

# Avoiding Market Dominance: Product Compatibility in Markets with Network Effects\*

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## **Abstract**

As is well-recognized, market dominance is a typical outcome in markets with network effects. A firm with a larger installed base offers a more attractive product which induces more consumers to buy its product which produces a yet bigger installed base advantage. Such a setting is investigated here but where firms have the option of making their products compatible. When firms have similar installed bases, they make their products compatible in order to expand the market. Nevertheless, random forces could result in one firm having a bigger installed base in which case the larger firm may make its product incompatible. We find that strategic pricing tends to prevent the installed base differential from expanding to the point that incompatibility occurs. This dynamic is able to neutralize increasing returns and avoid the emergence of market dominance.

# 1 Introduction

Products with network effects face the following conundrum: Consumers benefit most when one technology dominates but then the market is likely to be controlled by a single firm and thereby suffer from the usual deadweight welfare losses associated with market power. But dominance of a technology need not imply monopoly. It is possible to have the best of all worlds if competing firms choose to make their products compatible. In that case, consumers reap maximal benefits of network effects, while competition is preserved.

These issues are well-known and lie at the heart of the Microsoft case. Indeed, one proposed structural remedy was to divide the Windows monopoly into several companies which would initially have compatible (in fact, identical) products.<sup>1</sup> The argument for such a solution was explained by Levinson, Romaine, and Salop (2001, pp. 139-140):

The new Windows competitors would begin with totally compatible products. Although the new Windows companies subsequently *could* choose to drastically deviate from this standard and create highly incompatible products, they are unlikely to do so. ... Unlike the current system, where the Windows monopolist cannot be punished by the market for being incompatible with its much smaller rivals, one of three WinCos likely would be severely punished by applications developers and users if it created a significantly incompatible version. This would also reduce the likelihood of “tipping” back to a monopoly.

While this argument is plausible, it is far from obvious in light of the complexity and subtlety of dynamic competition in markets with network effects. A more formal analysis is required to understand the circumstances under which competitors would mutually decide to make their products compatible. There is, of course, an extensive literature on markets with network effects - superbly reviewed in Farrell and Klemperer (2006) - and some of that research endogenizes product compatibility, in particular, Katz and Shapiro (1986), Economides and Flyer (1998), Cremer, Rey, and Tirole (2000), and Tran (2006).<sup>2</sup>

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<sup>1</sup>These Microsoft clones have been colloquially dubbed “Baby Bills” which is a play on the term Baby Bells which is itself a colloquialism for the Regional Bell Operating Companies created with the break-up of the Bell System in 1984.

<sup>2</sup>Research by Farrell and Saloner (1992) and Choi (1996, 1997) allows consumers to

The standard model in that literature is a two stage structure; in the first stage, firms make compatibility decisions and, given products are or are not compatible, they then engage in price or quantity competition (for either one or two periods). Consistent with the Microsoft setting, both firms must agree for their products to be compatible. There are two primary forces that influence whether or not compatibility occurs in equilibrium. First, compatibility enhances the value of firms' products by increasing network effects. As this draws more consumers into the market, firms have a mutual interest in making their products compatible. Second, when firms have different installed bases, the larger firm loses an advantage with compatibility. In contrast, the smaller firm always prefers products to be compatible since it benefits through both effects. Existing work has shown that if firms are not too different - either in terms of installed bases or other traits - then products are compatible.

Though highly similar firms may then choose compatibility, the critical question not addressed by previous work is whether this is stable over time. Even if identical firms were to choose compatible products and thus achieve a symmetric equilibrium in the product market, randomness in demand and other shocks would surely lead to asymmetric market shares and thus asymmetric installed bases. At that point, the firm with the larger installed base may no longer desire compatibility. Of course, if firms are locked into their products being compatible then the larger firm cannot act on that desire. In practice, however, compatibility is not permanent; a firm can alter its technology and thereby make its product incompatible. The unanswered question then is: How stable are compatible products? Could a modest difference in installed bases induce the current market leader to choose incompatibility in a march towards dominance? Or are there forces that maintain incentives for compatibility?

To address these questions, we develop and numerically analyze an infinite horizon model of competition for a market with network effects. In each period, two firms decide on first compatibility and then price. Demand and customer turnover are stochastic which means that firms are very likely to end up with asymmetric installed bases even if they begin identical and choose compatible products. A Markov Perfect equilibrium is characterized and we assess the frequency with which market dominance occurs.<sup>3</sup>

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purchase converters which deliver compatibility. In our setting, converters are not available which would seem appropriate for the Microsoft case.

<sup>3</sup>The closest paper in style to ours is Markovich (2000). Though she also computes a Markov Perfect equilibrium for a market with network effects, she does not consider the decision of firms regarding compatibility which is central to our research questions. Other

The paper’s main result is the discovery of a dynamic that can neutralize increasing returns and prevent market dominance from emerging. As long as network effects are not too strong, firms that begin with comparably sized installed bases will choose to make their products compatible. Furthermore, if the installed base differential should grow - even to the point that the larger firm makes its product incompatible - the smaller firm prices aggressively to reduce the differential and thereby maintain or restore product compatibility. Since compatible products can then be stable in the long-run, this means that market dominance need not be the eventual outcome in markets with network effects.

## 2 Model

Though motivated by the proposed structural remedy for the Microsoft case, our interest lies more broadly in understanding the stability of competition in markets with network effects when compatibility is endogenous and firms can dynamically price to achieve dominance. This is reflected in the more general character of the model we have developed.

### 2.1 State Space and Firm Decisions

The model is cast in discrete time with an infinite horizon. Though our attention in this paper is limited to when there are just two firms, the model will be described for the more general case of  $N \geq 2$  firms. These firms sell to a sequence of heterogenous buyers with unit demands. At the start of a period, a firm is endowed with an installed base which represents consumers who have purchased its product in the past. Let  $b_i \in \{0, 1, \dots, M\}$  denote the installed base of firm  $i$  at the start of a period where  $M$  is the maximal size of the installed base.

Given  $(b_1, \dots, b_N)$ , firms engage in a two-stage decision process in which they choose compatibility in stage 1 and then price in stage 2. In stage 1, each firm decides whether or not to “propose compatibility” with each of the other firms. Let  $d_{ij} \in \{0, 1\}$  be the compatibility choice of firm  $i$  with respect to firm  $j$  where  $d_{ij} = 1$  means “propose compatibility.” To actually achieve compatibility requires that both firms propose it. Thus, the technologies of  $i$  and  $j$  are “compatible” iff  $d_{ij} \cdot d_{ji} = 1$ . Requiring both firms to consent is consistent with a number of markets including those involved

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dynamic analyses include Mitchell and Skrzypacz (2005) and Llobet and Manove (2006) though again they do not allow firms to choose compatibility.

in the Microsoft case. Furthermore, the analysis promises to be more interesting than when a firm can, by itself, make its product compatible. After compatibilities are determined, firms simultaneously choose price. Let  $p_i$  denote the price of firm  $i$ .

This is clearly a stylized modelling of compatibility but should serve our purposes well. Our primary interest is in understanding the incentives for compatibility and that means learning when firms *prefer* compatibility. We have then given them maximal flexibility by ignoring any technical constraints and assuming compatibility is costless to change. Furthermore, this modelling approach means that compatibility is not a state variable and this is important in keeping the dimensionality of the state space manageable.

## 2.2 Demand

Demand in each period comes from the replacement of some randomly selected old consumers (who previously purchased) with new consumers. There is one new consumer each period and her buying decision is based on the following discrete choice model. Let  $\epsilon_i$  be the idiosyncratic preference of the buyer for firm  $i$ 's product in the current period. The utility that the consumer gets from buying from firm  $i$  is

$$v_i + \theta g \left( b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j \right) - p_i + \sigma \epsilon_i.$$

$b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j$  is the effective installed base of firm  $i$  given the set of compatible technologies where  $\lambda \in [0, 1]$  allows for the value of the installed base of other compatible technologies to be worth less to consumers of firm  $i$ 's product.  $v_i$  is a measure of intrinsic product quality which is assumed to be common across firms:  $v = v_i$  and is also fixed over time.<sup>4</sup> Network effects are captured by the increasing function  $\theta g(\cdot)$  where  $\theta \geq 0$  is the parameter that controls the strength of network effects. We will refer to the sum of these two factors,  $v_i + \theta g(\cdot)$ , as quality.  $\sigma > 0$  is proportional to the standard deviation of  $\epsilon_i$  and thus measures the degree of horizontal differentiation between the products. The buyer can also choose to purchase an outside good with utility  $v_0 + \epsilon_0$ . As the intrinsic quality parameters only affect demand through the expression  $v_0 - v$ , wlog we set  $v = 0$ . The consumer's idiosyncratic preferences  $(\epsilon_0, \epsilon_1, \dots, \epsilon_N)$  are unobservable to firms.

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<sup>4</sup>A future extension is to allow firms to invest in quality in which case  $v_i$  will be endogenous.

A new consumer buys from the firm offering the highest current utility. To focus on the dynamics of firm behavior, we assume consumers make myopic decisions. Assuming  $(\epsilon_0, \epsilon_1, \dots, \epsilon_N)$  are independently extreme value distributed, the probability that firm  $i$  makes a sale to a new consumer is

$$\phi_i(p; d, b) \equiv \frac{\exp\left(\frac{\theta g(b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j) - p_i}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \sum_{j=1}^N \exp\left(\frac{\theta g(b_j + \lambda \sum_{k \neq j} d_{jk} d_{kj} b_k) - p_j}{\sigma}\right)},$$

where  $p$  is the vector of prices of all firms,  $d$  is the vector of compatibility choices, and  $b$  is the vector of installed bases. Note that if  $v_0 = -\infty$  then  $\phi_0(p; d, b) = 1 - \sum_{i=1}^N \phi_i(p; d, b) = 0$ , so the outside good is hopelessly unattractive and a consumer will buy from one of the  $N$  firms with probability one. In that case, expected market demand equals one in each period and, most importantly, is independent of firms' installed bases and any decisions regarding compatibility and price. Those decisions will only influence a firm's expected market share. The case of  $v_0 = -\infty$  is referred to as the case when market size (or demand) is fixed. When instead  $v_0$  is not infinitely negative then the expected market size is endogenous. In particular, a firm can increase its expected demand without necessarily decreasing the expected demand of its rivals by an equal amount.

### 2.3 Network Effects and Transition Probabilities

In modelling network effects, we will assume they are bounded in the sense that  $g(b_i) = g(m)$  if  $b_i \geq m$  for some  $m \leq M$ . Bounding the network effect is as specified in Cabral and Riordan (1994) though in their context was learning-by-doing. Several functional forms will be considered in order to derive robust results. These are provided below.

- *Linear*:  $g(b_i) = \frac{b_i}{m}$  if  $b_i \leq m$ .
- *Convex*:  $g(b_i) = \sin\left(\frac{\pi b_i}{2m} + \frac{3\pi}{2}\right) + 1$  if  $b_i \leq m$ .
- *Concave*:  $g(b_i) = \sin\left(\frac{\pi b_i}{2m}\right)$  if  $b_i \leq m$ .
- *S-shaped*:  $g(b_i) = \frac{\sin\left(\frac{\pi b_i}{m} + \frac{3\pi}{2}\right) + 1}{2}$  if  $b_i \leq m$ .

It is without loss of generality that the range of  $g(\cdot)$  is  $[0, 1]$ . As qualitative results were found to be similar across functional forms, we only report results when  $g$  is linear.

