]$

B. $\frac{1-\lambda m}{n-m}$ is the market share for firm $i$, $i \in \{m+1,\ldots,n\}$

6. Equilibrium conditions

(a) Firms $1,...m$ (multi-market firms). Given the punishment strategy, the optimal deviation is to simultaneously deviate in both markets. The condition is then:

$$\left(\frac{1}{1-\delta}\right)\left[\frac{\pi_A}{m} + \lambda\pi_B\right] \geq \pi_A + \pi_B.$$

(b) Firms $m+1,...,n$ (single-market firms):

$$\left(\frac{1}{1-\delta}\right)\left(\frac{1-\lambda m}{n-m}\right)\pi_B \geq \pi_B.$$

(c) For simplicity, assume $\pi_A = \pi_B$. The two conditions then become:

$$\left(\frac{1}{1-\delta}\right)\left(\frac{1}{m} + \lambda\right) \geq 2 \text{ and } \left(\frac{1}{1-\delta}\right)\left(\frac{1-\lambda m}{n-m}\right) \geq 1$$

i. First notice that if there are equal market shares in market B then $\lambda = \frac{1}{n}$ in which case the second inequality becomes $1 \geq (1-\delta)\, n$ which does not hold by assumption.

ii. It is then necessary that $\lambda < \frac{1}{n}$ which means that the multi-market firms will have a share of market B's demand which is less than that for the single-market firms.

(d) Combining these two conditions and re-arranging yields:

$$1 - (1-\delta)(n-m) \geq \lambda m \geq 2(1-\delta)\, m - 1$$

A necessary condition for such a value for $\lambda$ to exist is:

$$1 - (1-\delta)(n-m) > 2(1-\delta)\, m - 1 \Leftrightarrow 1 > \left(\frac{n+m}{2}\right)(1-\delta).$$

Recall that it was assumed:

$$n(1-\delta) > 1 > m(1-\delta)$$

Hence, $n$ cannot be too large.

7. Intuition

(a) The idea is to give additional share to the $n-m$ single-product firms to induce them to collude.

(b) What keeps the $m$ multi-market firms colluding in B in spite of a relatively low market share is the threat that if they deviate in B that it will not only induce a price war in B but also in A.

(c) Excess collusive capability which exists in A will be used to support collusion in B.

# 3 Selecting a Collusive Outcome

## 3.1 Introduction

### 3.1.1 Statement of Problem

1. Multiplicity of collusive outcomes

(a) If firms limit themselves to self-enforcing collusive outcomes (that is, equilibrium outcomes), there are typically many from which to choose.

(b) Suppose firms are symmetric.

i. A symmetric collusive outcome seems compelling.

ii. Among symmetric outcomes, it seems reasonable to settle on the Pareto superior outcome. This could be for the entire set of subgame perfect equilibria or for a class of SPE (e.g., trigger strategy equilibria).

(c) Suppose firms are asymmetric.

    i. Firms may have different cost or capacity or products.
    ii. An asymmetric outcome seems compelling but how asymmetric?
    iii. We need a mapping from firm traits to the set of price/quantity outcomes.

2. Selection is a bargaining problem.

    (a) Firms may disagree as to what price to set.
    (b) A more serious problem is agreeing to an allocation of quantity given a price

3. Why isn't joint profit maximization a reasonable selection criterion?

    (a) In the absence of side payments, it doesn't satisfy individual rationality.
        i. If firms were unconstrained in terms of making side payments, it would indeed make sense for them to maximize joint profit and then split the proceeds. Typically, firms cannot make side payments and, if they can, they are limited.
        ii. Without unconstrained side payments, a firm is interested in its own profit and not industry profit.
    (b) It is likely to produce counterfactuals.
        i. Assume firms have approximately constant marginal costs.
        ii. If firms are capacity unconstrained then the implication of joint profit maximization is that only the firms with the lowest cost produce. That is likely to be counterfactual.
    (c) British salt market (1980-84): "Thus we can conclude that the actual production allocation between British Salt and ICI Weston Point did not minimise total costs, and therefore did not maximise joint profit. An explanation for the non-maximisation of joint profit is of course that side-payments would have been required, and these would have been clear evidence of collusion." (Ray Rees, "Collusive Equilibrium in the Great Salt Duopoly," *Economic Journal*, July 1993.)

#### 3.1.2 Difficulty of Selection Problem in Practice

1. Inability to agree on market shares is often a source of collusion breakdown and price wars, perhaps more so than cheating.

2. Bromine (1885-1914) (Levenstein, *Explorations in Economic History*, 1996)

    (a) During the time of collusion, there were six price wars.

(b) The two severe price wars of 1891-92 and 1905-08 were not attributed to cheating in the Green-Porter sense but rather were perceived as an attempt to renegotiate the collusive outcome.

(c) Firms publicly announced their intent to sell below the existing collusive price.

3. Lysine (1992-95)

   (a) Entry by Archer Daniels Midland (ADM)

      i. Prior to the entry of ADM, the suppliers of lysine had been colluding.

      ii. ADM built a large low-cost plant and entered the industry aggressively. The U.S. lysine price fell from \$1.32/pound in the three months prior to ADM's entry down to \$0.68/pound 18 months later. By early-mid 1992, ADM's market share of the U.S. market was 71% and it had one-third of global sales.

      iii. In April 1992, representatives of ADM approached the other firms about forming a cartel and proposed that ADM's market share be one-third. Though the other cartel members did not accede to this demand, it is fairly clear that ADM's strategy was to grow market share to a target level and then propose that it be used as their quota in a new cartel.

   (b) First phase of collusion: November 1992 - March 1993

      i. Firms agreed on price but did not reach an agreement as to market share.

      ii. Price rose to \$0.98/month for three months but then the agreement broke down and price fell.

   (c) Second phase of collusion

      i. Agreement on shares occurred in late 1993 and thereafter collusion was successful.

      ii. The delay in reaching an agreement was partly due to different firms proposing different methods. Though they settled upon an allocation of market shares, Ajinomoto and Sewon preferred a market allocation scheme in which exclusive geographic markets were given to cartel members.

## 3.2 Theoretical Framework

### 3.2.1 Axiomatic Bargaining Theory

1. A *bargaining problem* for $n$ players is defined by a set $\Gamma \subset \Re^n$ and an element $v \in \Re^n$.

   (a) $\Gamma$ is compact and convex. [Utilities possibilities set or UPS]

(b) $v \in \Gamma$ and $\exists s \in \Gamma$ such that $s_i > v_i \; \forall i \in \{1, 2, \ldots, n\}$. [Threat point]

(c) The set of bargaining problems is denoted $\mathcal{B}$.

2. A *bargaining solution* is a function $f : \mathcal{B} \to \Re^n$.

3. Nash Bargaining Solution ($n = 2$)

(a) Axioms

**A1** (Invariance to Equivalent Utility Representations) Suppose the bargaining problem $\langle \Gamma', v' \rangle$ is obtained from $\langle \Gamma, v \rangle$ by the transformation: $s'_i = \alpha_i + \beta_i s_i$ where $\beta_i > 0$, $\forall i$. Then

$$f_i \left( \Gamma', v' \right) = \alpha_i + \beta_i f_i \left( \Gamma, v \right) \forall i.$$

**A2** (Symmetry) If $v_1 = v_2$ and $(s', s'') \in \Gamma$ iff $(s'', s') \in \Gamma$ then $f_1 (\Gamma, v) = f_2 (\Gamma, v)$.

**A3** (Pareto Efficiency) If $s, t \in \Gamma$ and $t_i > s_i \; \forall i$ then $f (\Gamma, v) \neq s$.

**A4** (Independence of Irrelevant Alternatives) If $S \subset T$ and $f (T, v) \in S$ then $f (S, v) = f (T, v)$.

(b) **Theorem** (Nash, 1950): There is a unique bargaining solution $f^{NBS} : \mathcal{B} \to \Re^2$ satisfying A1-A4 and it is given by

$$f^{NBS} (\Gamma, v) = \arg \max_{(v_1, v_2) \leq (s_1, s_2) \in \Gamma} (s_1 - v_1)(s_2 - v_2).$$

4. Nash Bargaining Solution (general $n$-player case):

$$f^{NBS} (\Gamma, v) = \arg \max_{v \leq s \in \Gamma} \Pi_{i=1}^n (s_i - v_i).$$

### 3.2.2 Two-Stage Process

1. The idea is that firms can only bargain over outcomes that are self-enforcing.

2. Self-enforcing bargaining game

(a) Deriving the UPS

i. Define $\Omega \subset \Re^n$ to be the set of subgame perfect equilibrium payoff vectors.

ii. The requirement is that $\Gamma \subseteq \Omega$.

iii. For example, $\Gamma$ is the set of payoff vectors associated with grim trigger strategy SPE.

(b) Deriving the threat point

i. If firms fail to come to an agreement then they revert to the non-collusive outcome.

ii. The threat point is then the static Nash equilibrium payoff vector,

$$(v_1, \ldots, v_n) = \left( \frac{\widehat{\pi}_1}{1-\delta_1}, \ldots, \frac{\widehat{\pi}_n}{1-\delta_n} \right).$$

3. Given this bargaining game, one applies a bargaining solution such as the Nash Bargaining Solution.

### 3.3 Application to the Asymmetric Bertrand Price Game

- Harrington, *International Journal of Industrial Organization*, 1989.

1. Model

   (a) Price game with homogeneous goods.

   (b) Common constant marginal cost of $c \geq 0$.

   (c) Market demand function

      i. $D : \Re_+ \rightarrow \Re_+$ is a continuous, bounded function.

      ii. $\exists \overline{P} > c$ such that $D(p) = 0$ iff $p \geq \overline{P}$.

      iii. $D(p)$ is decreasing in $p$ $\forall p \in \left[0, \overline{P}\right]$.

      iv. $(p-c)\,D(p)$ is strictly quasi-concave in $p$ $\forall p \in \left[0, \overline{P}\right]$.

   (d) Wlog, restrict each firm's set of feasible prices to $\left[0, \overline{P}\right]$.

   (e) Joint profit maximum

      i. $p^m$ is the joint profit maximizing price:

$$p^m \in (p-c)\,D(p)$$

      ii. $\pi^m \equiv (p^m - c)\,D(p^m)$.

   (f) Infinitely repeated game

      i. History is common knowledge (perfect monitoring)

      ii. A strategy is $\left\{\rho_i^t\right\}_{t=1}^{\infty}$ where $\rho_i^t : \left[0, \overline{P}\right]^{n(t-1)} \rightarrow \left[0, \overline{P}\right]$.

      iii. Heterogeneous discount factors, $0 < \delta_1 \leq \cdots \leq \delta_n < 1$.

2. Grim trigger strategy equilibria

   (a) Class of collusive outcomes

      i. $(p, q_1, \ldots, q_n)$ where $p \in [c, p^m]$ and $\sum_{i=1}^n q_i = D(p)$.

      ii. All firms charge a common price but, in contrast to the usual formulation, demand can be unequally split.

(b) Grim trigger strategy equilibria:

$$\rho_i^1 = p'$$

$$\begin{aligned} \rho_i^t &= \begin{cases} p' & \text{if } p_j^\tau = p' \ \forall \tau \le t-1, \ \forall j \\ c & \text{otherwise} \end{cases} \\ t &= 2, 3, \ldots; \ i = 1, 2, \ldots, n. \end{aligned}$$

where firm $i$'s quantity at a price of $p'$ is $q_i'$.

(c) Note that this encompasses the most severe punishment equilibrium since the static NE gives each firm its minimax payoff of zero.

3. Set of SPE outcomes

(a) Equilibrium conditions:

$$\frac{(p'-c)\, q_i'}{1-\delta_i} \ge \left(p'-c\right) D\left(p'\right) \ \forall i \ \Leftrightarrow q_i' \ge (1-\delta_i)\, D\left(p'\right).$$

(b) The set of self-enforcing outcomes:

$$\Delta \equiv \left\{ (p, q_1, \ldots, q_n) : q_i \ge (1-\delta_i)\, D(p)\, \forall i, \sum_{i=1}^{n} q_i = D(p), p \in [c, p^m] \right\}.$$

(c) **Theorem**: $\Delta$ includes elements with $p' \in (c, p^m]$ iff $\left(\frac{1}{n}\right) \sum_{i=1}^{n} \delta_i \ge \frac{n-1}{n}$.

(d) Collusion is sustainable iff the average discount factor is sufficiently high.