$\Delta(b_i)$  denotes the probability that the installed base of firm  $i$  depreciates. We specify  $\Delta(b_i) = 1 - (1 - \delta)^{b_i}$ , where  $\delta \in [0, 1]$  is the rate of depreciation. This specification captures the idea that the likelihood that a firm's installed base depreciates increases with the size of its installed base.  $\delta$  would be expected to be higher where consumer turnover is higher or products are short-lived so that consumers need to return to the market at a higher rate.

Letting  $q_i \in \{0, 1\}$  indicate whether or not firm  $i$  makes the sale, its installed base changes according to the transition function

$$\Pr(b'_i|b_i, q_i) = \begin{cases} 1 - \Delta(b_i) & \text{if } b'_i = b_i + q_i, \\ \Delta(b_i) & \text{if } b'_i = b_i + q_i - 1, \end{cases}$$

where, at the upper and lower boundaries of the state space, we modify the transition probabilities to be  $\Pr(M|M, 1) = 1$  and  $\Pr(0|0, 0) = 1$ , respectively.

### 3 Equilibrium

#### 3.1 Bellman Equation and Strategies

In working backwards through the compatibility and pricing decisions, we use the following notation:

- $V_i(b)$  denotes the net present value of future cash flows to firm  $i$  in state  $b$  *before* the compatibility decisions have been made.
- $U_i(d, b)$  denotes the net present value of future cash flows to firm  $i$  in state  $b$  *after* the compatibility decisions have been made and revealed to all firms.

We use  $d(b)$  and  $p(d, b)$  to denote the compatibility and pricing strategies in equilibrium. Given compatibility choices  $d$  and installed bases  $b$ , the net present value of future cash flows to firm  $i$  is given by

$$U_i(d, b) = \max_{p_i} \phi_i(p_i, p_{-i}(d, b); d, b)(p_i - c) + \beta \sum_{j=0}^N \phi_j(p_i, p_{-i}(d, b); d, b) \bar{V}_{ij}(b), \quad (1)$$

where  $p_{-i}(d, b)$  are the prices charged by firm  $i$ 's rivals in equilibrium (given installed bases and compatibility choices),  $c \geq 0$  is the marginal cost of production,  $\beta \in [0, 1)$  is the discount factor, and  $\bar{V}_{ij}(b)$  is the continuation value to firm  $i$  given that firm  $j$  wins the current consumer.

Given any feasible vector of compatibility choices  $d$ , differentiating the RHS of equation (1) with respect to  $p_i$  and using the properties of logit demand yields the first-order condition

$$-\frac{1}{\sigma}\phi_i(1 - \phi_i)(p_i - c + \beta\bar{V}_{ii}) + \phi_i + \beta \sum_{j \neq i} \frac{1}{\sigma}\phi_i\phi_j\bar{V}_{ij} = 0. \quad (2)$$

The pricing strategies  $p(d, b)$  are the solution to the system of first-order conditions.

Folding back from pricing to compatibility decisions, given installed bases  $b$ , the net present value of future cash flows to firm  $i$  is given by

$$V_i(b) = \max_{d_i \in \{0,1\}^{N-1}} U_i(d_i, d_{-i}(b), b), \quad (3)$$

where  $d_i = (d_{i1}, \dots, d_{ii-1}, d_{ii+1}, \dots, d_{iN})$  and  $d_{-i}(b)$  are the compatibility choices of firm  $i$ 's rivals in equilibrium (given installed bases). Since firm  $i$  has  $2^{N-1}$  feasible compatibility choices the complexity of the problem is increasing exponentially in the number of firms.

### 3.2 Computation

We focus attention on Markov perfect equilibria (MPE). Given that firms are *ex ante* identical, we impose the restriction that the MPE is symmetric. Moreover, we follow the majority of the literature on numerically solving dynamic stochastic games (Pakes and McGuire, 1994, 2001) by restricting attention to pure strategies.

As with many other dynamic models, the multiplicity of MPE is a concern. Unfortunately, it is not practical to compute all of them using homotopy methods as in Besanko et al (2005) because our game is more complex (compatibility and pricing decisions). In what follows we therefore propose an algorithm that computes a particular kind of equilibrium, namely the limit of a finite-horizon game as the horizon grows to infinity. This is a widely used selection criterion in the theoretical literature on dynamic games but, to the best of our knowledge, has never been applied to numerically solving such games.

The idea is as follows: Given continuation values that encapsulate the value of future play and installed bases, it is *as if* firms are playing a two-stage game of making first compatibility and then pricing decisions. In

the last period of a finite-horizon game, the continuation values are zero. Hence, we can solve for the subgame-perfect equilibrium of the two-stage game. In the previous-to-last period, the continuation values are given by the equilibrium payoffs of the last period. Continuing this line of thought, we can construct an algorithm that computes the limit of a finite-horizon game by iterating backwards in time.

We set  $N = 2$  in what follows. This greatly simplifies the computations because there are only two possible outcomes of firms' compatibility decisions: either the products are compatible or they are not. Moreover, with  $N = 2$  it is clear that the products are compatible in equilibrium if and only if both firms prefer it.

Our algorithm is iterative. It takes a value function  $\tilde{V}_i(b)$  for each firm  $i$  as the starting point for an iteration and generates an updated value function  $V_i(b)$  along with policy functions  $p_i(d, b)$  and  $d_i(b)$ . The initial value is  $\tilde{V}_i(b) = 0$ ,  $i = 1, 2$  for all  $b$ , thereby ensuring that the continuation values  $\overline{V}_{ij}(b)$  are zero.

Each iteration cycles through the state space in some predetermined order. Given a state  $b$ , it solves for the subgame-perfect Nash equilibrium of the two-stage game in that state, holding fixed the continuation values. Specifically, the algorithm process is as follows:

1. Solve the system of first-order conditions (2) given first  $d_{12}(b) = d_{21}(b) = 0$  and then  $d_{12}(b) = d_{21}(b) = 1$ . In doing so, use  $\tilde{V}_i(b)$  to compute  $\overline{V}_{ij}(b)$ ,  $i, j = 1, 2$ . This yields prices given compatibility decisions, i.e.,  $p_i(0, 0, b)$  and  $p_i(1, 1, b)$ ,  $i = 1, 2$ .
2. Substitute  $p_i(0, 0, b)$  and  $p_i(1, 1, b)$ ,  $i = 1, 2$ , into equation (1). This yields payoffs given compatibility decisions, i.e.,  $U_i(0, 0, b)$  and  $U_i(1, 1, b)$ ,  $i = 1, 2$ .
3. Determine firms compatibility decisions as

$$d_{ij}(b) = \begin{cases} 0 & \text{if } U_i(0, 0, b) \geq U_i(1, 1, b), \quad i = 1, 2, j \neq i \\ 1 & \text{otherwise} \end{cases}$$

4. Substitute  $d_{ij}(b)$ ,  $i = 1, 2, j \neq i$ , into equation (3). This yields payoffs, i.e.,  $V_i(b)$  and  $V_i(b)$ ,  $i = 1, 2$ .

Once the computations for one state are completed, the algorithm moves on to another state. After all states have been visited, the algorithm updates the current guess for the value function by setting  $\tilde{V}_i(b) = V_i(b)$ ,  $i = 1, 2$  for

all  $b$ . This completes the iteration.<sup>5</sup> The algorithm continues to iterate until the relative change in the value and the policy functions from one iteration to the next is below a pre-specified tolerance. This procedure always converged.

As already noted, multiplicity of MPE is a concern. One source of multiplicity is associated with the compatibility stage. Since products are compatible between firms  $i$  and  $j$  if and only if both firms propose compatibility then, for any state, there is always an equilibrium outcome in which a pair of firms' products are incompatible. Given, say, firm  $j$  chooses not to propose compatibility, firm  $i$ 's payoff is the same regardless of whether or not it proposes compatibility since products will be incompatible in either event. When it is also an equilibrium for products to be compatible, that equilibrium will be selected because: 1) our interest is in exploring the implications of product compatibility; and 2) the equilibrium with compatible products Pareto dominates the one with incompatible products (except when compatibility does not matter, such as when  $\lambda = 0$ ). In the event that a firm is indifferent about whether or not to make its product compatible, we assume it proposes incompatibility.<sup>6</sup> At least when there are just two firms (which is the market structure of focus in this paper), this selection criteria takes care of multiplicity issues at the compatibility stage. This procedure always converged and resulted in a unique equilibrium.

After solving for MPE value and policy functions, the analysis is two-pronged. The first exercise is to explore the policy function towards characterizing the set of states such that firms have compatible technologies. Of particular interest is to understand the degree of firm asymmetries - in terms of installed bases - for which firms choose compatibility and how that degree of asymmetry depends on the rate of consumer replacement, the extent of spillovers from compatibility, and the strength of network effects. Thus, we will be solving the model for the policy function for various parameter configurations. The parameters are summarized in Table 1 along with the different values used in the numerical analysis. The key parameters are  $\theta$ ,  $\lambda$ ,  $\delta$ , and  $v_0$ .<sup>7</sup>

The second set of exercises explores the determinants of market outcomes and, in particular, the frequency with which market dominance emerges.

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<sup>5</sup>Our procedure is a Gauss-Jacobi scheme; see Judd (1998) for an extensive discussion of Gauss-Jacobi and Gauss-Seidel methods.

<sup>6</sup>Experimentation with the tie-breaking rule revealed that it does not make a difference for our results.

<sup>7</sup>As an initial check that the parameter values are reasonable, we compared the demand elasticities for our model with empirical estimates for products with network effects and find they are comparable in size. These results are available on request.

Our approach is to use the equilibrium policy function to calculate the probability distribution over the industry's state tomorrow given its state today; that is, we use the policy function to construct the transition matrix of the Markov process that characterizes industry dynamics in the equilibrium of the game. This allows us to use tools from stochastic process theory to analyze industry dynamics rather than rely on simulation. Both the short-run (transient) dynamics and the long-run (limit or ergodic) distribution of the Markov process are characterized.

## 4 Static Equilibrium

Prior to characterizing equilibria for the dynamic game, it is useful to first understand the incentives for compatibility in the static model. The static equilibrium is derived by setting  $\beta = 0$  in which case firms choose price to maximize current profit. Installed bases matter only because of how they affect the current value that consumers attach to firms' products; they are not an instrument to later dominance.

Representative of our findings is Figure 1 where we have plotted firm 1's equilibrium price as a function of its own base,  $b_1$ , and its rival's base,  $b_2$ . Also reported is the compatibility region; that is, the states for which both firms prefer compatibility and thus their products are compatible. When compatibility affects market demand ( $\lambda = .5$  or  $\lambda = 1$ ), products are compatible when firms' installed bases are sufficiently similar in size. The forces at work are basically the same as those in other static models that allow for compatibility choice and are most clearly identified in Cremer, Rey, and Tirole (2000). We review and elaborate upon them below.

Holding price fixed, there are two quantity effects from firms making their products compatible. Compatibility raises firm  $i$ 's effective installed base from  $b_i$  to  $b_i + \lambda b_j$  which then increases the value that consumers attach to its product by  $g(b_i + \lambda b_j) - g(b_i)$ . Each firm's product is more attractive relative to the outside option. Firms then have a mutual interest in having compatible products because both benefit from drawing more consumers into the market. This we refer to as the *market expansion effect*.

A second quantity effect arises when firms have different installed bases. In that situation, compatibility reduces the quality differential between their products which, generally, harms the firm with a bigger installed base. In other words, the larger firm has an edge because of its installed base and that edge is partially (when  $\lambda = .5$ ) or fully (when  $\lambda = 1$ ) lost when products are made compatible. We call this the *business gift effect* as it means

enhancing the business stealing effect of one's rival. As shown in the appendix, sufficient conditions for the business gift effect to harm the firm with the larger installed base are that  $g$  is linear or concave and/or  $\lambda \simeq 1$ . If  $g$  is sufficiently convex and  $\lambda \ll 1$ , it is possible that the business gift effect instead harms the firm with the smaller installed base. Since our reported results are for when  $g$  is linear, the business gift effect then harms the larger firm and benefits the smaller firm.

Supplementing these quantity effects are price effects which can best be understood through the following decomposition when  $\lambda = 1$ . Suppose the initial state is  $(b_1, b_2) = (b', b'')$  where  $b' < b''$ . Compatibility can then be decomposed into two parts: it causes firms' effective installed bases to shift from  $(b', b'')$  to  $(b'', b'')$  and then from  $(b'', b'')$  to  $(b' + b'', b' + b'')$ . As the first shift only improves the smaller firm's quality, its price rises and the larger firm's price falls.<sup>8</sup> The second shift causes both firms' prices to increase as the quality of their products rises relative to the outside good.<sup>9</sup> The first price effect is ambiguous as to how it impacts profitability though the second price effect amplifies the market expansion effect and thus further enhances the value to making products compatible.

We can now use the market expansion and business gift effects to explain why compatibility emerges when firms' installed bases are sufficiently similar in size. Suppose firms have identical installed bases and recall that firms are static profit-maximizers in this exercise. Both firms experience higher profit by having compatible products because they take demand away from the outside good (which is the market expansion effect) and neither firm loses any advantage over its competitor since relative quality is unaffected (that is, there is no business gift effect). Now suppose firms' bases are close but not identical. With compatibility, the larger firm loses only a small relative quality advantage over the smaller firm (since similar bases means similar qualities) but there is a discrete jump in quality with compatibility. Hence, the market expansion effect exceeds the business gift effect when a firm's installed base is slightly larger than that of its rival. Obviously, the firm with a smaller base is better off with compatible products. This explains why there is an area around the diagonal in which firms agree to make their products compatible, as can be seen in Figure 1. Now move the bases

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<sup>8</sup>For our demand structure, Anderson et al (1992, p. 266) prove that a firm's equilibrium price is increasing in its quality (or installed base). Since prices are strategic complements then a firm's equilibrium price is decreasing in the other firm's quality.

<sup>9</sup>If firms have a common quality (which is comprised of both intrinsic quality and network effects) then the symmetric equilibrium price can be shown to be increasing in that common quality as long as  $\exp v_0 > 0$  (that is, market size is variable).

farther off of the diagonal. The business gift effect rises in importance - as the larger firm gives up a greater quality advantage - until it exceeds the market expansion effect; at that point the larger firm prefers that products be incompatible.<sup>10</sup>

This explanation is confirmed when one examines the case when there is no outside good ( $v_0 = -\infty$ ). Since the market expansion effect vanishes, only the business gift effect is operative which would argue that the larger firm would never want to have compatible products. Indeed, when the market size is fixed, compatibility never occurs in equilibrium as long as firms have different installed bases.

## 5 Dynamic Equilibrium

### 5.1 With Outside Good

We begin by assuming  $v_0 = 0$  so that market demand is not fixed. Depending on the parameter values, four qualitatively distinct equilibrium policy functions were found.<sup>11</sup> These are referred to as Flat, Rising, Trenchy, and Dual Trenchy. Based upon visual inspection, Table 2 reports the type of equilibrium for an array of values for  $\delta$ ,  $\theta$ , and  $\lambda$ .

A *Flat* equilibrium is a modest perturbation of the policy function for a static equilibrium. Not surprisingly, it arises when dynamic effects are minimal - either the network effect is weak ( $\theta$  is low), spillovers are absent ( $\lambda = 0$ ), and/or customer turnover is high ( $\delta$  is high). Note that when  $\delta$  is high, there is little point in competing aggressively for customers in order to build an installed base since a firm's gains are likely to fritter away due to customer turnover; in other words, the return to investment is low when the depreciation rate is high.

A *Rising* equilibrium is characterized by a fairly monotonic policy function for which price is increasing in a firm's own base but relatively insensitive to its rival's base; an example is shown in Figure 2. Products are typically incompatible. This equilibrium arises when compatibility does not impact demand ( $\lambda = 0$ ) or when the rate of customer turnover rate is very low. Also reported in Figure 2 are the limit (or ergodic) distribution and the transient distributions - for 5, 15, and 25 periods - starting from an initial state in which  $(b_1, b_2) = (0, 0)$ . Though the limit distribution is unimodal

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<sup>10</sup>We have indeed confirmed that where incompatibility occurs, the smaller firm prefers to have compatible products but it is vetoed by the larger firm.

<sup>11</sup>Recall that we always found the equilibrium to be unique so, for any parameter configuration, only one of these equilibrium types is observed.

and rather tight, this is misleading as, with such a low rate of customer turnover, each firm will frequently achieve the maximal installed base of 20 in the long run. Over shorter horizons, the distribution is rather dispersed and it is quite common for the market outcome to be skewed. This is seen in the transient distributions.

For the primary dynamic forces of our model to be at work, the relevant part of the parameter space is when compatibility matters, network effects are not weak, and the rate of customer turnover is not too high. In that part of the parameter space, two types of equilibria are dominant: *Trenchy* and *Dual Trenchy*. These are by far the most insightful for learning about dynamic competition and will be the focus of our attention for the remainder of the analysis.

### 5.1.1 *Trenchy* Equilibrium

A *Trenchy* equilibrium has the following properties: i) intense price competition when firms' installed bases are of comparable size; ii) the limit distribution for installed bases is bimodal with a lot of mass at highly asymmetric states; and iii) products are generally incompatible. An example of a *Trenchy* equilibrium is shown in Figure 3. The policy function for a *Trenchy* equilibrium is characterized by a deep trench along and around the diagonal. In Figure 3, price is actually negative - below marginal cost - for some states near the diagonal. Sufficiently off of the diagonal, price is relatively high. This equilibrium is similar to that found in models with increasing returns such as arises with advertising (Doraszelski and Markovich, 2005) and learning-by-doing (Besanko et al, 2005).<sup>12</sup>

When firms have sufficiently disparate installed bases, dynamic competition largely ceases as reflected in relatively high prices (these are the plateaus off of the diagonal). Due to network effects, the profitable strategy for the smaller firm is to accept having a low market share. If instead it were to try to supplant the larger firm, it would need to price at a considerable discount in light of the quality disadvantage emanating from a smaller installed base and that products are incompatible. Furthermore, low pricing would have to continue for an extended period of time in order to eliminate the installed base differential. Since such an aggressive strategy is not profitable and thus not pursued, prices are high and the larger firm reaps large profits due to its high market share by virtue of having a better product (which comes from a bigger installed base and network effects).

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<sup>12</sup>This is also the case with capacity investment with price competition (Besanko and Doraszelski, 2004; Chen, 2005) though it is not an increasing returns story.

It is this “prize” to a firm with a significant installed base advantage that causes competition to be so intense when firms have comparable installed bases. A firm knows that if it were to gain such an advantage that the other (smaller) firm would accept its position in the market and the larger firm would reap high profits. We then have a deep trench along and around the diagonal where prices are very low. Each firm’s conduct focuses on fighting its rival to become the dominant firm. Note that for states in the trench, firms’ products are incompatible except possibly when  $b_1 = b_2$ .<sup>13</sup>

As shown in Figure 3, the limit distribution on installed bases is bimodal, which indicates that it is quite likely market dominance will emerge. Once one of the firms gains an advantage in terms of installed bases, the strength of network effects transforms it into a long-run advantage. The movement towards skewed outcomes is apparent by following the transient distribution over time; more and more mass is dispersed away from the diagonal. The pricing behavior of firms contributes to the emergence and persistence of market dominance since the firm with the smaller installed base generally accepts its position by not pricing aggressively. The Trenchy equilibrium embodies the quintessential property of network effects which is that the market “tips” to one firm dominating as soon as it has an advantage.

As reported in Table 2, a Trenchy equilibrium occurs when the network effect is strong - it does not occur for  $\theta \in \{1, 2\}$  but does arise when  $\theta \in \{3, 4\}$  - and customer turnover is modest ( $\delta$  is low). For a firm to price aggressively and forego current profit, the prospect of future dominance by building its installed base must be sufficiently great. This requires that the network effect is sufficiently strong and the installed base does not deteriorate too rapidly.

**Result 1 ("Fight-Your-Rival" Equilibrium)** *When the network effect is strong and customer turnover is modest, equilibrium is characterized by incompatible products, intense price competition when firms’ installed bases are of comparable size, and tipping towards market dominance when one firm gains an advantage in terms of its installed base.*

### 5.1.2 Dual Trenchy Equilibrium

The *Dual Trenchy* equilibrium is new to the increasing returns literature and arises solely because of the option of compatibility. A Dual Trenchy

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<sup>13</sup>Note that products are not always compatible on the diagonal. Incompatibility occurs when firms price *below* marginal cost because of their eagerness to increase their installed bases. As a result, compatibility would *reduce* current profit because it increases demand and each unit sold is at a loss.

equilibrium has the following properties: i) high prices when firms' installed bases are of similar or highly disparate size but intense price competition when modestly different; ii) the transient and limit distributions for installed bases are unimodal with a lot of mass at reasonably symmetric states; and iii) products are compatible when firms' installed bases are comparable. Examples are provided in Figure 4 - when  $(\delta, \theta, \lambda) = (.01, 3, 1)$  - and Figure 5 - when  $(\delta, \theta, \lambda) = (.08, 3, 1)$ .

Let us explore compatibility and pricing in three scenarios: when the installed base differential is large, modest, and small.

**Large installed base differential.** When the differential is large, the outcome is basically the same as with a Trenchy equilibrium. Products are incompatible and the firm with the larger installed base dominates the market due to network effects. The smaller firm is resigned to its inferior position in the market and thus dynamic competition is minimal. Prices are relatively high as a result.

**Small installed base differential.** When firms have installed bases that are identical or highly comparable in size, prices are also relatively high though now products are compatible. Recall from our examination of the static equilibrium that compatibility reduces the quality differential emanating from firms having different installed bases, which is detrimental to the firm with a bigger installed base. At the same time, it enhances both firms' product quality and thereby expands the market. The former effect we referred to as the business gift effect and the latter as the market expansion effect. Due to these two effects, there was a region around the diagonal for which both firms choose to make their products compatible. These forces are still present in the dynamic equilibrium and are partly at work in generating the compatibility region for a Dual Trenchy equilibrium.

Though products are compatible, this need not imply the absence of price competition. For a Trenchy equilibrium, firms often make their products compatible when they have identical installed bases and, at the same time, price very low in order to acquire an advantage in its installed base. Such dynamic price competition is not observed for a Dual Trenchy equilibrium when the installed base differential is small. To see why, suppose  $\lambda = 1$  and firms begin with identical installed bases. Regardless of which firm (if any) wins today's customer and thereby expands its installed base, firms expect their products to be compatible tomorrow. This follows from the compatibility region encompassing asymmetric as well as symmetric states.