(e) Proof: Suppose $p' \in (c, p^m]$ and sum up the $n$ equilibrium conditions:

$$\begin{aligned} \sum_{i=1}^{n} q_i' &\ge \sum_{i=1}^{n} (1-\delta_i)\, D\left(p'\right) \Leftrightarrow D\left(p'\right) \ge \sum_{i=1}^{n} (1-\delta_i)\, D\left(p'\right) \Leftrightarrow \\ 1 &\ge \sum_{i=1}^{n} (1-\delta_i) \Leftrightarrow \left(\frac{1}{n}\right) \sum_{i=1}^{n} \delta_i \ge \frac{n-1}{n}. \end{aligned}$$

If $\left(\frac{1}{n}\right) \sum_{i=1}^{n} \delta_i < \frac{n-1}{n}$ then one of the equilibrium conditions must be violated. Thus, $\left(\frac{1}{n}\right) \sum_{i=1}^{n} \delta_i \ge \frac{n-1}{n}$ is a necessary condition. To show sufficiency, consider the following collusive outcome

$$\left(p', (1-\delta_1)\, D\left(p'\right) + \theta, \ldots, (1-\delta_n)\, D\left(p'\right) + \theta\right)$$

where

$$\theta \equiv \left(\frac{D\left(p'\right)}{n}\right) \left[1 - \sum_{i=1}^{n} (1-\delta_i)\right].$$

Note that this is a legitimate outcome since

$$\sum_{i=1}^{n} (1-\delta_i) \left( D \left( p' \right) + \theta \right) = D \left( p' \right).$$

Next note that if $\theta > 0$ then all of the equilibrium conditions are satisfied. Finally, if $\left(\frac{1}{n}\right) \sum_{i=1}^{n} \delta_i \geq \frac{n-1}{n}$ then it is indeed true that $\theta > 0$.

4. NBS

   (a) Nash Bargaining Program

$$\max_{(p,q_1,\ldots,q_n)\in\Delta} \prod_{i=1}^{n} \left( \frac{(p-c)\,q_i}{1-\delta_i} \right),$$

   where the threat point is zero. Next note that

$$\max_{(p,q_1,\ldots,q_n)\in\Delta} \prod_{i=1}^{n} \left( \frac{1}{1-\delta_i} \right) \prod_{i=1}^{n} (p-c)\,q_i = \prod_{i=1}^{n} \left( \frac{1}{1-\delta_i} \right) \max_{(p,q_1,\ldots,q_n)\in\Delta} \prod_{i=1}^{n} (p-c)\,q_i.$$

   Thus, discount factors only matter in that they affect $\Delta$.

   (b) **Theorem**: If $\left(\frac{1}{n}\right) \sum_{i=1}^{n} \delta_i \geq \frac{n-1}{n}$ then the NBS is $p^* = p^m$ and

$$\begin{aligned} q_i^* &= (1-\delta_i)\, D(p^m), \ i = 1, \ldots, \widehat{k} \\ q_i^* &= \left( \frac{1}{n-\widehat{k}} \right) [D(p^m) - \sum_{j=1}^{\widehat{k}} (1-\delta_i)\, D(p^m)], \ i = \widehat{k}+1, \ldots, n; \end{aligned}$$

   where

$$\begin{aligned} \widehat{k} &= \min k : \left\{ \left( \frac{1}{n-k} \right) [D(p^m) - \sum_{j=1}^{k} (1-\delta_i)\, D(p^m)] \geq (1-\delta_i)\, D(p^m), \right. \\ &\quad \left. \text{for } i = k+1, \ldots, n; \ k \in \{0, 1, \ldots, n-1\} \right\}. \end{aligned}$$

   (c) **Theorem**: $q_1^* \geq \cdots \geq q_n^*$. [Less patient firms produce more.]

5. Interpretation

   (a) Standard in the Bertrand price game, if any price above cost can be sustained than all prices can be sustained. Thus, Pareto efficiency implies that the collusive price is that which maximizes joint profit.

   (b) Suppose all equilibrium conditions are satisfied when firms have equal market shares.

      i. This occurs when the lowest discount factor, $\delta_1$, is sufficiently high.

      ii. Then $\widehat{k} = 0$ in which case the NBS has equal shares.

(c) Suppose all equilibrium conditions are not satisfied when firms have equal market shares.

   i. This occurs when $\delta_1$ is sufficiently low but still $\left(\frac{1}{n}\right)\sum_{i=1}^{n}\delta_i \geq \frac{n-1}{n}$.
   ii. In this case, $\widehat{k} \in \{1,\ldots,n-1\}$ and firms are treated in one of two ways.
   iii. The most impatient firms, $i = 1,\ldots,\widehat{k}$, receive a quota that is just sufficient to induce them to price at $p^m$.
   iv. The more patient firms, $i = \widehat{k}+1,\ldots,n$, equally divide up the remaining demand.

6. Duopoly case

   (a) Figure 1 - Constrained solution
      i. Triangle ADE is $\Delta$.
         A. All quantity vectors to the right of line AD satisfy firm 1's equilibrium condition.
         B. All quantity vectors to the left of line AE satisfy firm 2's equilibrium condition.
      ii. The parabola represents pairs of payoffs for which the products of those pairs is the same.
      iii. The NBS is achieved at point E.
   (b) Figure 2 - Unconstrained solution
      i. The solution has equal market shares.
      ii. However, note that $\delta_1 \neq \delta_2$ so that $\Delta$ is asymmetric.

### 3.4 Market Sharing Rules in Practice

1. Historical market share: Citric acid (1991-95) (Connor, 2001)

   (a) Firms agreed to raise prices and allowed each company to offer a 3% discount to their five biggest customers.
   (b) 1991 collusive market shares were set at each firm's historical market share over 1988-1990
      i. Haarman & Reimer (a subsidiary of Bayer) - 34%
      ii. Archer Daniels Midland - 27.5%
      iii. Jungbunzlauer - 24%
      iv. Hoffman-La Roche - 14.5%

2. Graphical example of market sharing rule based on history

   (a) Infinitely repeated quantity game
      i. $P(Q) = 100 - Q$

ii. $C_1(q) = 35q$, $C_2(q) = 45q$

iii. Best reply functions:

$$q_1 = 32.5 - 0.5q_2$$

$$q_2 = 27.5 - 0.5q_1$$

(b) Grim trigger strategy equilibrium conditions.

i. Firm 1

$$\frac{(100 - q_1^c - q_2^c - 35)\, q_1^c}{r} \geq \frac{(100 - (32.5 - 0.5q_2^c) - q_2^c - 35)\,(32.5 - 0.5q_2^c)}{(1+r)} + \frac{625}{(1+r)\, r},$$

ii. Firm 2

$$\frac{(100 - q_1^c - q_2^c - 45)\, q_2^c}{r} \geq \frac{(100 - q_1^c - (27.5 - 0.5q_1^c) - 45)\,(27.5 - 0.5q_1^c)}{(1+r)} + \frac{225}{(1+r)\, r}.$$

iii. The gray area in the figure is the set of values for $q_1^c$ and $q_2^c$ which satisfy these two equations. That is, if $q_1^c$ and $q_2^c$ lie in the gray area then firms can sustain those quantities as part of a collusive equilibrium.

(c) Selection of an outcome

i. Suppose firms have decided on raising price from 60 (the Cournot price) to 70 which means industry supply must be reduced from 40 to 30. The line running from the horizontal axis at $q_1 = 30$ to the vertical axis at $q_2 = 30$ represents all pairs of quantities for which the total supply is 30. The bold portion of that line is the part which intersects the gray area and thus represents all of the values for $q_1^c$ and $q_2^c$ which are sustainable and produce a price of 70. We've then narrowed the problem down to firms choosing from a pair of quantities from the bold line.

ii. Using historical precedent, the collusive market shares are set equal to recent (non-collusive) market shares. If, prior to forming a cartel, the non-collusive solution prevailed then any then any pair of quantities on the line running from (0,0) to (25,15) results in the historical market share of 62.5% for firm 1 and 37.5% for firm 2.

iii. If firms have agreed upon a collusive price of 70, the solution is then the intersection which yields $q_1^c = 18.75$ and $q_2^c = 11.25$.

3. Capacities

(a) Norwegian cement (1923-68) (Röller and Steen, 2003)

i. This is a legal cartel that was cartelized by means of a common sales office.

ii. The sales office determined the total domestic supply.

iii. The sharing rule had domestic supply divided according to each firm's share of industry capacity.

(b) German coal cartel (1920s and 1930s) (Scherer, 1980)

   i. Output shares were allocated on the basis of capacity.

   ii. There is evidence that capacity exceeded peak demand by 25% because of competition in investment so as to increase a firm's collusive market share.

4. School milk auctions (Pesendorfer, 2000)

   (a) Background

      i. Procurement auctions for school milk from Florida and Texas, 1980-91.

      ii. Firms were convicted of bid rigging; ten in Florida and seven in Texas.

      iii. On average, 239 contracts were bid on each year in Florida, and 136 in the Dallas-Fort Worth area of Texas.

      iv. Average number of bidders in an auction

         A. Florida: 3.08 cartel bidders and 0.58 non-cartel bidders

         B. Texas: 2.05 cartel bidders and 0.55 non-cartel bidders.

   (b) Cartel implementation scheme

      i. Market shares fluctuated much more in Florida than in Texas.

      ii. This is consistent with the Florida cartel using side payments in which case the cartel member with the lowest cost could be the one designated to bid competitively for the contract.

      iii. In the case of Texas, market shares were much more stable which is consistent with a contract allocation scheme. This would be less efficient.

      iv. There is legal evidence that the Florida cartel did use side payments but there is no evidence that the Texas cartel did.

5. Can we derive these market sharing rules from some primitive assumptions?

### 3.5 Market Sharing Rule and Capacity Investment

- Davidson and Deneckere (*International Economic Review*, 1990)

1. Introduction

   (a) If one is to explore investment or endogenize firm traits then there must be a solution to the problem of how firms' traits determine the collusive outcome.

   (b) Main insight

      i. Excess capacity is conducive to collusion.

      ii. Market sharing rule can dissipate collusive profits through excessive capacity investment.

2. Model

   (a) Two-stage game

      i. Stage 1 has two firms simultaneously invest in infinitely durable capacity.

      ii. Stage 2 - Given capacities are common knowledge, the two firms engage in an infinitely repeated capacity-constrained price game with perfect monitoring.

   (b) Capacity-constrained price game

      i. Firms offer homogeneous products and simultaneously choose price.

      ii. Consumers demand from the firms with the lowest price (if there is more than one firm with the lowest price then demand is equally divided among them).

      iii. Firms supply so as to meet demand.

   (c) Production and capacity cost functions

      i. At zero marginal cost, a firm is constrained to producing less than or equal to its capacity, $K_i$.

      ii. Capacity costs $c > 0$ per unit.

   (d) Market demand function

      i. $D(\cdot) : \Re_+ \rightarrow \Re_+$ is continuous, nonincreasing, and bounded.

      ii. $\exists$ finite $\overline{P} > c$ such that $D(P) = 0$ iff $P \geq \overline{P}$.

   (e) Firm demand function

      i. Firms produce so as to meet demand

      ii. When $p_i \neq p_j$,

$$D_i(p_1, p_2) = \begin{cases} D(p_i) \ \text{ if } p_i < p_j \\ \\ \max\{D(p_i) - K_j, 0\} \ \text{ if } p_i > p_j \end{cases}$$

      iii. When $p_i = p_j$ then firms share the market in a manner to be specified below.

3. Class of SPE

   (a) Semi-collusive outcomes

      i. Firms collude in price (and quantity) with a stationary outcome. This must result in a *function* which maps the space of capacity pairs into the space of price and quantity outcomes. Let $\pi_i^*(K_1, K_2)$ be the profit of firm $i$ associated with the collusive price and market shares when firms capacities are $(K_1, K_2)$.

ii. Given that mapping, each firm chooses its capacity to maximize its stage 2 payoff less the cost of capacity. A pair of capacities, $(K_1^*, K_2^*)$, must then satisfy:

$$\begin{aligned}\frac{\pi_1^*(K_1^*, K_2^*)}{1-\delta} - cK_1^* &\geq \frac{\pi_1^*(K_1, K_2^*)}{1-\delta} - cK_1, \forall K_1 \\ \frac{\pi_2^*(K_1^*, K_2^*)}{1-\delta} - cK_2^* &\geq \frac{\pi_2^*(K_1^*, K_2)}{1-\delta} - cK_2, \forall K_2.\end{aligned}$$

(b) Grim trigger strategy equilibria for stage 2

i. Deviation in price in stage 2 results in infinite reversion to the static Nash equilibrium.

ii. This equilibrium is characterized in Kreps and Scheinkman (*Bell Journal of Economics*, 1983).

A. If both capacities are sufficiently small then firms choose the price for the (symmetric) Nash equilibrium for the quantity game.

B. If both capacities are sufficiently large then firms choose price equal to marginal cost of zero.

C. Otherwise, the equilibrium is in mixed strategies so firms randomize in their prices.

(c) Market sharing rule

i. Allocation of demand when firms choose the same price.

$$q_i = \frac{\min\{K_i, D(0)\}}{\min\{K_1, D(0)\} + \min\{K_2, D(0)\}}.$$

ii. A firm's collusive market share equals its capacity market share except that capacities in excess of the maximum amount of demand do not count.

4. Collusion must result in identical prices.

(a) **Result**: A collusive outcome for the stage 2 game that is sustainable by grim trigger strategy equilibria must have firms charging the same price.

(b) The stage 2 cartel problem is to find the highest price (less than or equal to $p^m$) which is sustainable.

5. Role of capacity in determining the set of collusive outcomes (Brock and Scheinkman, *Review of Economic Studies,* 1985; Benôit and Krishna, *Review of Economic Studies,* 1987)

(a) Consider the stage 2 game in which capacities are exogenous.