Thus, a firm who gains a small installed base advantage does not anticipate gaining a quality advantage in the near term because compatibility is maintained. Quite contrary to acquiring sales to dominate, there could be a short-run free-rider effect because a firm that lowers price and increases the probability of a sale benefits the future quality of both firms' products compared to the outside option. When the compatibility region is extensive, firms anticipate compatible products in the near future when the installed base differential is relatively low; this stifles dynamic price competition.

**Modest installed base differential.** The most intriguing region is when firms have modestly different installed bases in which case prices are low as reflected in the dual trenches in the policy function. As explained below, pricing behavior is largely driven by dynamics associated with endogenous product compatibility. Whether or not products are compatible tomorrow depends on firms' installed bases tomorrow; only if they are sufficiently similar in size will firms mutually decide to have compatible products. Of course, tomorrow's state depends on today's pricing. With a Dual Trenchy equilibrium, pricing is then driven not only by the prospect of dominating the market - a force that is ever present in a market with network effects - but also by the strength of firms' desire to maintain product compatibility.

To explore the incentives for compatibility, first note that the smaller firm almost always prefers compatible products - as it is benefitted both by the market expansion and business gift effects - while the larger firm prefers compatible products only when the installed base differential is sufficiently small. Thus, when products are incompatible, it is the larger firm that prevents it.<sup>14</sup> Let us begin by examining how the smaller firm's desire for compatibility influences its pricing behavior.

Corresponding to Figures 4 and 5, Figures 6 and 7 report firm 1's price for different states. The states for which firms' products are compatible are shaded, while negative (below cost) prices are boxed. Prices are high when firms have comparable bases (that is, near the diagonal). As the state moves farther off of the diagonal - so that the difference in firms' bases increases - the smaller firm lowers its price. It does so even though firms' products are of equal quality (due to compatibility and  $\lambda = 1$ ). In particular, the smaller firm significantly drops its price when the state approaches the (interior)

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<sup>14</sup>This was verified numerically. For the 62,160 different combinations of parameters and asymmetric states, the smaller firm prefers compatible products in 57,715 of them and prefers incompatible products in only five instances (with the remainder there is indifference). For the larger firm, there are 11,172 instances in which it prefers compatible products and 46,548 instances in which it prefers incompatible products.

border of the compatibility region. Its intent is to increase expected sales and thereby reduce the installed base differential. In Figure 6, firm 1's price drops from 0.8 to 0.2 when the state moves from  $(b_1, b_2) = (4, 6)$  to  $(4, 7)$  where  $(4, 7)$  is just on the interior of the compatibility region. Just outside of the compatibility region, the smaller firm drops price even more; when the state moves from  $(4, 7)$  to  $(4, 8)$ , firm 1's price drops from 0.2 to -0.7. In Figure 7, firm 1's price drops from 1.3 to 1.0 as it goes from  $(4, 9)$  to  $(4, 10)$  (just on the interior of the compatibility region) but from 1.0 to -0.1 as it goes from  $(4, 10)$  to  $(4, 11)$  (just on the exterior of the compatibility region). The smaller firm is trying to add to its base in order to move the state back into the region where the larger firm prefers compatibility. Compared to when the state is just inside the compatibility region, this task is made more difficult because products are no longer compatible which means the smaller firm suffers from a quality disadvantage. To compensate for that disadvantage, it needs to sell its product at an even bigger discount to the larger firm's product.

In sum, the smaller firm is pricing aggressively in order to keep the differential in installed bases sufficiently small. Its intent is to pacify, rather than fight, the larger firm so that the larger firm will "make nice" (by having a compatible product) rather than "make mean" (by pursuing monopolization through incompatible products).

Though the larger firm also drops price around the border of the compatibility region, that apparently is a response to the smaller firm's pricing behavior - as their prices are strategic complements - rather than an attempt to monopolize. For example, in Figure 6, a movement in the state from  $(4, 6)$  to  $(4, 7)$  results in the smaller firm dropping price from 0.8 to 0.2, while the larger firm's price only falls from 1.5 to 1.2. Examination of the value function shows that, along the border of the compatibility region, the large firm only slightly prefers compatibility which is why it is willing to price much higher than the smaller firm even though it might mean the state moves out of the compatibility region. In contrast, the smaller firm strongly prefers compatibility, which explains why it is willing to price so low in order for the state to remain in that region.

It is worth emphasizing that this pricing behavior is quite distinct from what static demand effects would produce. Within the compatibility region, the relative quality of firms' products is identical since products are compatible and  $\lambda = 1$ . In a static model, prices would then be identical while we find they are different (in particular, the smaller firm has a lower price). Second, price falls sharply just outside of the compatibility region but eventually rises as the installed base differential becomes sufficiently large. That

is also in support of our dynamic story as a firm's static equilibrium price monotonically declines as its relative quality falls.

A Dual Trenchy equilibrium occurs when network effects are neither weak nor strong and the effect of compatibility on demand is significant. It is typical when  $\theta \in \{1, 2\}$  but also occurs when  $\theta \in \{3, 4\}$  as long as  $\delta$  is not too low. If the network effect is weak then pricing is largely uninfluenced by dynamic considerations, while if it is strong then the ability to translate a small installed base advantage into long-run dominance deters the larger firm from making its product compatible. Obviously, the impact of compatibility on demand has to be sufficiently strong for the larger firm to prefer compatibility over a dominance strategy.

**Result 2 ("Pacify-Your-Rival" Equilibrium)** *When the network effect is modestly strong and the effect of compatibility on demand is strong then equilibrium is characterized by compatible products, mild price competition, and an absence of market dominance.*

## 5.2 Without Outside Good

When  $v_0 = -\infty$ , each consumer buys from either firm 1 or firm 2 as the outside option is wholly unattractive. Firms are then competing over a market of fixed size which means there is no market expansion effect. Higher quality for a firm's product may attract consumers who would have otherwise bought from one's rival but will not increase the total number of consumers who are purchasing. This is an environment expected to breed intense competition.

In eliminating the market expansion effect, the first notable consequence is that firms now never choose to make their products compatible, except possibly when their installed bases are identical. This is a property that is quite predictable in light of the preceding discussion. With a fixed market size, each firm is only interested in having higher quality *relative* to its rival. Compatibility is then always to the detriment of the larger firm as it shifts demand to its rival.

Though a Dual Trenchy equilibrium does not then arise when  $v_0 = -\infty$ , the other types of equilibria are still present. A Trenchy equilibrium occurs for a wider array of parameter values relative to when market demand is elastic. When  $v_0 = 0$ , firms do not compete aggressively for dominance when  $\theta \in \{1, 2\}$  (that is, there are no Trenchy equilibria). However, a Trenchy equilibrium does occur when  $v_0 = -\infty$ ; it arises for  $\delta \in \{.04, \dots, .07\}$  when  $\theta = 1$  and  $\delta \in \{.04, \dots, .12\}$  when  $\theta = 2$ . Fixing the market size then

intensifies competition, as firms compete to gain an advantage in terms of their installed base, and provides almost no basis for compatibility.

## 6 Product Compatibility and Market Dominance

One of the central questions of this research project is understanding to what extent endogenous product compatibility can prevent market dominance from emerging. If the transient and limit distributions with respect to installed bases are heavily skewed - putting a lot of mass on relatively asymmetric outcomes - then market dominance is likely to occur. The extent to which compatibility is feasible can be measured by the parameter  $\lambda$ . Firms effectively do not have the option of compatible products when  $\lambda = 0$  as compatibility does not impact demand. As products are almost never compatible when the market size is fixed, the results reported here are for when  $v_0 = 0$ . Furthermore, variable market demand would seem to be the more empirically relevant scenario.

The pricing behavior identified in the previous section creates a "compatibility dynamic" which has the potential for maintaining some balance in the market and avoiding dominance. Suppose firms begin with identical or near-identical installed bases. They generally will find it optimal to make their products compatible in order to expand the market. With products of similar quality, firms charge similar prices. At that point, their expected market shares are comparable. Though, in expectation, future installed bases remain similar in size, random shocks to demand and customer turnover could result in one of the firms gaining a significant advantage in terms of installed bases. If that differential becomes large enough, products will no longer be compatible and firms will price in a manner to perpetuate such a skewed market structure. However, in a Dual Trenchy equilibrium, there are forces preventing a slight advantage from growing into a large one. When firms' installed bases differ and are near the boundary of the compatibility region, the smaller firm prices aggressively in order to increase its installed base and thereby shift the state back towards symmetry. When the state is close to but outside of the compatibility region - so that the larger firm chooses to make its product incompatible - the smaller firm offers its product at an even larger discount so as to shift the state back into the compatibility region and eliminate the quality differential. The strategic pricing behavior of the smaller firm in the vicinity of the boundary of the compatibility region acts to keep the state within that region and thus works against market dominance.

The compatibility dynamic is revealed by reporting the resultant force, which measures the direction of movement of the state. Its expected movement is determined by computing the probability-weighted average of the difference between next and this period's state. Figure 8 shows the resultant forces for the three parameter configurations in Figures 3-5. Figure 8a has a Trenchy equilibrium and, therefore, products are incompatible (except perhaps on the diagonal). Once the state is off of the diagonal, so that firms have different installed bases, the state moves away from symmetry as the larger firm builds on its advantage. Increasing returns is at work. Figures 8b and 8c have a Dual Trenchy equilibrium and nicely show how the increasing returns dynamic can be countered by the compatibility dynamic. In Figure 8b, the state is moving up and towards the diagonal as long as the installed base differential is not too large. If it does become great then increasing returns kick in and the larger firm grows yet larger. In Figure 8c, where the rate of customer turnover is higher, there is a strong attraction to the diagonal for a wide range of states. In both of these cases, we are observing forces that move the state backs towards symmetry once it has moved away.

The real test of this dynamic is examining how the option of compatibility impacts the distribution on installed bases. Let us begin with a few illustrative examples and then present more systematic evidence. For two different parameter configurations, Figure 9 reports the limit distributions when  $\lambda = 0$  and  $\lambda = 1$  and, for the latter, the set of states for which products are compatible. In Figure 9a, the network effect is moderate ( $\theta = 2$ ) and thus the limit distribution is unimodal even when compatibility is not an option. Market dominance is not likely to emerge in that case. As compatibility becomes a possibility ( $\lambda = 1$ ), a unimodal distribution persists with more mass pushed towards symmetric outcomes. Introducing the option of compatibility makes it more likely that a roughly symmetric state occurs though does not have a sizable impact.

As the strength of networks effects is increased to  $\theta = 3$ , endogenous product compatibility makes a striking difference; see Figure 9b. When  $\lambda = 0$ , the limit distribution is significantly bimodal. Without the prospect of compatibility, it is very likely that one of the firms will dominate the market. Allowing for products to be compatible has a dramatic effect as the distribution shifts to being unimodal with a lot of mass around the diagonal. Firms are choosing to make their products compatible unless the state is reasonably asymmetric.<sup>15</sup> Introducing the option of compatibility

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<sup>15</sup>For the case of a very strong network effect ( $\theta = 4$ ), which is not shown, endogenous compatibility does not matter as, even when  $\lambda = 1$ , products are incompatible and a

makes it vastly less likely that market dominance will emerge.

Table 3 and Figure 10 provide a broader set of confirming results. Focusing on  $\delta \leq .15$ , Table 3 reports the mode of the limit distribution.<sup>16</sup> Highlighted are parameter values for which a bimodal distribution occurs when compatibility is not an option ( $\lambda = 0$ ) and a unimodal distribution occurs when compatibility is an option ( $\lambda = 1$ ). For example, when  $(\delta, \theta) = (.07, 3)$ , the lack of compatibility results in a highly skewed mode in which one firm has an installed base of 15 units and the other has only one unit. When firms have the option of product compatibility, the mode is symmetric with each having eight units. Figure 10 reports the expected long-run Herfindahl index using the limit distribution over states. To the extent that the long-run Herfindahl index exceeds 0.5, asymmetries arise and persist. If the customer turnover rate is not too low, the option of compatibility reduces market concentration and sometimes significantly so.

To summarize, endogenous product compatibility can neutralize the usual increasing returns mechanism associated with network effects. The trick is keeping the differential in installed bases sufficiently modest so that the larger firm chooses to make its product compatible. The burden of ensuring the differential is kept low falls on the smaller firm, whose incentive for compatibility is much greater, and is reflected in aggressive pricing when the installed base differential becomes too large. Whenever the state ends up close to the boundary of the compatibility region, the smaller firm significantly lowers its price to increase its installed base. If it should fail and the larger firm becomes yet larger, then the asymmetric market structure persists as both firms price relatively high. Nevertheless, such an occurrence tends to be rather unlikely because of the strategic pricing of the smaller firm and in this manner market dominance is avoided in spite of the presence of network effects. Compatible products can be stable and, as a result, both firms can have significant market shares in the long-run.

**Result 3 (Avoidance of Market Dominance)** *Having the option of product compatibility can result in a market achieving a relatively symmetric outcome when, in the absence of that option, there would have been market dominance.*

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bimodal distribution arises. Though this has not been established, it is possible that a bimodal distribution always arises by making the network effect sufficiently large.

<sup>16</sup>A bimodal distribution never occurs for  $\delta > .15$ .

## 7 Strength of Network Effects

Figure 11 reports the long-run Herfindahl index and shows, consistent with previous work, that market concentration is increasing in the strength of the network effect. This is true whether or not firms have the option of compatibility though that option does tend to reduce concentration. Where there is a big increase in concentration from a stronger network effect - such as when  $\theta$  rises from 2 to 3 for  $(\delta, \lambda) = (.08, 0)$  and from 3 to 4 for  $(\delta, \lambda) = (.08, 1)$  - it is because the equilibrium is switching to a Trenchy equilibrium. Indeed, inspection of Table 2 shows that a Trenchy equilibrium is more likely when  $\theta$  is higher.

While these general results are useful, more insight may be had by exploring how the equilibrium policy function changes with respect to the strength of network effects. Consider Figure 12. When network effects are weak ( $\theta = 1$ ), there is a mild Dual Trenchy (almost Flat) equilibrium with a large compatibility region. The limit distribution is unimodal. As  $\theta$  is increased, the dual trenches deepen and the compatibility region shrinks. The larger firm increasingly prefers a monopolization strategy rather than enhancing current demand through compatibility. This causes the compatibility region to shrink. Because a dominance strategy is more appealing to the larger firm, it is all the more important for the smaller firm to prevent the gap in bases from widening too much. This induces it to price lower along the border of the compatibility region; hence, the trench deepens as  $\theta$  rises. While the limit distribution does become more dispersed as the network effect rises - indicating that it is more likely that installed bases will be highly asymmetric - the effect is weak. The real impact of a stronger network effect is to induce the smaller firm to price more aggressively in order to ensure that products are compatible. This highlights the role of the compatibility dynamic in that market dominance is only mildly increasing when the network effect is strengthened.

Figure 13 considers the same parameter configuration as in Figure 12 except that  $\lambda = .5$  so that compatibility does not have as much an impact on product quality. Again, the equilibrium is a mild Dual Trenchy when  $\theta = 1$ . As  $\theta$  rises, the plateau between the dual trenches shrinks as the compatibility region declines in size. In fact, when  $\theta$  goes from 3 to 4, the plateau disappears and the dual trenches are replaced with a single trench around the diagonal. In other words, the equilibrium has morphed from Dual Trenchy to Trenchy. Except for when firms' bases are low and similar, products are no longer compatible when  $\theta = 4$ . In this case, the limit distribution does go through a drastic change as it moves from being unimodal

to bimodal. Having sufficiently increased the strength of the network effect, market dominance now emerges with great frequency.

In exploring the relationship between the network effect and product compatibility, we observe in Figures 12 and 13 that the compatibility region shrinks as  $\theta$  rises. This is confirmed to hold more generally. There are two ways in which an enhanced network effect reduces the incentive of the larger firm to make its product compatible. First, a higher value for  $\theta$  makes it more likely that a current advantage - in terms of the installed base - will result in future market dominance. A firm with a bigger base is then more inclined to want to build on its competitive advantage, which it can do by making its product incompatible. Second, a rise in  $\theta$  improves the quality of firms' products which makes them more attractive compared to the outside good. This serves to weaken the size of the market expansion effect coming from making products compatible. Both these effects work towards reducing the incentive for compatibility and making it more likely that firms will compete aggressively.

In spite of the compatibility region shrinking as the network effect strengthens, it is not always the case that products are less likely to be compatible. Figure 14 reports the long-run probability that products are compatible using the probability distribution over states given by the limit distribution. When the customer turnover rate is relatively low, the frequency with which products are compatible is generally lower when the network effect is stronger. In some cases, the frequency drops to zero which reflects a shift of the equilibrium from Dual Trenchy to Trenchy.

However, when the customer turnover rate is modestly high ( $\delta \geq .1$ ), products are *more* likely to be compatible when the network effect is stronger. Consider the case of  $\delta = .14$  where the probability of compatible products rises from .68 to .83 as  $\theta$  increases from 1 to 4. This is in spite of the fact that, as shown in Figure 12, the set of states for which products are compatible is shrinking. The resolution of this riddle lies in the policy functions. Because the network effect is stronger, the smaller firm is more aggressive in keeping the installed base differential relatively low as it fears the larger firm may shift to a monopolization strategy. This is reflected in the deeper trench in the policy function when comparing  $\theta = 1$  and  $\theta = 4$ . It is this aggressive pricing behavior that makes it more likely that the state remains in the compatibility region when  $\theta = 4$  than when  $\theta = 1$  even though the region is smaller.

Note that this surprising comparative static occurs as long as the equilibrium is Dual Trenchy. But for a Dual Trenchy equilibrium to persist as  $\theta$  is increased (and not transform into a Trenchy equilibrium), it is necessary

that the customer turnover rate not be too low. That is why  $\delta$  must be sufficiently high for a stronger network effect to increase the frequency of compatible products.

**Result 4 (Strength of Network Effect)** *A stronger network effect increases market concentration. A stronger network effect decreases the frequency of compatible products when the customer turnover rate is low and increases the frequency of compatible products when the customer turnover rate is high.*

## 8 Concluding Remarks

The main contribution of this paper is identifying the compatibility dynamic which can prevent market dominance in markets with network effects. When firms have comparably-sized installed bases, they choose to make their products compatible in order to expand market size. This occurs at a cost to the firm with the larger installed base since its quality advantage over the other firm is diminished when products are compatible. However, as long as installed bases are sufficiently similar in size, the reduction in relative quality is small relative to the rise in absolute quality so the larger firm prefers to have compatible products (and the smaller firm always prefers it). The challenge to compatibility being stable is that, due to the randomness in demand and customer turnover, the differential in firms' installed bases could grow to the point that the larger firm chooses to pursue a dominance strategy and thus makes its product incompatible. However, there are strong forces preventing a slight advantage from growing into a considerable one. When firms' installed bases differ and are near the boundary of the compatibility region, the smaller firm prices aggressively in order to increase its installed base and thereby shift the state back towards symmetry. When the state is close to but outside of the compatibility region - so that the larger firm chooses to make its product incompatible - the smaller firm offers its product at an even larger discount so as to shift the state back into the compatibility region and eliminate the quality differential. Strategic pricing tends to keep the installed base differential from expanding to the point that incompatibility occurs. Compatible products are then stable. The compatibility dynamic is able to neutralize increasing returns and result in long-run market structures that are not characterized by a single dominant firm.

This research project will continue in two directions. First is to extend the model to allow for three firms. In most models, the extension from

duopoly to triopoly is straightforward and uninteresting. To the contrary, the triopoly extension for our model poses modelling challenges and is likely to produce additional insight. As regards modelling challenges, there are likely to be multiple equilibria as different collections of firms could choose to have compatible products. In the duopoly model, such existed in that there is always an equilibrium without compatible products. However, if there is an equilibrium with compatible products, it was selected because of both weak dominance and the Pareto criterion. When there are three firms, it is no longer clear that these selection criteria will give us a unique solution. For example, suppose when firms have identical installed bases, there is an equilibrium in which only two firms' products are compatible and no equilibrium in which all three are compatible. There are, of course, three such equilibria in the triopoly case and they are not Pareto-ranked. This is a modelling challenge for the triopoly case.

More substantively is to explore whether compatibility and the avoidance of market dominance is easier with three firms. Recall for the duopoly case that products are almost never compatible when the market size is fixed. Is it possible that the presence of a third firm could induce compatibility in such an environment? Note that from the perspective of two firms, the remaining third firm is an outside option though one whose value is endogenous. As products were compatible when an outside option was present, this suggests that some compatibility may emerge in a broader set of circumstances where there are three firms.

The second research direction is to enrich the model by allowing firms to innovate. Prior to deciding on compatibility and price, each firm invests in R&D; the outcome of which is stochastic and affects the intrinsic quality of the good. Does the option of product compatibility reduce innovation as a firm can free ride as long as products are compatible? Is market dominance more likely when firms can innovate? Does innovation offset the compatibility dynamic and allow increasing returns to flourish? Can a policy of mandatory compatibility be socially desirable? These are some of questions that will be addressed.

## 9 Appendix: Analysis of Market Expansion and Business Gift Effects

To parse out the market expansion and business gift effects, consider the effect on firm 1's demand from having compatible products:

$$\begin{aligned} & \phi_1(p_1, p_2; (1, 1), (b_1, b_2)) - \phi_1(p_1, p_2; (0, 0), (b_1, b_2)) \\ &= \frac{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)} \\ & \quad - \frac{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)} \end{aligned}$$

This can be re-arranged to:

$$\begin{aligned} & \phi_1(p_1, p_2; (1, 1), (b_1, b_2)) - \phi_1(p_1, p_2; (0, 0), (b_1, b_2)) \\ &= \left[ \frac{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)} \right] \times \\ & \quad \left[ \frac{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right)}{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)} \right] \\ & \quad - \left[ \frac{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)} \right] \times \\ & \quad \left[ \frac{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right)}{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)} \right] \\ &= \Delta^c(b_1, b_2, \lambda) \Omega^c(b_1, b_2, \lambda) - \Delta^{in}(b_1, b_2) \Omega^{in}(b_1, b_2). \end{aligned}$$

$\Delta^c(b_1, b_2, \lambda)$  and  $\Delta^{in}(b_1, b_2)$  measure the market size, for the case of compatible and incompatible products respectively, in terms of total demand for these two firms as a proportion of total demand including the outside option. The market expansion effect from making products compatible is

then necessarily positive when:

$$\begin{aligned} \Delta^c(b_1, b_2, \lambda) - \Delta^{in}(b_1, b_2) = & \\ & \frac{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)} - \\ & \frac{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)}{\exp\left(\frac{v_0}{\sigma}\right) + \exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)} > 0. \end{aligned}$$

This is true as long as  $\lambda > 0$  and  $g' > 0$ .