(b) Main insight is that more excess capacity is conducive to collusion.

i. When a firm has more capacity, its gain from cheating is higher since it can supply a bigger part of market demand when it undercuts the collusive price. This serves to make collusion more difficult.

ii. But it is also true that the future loss from cheating is higher since more capacity means that the static Nash equilibrium profits tend to be lower. This serves to make collusion less difficult.

iii. Generally, the latter effect dominates so that more capacity allows for a higher collusive price.

(c) Example

i. For the unconstrained capacity case, let $q^m$ denote $\frac{1}{n}$ of the monopoly quantity and $\widehat{q}$ denote the symmetric Nash equilibrium for the quantity game.

ii. Assume $K_i = k$, $\forall i$, where $k \in (q^m, \widehat{q})$. In this case, the (symmetric) Nash equilibrium to the capacity constrained price game is for each firm to price at $P(nK')$ and supply $K'$. $P(\cdot)$ is the inverse market demand function.

iii. Consider the infinitely repeated capacity constrained price game and consider supporting a collusive outcome of all firms pricing at $p^m = P(nq^m)$ using infinite reversion to the static NE as the punishment.

iv. Optimal deviation entails pricing arbitrarily below $p^m$ and meeting all demand subject to one's capacity constraint. Assuming $k < nq^m$ then the equilibrium condition is

$$\frac{p^m q^m}{1-\delta} \geq p^m k + \left(\frac{\delta}{1-\delta}\right) P(nk)\, k.$$

v. The gain to cheating, $p^m k$, is increasing in a firm's capacity; thereby making collusion more difficult.

vi. The loss to cheating, $\left(\frac{\delta}{1-\delta}\right) P(nk)\, k$, is decreasing in a firm's capacity as its derivative is

$$\left(\frac{\delta}{1-\delta}\right) \left[P'(nk)\, nk + P(nk)\right].$$

Given $k > q^m$ then, by the usual concavity of the profit function, $P'(nk)\, nk + P(nk) < 0$.

6. Capacity has two effects

(a) Capacity influences the set of collusive prices.

(b) Capacity influences a firm's share of collusive quantity according to the market sharing rule.

7. **Theorem**: There exists a unique cartel price, $p^*(K_1, K_2)$, and there exists critical discount factors $\underline{\delta}(K_1, K_2)$ and $\overline{\delta}(K_1, K_2)$ such that

(a) if $\delta \geq \overline{\delta}(K_1, K_2)$ then $p^*(K_1, K_2) = p^m$

(b) if $\delta \in \left(\underline{\delta}(K_1, K_2), \overline{\delta}(K_1, K_2)\right)$ then $p^*(K_1, K_2) \in (0, p^m)$ and $p^*$ is increasing in $\delta$

(c) if $\delta < \overline{\delta}(K_1, K_2)$ then collusion is not sustainable.

8. Vasconcelos (2004)

   (a) This paper explores the effect of mergers on the collusive price.

   (b) Each firm has an asset which reduces marginal cost and the sharing rule is determined by a firm's share of total capital. A merger influences the distribution of this capital as well as the number of firms and thereby affects collusion.

   (c) A merger that increases inequality in the distribution of capital is shown to result to make collusion more difficult.

# 4 Detection of Cartels

## 4.1 Introduction

### 4.1.1 What do Cartel Price Paths Look Like?

1. Examination of cartel price series

   (a) Graphite electrodes

   (b) Citric acid

   (c) Vitamin C

2. Inadequacy of existing theory for understanding collusion

   (a) Existing theory doesn't predict anything like these pricing dynamics.

   (b) Previous models have left out the task of eluding detection and have exclusively focused on the issue of raising price in an internally stable manner.

### 4.1.2 Cartel Discovery

1. Process of detection

   (a) Suspicions - someone suspects there is collusion.

   (b) Investigation of a complaint - antitrust authorities determine whether there is sufficient evidence to warrant a formal investigation.

   (c) Prosecution - Antitrust authorities and/or plaintiffs develop a legal case to establish the presence of a price-fixing conspiracy and measure damages.

2. Antitrust authorities are not actively engaged in detection:

(a) “As a general rule, the [Antitrust] Division follows leads generated by disgruntled employees, unhappy customers, or witnesses from ongoing investigations. As such, it is very much a reactive agency with respect to the search for criminal antitrust violations. ... Customers, especially federal, state, and local procurement agencies, play a role in identifying suspicious pricing, bid, or shipment patterns.” [McAnney, Joseph W., “The Justice Department’s Crusade Against Price-Fixing: Initiative or Reaction?,” *Antitrust Bulletin*, Fall 1991, 521-542 (pp. 529, 530).]

(b) ”The proactive efforts [of the Department of Justice] are a targeted and focused undertaking, directed at markets in industries where the Division has information that collusion has occurred or where the Division has had leads or prosecution in adjacent industries.” [Gary R. Spratling, ”Detection and Deterrence: Rewarding Informants for Reporting Violations,” *George Washington Law Review*, October/December 2001.]

3. Hay and Kelley (1974)

   (a) Data is from Section 1 cases over January 1963 to December 1972 that ended in guilt or a *nolo contendere* plea.

   (b) 13 out of 49 cases were due to a complaint by a customer or a government agency.

   (c) 12 were due to a Grand Jury investigation of an unrelated case.

4. Recent examples

   (a) Customer detection

   - Graphite electrodes - the investigation began after a complaint from a steel manufacturer.
   - Stainless steel - buyers complained to the European Commission about the rapid increase in prices.

   (b) Employees

   - Lysine - an employee of ADM divulged the existence of a lysine price-fixing conspiracy and became an FBI informant.

   (c) Other investigations

   - Sodium gluconate - prosecutions grew out of information received from defendants in the lysine and citric acid investigations.

   (d) Other companies

   - Heavy-lift marine construction services - A U.S. company tipped off the authorities of bid-rigging based on information it had received due to a joint venture it owned with a Dutch partner that was a member of the cartel.

### 4.1.3 U.S. Corporate Leniency Program

1. 1978 policy

   (a) In 1978, the U.S. Department of Justice (DoJ) established a program whereby corporations and individuals who were engaging in illegal antitrust activity (such as a price-fixing) could receive lenient treatment if they fully cooperated in an investigation.

   (b) Leniency means not being criminally charged for the activity being reported which allows a corporation to avoid government fines (though it is still liable for private damages) and an individual to escape fines and prison sentences.

   (c) In spite of the potential appeal of amnesty, the program was rarely used, averaging only about one per year.

   (d) A likely reason is that its design left considerable uncertainty as to whether an application for leniency would be approved. In particular, leniency would be denied if the government could have "reasonably expected" that it would have learned of the cartel without the applicant's assistance.

2. 1993 revision of policy

   (a) It laid out a much clearer set of conditions for a leniency application to be approved which served to reduce uncertainty.

   (b) It allowed amnesty in cases for which an investigation had been started. While firms are unlikely to apply for leniency when the authorities do not even have a hint that collusion is occurring, there is a much stronger incentive if the authorities suspect a cartel exists in which case the prospect of prosecution may be imminent.

   (c) One of the conditions for leniency is that the DoJ "has not received information about the illegal activity being reported from any other source." This meant that amnesty is limited to one firm per cartel which can create a "race for the courthouse" as a firm may apply for leniency simply out of fear that another firm will beat them to it.

   (d) Impact

      i. Soon after this revision, applications were coming in at the rate of two per month.

      ii. But usage of the corporate leniency program only means that it's a good deal for firms. Has it proven to be a useful instrument to identify, prosecute, and deter collusion?

3. Has the corporate leniency program aided in discovery?

   (a) To my knowledge, there are very few cases in which a firm employee came forward to provide information on a cartel when there were no suspicions about collusion.

(b) The corporate leniency program is probably critical in prosecution.

(c) Indirectly, it has contributed to detection through the Omnibus question.

i. At the conclusion of a witness interview or grand jury investigation, the following question is posed: "Do you have any information whatsoever, direct or indirect, relating to (description of conduct: e.g., price-fixing, bid-rigging, market allocation) with respect to other products in this industry or in any other industry?"

ii. The witness must answer the question and must do so truthfully, or he would not only lose whatever protection he would otherwise have had for his statements, but also would be subject to the penalties of perjury or making false statements or declarations.

## 4.2 Eluding Detection

### 4.2.1 Model

1. Stage game

(a) Symmetric oligopoly price game including

i. Differentiated products price game

A. $\pi(P_i, P_{-i}) : \Omega^2 \rightarrow \Re$ is continuously differentiable in its own price, $P_i$, and in the common price of rivals, $P_{-i}$.

B. It is quasi-concave in $P_i$.

ii. Bertrand price game - homogeneous products with constant marginal cost.

(b) $\exists$ unique symmetric equilibrium price, $\widehat{P}$. Let $\widehat{\pi} \equiv \pi\left(\widehat{P}, \widehat{P}\right)$.

(c) $\pi(P) \equiv \pi(P, P)$ is quasi-concave in $P$ and $\exists P^m > \widehat{P}$ such that $\pi(P^m) > \pi(P)\ \forall P \neq P^m$.

2. Infinite horizon game of perfect monitoring.

3. Sequence of Events

(a) Firms decide whether to form a cartel.

i. If they decide not to form a cartel, they receive a payoff of $\widehat{\pi}/(1-\delta)$.

ii. If they decide to form a cartel, they choose price.

(b) Suppose a cartel is active as of period $t$.

i. Firms agree to a common price $P^t$ and each realizes profit of $\pi\left(P^t\right)$.

ii. With some probability, the cartel is detected.

A. Each firm pays a penalty of $X^t + F$ and receives $\widehat{\pi}$ in all future periods.

B. $X^t$ is accumulated damages at the end of period $t$.

C. $F$ is the level of fines.

iii. If the cartel is not detected then collusion continues to period $t+1$.

4. Evolution of damages

(a) $X^t$ is accumulated damages as of $t$.

$$X^t = \beta X^{t-1} + \gamma x\left(P^t\right)$$

(b) $x\left(P^t\right)$ is the level of damages incurred in period $t$.

(c) $x : \Omega \to \Re_+$ is bounded, continuous, and non-decreasing.

(d) Damages are assessed only in periods for which the cartel is active and effective.

(e) Private damages in the U.S.

i. Current U.S. antitrust practice:

$$x\left(P^t\right) = \left(P^t - \widehat{P}\right) D\left(P^t\right),$$

where $D(P)$ is firm demand and $\widehat{P}$ is the "but for" price.

ii. If a firm is found guilty in court then $\gamma = 3$.

iii. Most cases are settled out of court and $\gamma = 1$ is typical.

(f) Government fines

i. Penalty is a fraction of sales

$$x\left(P^t\right) = P^t D\left(P^t\right).$$

ii. U.S. sentencing guidelines (for government fines) has $\gamma = .2$ as the base multiple.

(g) $1 - \beta \in (0, 1)$ is the depreciation rate of damages.

5. Modelling detection

(a) Motivation

i. Focus on how prices affect detection through the "creation of suspicions."

ii. Buyers (typically, industrial buyers) are assumed not to be consciously looking for collusion.

iii. Buyers become "suspicious" when observed pricing is anomalous.

A. High prices

B. Large price increases or even decreases.

(b) Reduced form model of detection: $\phi\left(\underline{P}^t, \underline{P}^{t-1}\right)$ is the probability of detection in period $t$.

i. $\phi : \Omega^{2n} \to [0, 1]$ is continuous.

ii. For any $\underline{P}^{t-1}$, $\phi\left(\underline{P}^t, \underline{P}^{t-1}\right)$ is minimized when $\underline{P}^t = \underline{P}^{t-1}$. [minimized at a zero price change]

iii. If $\underline{P}'' \geq \underline{P}' \geq \underline{P}^o$ then $\phi\left(\underline{P}'', \underline{P}^o\right) \geq \phi\left(\underline{P}', \underline{P}^o\right)$. [non-decreasing in price increases]

### 4.2.2 Equilibrium

1. Optimal Symmetric Subgame Perfect Equilibrium (OSSPE)

   (a) State variables: $P^{t-1}$, $X^{t-1}$.

   (b) Initial conditions: $\left(P^0, X^0\right) = \left(\widehat{P}, 0\right)$.

   (c) Punishment path

      i. Deviation from the collusive price path results in a Markov Perfect Equilibrium
      $$V_i^{mpe}\left(\left(P^t, \ldots, P_i, \ldots, P^t\right), \beta X^{t-1}\right).$$

      ii. Assumed or derived property:
      $$\widehat{\pi}/\left(1-\delta\right) \geq V_i^{mpe}\left(\underline{P}^{t-1}, X^{t-1}\right) \geq \left(\widehat{\pi}/\left(1-\delta\right)\right) - \beta X^{t-1} - F$$

   (d) Example: Bertrand price game

      i. Unique MPE is marginal cost pricing.

      ii. $V_i^{mpe}\left(\underline{P}^{t-1}, X^{t-1}\right)$ equals $\widehat{\pi}/\left(1-\delta\right)$ less the expected present value of penalties.