The business gift effect concerns the impact of compatibility on each firm's share of market demand (excluding the outside option). It is measured by:

$$\begin{aligned} \Omega^c(b_1, b_2, \lambda) - \Omega^{in}(b_1, b_2) = & \\ & \frac{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right)}{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)} - \\ & \frac{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right)}{\exp\left(\frac{\theta g(b_1) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2) - p_2}{\sigma}\right)}. \end{aligned}$$

Since  $\Omega^c(b_1, b_2, 0) = \Omega^{in}(b_1, b_2)$  then

$$\Omega^c(b_1, b_2, \lambda) = \Omega^{in}(b_1, b_2) + \int_0^\lambda \left( \frac{\partial \Omega^c(b_1, b_2, \lambda')}{\partial \lambda'} \right) d\lambda'.$$

Given that

$$\begin{aligned} \frac{\partial \Omega^c(b_1, b_2, \lambda)}{\partial \lambda} = & \left\{ \frac{\exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right)}{\left[ \exp\left(\frac{\theta g(b_1 + \lambda b_2) - p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2 + \lambda b_1) - p_2}{\sigma}\right) \right]^2} \right\} \times \\ & \left( \frac{\theta}{\sigma} \right) [b_2 g'(b_1 + \lambda b_2) - b_1 g'(b_2 + \lambda b_1)], \end{aligned}$$

then

$$\text{sign} \left\{ \frac{\partial \Omega^c(b_1, b_2, \lambda)}{\partial \lambda} \right\} = \text{sign} \{ b_2 g'(b_1 + \lambda b_2) - b_1 g'(b_2 + \lambda b_1) \}. \quad (4)$$

Thus, if  $b_2g'(b_1 + \lambda'b_2) - b_1g'(b_2 + \lambda'b_1) > (<) 0 \forall \lambda' \in (0, \lambda)$  then  $\Omega^c(b_1, b_2, \lambda) - \Omega^{in}(b_1, b_2) > (<) 0$ .

The business gift effect is said to apply to the firm with the larger installed base when its market share declines with compatibility. That is, if  $b_1 > b_2$  then  $\Omega^c(b_1, b_2, \lambda) - \Omega^{in}(b_1, b_2) < 0$ . Using (4), a sufficient condition is:

$$\text{If } b_1 > b_2 \text{ then } b_2g'(b_1 + \lambda b_2) < b_1g'(b_2 + \lambda b_1) \quad \forall \lambda \in (0, 1). \quad (5)$$

A sufficient condition for (5) is:

$$g'(b_2 + \lambda b_1) \geq g'(b_1 + \lambda b_2) \quad \forall \lambda \in (0, 1). \quad (6)$$

Since  $b_1 > b_2$  implies  $b_1 + \lambda b_2 \geq b_2 + \lambda b_1$ , (6) holds when  $g$  is linear or concave. Thus, if there is a constant or diminishing marginal effect of the installed base on a product's value then the business gift effect harms the firm with the larger installed base and benefits the firm with the smaller installed base.

The business effect also applies to the larger firm when spillovers are complete (that is,  $\lambda = 1$ ):

$$\begin{aligned} & \Omega^c(b_1, b_2, 1) - \Omega^{in}(b_1, b_2) < 0 \\ \Leftrightarrow & \frac{\exp\left(\frac{\theta g(b_1+b_2)-p_1}{\sigma}\right)}{\exp\left(\frac{\theta g(b_1+b_2)-p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2+b_1)-p_2}{\sigma}\right)} < \frac{\exp\left(\frac{\theta g(b_1)-p_1}{\sigma}\right)}{\exp\left(\frac{\theta g(b_1)-p_1}{\sigma}\right) + \exp\left(\frac{\theta g(b_2)-p_2}{\sigma}\right)} \\ \Leftrightarrow & g(b_2) < g(b_1) \Leftrightarrow b_2 < b_1 \end{aligned}$$

because  $g' > 0$ .

In sum, product compatibility reduces the market share of the firm with the larger installed base when either  $g$  is linear or concave and/or  $\lambda$  is sufficiently close to one. For the business gift effect to instead imply that the smaller firm's market share is reduced with compatible products, necessary conditions are that  $\lambda \ll 1$  and  $g$  is sufficiently convex.

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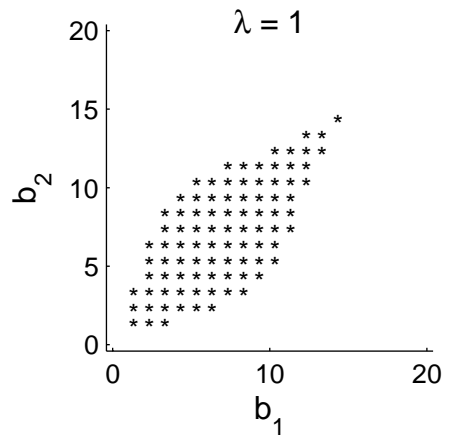
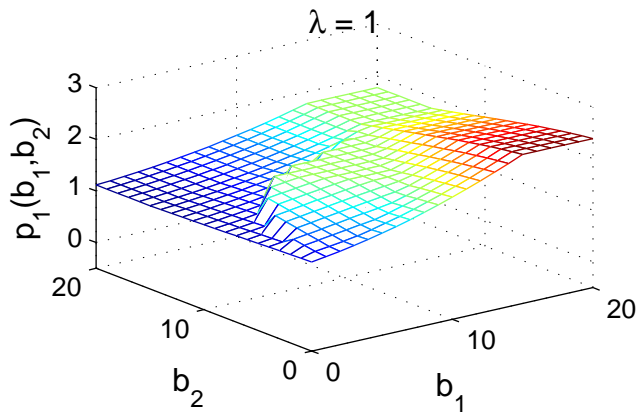
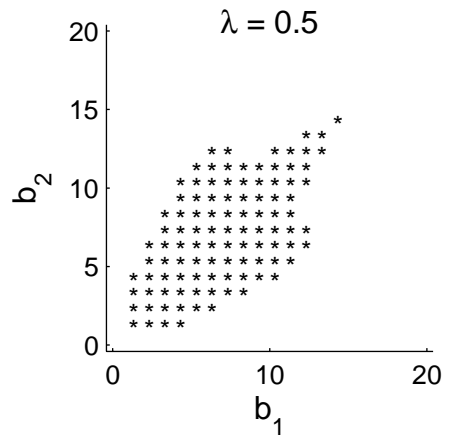
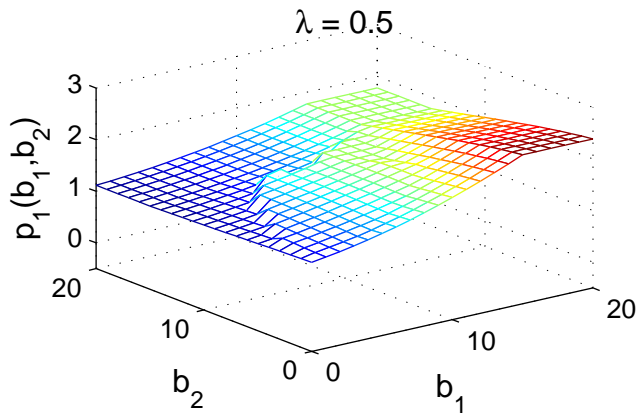
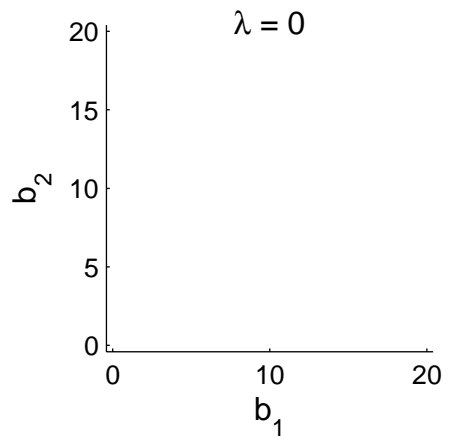
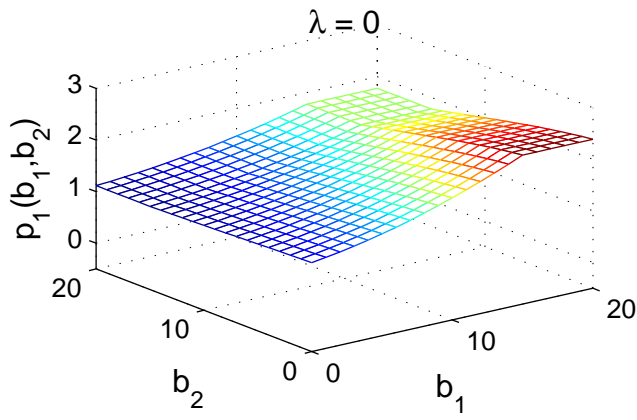
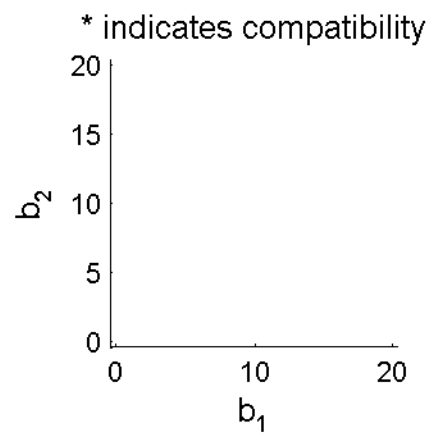
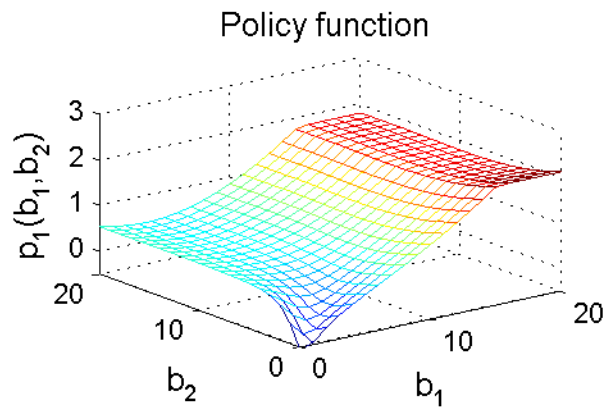
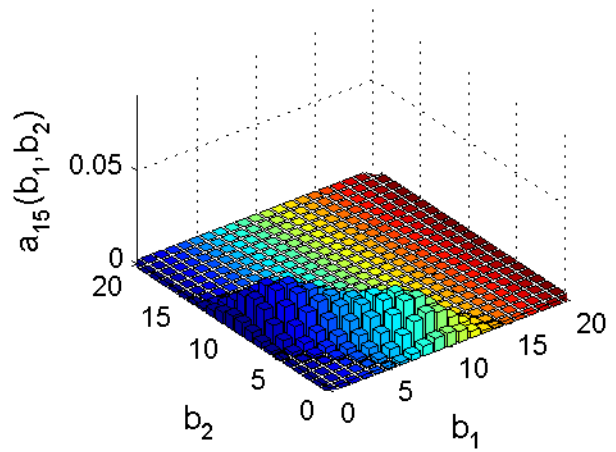
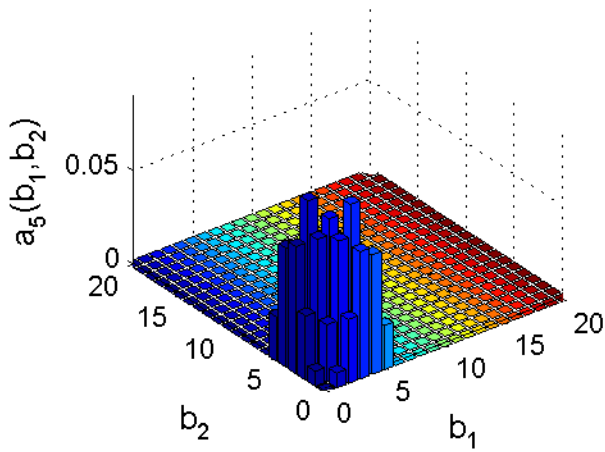


Figure 1. Policy function and compatibility region (static equilibrium).  $v_0 = 0$ ,  $\sigma = 1$ ,  $\theta = 3$ .



Transient distribution after 5 periods

Transient distribution after 15 periods



Transient distribution after 25 periods

Limit distribution

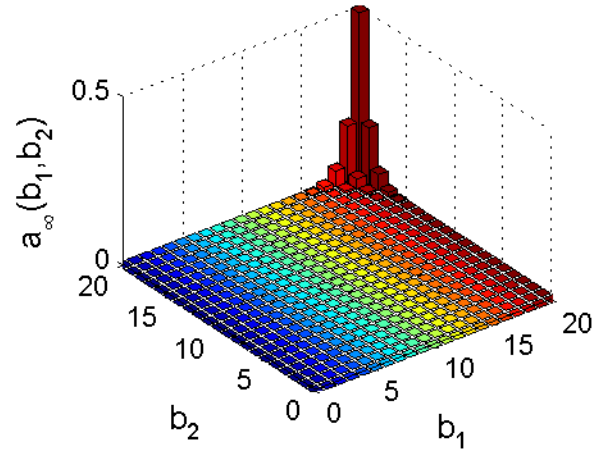
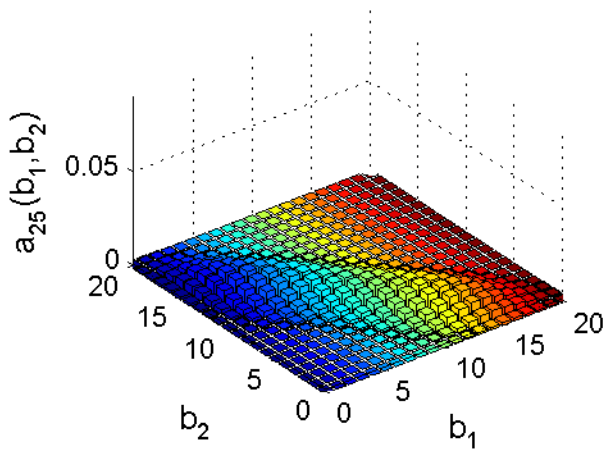


Figure 2.  $v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.01$ ,  $\theta = 3$ ,  $\lambda = 0$ .

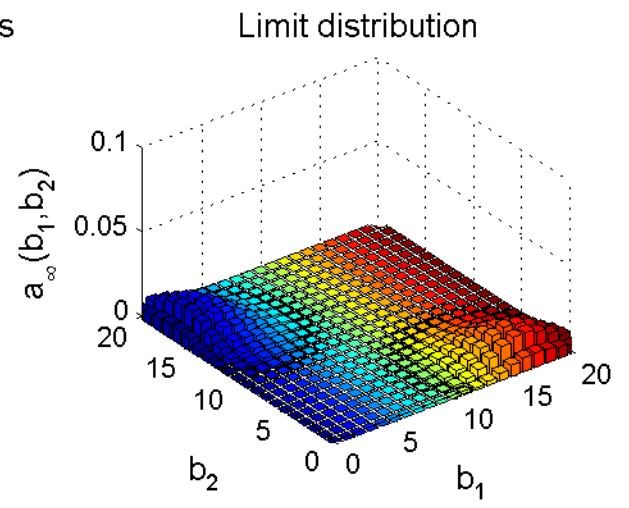
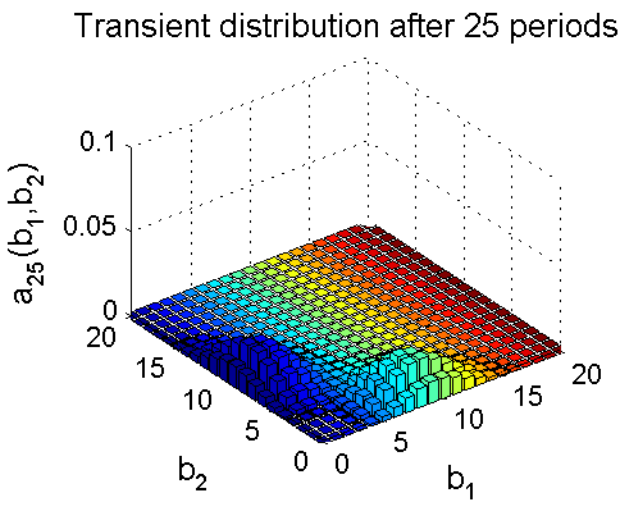
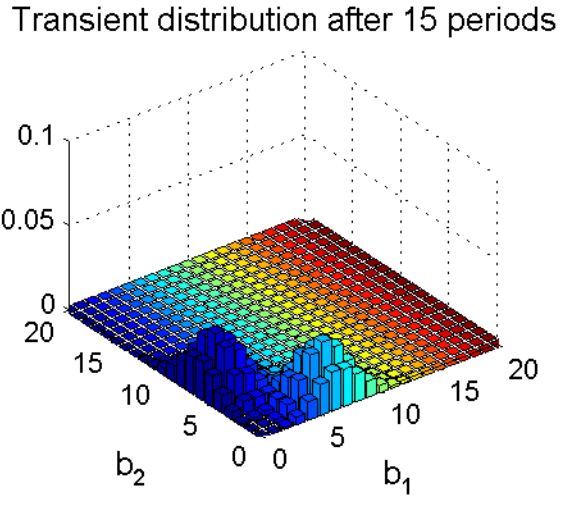
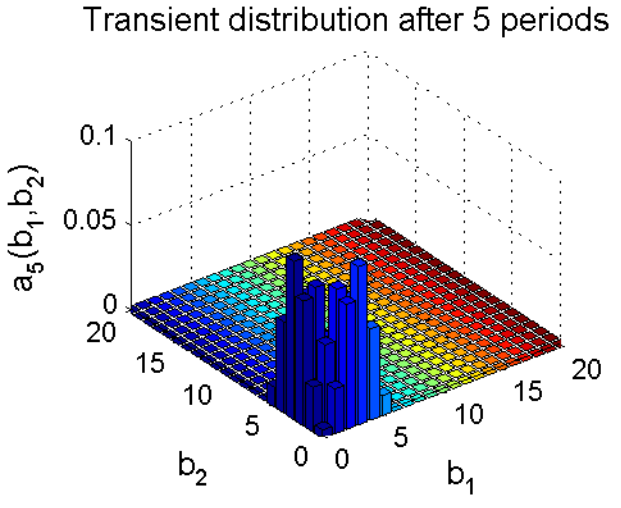
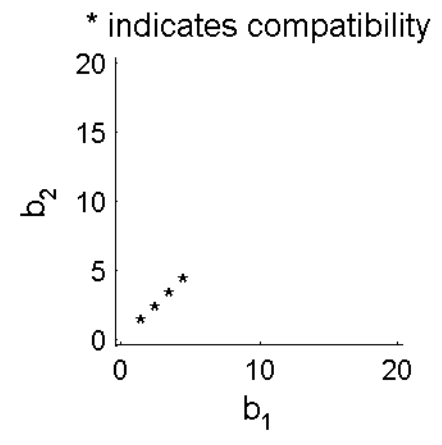
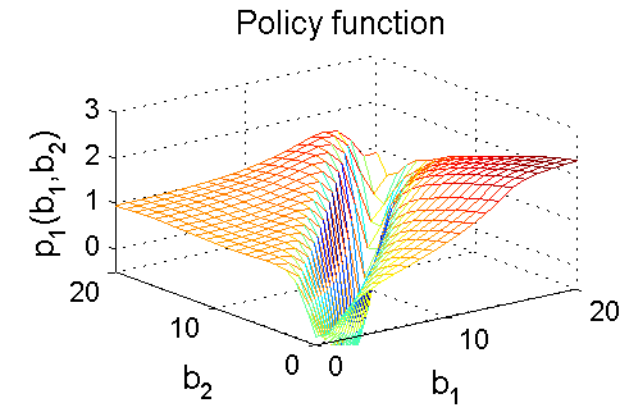
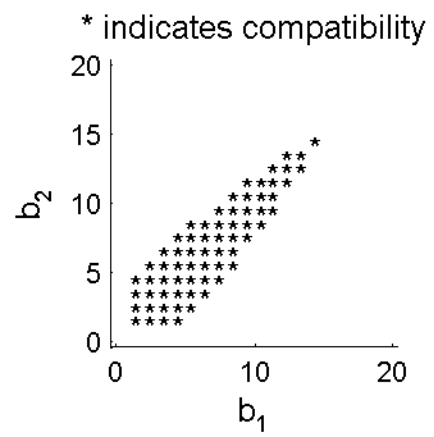
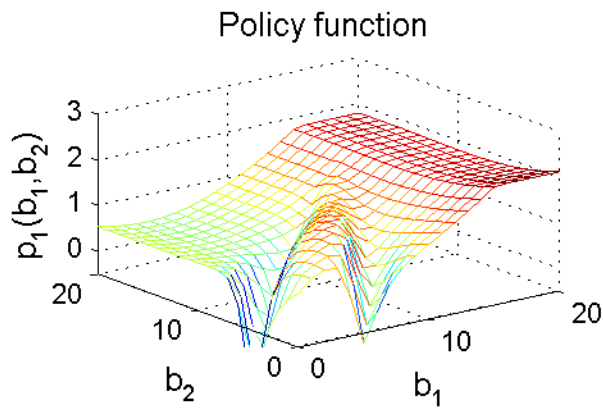
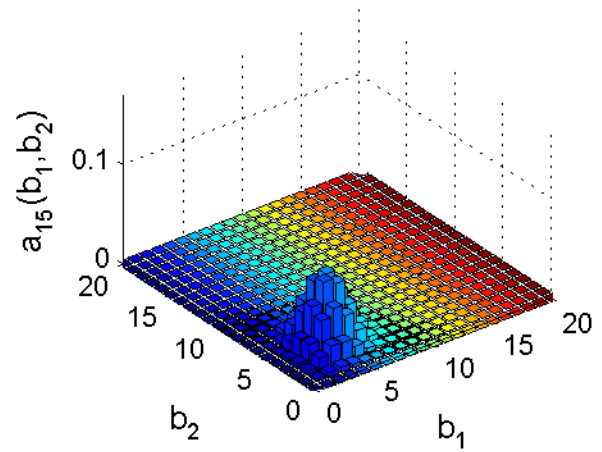
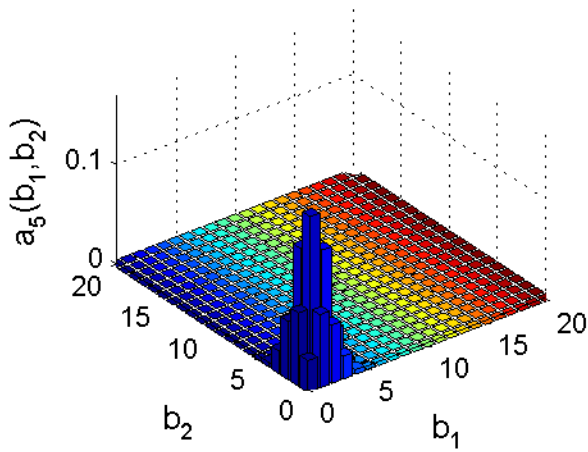


Figure 3.  $v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.06$ ,  $\theta = 3$ ,  $\lambda = 1$ .