2. Cartel's constrained value function:

$$\begin{aligned} V\left(P^{t-1}, X^{t-1}\right) &= \max_P \pi\left(P\right) + \delta\phi\left(P, P^{t-1}\right)\left[\left(\widehat{\pi}/\left(1-\delta\right)\right) - \beta X^{t-1} - \gamma x\left(P\right) - F\right] \\ &\quad + \delta\left[1 - \phi\left(P, P^{t-1}\right)\right] V\left(P, \beta X^{t-1} + \gamma x\left(P\right)\right) \end{aligned}$$

   subject to

$$\begin{aligned} &\pi\left(P\right) + \delta\phi\left(P, P^{t-1}\right)\left[\left(\widehat{\pi}/\left(1-\delta\right)\right) - \beta X^{t-1} - \gamma x\left(P\right) - F\right] \\ &\quad + \delta\left[1 - \phi\left(P, P^{t-1}\right)\right] V\left(P, \beta X^{t-1} + \gamma x\left(P\right)\right) \quad \geq \\ &\max_{P_i} \pi\left(P_i, P\right) + \delta\phi\left(\left(P\left|P_i\right.\right), P^{t-1}\right)\left[\left(\widehat{\pi}/\left(1-\delta\right)\right) - \beta X^{t-1} - F\right] \\ &+ \delta\left[1 - \phi\left(\left(P\left|P_i\right.\right), P^{t-1}\right)\right] V_i^{mpe}\left(\left(P\left|P_i\right.\right), \beta X^{t-1}\right), \end{aligned}$$

   where $\left(P\left|P_i\right.\right) \equiv \left(P, \ldots, P_i, \ldots, P\right)$.

**Incentive compatibility constraints are not binding**

1. Additional assumptions

   (a) $\exists\widehat{\phi} : \Re \rightarrow [0,1]$ such that
   $$\phi\left(P^t, P^{t-1}\right) = \widehat{\phi}\left(P^t - P^{t-1}\right) \ \forall\left(P^t, P^{t-1}\right).$$

   (b) If $x \geq y \geq 0$ then $\widehat{\phi}\left(x\right) \geq \widehat{\phi}\left(y\right)$.

   (c) $\widehat{\phi}\left(x\right) \geq \widehat{\phi}\left(0\right) \ \forall x \in \Re$ and $\widehat{\phi}\left(0\right) \in [0,1)$.

(d) $\exists\varepsilon > 0$ such that $\widehat{\phi}$ is continuously differentiable in an $\varepsilon$-ball around $P'$ and $\widehat{\phi}'(0) = 0$. [Though we state $\widehat{\phi}'(0) = 0$ as an assumption, it actually follows from the other assumptions.]

(e) $\pi(P) - \delta\widehat{\phi}(0)\left[\left(\frac{\gamma x(P)}{1-\beta}\right) + F\right] > \widehat{\pi}\ \forall P \in \left(\widehat{P}, P^m\right]$. [Sufficient condition is that $\gamma$ and $F$ are not too high.]

(f) $\exists P^* \in \left(\widehat{P}, P^m\right]$ such that

$$\pi'(P) - \left[\delta\widehat{\phi}(0) / \left(1 - \delta\beta\left(1 - \widehat{\phi}(0)\right)\right)\right]\gamma x'(P) \gtreqless 0 \text{ as } P \lesseqgtr P^*.$$

[This holds if this income function, profit less some multiple of damages, is quasi-concave.]

2. **Theorem**: An OSSPE price path, $\left\{\overline{P}^t\right\}_{t=1}^{\infty}$, is non-decreasing over time.

   (a) In raising price, the cartel balances off higher current profit and a higher probability of detection.

   (b) A bigger price increase means a higher probability of detection but, in the event detection doesn't occur, a higher future payoff since the cartel's inherited price is higher.

3. Long-run cartel price: $\lim_{t\to\infty} \overline{P}^t = P^*$.

   (a) One can show that $\left\{\overline{P}^t\right\}_{t=1}^{\infty}$ is bounded which means that $P^*$ exists.

   (b) $P^*$ is defined by:

$$\pi'(P^*) - \left[\frac{\delta\widehat{\phi}(0)}{1 - \delta\beta\left(1 - \widehat{\phi}(0)\right)}\right]\gamma x'(P^*) = 0.$$

   (c) Method of derivation (variational approach)

      i. Consider a price path in which $P^t = \overline{P}^t$ for $t < T$ and $P^t = \overline{P}^t + \varepsilon$ for $t \geq T$.

      ii. The cartel's payoff is continuous in $\varepsilon$ and equals the payoff from $\left\{\overline{P}\right\}_{t=T}^{\infty}$ when $\varepsilon = 0$.

      iii. Optimality requires that if the derivative of the payoff with respect to $\varepsilon$ is defined then it equals 0 at $\varepsilon = 0$.

      iv. Taking this derivative and evaluating it at $\varepsilon = 0$ as $T \to \infty$, it is indeed defined because $\widehat{\phi}'(0)$ exists. Furthermore, it is equal to:

$$\frac{\pi'\left(\overline{P}\right) - \overline{\Delta}\gamma x'\left(\overline{P}\right)}{1 - \delta\left(1 - \widehat{\phi}(0)\right)} \text{ where } \overline{\Delta} \equiv \frac{\delta\widehat{\phi}(0)}{1 - \delta\beta\left(1 - \widehat{\phi}(0)\right)}.$$

v. Optimality then requires that $\pi'\left(\overline{P}\right) - \overline{\Delta}\gamma x'\left(\overline{P}\right) = 0$ which, by A11, implies $\overline{P} = P^*$.

4. Properties of long-run cartel price

   (a) Assume

      i. profit function is concave, $\pi'' < 0$

      ii. damage function is strictly increasing, $\gamma x' > 0$

      iii. minimum probability of detection is positive, $\widehat{\phi}(0) > 0$

   (b) Then $P^* < P^m$ so that the cartel price is bounded below the simple monopoly price in all periods.

   (c) If $\gamma = 0$, so that the only penalty is fixed fines, then $P^* = P^m$.

      i. At the steady-state, fixed fines do not constrain the cartel's price.

      ii. In the long run, price settles down so that price changes converge to zero. Given that $\widehat{\phi}'(0) = 0$, marginal changes in price have no first-order effect on the probability of detection though continue to have a first-order effect on the potential penalty through the damage function. Thus, factors that influence the relationship between price and the size of the penalty - the discount factor, the rate of depreciation of damages, the damage multiple, and the damage function - all influence the long-run price. As a result, if there are only fines and no damages then, as price changes go to zero, marginal changes in price have no effect on the expected penalty so that the cartel price converges to the simple monopoly price.

      iii. The independence of the steady-state cartel price with respect to fixed penalties is in contrast to static models of collusive pricing in the presence of antitrust laws and represents a unique implication of a dynamic approach. In those models, there is an equivalence between fines and damages in the sense that any price resulting for some damage multiple could alternatively be generated through an appropriately selected fine. In contrast, when detection depends on price changes in a dynamic model, price is bounded below the simple monopoly price when penalties include damages but converges to the simple monopoly price when damages are not deployed. Thus, if antitrust policy is intended to constrain cartel prices in the steady-state, it is essential that penalties be responsive to the price charged.

5. Comparative statics

   (a) Assume linear demand and cost functions.

   (b) $P^*$ is

      i. decreasing in the damage multiple, $\gamma$

      ii. decreasing in the probability of detection, $\widehat{\phi}$

      iii. *decreasing* in the but for price, $\widehat{P}$, if $x\left(P^t\right) = \left(P^t - \widehat{P}\right) D\left(P^t\right)$.

**Incentive compatibility constraints are binding**

1. Research strategy
   (a) Use numerical analysis so as to identify possible equilibrium price paths.
   (b) Prove results for special cases of the model so as to understand the numerical results.
2. Numerical Results
   (a) Assumptions
      i. Demand with differentiated products:
      $$D(P_i, P_{-i}) = a - bP_i - dP_{-i}, \text{ where } b > d > 0.$$
      ii. Cost:
      $$C(q) = cq, \text{ where } a > c \geq 0.$$
         A. Detection function:
         $$\phi\left(f^t, f^{t-1}\right) = \begin{cases} \min\left\{\alpha_0 + \alpha_1^u \left(f^t - f^{t-1}\right)^2, 1\right\} & \text{if } f^t \geq f^{t-1} \\ \min\left\{\alpha_0 + \alpha_1^d \left(f^t - f^{t-1}\right)^2, 1\right\} & \text{if } f^t < f^{t-1} \end{cases}$$
         where
         $$f^t \equiv \sum_{i=1}^{n} \left( \frac{D\left(P_i^t, P_{-i}^t\right)}{\sum_{j=1}^{n} D\left(P_j^t, P_{-j}^t\right)} \right) P_i^t.$$
   (b) Parameter values
      i. 13 parameters
      ii. Considered 32 parameter configurations of which 26 resulted in cartel formation.
   (c) Summary of findings
      i. Among the 26 cases, there are two qualitatively distinct cartel price paths.
      ii. Montonically increasing price path
         A. Cartel gradually raises price - so as to avoid detection - and price achieves some steady-state level which is typically below the monopoly price because it isn't worth it for the cartel to risk detection by further raising price or it isn't feasible for the cartel to do so.
         B. Thus, the monotonicity of price when ICCs do not bind can still occur when ICCs bind.
      iii. Non-monotonic price path

A. Cartel price path initially increases and then declines; approaching its steady-state level from above.
B. A representative example is shown in the figure where price rises from 20 to over 45 during the first ten periods and then the cartel gives up about 10% of its price increase as it falls to its steady-state level.

3. Analytical Results - Model 1: Exogenous penalties ($\gamma = 0$), endogenous probability of detection.

   (a) **Theorem**: An OSSPE price path is non-decreasing over time.
   (b) Proof strategy
      i. Show that if it is incentive compatible to raise price to some level then it is incentive compatible to keep it there.
      ii. Price undercutting lowers the probability of detection more when the cartel is raising price than when they are keeping it constant.
      iii. Role of fixed penalties.

4. Analytical Results - Model 2: Endogenous penalties ($\gamma \geq 0$), exogenous probability of detection

   (a) Assumptions
      i. $\phi\left(\underline{P}', \underline{P}^o\right) = \phi^o \ \forall \underline{P}', \underline{P}^o \in \Omega^n$.
      ii. $\rho\left(\tau\right)$ is the probability of detection $\tau$ periods after the last cartel meeting.
      iii. $\rho\left(0\right) = \phi^o$ and $\rho\left(\tau\right)$ is non-increasing in $\tau$.
   (b) Key property is that penalties are more likely when firms are colluding than when they are not.
   (c) **Theorem**: If $\left\{\overline{P}^t\right\}_{t=1}^{\infty}$ is an OSSPE price path then $\overline{P}^1 > P^0$ and $\left\{\overline{P}^t\right\}_{t=1}^{\infty}$, is non-increasing $\forall t \geq 1$.
   (d) Proof strategy
      i. Both the collusive payoff and the payoff to cheating are decreasing in damages.
      ii. The collusive payoff is more sensitive to damages than the payoff to cheating.
      iii. Lemma: On an OSSPE path, damages are non-decreasing over time.
      iv. Incentive compatibility constraints are then tightening over time.
      v. The cartel must lower price to reduce the incentive to deviate.

5. Discussion of collusive price paths

   (a) If the ICCs are not binding, the cartel price path rises over time.

(b) When ICCs bind, the central issue is whether the cartel will, at some point, be forced to lower price so as to maintain cartel stability. Both the sensitivity of detection to price movements and the sensitivity of penalties to price levels are pertinent to this issue.

(c) Effect of price on the probability of detection

   i. If a firm did not find it optimal to cheat when other firms were raising price then it is not optimal for them to cheat when other firms are keeping price constant.

   ii. Thus, higher prices are easier to sustain as lagged price is higher.

(d) Effect of price on penalties can have the opposite effect - collusion may be more difficult over time.

   i. As penalties grow, cartel members become increasingly concerned with the prospects of detection. If detection is less likely when collusion stops, there is an added incentive for a firm to cheat. With rising penalties as firms collude longer, the cartel must lower price so as to counterbalance this increased desire to deviate.

   ii. The extent to which rising penalties make the cartel less stable then depends on whether cheating - with the ensuing collapse of the cartel - makes detection more or less likely. If the probability of detection is sufficiently insensitive to the price decline that would ensue in the post-cartel periods then a firm reduces the probability of paying antitrust penalties by cheating and causing the cartel to dissolve. In that case, this dynamic forces price down.

   iii. If instead a post-cartel price war is likely to trigger detection, rising penalties serve to stabilize the cartel. Firms increasingly prefer to maintain relatively stable cartel prices than to risk detection by inducing a price war.

(e) Balancing two forces

   i. When the probability of detection is sufficiently sensitive to price increases, the cartel will gradually raise price for reasons that are clear. If, in addition, detection is sufficiently sensitive to price decreases then collusion will become easier over time which allows further price hikes; so the price path is always increasing. Collusion is becoming easier because penalties are growing - so avoiding detection is increasingly important - and the best way to avoid detection is to maintain moderately rising prices rather than experience a price war.

   ii. When instead detection is fairly insensitive to price decreases then the price path, after initially rising, will eventually fall. The growing penalties make cheating increasingly attractive - as it brings collusion to an end and reduces the chances of having to pay these penalties - and the cartel must lower price as a result. Thus, the second dynamic eventually comes to dominate the pricing dynamics.

6. Impact of Antitrust Laws

   (a) Assumptions

      i. $\exists \widetilde{P} \in \left(\widehat{P}, P^m\right)$ such that $\pi(P)/(1-\delta) \gtreqqless \pi(\psi(P), P) + \delta(\widehat{\pi}/(1-\delta))$ as $P \lesseqqgtr \widetilde{P}$. [Without antitrust laws, $\widetilde{P}$ is charged.]

      ii. $\phi : \Omega^{2n} \rightarrow \Re_+$ is continuously differentiable.

      iii. $\phi(P, P) = 0 \; \forall P \in \Omega$.

   (b) **Theorem**: If $\left\{\overline{P}^t\right\}_{t=1}^{\infty}$ is an OSSPE price path then $\lim_{t\rightarrow\infty} \overline{P}^t > \widetilde{P}$.

   (c) Antitrust laws may initially lower the cartel price but must eventually raise it.

   (d) Intuition for $\lim_{t\rightarrow\infty} \overline{P}^t > \widetilde{P}$.

      i. As $\phi(P, P) = 0$ then, as the price path converges to its steady-state level, the collusive payoff is the same as when there is no antitrust authority.

      ii. Example: homogeneous products. Pricing at $\widehat{P}$ forever is the unique MPE.

      iii. In the event of a deviation, there is a discrete change in price to firms pricing at $\widehat{P}$. Thus, the probability of detection is positive and this lowers the punishment payoff below $\widehat{\pi}/(1-\delta)$.

   (e) Intuition for antitrust policy making collusion easier: In the steady-state, detection can best be avoided by maintaining stable prices. This makes a firm less inclined to cheat on the cartel and thus the cartel can support higher prices.

## 4.3 Distinguishing Collusion and Competition

### 4.3.1 Methods for Distinguishing Collusion and Competition

1. Methods vary according to

   (a) the type of data that is available

   (b) whether there is prior information about collusion

   (c) reduced form or structural estimation methods

2. Method A: Does the behavior of the suspected colluding firms differ from that of competitive firms?

   (a) Data requirements

      i. Data during the time of suspected collusion.

      ii. Prior information as to which firms might be colluding.

      iii. Competitive benchmark which requires that the suspected firms are a proper subset of all firms.

(b) Implementation

   i. Estimate reduced form price equations by regressing price on cost and demand conditions.

      A. Estimate a price equation for (suspected) cartel members.

      B. Estimate a price equation for (unsuspected) competitive or non-cartel firms.

   ii. Test these equations to determine whether they are statistically different.

   iii. If they are statistically different

      A. check that competitive firms act in a manner consistent with a competitive model

      B. check that colluding firms act in a manner consistent with some model of collusion such as complementary bidding or bid rotation.

3. Method B: Is the observed behavior inconsistent with competition?

   (a) Data during the time of suspected collusion is required.

   (b) Theoretically identify properties that competitive behavior must satisfy.

      i. Empirical work thus far has been conducted in an auction setting and takes advantage of the assumption of Independent Private Values (after conditioning on publicly available information).

      ii. Bajari and Ye (2003)

         A. Firms' bids must be independent and exchangeable.

         B. Independence means that, after controlling for publicly available information, the unexplained part of one firm's bid should be independent of the unexplained part of another firm's bid.

         C. Exchangeability implies that the estimated coefficients in the bid equations are the same.

      iii. Porter and Zona (1993) use the property that the estimated ranking of firms' bids must be the same regardless of the subsets of firms used in the estimation.

      iv. These properties pertain to the relationship between bids among collections of firms.

   (c) Implementation

      i. Estimate a pricing equation for each firm.

      ii. Test whether the property holds for various (perhaps all) subsets of firms.

      iii. Example: independence of bids (Bajari and Ye, 2003)

         A. Calculate the residuals of each firm's bid function, $\epsilon_{i,t}$.

         B. Test the hypothesis that the coefficient of correlation for $\epsilon_{i,t}$ and $\epsilon_{j,t}$ is zero.

(d) Inferences drawn

   i. Suppose the test fails (e.g., the hypothesis of independence is rejected).

      A. The model is then misspecified and collusion is one possible source of misspecification.

      B. Is the failure of the test consistent with some model of collusion? E.g., bids may not be independent if cartel members submit identical bids or some submit complementary bids.

   ii. Suppose the test doesn't fail. Firms could still be colluding as it is always possible for firms to beat these test.

4. Method C: Does a competitive model or a collusive model better fit the data?

   (a) Data during the time of suspected collusion is required.

   (b) Empirical strategy

      i. Specify and estimate a (structural) competitive model using price data and cost and demand shifters.

      ii. Specify and estimate a (structural) collusive model using price data and cost and demand shifters.

   (c) Implementation

      i. Nest the two models and test which model performs better the log likelihood criterion (Baldwin et al, 1997).

      ii. Bayesian analysis using the various candidate models (Bajari and Ye, 2003).

5. Method D: Has firm behavior become less competitive over time?

   (a) Data requirements

      i. Data over time which includes periods of suspected collusion and periods of competition is required.

      ii. Data from competitive regime is needed to provide a competitive benchmark.

   (b) Empirical strategy is to identify a structural break in firms' pricing equations associated with the formation of a cartel.

   (c) Implementation - not done yet

6. Comparison of approaches

   (a) Prior information

      i. Methods A and D require a competitive benchmark.

         A. Method A requires some prior information about which firms might be colluding.

B. Method D requires some prior information as to when firms might be colluding.

ii. Methods B and C do not require any such prior information.

iii. Method A is a second stage tool in that it can confirm suspicions about collusion but the suspicions must already be there.

iv. Methods B, C, and D could be used as a first stage tool in that it can identify collusion without any suspicions.

(b) Cartel is all inclusive or not.

i. Method A requires that the cartel be a strict subset of all firms so that there is a competitive benchmark in the form of colluding firms.

ii. Methods B, C, and D work even if all firms are colluding.

(c) Can the cartel the beat the test.

i. Methods A, B, and C can find no evidence of collusion even if there is collusion because the cartel can always beat the test.

A. E.g., if cartel members' bids are an affine transformation of their competitive bids then these methods will not find any evidence of collusion. There exist costs which rationalize cartel members' bids in the context of a competitive model.

B. Lack of information about firms' costs allow the cartel to beat the test.

ii. Tentative claim that Method D cannot be beat because the cartel must raise price relative to what it priced at prior to collusion. Still, behavior could be rationalized in the context of a competitive model if there is a structural break in, for example, the stochastic process determining cost.

(d) Room for misspecification and omitted variables.

i. Methods A and B are the least dependent on misspecification though Method B can be subject to problems of omitted variables. E.g., the test for independence requires that one has controlled for common factors that would correlate firms' costs.

ii. Method C relies upon a proper specification of the competitive model and, even more problematic, the collusive model.

iii. Method D need not require specification of a collusive model but, if it is to distinguish a structural break for cost and demand reasons, it may need to specify a collusive model.

### 4.3.2 Porter and Zona (1993)

1. Market

(a) New York State Department of Transportation (DOT) solicits bids and awards highway construction contracts to the lowest responsible bidder using a first-price sealed bid auction.

(b) DOT awarded about $120M in highway contracts over April 1979 to March 1985 in Nassau and Suffolk counties on Long Island.

  i. This comprised 186 contracts with 161 contracts less than $1M.
  ii. Though 66 firms bid on at least one contract, only 22 firms submitted bids for contracts exceeding $1M and 45% of the bids on those jobs were submitted by four firms.

2. Prior information

  (a) Identifying "suspected" ring members

    i. In 1984, one of the largest firms was convicted on rigging bids on a Long Island highway construction project. Four other firms were listed as unindicted conspirators. This case predates the sample.
    ii. These five firms constitute a candidate cartel.
    iii. Comment

      A. One should also explore whether, in the absence of collusion, the suspected firms would have competed. If they would not have then there is little rationale for them colluding.
      B. Observed participation may not be appropriate because it could be the product of collusion. E.g., there could be minimal interaction because of a collusive scheme not to participate when a firm has been designated as the one to submit a bid.
      C. Better evidence would be the proximity of equipment and offices, proximity of jobs won, etc.

  (b) Identifying the form of collusion.

    i. The "cartel" members frequently bid on the same job.

      A. These firms submitted 54% of the bids on larger jobs and were awarded 37% of the contracts.
      B. On paving jobs, the probability that another cartel firm bid given a bid by a cartel firm was 88% for firm 1 (thus, in 88% of the paving jobs that firm 1 submitted a bid, another of these cartel firms also submitted a bid), 79% for firm 2, 89% for firm 3, 67% for firm 4, and 100% for firm 5.
      C. Of the 48 paving jobs that firm 1 submitted a bid, firm 2 submitted a bid in 37 of them, firm 3 in 34, firm 4 in just 1, and firm 5 in 11.

    ii. Cartel firms bidding on the same project could be part of a phantom or complementary bidding strategy.

3. Reduced form model of bidding

  (a) Theoretical framework

i. Firm $i$'s expected payoff for job $t$ from bidding $b$ is

$$(b - c_{it}) \varphi_{it}(b),$$

where $c_{it}$ is its cost and $\varphi_{it}(\cdot)$ is the probability of winning.

ii. The equilibrium bid, $b_{it}$, satisfies the first-order condition:

$$\varphi_{it}(b_{it}) + (b_{it} - c_{it}) \frac{\partial \varphi_{it}(b_{it})}{\partial b} = 0.$$

(b) Assume bidding behavior satisfies the log-linear bidding rule:

$$\log(b_{it}) = \alpha_t + \beta \mathbf{X}_{it} + \epsilon_{it},$$

where $\alpha_t$ is an auction-specific effect, $\mathbf{X}_{it}$ is a vector of observable variables affecting this firm's probability of winning and its observable costs for this job, and $\epsilon_{it}$ represents private information. Note that $\beta$ is assumed to be constant across jobs and firms.

4. Data

(a) Only paving contracts from Nassau and Suffolk counties between April 1979 and March 1985. Focus on those auctions in which both cartel and competitive firms submitted bids.

i. A total of 575 bids for 116 projects.

ii. Competitive firms submitted 319 bids on 75 projects in which at least two competitive bids were submitted. Jobs receiving one or zero competitive bids were dropped for econometric reasons.

iii. Cartel firms submitted 157 bids in those 75 auctions.

(b) Various data sets to be used.

i. Data set 1 has all 75 auctions and is used to estimate the parameters to the bidding rule using OLS.

ii. Data set 2 uses the same data but uses the ranking of competitive bids rather than the bid level.

iii. Data set 3 explores the ranking of cartel bids and is comprised of the 50 auctions with two or more cartel bids.

(c) Exogenous variables

i. $BACKLOG_{it}$ is backlog of firm $i$ at the time of auction $t$ as measured by dollar value of contracts won but not yet completed.

ii. $CAP_i$ is capacity of firm $i$ as measured by the maximum backlog (also has $CAP_i$ squared)

iii. $UTIL_{it} \equiv BACKLOG_{it}/CAP_i$ is the firm's utilization rate (also has $UTIL_{it}$ squared).

iv. $ISLAND_I = 1$ iff the firm's headquarters is on Long Island and measures geographic proximity to a job.

5. Bid levels

   (a) Empirical strategy

      i. Run model with bids from all firms, bids from competitive firms, and bids from cartel firms.
      ii. Determine whether bidding by suspected cartel firms is distinct from that of competitive firms.

   (b) Empirical estimates

      i. Reduced form estimates for bid function for competitive firms: bid is
         A. initially decreasing and then increasing in utilization
         B. initially decreasing and then increasing in capacity though for all but one firm it is decreasing throughout
         C. All of the variables are highly significant.
      ii. Reduced form estimates for bid function for cartel firms: bid is
         A. not statistically significantly related to utilization
         B. initially increasing and then decreasing in capacity

   (c) Comparing the two models

      i. A Chow test allows one to compare the estimated coefficients using all of the data with the coefficients using only competitive data and cartel data separately.
      ii. One can reject that the estimated coefficients are the same.

   (d) Conclusions

      i. The model fits the competitive data quite well.
      ii. Bids from cartel firms are statistically different from those of competitive firms.

6. Ranking of bids

   (a) Empirical strategy

      i. If the cartel engages in phantom bidding then some cartel bids are determined by the model of bidding behavior and, in particular, are unaffected by factors influencing the probability of winning since they are designed not to win. Though the authors do not put forth a specific model of phantom bidding, bids are presumed to be higher than the bid of the cartel member chosen for this job.
      ii. The low cartel bidder (that is, the designated cartel firm) and the competitive firms bid so as to balance the level of the bid and the probability of winning. With phantom bidding, the other cartel firms

may not. For example, the ordering of the bids of the low cartel bidder and the competitive firms should be determined by cost, while the bids of the other cartel firms may not.

(b) Empirical model

i. Multinomial logit model is used to model the ranking of bids. The likelihood of the observed ranking for auction $t$ is

$$\Pr\left(b_{r_1 t} < \cdots < b_{r_{n_i} t}\right) = \prod_{i=1}^{n_t} \frac{e^{\beta Z_{r_i t}}}{\sum_{j=i}^{n_t} e^{\beta Z_{r_j t}}}$$

where $Z_{r_i t}$ are the exogenous variables for the firm with the $i^{th}$ lowest bid.

ii. The likelihood of observing the rankings for all auctions is

$$L(\beta) = \prod_{t=1}^{T} \prod_{i=1}^{n_t} \frac{e^{\beta Z_{r_i t}}}{\sum_{j=i}^{n_t} e^{\beta Z_{r_j t}}}.$$

Maximum likelihood is using to solve for $\beta$.

iii. Given the multiplicative separability, the model (if correctly specified) can be estimated using any subset of bids. For example, compare using the lowest bidder

$$L(\beta) = \prod_{t=1}^{T} \frac{e^{\beta Z_{r_1 t}}}{\sum_{j=1}^{n_t} e^{\beta Z_{r_j t}}}$$

and the remaining higher bidders

$$L(\beta) = \prod_{t=1}^{T} \prod_{i=2}^{n_t} \frac{e^{\beta Z_{r_i t}}}{\sum_{j=i}^{n_t} e^{\beta Z_{r_j t}}}.$$

(c) Hypotheses

i. Under the null hypothesis of no phantom bidding, the estimated coefficients using the lowest cartel bid should be the same as those using the cartel bids excluding the lowest.

ii. Of course, rejection of the likelihood ratio test between those estimates could be due to sources of model misspecification other than phantom bidding.

(d) Results

i. Competitive bidders - Comparing the estimates using the lowest competitive bid and using the higher competitive bids, one cannot reject the null hypothesis of no model misspecification.

ii. Cartel bidders - Comparing the estimates using the lowest cartel bid and using the higher cartel bids, one can reject the null hypothesis of no model misspecification at the 94% level.

# 5 Future Challenges

## 5.1 Theoretical Challenges

- Dealing with the multiplicity of equilibrium outcomes so as to have a predictive theory of collusion when firms are asymmetric.

- Developing a model of bargaining within a cartel that can explain why some cartels are unstable.

- Developing a framework that distinguishes between tacit and explicit collusion.

  - Equilibrium theory is good at characterizing what are stable firm behavioral rules. It is lacking in terms of describing how firms come to be at a particular equilibrium.
  - Equilibrium approach to the explicit/tacit collusion dichotomy.
    - Firms have private information; e.g, about cost, capacity, demand.
    - Explicit collusion allows firms to send arbitrary messages to each other to convey their private information. If these messages are informative (that is, firms have an incentive to reveal the truth) then prices and quantities can be responsive to this information.
    - Tacit collusion doesn't allow messages. Firms can only choose prices and quantities and if there is to be any information exchange, it must be through those instruments. Of course, firms can signal through pre-announcing price changes. Firms were able to find clever ways in which to signal in the airline industry and in the spectrum auctions.

- Developing a theory of cartel meetings.

  - In some cartels, meetings are very frequent even though they increase the chances of detection.
  - The meetings are not just informational exchange as typically modelled but also an opportunity to explain behavior; e.g., the sugar cartel.

- Endogenizing the detection of cartels.

- Developing a theory of cartel formation.

- Anecdotal evidence suggests that cartel formation is preceded by a sharp decrease in price.
  - Graphite electrodes (1992-97) - see price series
  - Citric acid (1992-95) - see price series
  - Commercial explosives (1988-92) - Conspiracy began when explosive prices were depressed as a result of a price war between two brothers who owned competing companies. Eventually, these brothers sold their companies, and a few of the remaining explosive manufacturers agreed to fix prices and rig bids to increase prices from their depressed state.
- Who should be included in the cartel? With cartels being legal, it is best to include all firms. But if they are illegal, additional members run the risk of discovery. Also consider the corporate leniency program in that light.

### 5.2 Policy Challenges

- Developing a method for detecting cartels. Currently, antitrust authorities are passive in that they react to complaints. In contrast, the Securities and Exchange Commission is active in detecting insider trading and the Internal Revenue Service is active in detecting tax fraud.

- Developing harsher penalties. In many cases, collusion is ex post profitable even when a cartel is successfully prosecuted. Damages are often single damages and government fines, though they have become more substantial, need not be large compared to the additional profits had.

- Prosecuting tacit collusion. Current case law makes it hard to prosecute collusion when there is not evidence of explicit communication even though tacit collusion may cause serious damage.

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# 7 Appendix: Static Equilibrium Models of Oligopoly

## 7.1 A Little Game Theory

1. Modelling a game - Strategic (or Normal ) Form of a Game
   (a) Set of players, $\{1, 2, \ldots, n\}$, are those agents making choices.
   (b) Strategy sets of players: $S_i, i \in \{1, 2, \ldots, n\}$.
      i. A strategy is a fully-specified decision rule as to how to play the game.
      ii. A strategy set is the set of feasible strategies for a player.
   (c) Payoff functions of players, $V_i : S_1 \times \cdots \times S_n \rightarrow \Re, i \in \{1, 2, \ldots, n\}$.
      i. A payoff function assigns a real number to each strategy profile where a strategy profile is an $n$-tuple of strategies; one for each player.
      ii. A payoff function is typically a composition of an outcome function, which maps from strategy profiles into outcomes (e.g., bundles of goods or money), and a utility function, which maps from outcomes into utils.
      iii. Example - First Price Sealed Bid Auction
         A. Description - all bidders submit written bids and the bidder who submits the highest bid wins the item and pays a price equal to her bid.
         B. The rules of the auction describe how bids (or strategies) map into who wins and how much each bidder pays (or the outcome).
         C. A bidder's utility function maps the outcome into utils.
2. Solving a game
   (a) Introduction
      i. Our objective is to describe what strategies players will choose and what the resulting outcome will be.
      ii. The strategic form defines the structure or environment *as it is perceived by players*. It is insufficient for achieving a description as to how players will behave.
      iii. We need to make behavioral assumptions as to what properties behavior satisfies. This takes the form of a solution concept.
   (b) Nash equilibrium
      i. Definition: $(s_1^*, \ldots, s_n^*) \in S_1 \times \cdots \times S_n$ is a *Nash equilibrium* iff

$$V_i(s_1^*, \ldots, s_n^*) \geq V_i\left(s_1^*, \ldots, s_{i-1}^*, s', s_{i+1}^*, \ldots, s_n^*\right) \forall s' \in S_i, \forall i \in \{1, 2, \ldots, n\}.$$

      ii. Interpretation
         A. Given a player's beliefs as to other players' strategies, his strategy maximizes his payoff.

B. A player's beliefs as to other players' strategies is accurate.

C. These conditions hold for all players.

iii. Comments

A. Definition: A player is *rational* if his behavior maximizes his utility given his beliefs over unknown variables.

B. All players are rational does not imply that players act according to a Nash equilibrium.

C. Definition: An event E is *common knowledge* to all players if: all players know E (i.e., player 1 knows E, player 2 knows E, ..., player $n$ knows E), all players know that all other players know E (e.g., player 1 knows that player 2 knows E), all players know that all other players know that all other players know E (e.g., player 1 knows that player 2 knows that player 3 knows E), ad infinitum.

D. Rationality is common knowledge does not imply that players act according to a Nash equilibrium.

(c) Existence of Nash equilibrium

i. Assumptions

A. **A1**: The number of players is finite.

B. **A2**: $S_i$ is a compact convex subset of a finite-dimensional Euclidean space, $\forall i$.

C. **A3**: $V_i$ is continuous everywhere. $\forall i$.

D. **A4**: $V_i$ is quasi-concave in $s_i$, $\forall i$.

ii. **Theorem**: If A1-A4 hold then a Nash equilibrium exists.then it is quasi-concave.

## 7.2 Quantity Competition

### 7.2.1 Existence of Best Reply Function

1. Basic structure

(a) Firms simultaneously choose quantities.

(b) Price is set so as to clear the market.

2. Inverse demand function, $P(\cdot)$

(a) Firms' products are homogeneous

(b) Assumptions

**A1** $P(\cdot) : \Re_+ \rightarrow \Re_+$ is continuous and bounded $\forall Q \geq 0$.

**A2** $\exists$ finite $\overline{Q} > 0$ such that $P(Q) = 0 \forall Q \geq \overline{Q}$.

A2 allows us to bound the strategy space of firms.

(c) Example: $P(Q) = \max\{a - bQ, 0\}$ where $a, b > 0$

3. Firm cost function, $C_i(\cdot)$

   (a) Assumptions

   **A3** $C_i(\cdot) : \Re_+ \to \Re_+$ is continuous $\forall q \geq 0$.

   (b) A3 rules out avoidable fixed cost which create a discontinuity at $q = 0$.

4. Firm profit function

$$\pi_i(q_1, \ldots, q_n) \equiv \pi_i(q_i, Q_{-i}) \equiv P(q_i + Q_{-i}) q_i - C_i(q_i)$$

   where $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) A1-A3 imply that $\pi_i$ is continuous and bounded from above.

5. Strategic form

   (a) Set of players is the $n$ firms

   (b) Strategy set for each firm is $\left[0, \overline{Q}\right]$

   (c) Payoff function of firm $i$ is $\pi_i(q_1, \ldots, q_n)$

6. Existence of best reply function

   - Note: A best reply correspondence is what exists but let $\psi_i(\cdot)$ be a selection from the possible set of values.

   **Theorem 1** By A1-A3, $\exists \psi_i(\cdot) : R_+ \to R_+$ such that

$$\pi_i(\psi_i(Q_{-i}), Q_{-i}) \geq \pi_i(q_i, Q_{-i}) \forall q_i \in \left[0, \overline{Q}\right], \forall Q_{-i}.$$

   **Proof** Maximization of a continuous function over a compact convex set, $\left[0, \overline{Q}\right]$.

### 7.2.2 Existence and Uniqueness of Equilibrium

1. There are various assumptions that ensure equilibrium. We will begin by assuming sufficient structure to utilize a standard existence theorem in game theory and then consider some weaker assumptions.

2. Assumptions

   **A4** $P(\cdot)$ is twice continuously differentiable and $P'(Q) < 0 \forall Q \in \left(0, \overline{Q}\right)$.

   **A5** $C_i(\cdot)$ is twice continuously differentiable and $C_i'(q) \geq 0 \forall q > 0$.

   **A6** $\pi_i(q_i, Q_{-i})$ is quasi-concave in $q_i \forall q_i$.

3. Existence of equilibrium

**Theorem 2** By A2 and A4-A6, a Nash equilibrium exists; that is, there exists $(\widehat{q}_1, \ldots, \widehat{q}_n) \in [0, \overline{Q}]^n$ such that

$$\widehat{q}_i \in \psi_i \left( \sum_{j \neq i} \widehat{q}_j \right) \forall i.$$

**Proof** Sufficient conditions for the existence of a pure-strategy Nash equilibrium are: i) finite number of players; ii) compact and convex strategy sets; and iii) each player's payoff function is continuous and quasi-concave in its own strategy.

4. Uniqueness of equilibrium

   (a) Sufficient conditions for the equilibrium to be unique.

      i. A4

      ii. $P(\cdot)$ is log-concave

         A. This means that $\ln P(\cdot)$ is concave which holds iff

$$\begin{aligned} P''(Q)P(Q) - [P'(Q)]^2 &\leq 0 \forall Q \text{ s.t. } P(Q) > 0 \Leftrightarrow \\ P''(Q) &\leq [P'(Q)]^2 / P(Q). \end{aligned}$$

         B. Contrast this to requiring $P(\cdot)$ to be concave which means

$$P''(Q) \leq 0 \forall Q \text{ s.t. } P(Q) > 0.$$

      iii. $P' - C_i'' < 0 \forall i$

   (b) These conditions are also sufficient for quasi-concavity of the profit function.

5. Existence of an interior equilibrium

**A7** $P(0) > C_i'(0)$ for some $i$.

**Theorem** By A2 and A4-A7, a Nash equilibrium, $(\widehat{q}_1, \ldots, \widehat{q}_n)$, exists and $\widehat{q}_i > 0$ for some $i$.

**Proof** Suppose not so that $\widehat{q}_j = 0 \forall j$. Let firm $i$ have: $P(0) > C_i'(0)$. First note that:

$$\pi_i \left( \overline{q}_i, \sum_{j \neq i} \widehat{q}_j \right) = P(\overline{q}_i)\overline{q}_i - \int_0^{\overline{q}_i} C_i'(q)\, dq = \int_0^{\overline{q}_i} [P(q) - C_i'(q)]\, dq$$

By $P(0) > C_i'(0)$ and continuity of $P$ and $C_i'$, it follows that this expression is positive for $\overline{q}_i$ sufficiently close to zero. Since $\pi_i \left( \widehat{q}_i, \sum_{j \neq i} \widehat{q}_j \right) = 0$, this contradicts $(\widehat{q}_1, \ldots, \widehat{q}_n)$ being an equilibrium. We conclude that our supposition that $\widehat{q}_j = 0 \forall j$ is false. ■

### 7.2.3 Properties of the Best Reply Function and Equilibrium

1. Assumptions

   **A8** If $P(Q) > 0$ then $P'(Q) + q_i P''(Q) < 0$ (decreasing marginal revenue curve)

   **A9** $P' - C'' < 0 \forall i$

   (a) These assumptions imply $\pi_i(q_i, Q_{-i})$ is strictly concave in $q_i$ $\forall q_i \in \left[0, \overline{Q} - Q_{-i}\right]$, $\forall Q_{-i}$.

2. Best reply correspondence is a downward sloping, differentiable function

   **Theorem** By A2, A4-A5, and A8-A9, $\psi_i(Q_{-i})$ is differentiable in $Q_{-i}$ $\forall Q_{-i} \in \left\{Q'_{-i} \left| Q'_{-i} > 0 \text{ and } \psi_i\left(Q'_{-i}\right) > 0\right.\right\}$ and if $\psi_i(Q_{-i}) > 0$ then $\psi'_i(Q_{-i}) < 0$.

   **Proof** (We will only prove that the best response function is downward sloping when it is positive.) Given strict concavity of the profit function, an interior optimum for a firm is defined by the first-order condition:

$$\frac{\partial \pi_i(\psi_i(Q_{-i}), Q_{-i})}{\partial q_i} = 0$$

   Taking the total derivative with respect to $Q_{-i}$ :

$$\frac{\partial^2 \pi_i(\psi_i(Q_{-i}), Q_{-i})}{\partial q_i^2}\psi'_i(Q_{-i}) + \frac{\partial^2 \pi_i(\psi_i(Q_{-i}), Q_{-i})}{\partial q_i \partial Q_{-i}} = 0 \Leftrightarrow$$

$$\psi'_i(Q_{-i}) = -\frac{\frac{\partial^2 \pi_i(\psi_i(Q_{-i}), Q_{-i})}{\partial q_i \partial Q_{-i}}}{\frac{\partial^2 \pi_i(\psi_i(Q_{-i}), Q_{-i})}{\partial q_i^2}}$$

$$= -\frac{P'(\psi_i(Q_{-i}) + Q_{-i}) + \psi_i(Q_{-i}) P''(\psi_i(Q_{-i}) + Q_{-i})}{2P'(\psi_i(Q_{-i}) + Q_{-i}) + \psi_i(Q_{-i}) P''(\psi_i(Q_{-i}) + Q_{-i}) - C''_i(\psi_i(Q_{-i}))}$$

   Since the denominator is negative by A8-A9 and the numerator is negative by A8, $\psi'_i(Q_{-i}) < 0$. ■

   (a) Intuition

      i. Firm $i$'s demand curve is $P_i(q_i; Q_{-i}) = P(q_i + Q_{-i})$

      ii. As $Q_{-i}$ increases, this shifts in the firm's demand curve and its marginal revenue curve

      iii. Under normal assumptions, a weaker demand causes the optimal quantity to fall

3. Equilibrium price

(a) From the foc, interior equilibrium quantities are defined by:

$$\frac{\partial \pi_i \left( \widehat{q}_i, \sum_{j \neq i} \widehat{q}_j \right)}{\partial q_i} = 0 \forall i \Leftrightarrow$$

$$P \left( \sum_{j=1}^{n} \widehat{q}_j \right) + P' \left( \sum_{j=1}^{n} \widehat{q}_j \right) \widehat{q}_i - C_i' \left( \widehat{q}_i \right) = 0 \Leftrightarrow P \left( \sum_{j=1}^{n} \widehat{q}_j \right) > C_i' \left( \widehat{q}_i \right)$$

(b) Hence, price exceeds marginal cost for all firms that are producing a positive amount.

**Example**

1. Model

   (a) Linear demand
   $$P(Q) = a - bQ.$$

   (b) Linear cost functions
   $$C_i(q) = c_i q.$$

   (c) Parametric assumptions

      i. $a > c_i \geq 0 \forall i$, $b > 0$.

      ii. $|\max\{c_1, \ldots, c_n\} - \min\{c_1, \ldots, c_n\}|$ is sufficiently small so that the unique equilibrium is interior.

2. Equilibrium

$$\begin{aligned} \widehat{q}_i &= \frac{a + \sum_{j \neq i} c_j - n c_i}{b(n+1)} \\ \widehat{P} &= \frac{a + \sum_{j=1}^{n} c_j}{n+1} \\ \widehat{\pi}_i &= \frac{\left( a + \sum_{j \neq i} c_j - n c_i \right)^2}{b(n+1)^2}. \end{aligned}$$

**Herfindahl-Hirschman Index**

1. What is a concentration index?

   (a) It is a summary statistic of how market share is allocated among firms.

   (b) Its intent is to measure market power which is, roughly speaking, the ability of firms to profitably price above marginal cost.

2. Defining the Herfindahl-Hirschman Index

(a) Assume an interior equilibrium exists

(b) The HHI is defined by

$$HHI = \sum_{i=1}^{n} s_i^2 \text{ where } s_i = \frac{q_i}{\sum_{j=1}^{n} q_j}$$

(c) Properties

i. $HHI \in [0,1]$

ii. Higher $HHI$ corresponds to a more concentrated industry.

iii. Shifting market share from a smaller to a larger firm raises the $HHI$. (*Prove this claim.*)

iv. When market shares are equal, $HHI = \frac{1}{n}$ and thus is decreasing in $n$

3. Relation of HHI to Nash equilibrium for quantity game

(a) FOC

$$P\left(\sum_{j=1}^{n} \widehat{q}_j\right) + P'\left(\sum_{j=1}^{n} \widehat{q}_j\right)\widehat{q}_i - C_i'\left(\widehat{q}_i\right) = 0 \Leftrightarrow \widehat{P} - C_i' = -\widehat{P}'\widehat{q}_i \Leftrightarrow$$

(b) Derivation:

$$\frac{\widehat{P} - C_i'}{\widehat{P}} = \frac{-\widehat{P}'\widehat{q}_i}{\widehat{P}} \Leftrightarrow \frac{\widehat{P} - C_i'}{\widehat{P}} = \frac{-\widehat{P}'\widehat{Q}\left(\widehat{q}_i/\widehat{Q}\right)}{\widehat{P}} \Leftrightarrow \frac{\widehat{P} - C_i'}{\widehat{P}} = \frac{s_i}{\eta}$$

where $\eta = \left|\frac{P}{QP'}\right|$ is the absolute value of the market demand elasticity. Summing across all firms and weighting each by its market share:

$$\sum_{i=1}^{n} s_i\left(\frac{\widehat{P} - C_i'}{\widehat{P}}\right) = \sum_{i=1}^{n} s_i\left(\frac{s_i}{\eta}\right) \Leftrightarrow \sum_{i=1}^{n} s_i\left(\frac{\widehat{P} - C_i'}{\widehat{P}}\right) = \frac{HHI}{\eta}$$

(c) An elasticity-adjusted $HHI$ is the industry price-cost margin where a firm's PCM is weighted by its market share. Thus, a higher HHI corresponds to a higher industry PCM.

**Effect of the Number of Firms**

1. Assumptions: A2, A4-A5, and A8-A10.

   **A10** $C_i(\cdot) = C(\cdot)\,\forall i$ (symmetry)

2. Equilibrium quantity is defined by

$$\widehat{q}(n) = \psi\left((n-1)\,\widehat{q}(n)\right)$$

3. Firm quantity is decreasing in the number of firms. Taking the total derivative wrt $n$ :

$$\widehat{q}'(n) = \psi'((n-1)\widehat{q}(n))\left[\widehat{q}(n) + (n-1)\widehat{q}'(n)\right] \Leftrightarrow$$

$$\widehat{q}'(n) = \frac{\psi'((n-1)\widehat{q}(n))\widehat{q}(n)}{1-(n-1)\psi'((n-1)\widehat{q}(n))} < 0$$

This is negative because $\psi' < 0$.

4. Industry quantity is increasing in the number of firms

$$\begin{aligned} \frac{dn\widehat{q}(n)}{dn} &= \widehat{q}(n) + n\widehat{q}'(n) \\ &= \widehat{q}(n) + n\left[\frac{\psi'((n-1)\widehat{q}(n))\widehat{q}(n)}{1-(n-1)\psi'((n-1)\widehat{q}(n))}\right] \end{aligned}$$

Working through the algebra, this is positive iff $\psi' > -1$. Recall that

$$\psi' = -\frac{P' + \psi_i P''}{2P' + \psi_i(Q_{-i})P'' - C_i''} > -1 \Leftrightarrow C'' > P'$$

which is true by A9. We conclude that industry quantity is increasing in $n$ and thus equilibrium price is decreasing in $n$.

5. Firm equilibrium profit is decreasing in the number of firms

$$\begin{aligned} \frac{d\pi_i}{dn} &= \frac{\partial\pi_i}{\partial q_i}\widehat{q}'(n) + \frac{\partial\pi_i}{\partial Q_{-i}}\left[\widehat{q}(n) + (n-1)\widehat{q}'(n)\right] \\ &= \frac{\partial\pi_i}{\partial Q_{-i}}\left[\widehat{q}(n) + (n-1)\widehat{q}'(n)\right] \end{aligned}$$

as $\frac{\partial\pi_i}{\partial q_i} = 0$. Next note that $\frac{\partial\pi_i}{\partial Q_{-i}} = P'q_i < 0$. Finally, we have already shown that $\widehat{q}(n) + n\widehat{q}'(n) > 0$ which implies $\widehat{q}(n) + (n-1)\widehat{q}'(n) > 0$. Thus, $\frac{d\pi_i}{dn} < 0$.

**Pareto-Inefficiency of Outcome**

1. Assume A2, A4-A5, A8-A11

   **A11** $\pi_i(q,(n-1)q)$ is strictly concave in $q$.

2. Joint profit maximum

$$\max_q \pi_i(q,(n-1)q)$$

$$\frac{d\pi_i(q^m,(n-1)q^m)}{dq} = 0$$

3. Result: $\widehat{q} > q^m$

$$\frac{d\pi_i\left(\widehat{q},(n-1)\,\widehat{q}\right)}{dq} = \frac{\partial\pi_i\left(\widehat{q},(n-1)\,\widehat{q}\right)}{\partial q_i} + \frac{\partial\pi_i\left(\widehat{q},(n-1)\,\widehat{q}\right)}{\partial Q_{-i}}(n-1)$$

$$= \frac{\partial\pi_i\left(\widehat{q},(n-1)\,\widehat{q}\right)}{\partial Q_{-i}}(n-1) < 0$$

Since

$$\frac{d\pi_i\left(q^m,(n-1)\,q^m\right)}{dq} > \frac{d\pi_i\left(\widehat{q},(n-1)\,\widehat{q}\right)}{dq},$$

it follows by A11 that $\widehat{q} > q^m$.

4. Discussion

(a) From the perspective of the firms, the Nash equilibrium is Pareto-inefficient. All firms could have higher profit if they all reduced quantity below $\widehat{q}$ and toward $q^m$.

(b) There is a negative externality. When a firm raises its quantity, it fails to take into account the negative effect on other firms' products. From a collective perspective, firms produce too much.

(c) Generically, Nash equilibria are Pareto-inefficient.

## 7.3 Price Competition with Homogeneous Products

### 7.3.1 Standard Model

1. Basic structure

(a) $n$ firms offer homogeneous products.

(b) Firms simultaneously choose price.

(c) Consumers demand from the firms with the lowest price (if there is more than one firm with the lowest price then demand is equally divided among them).

(d) Firms supply so as to meet demand.

2. Demand function

(a) $D(\cdot) : \Re_+ \to \Re_+$ is continuous, nonincreasing, and bounded.

(b) $\exists$ finite $\overline{P} > c$ such that $D(P) = 0$ iff $P \geq \overline{P}$.

3. Cost function

(a) $C_i(q) = cq, \forall q \in [0, K], \forall i$

(b) Firms are constrained to producing less than or equal to capacity, $K$.

### 7.3.2 No Capacity Constraints

1. Assume $K$ is sufficiently high so that firms are unconstrained

2. Strategic form

    (a) Set of players is the $n$ firms

    (b) Strategy set for each firm is $\left[0, \overline{P}\right]$

    (c) Payoff function of firm $i$ is $\pi_i(p_1, \ldots, p_n)$ where

    $$\pi_i(p_1, \ldots, p_n) = \begin{cases} (p_i - c)\,\mu(p_1, \ldots, p_n)\, D(p_i) & \text{if } p_i = \min\{p_1, \ldots, p_n\} \\ 0 \text{ if } p_i > \min\{p_1, \ldots, p_n\} \end{cases}$$

    where

    $$\mu(p_1, \ldots, p_n) \equiv \frac{1}{|\{j \in \{1, \ldots, n\} \,| p_j = \min\{p_1, \ldots, p_n\}\}|}$$

3. Equilibrium

    (a) We cannot apply the standard existence theorem as the payoff function is not continuous.

    (b) An equilibrium will not be defined by first-order conditions as the firm's payoff function is not differentiable everywhere

    **Theorem** $(\widehat{p}_1, \ldots, \widehat{p}_n)$ is a Nash equilibrium iff $\widehat{p}_i \geq c \forall i$ and

    $$|\{j \in \{1, \ldots, n\} \,| p_j = c\}| \geq 2.$$

    **Corollary** The unique symmetric Nash equilibrium is $c$.

    (c) Discussion

        i. Equilibrium outcome is perfectly competitive outcome when there are at least two firms.

        ii. Price is independent of the number of firms when there are at least two firms.

        iii. Pricing at marginal cost is weakly dominated by pricing above marginal cost.

        iv. A NE without weakly dominated strategies does not exist.

### 7.3.3 Binding Capacity Constraints

1. Non-existence of equilibrium

    (a) Assumptions

        i. $D(\cdot)$ is continuously differentiable.

ii. Static interior NE quantity for the quantity game exists,

$$\widehat{q} \in \arg\max \left[D^{-1}\left(q + (n-1)\widehat{q}\right) - c\right] q,$$

and is defined by

$$\left[\frac{\partial D^{-1}(n\widehat{q})}{\partial Q}\right]\widehat{q} + \left[D^{-1}(n\widehat{q}) - c\right] = 0$$

iii. Firm profit is decreasing for all symmetric quantity vectors exceeding the static NE:

$$\left[\frac{\partial D^{-1}(nq)}{\partial Q}\right] q + \left[D^{-1}(nq) - c\right] < 0 \text{ if } q > \widehat{q}$$

iv. $K \in \left(\widehat{q}, \frac{D(c)}{n-1}\right)$.

(b) Proof of non-existence of a pure-strategy Nash equilibrium.

i. Suppose we conjecture that there exists a pure strategy equilibrium, $(\widehat{p}_1, \ldots, \widehat{p}_n)$.

ii. First note that $\widehat{p}_i > c \forall i$. Suppose not so that $\widehat{p}_i = c$ for some $i$ (it's clear that price is not below marginal cost). This firm earns zero profit. This can only be optimal if a firm's demand is zero for prices above $c$. This is true iff the set of other firms that price at $c$ have total capacity of at least $D(c)$. Since $K < \frac{D(c)}{n-1}$ then the total capacity of all other firms is less than $D(c)$ so that is not possible. Contradiction.

iii. Suppose there are $m$ lowest priced firms, each with a price of $p' > c$

A. Suppose $D(p') > mK$. Any one of these firms can raise profit by increasing its price slightly as it will still be able to sell $K$ since there is excess demand.

B. Suppose $D(p') < mK$. One of these firms could raise profit by setting a slightly lower as its demand rises from $\frac{D(p')}{m}$ to $K$.

C. It follows that $D(p') = mK$. If $m < n$ then $n-m$ firms are earning zero profit as the lowest priced firms are able to meet all demand. But one of these firms could earn positive profit by pricing between $c$ and $p'$. Hence, $m = n$.

iv. It follows that all firms price at the same level of $p' > c$ and that $D(p') = nK$.

A. No firm want to sets a lower price as they have zero excess capacity.

B. In considering a higher price, first note that

$$\pi_i = (p_i - c)\left[D(p_i) - (n-1)K\right] \text{ if } p_i \geq p'$$

$$\frac{\partial \pi_i}{\partial p_i} = D(p_i) - (n-1)K + (p_i - c)D'(p_i) \text{ if } p_i \geq p'$$

It is optimal to raise price when all firms price at $p'$ if

$$K + \left(p' - c\right) D'\left(p'\right) > 0$$

where we use the fact that $D(p') = nK$. This is equivalent to

$$K + \left[D^{-1}(nK) - c\right] \left(\frac{\partial D^{-1}(nK)}{\partial Q}\right)^{-1} > 0$$

$$K \left(\frac{\partial D^{-1}(nK)}{\partial Q}\right) + \left[D^{-1}(nK) - c\right] < 0$$

which is true as $K > \widehat{q}$. ■

2. Mixed-strategy equilibrium

   (a) A little more game theory

      i. Strategic form game

         A. Set of players, $\{1, 2, \ldots, n\}$.

         B. Strategy sets of players: $S_i, i \in \{1, 2, \ldots, n\}$.

         C. Payoff functions of players,

$$V_i : S_1 \times \cdots \times S_n \to R, i \in \{1, 2, \ldots, n\}.$$

      ii. **Definition**: If $S_i$ is player $i$'s strategy set then a *mixed strategy* is a probability distribution over $S_i$.

      iii. **Definition**: A *pure strategy* is an element of $S_i$.

      iv. Nash equilibrium

         A. Let $\phi_i : S_i \to [0, 1]$ be a cdf on $S_i$.

         B. Let $\Phi_i$ be the set of cdfs with support $S_i$.

         C. With mixed strategies, player $i$ acts to maximize his expected payoff:

$$W_i(\phi_1, \ldots, \phi_n) \equiv \int_{s_1 \in S_1} \cdots \int_{s_n \in S_n} V_i(s_1, \ldots, s_n)\, d\phi_1 \cdots d\phi_n$$

         D. $(\phi_1^*, \ldots, \phi_n^*)$ is a Nash equilibrium iff

$$W_i(\phi_1^*, \ldots, \phi_n^*) \geq W_i\left(\phi_1^*, \ldots, \phi_{i-1}^*, \phi, \phi_{i+1}^*, \ldots, \phi_n^*\right) \forall \phi \in \Phi_i, \forall i$$

         E. If $\phi_i^*$ is part of a Nash equilibrium then all pure strategies assigned positive probability yield the same expected payoff.

      v. **Existence Theorem** (Nash, 1950): If $n$ is finite and $S_i$ is nonempty and finite $\forall i$ then there exists a Nash equilibrium in mixed strategies.

      vi. **Existence Theorem** (Glicksberg, 1952): If $n$ is finite, $S_i$ is a nonempty compact subset of a metric space $\forall i$, and $V_i$ is continuous in all strategies $\forall i$ then there exists a Nash equilibrium in mixed strategies.

(b) Assumptions

i. Two firms

ii. Linear demand: $D(P) = a - P$

iii. Zero marginal cost

iv. $K \in \left(\frac{a}{3}, a\right)$

(c) Conjectures

i. $\phi$ is cdf on a firm's price

ii. $\phi$ is continuous (no mass points) with support $\left[\underline{p}, \overline{p}\right]$

iii. $a - p > K \forall p \in \left[\underline{p}, \overline{p}\right] \Leftrightarrow a - \overline{p} > K$ (there is excess demand at the highest price)

iv. $K > a - K - p \forall p \in \left[\underline{p}, \overline{p}\right] \Leftrightarrow \underline{p} > a - 2K$ (there is excess supply when both firms price at the lowest price)

(d) Firm 1's expected profit from a price of $p_1$ given firm 2 uses $\phi$ :

$$E\left[\pi_1\left(p_1\right)\right] = p_1\left[\left(1 - \phi\left(p_1\right)\right) K + \phi\left(p_1\right)\left(a - K - p_1\right)\right]$$

i. With probability $1 - \phi\left(p_1\right)$, $p_1 < p_2$ and firm 1 has demand exceeding its capacity

ii. With probability $\phi\left(p_1\right)$, $p_1 > p_2$ and firm 1 has residual demand of $a - K - p_1$

iii. $p_1 = p_2$ with prob. zero

(e) If $\phi$ is a symmetric NE then it must be true that

$$E\left[\pi_1\left(p_1\right)\right] = \widehat{\pi} \forall p_1 \in \left[\underline{p}, \overline{p}\right], \text{ for some } \widehat{\pi}$$

(f) Solve this equation for $\phi\left(p_1\right)$ :

$$p\left[\left(1 - \phi(p)\right) K + \phi(p)(a - K - p)\right] = \widehat{\pi} \Leftrightarrow$$

$$\phi(p) = \frac{K - \left(\frac{\widehat{\pi}}{p}\right)}{p + 2K - a}$$

(g) Claim: $\overline{p} = \frac{a-K}{2}$

i. If $p_1 = \overline{p}$ then with prob. one, firm 1 is the high priced firm so that its profit is $\left(a - K - p_1\right) p_1$

ii. It must then be true that

$$\overline{p} \in \arg\max\left(a - K - p_1\right) p_1 \Rightarrow \overline{p} = \frac{a - K}{2}$$

(h) It follows that

$$\phi\left(\frac{a - K}{2}\right) = 1 \Leftrightarrow \frac{K - \left(\frac{\widehat{\pi}}{\frac{a-K}{2}}\right)}{\frac{a-K}{2} + 2K - a} = 1 \Leftrightarrow \widehat{\pi} = \frac{(a - K)^2}{4}$$

(i) We can then define the cdf:

$$\phi(p) = \frac{K - \left(\frac{(a-K)^2}{4p}\right)}{p + 2K - a}$$

Setting this expression equal to zero, we derive $\underline{p} = \frac{(a-K)^2}{4K}$.

(j) Confirm conjectures

$$a - \overline{p} > K \Leftrightarrow a - \frac{a-K}{2} > K \Leftrightarrow K > a$$

$$\underline{p} > a - 2K \Leftrightarrow \frac{(a-K)^2}{4K} > a - 2K \Leftrightarrow K > \frac{a}{3}$$

### 7.3.4 Endogenous Capacity

- Kreps and Scheinkman (BJE, 1983)

1. Model

   (a) Extensive form

      i. Stage 1 - two firms make simultaneous capacity decisions where $K_i$ denotes the capacity of firm $i$

      ii. Stage 2 - with capacities common knowledge, firms make simultaneous price decisions with homogeneous products (capacity-constrained price game)

   (b) Inverse demand function

      i. $P(\cdot) : \Re_+ \to \Re_+$ is twice continuously differentiable, strictly decreasing, and concave

      ii. $\exists$ finite $\overline{Q} > 0$ such that $P(Q) = 0 \forall Q \geq \overline{Q}$.

   (c) Firm demand function

      i. Homogeneous products

      ii. Parallel rationing rule specifies that, if there is excess demand for some firm's product, those consumers with the highest valuation are served first

$$P_1^{-1}(p_1, p_2) = \begin{cases} P^{-1}(p_1) \text{ if } p_1 < p_2 \\ \max\left\{\frac{P^{-1}(p_1)}{2}, P^{-1}(p_1) - K_2\right\} \text{ if } p_1 = p_2 \\ \max\left\{P^{-1}(p_1) - K_2, 0\right\} \text{ if } p_1 > p_2 \text{ and } P^{-1}(p_2) > K_2 \\ 0 \text{ if } p_1 > p_2 \text{ and } P^{-1}(p_2) \leq K_2 \end{cases}$$

   (d) Capacity cost function

      i. $b(\cdot) : \Re_+ \to \Re_+$ is twice continuously differentiable

ii. $b' > 0, b'' > 0$

iii. $P(0) > b'(0)$

(e) Zero production cost

(f) A strategy is $(K_i, \Phi_i)$ where $\Phi_i : R_+^2 \to \{\text{cdfs over } R_+\}$

2. Subgame perfect equilibrium

(a) Solving stage 2 game (multiplicity? existence?)

i. For arbitrary $(K_1, K_2)$, solve for Nash equilibrium (in mixed strategies)

ii. $E[R_i(K_1, K_2)]$ is expected Nash equilibrium payoff

iii. $E[R_i(K_1, K_2)]$ is continuous in $(K_1, K_2)$

(b) Solving stage 1 game for pure-strategy symmetric equilibrium

i.

$$\widehat{K} \in \arg\max E\left[R_1\left(K_1, \widehat{K}\right)\right] - b(K_1)$$

ii. Stage 2 equilibrium involves pure strategies with mass at $\widehat{p}$:

$$P\left(2\widehat{K}\right) = \widehat{p}$$

3. Equilibrium outcome

(a) Define symmetric NE for quantity game with capacity cost function as production cost function:

$$\widehat{q} \in \arg\max P(q_1 + \widehat{q})\, q_1 - b(q_1)$$

(b) The unique SPE outcome has firms choosing capacity equal to the NE quantities for the Q game in stage 1, $\widehat{K} = \widehat{q}$, and pricing in stage 2 so that demand equals capacity. The outcome is then the same as that for the quantity game.

(c) This provides a justification for applying the quantity game to industries in which firms are choosing price.

4. Discussion

(a) Intuition

i. If $K_j = \widehat{q}$ then the stage 2 equilibrium involves both firms producing at capacity (and pricing so as to clear the market) when $K_i \leq \widehat{q}$. From the perspective of firm $i$, it is as if it takes the other firms' quantity as fixed when it considers $K_i \leq \widehat{q}$. By the definition of $\widehat{q}$, we know that $\widehat{q}$ is an optimal response. Hence, a capacity of $\widehat{q}$ is superior to any lower capacity.

ii. Now consider $K_i > \widehat{q}$. In this case, the stage 2 equilibrium involves randomized pricing and one can show that the expected payoff is less than that from a capacity of $\widehat{q}$.

(b) Result is not robust to the rationing rule

i. Proportional rationing rule specifies that, if there is excess demand for some firm's product, consumer are served randomly independent of their valuation

$$P_1^{-1}(p_1, p_2) = \begin{cases} P^{-1}(p_1) \text{ if } p_1 < p_2 \\ \max\left\{\frac{P^{-1}(p_1)}{2}, P^{-1}(p_1) - K_2\right\} \text{ if } p_1 = p_2 \\ \left[\frac{P^{-1}(p_2)-K_2}{P^{-1}(p_2)}\right] P^{-1}(p_1) \text{ if } p_1 > p_2 \text{ and } P^{-1}(p_2) > K_2 \\ 0 \text{ if } p_1 > p_2 \text{ and } P^{-1}(p_2) \leq K_2 \end{cases}$$

ii. Davidson and Deneckere (RJE, 1986)

A. SPE outcome need not be the Cournot solution

B. Equilibrium may entail mixed pricing strategies in stage 2

C. Numerical results suggest that the outcome is more competitive than the Cournot solution

iii. Note that the parallel rationing rule results in the same residual demand curve as occurs in the quantity game.