Transient distribution after 5 periods

Transient distribution after 15 periods



Transient distribution after 25 periods

Limit distribution

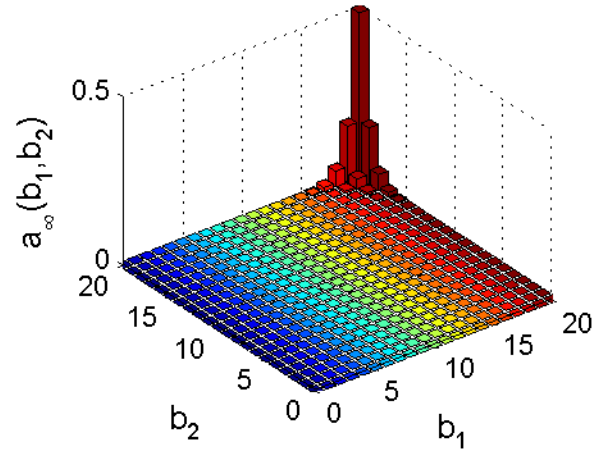
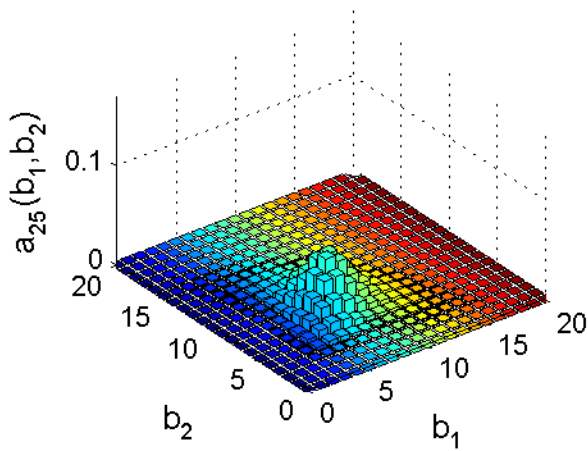
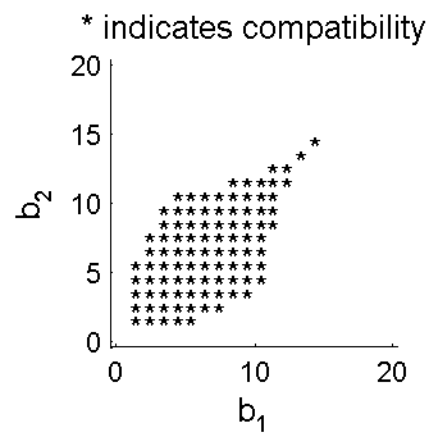
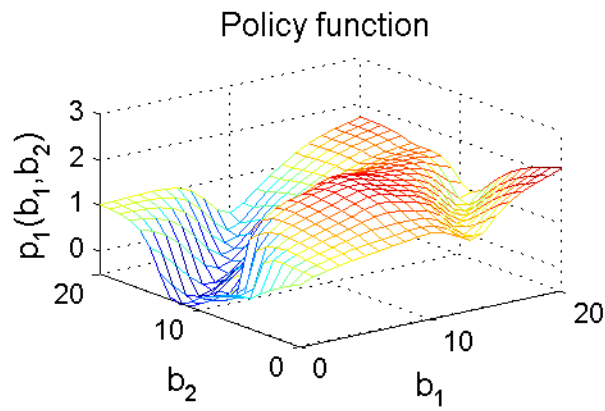
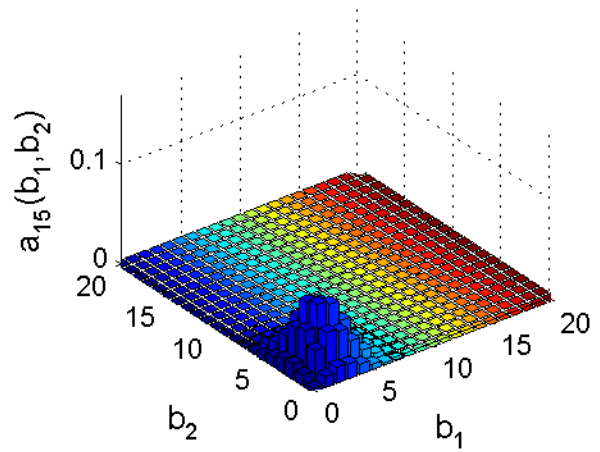
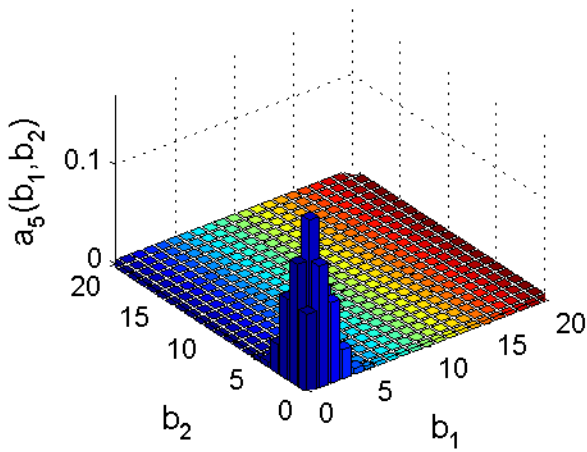


Figure 4.  $v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.01$ ,  $\theta = 3$ ,  $\lambda = 1$ .



Transient distribution after 5 periods

Transient distribution after 15 periods



Transient distribution after 25 periods

Limit distribution

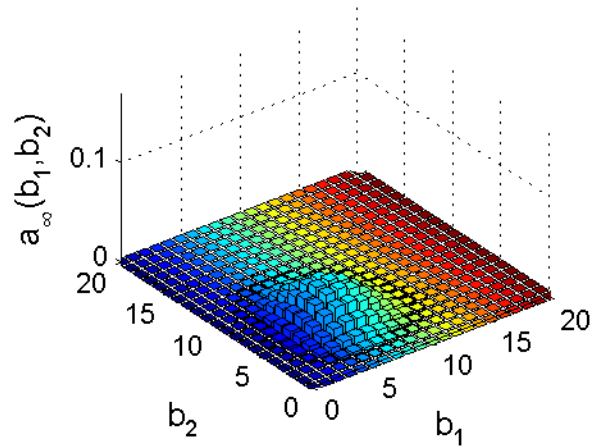
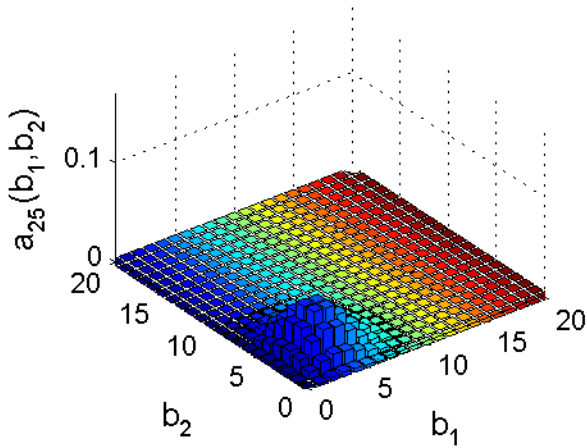


Figure 5.  $v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.08$ ,  $\theta = 3$ ,  $\lambda = 1$ .

Figure 6. Price function  $p_1(b_1, b_2)$ .  $v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.01$ ,  $\theta = 3$ ,  $\lambda = 1$ .

Compatibility is indicated by shaded background, negative prices are boxed.

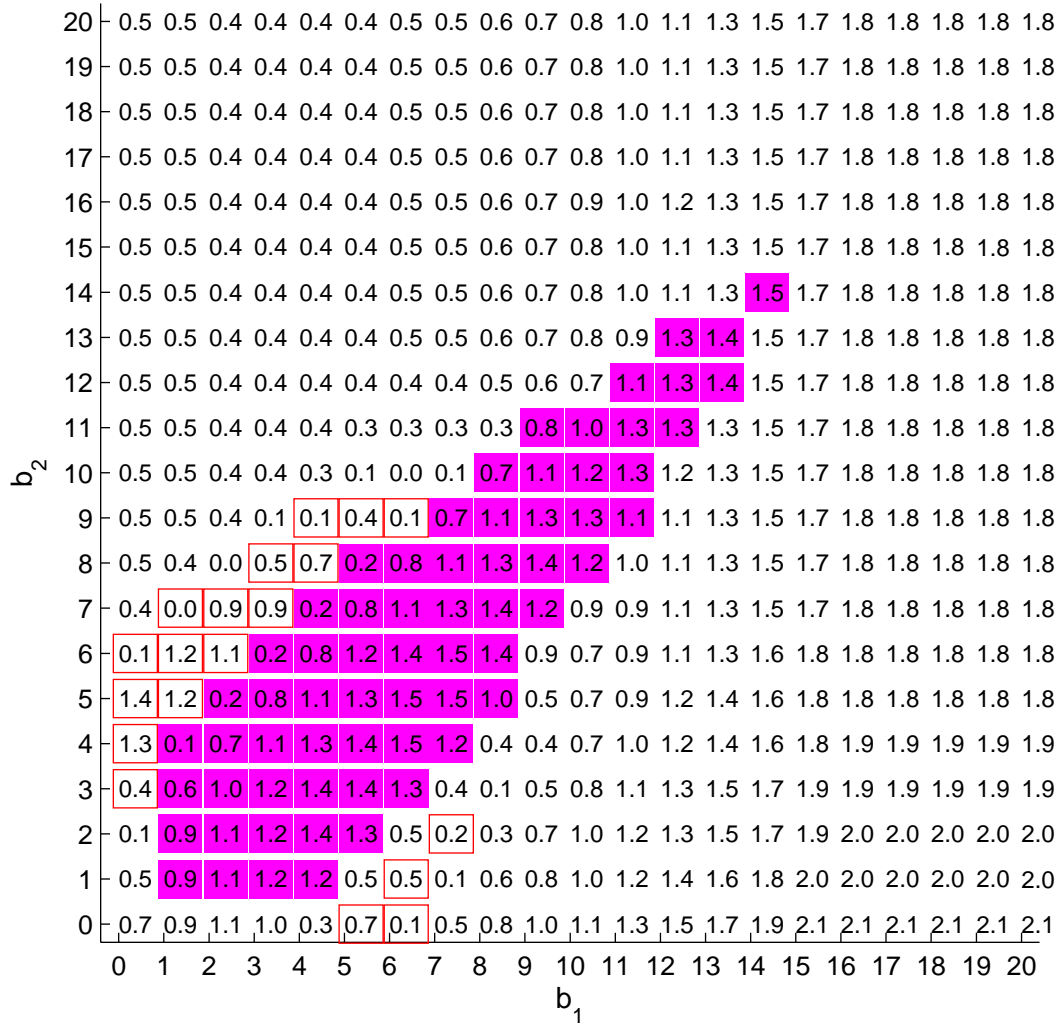
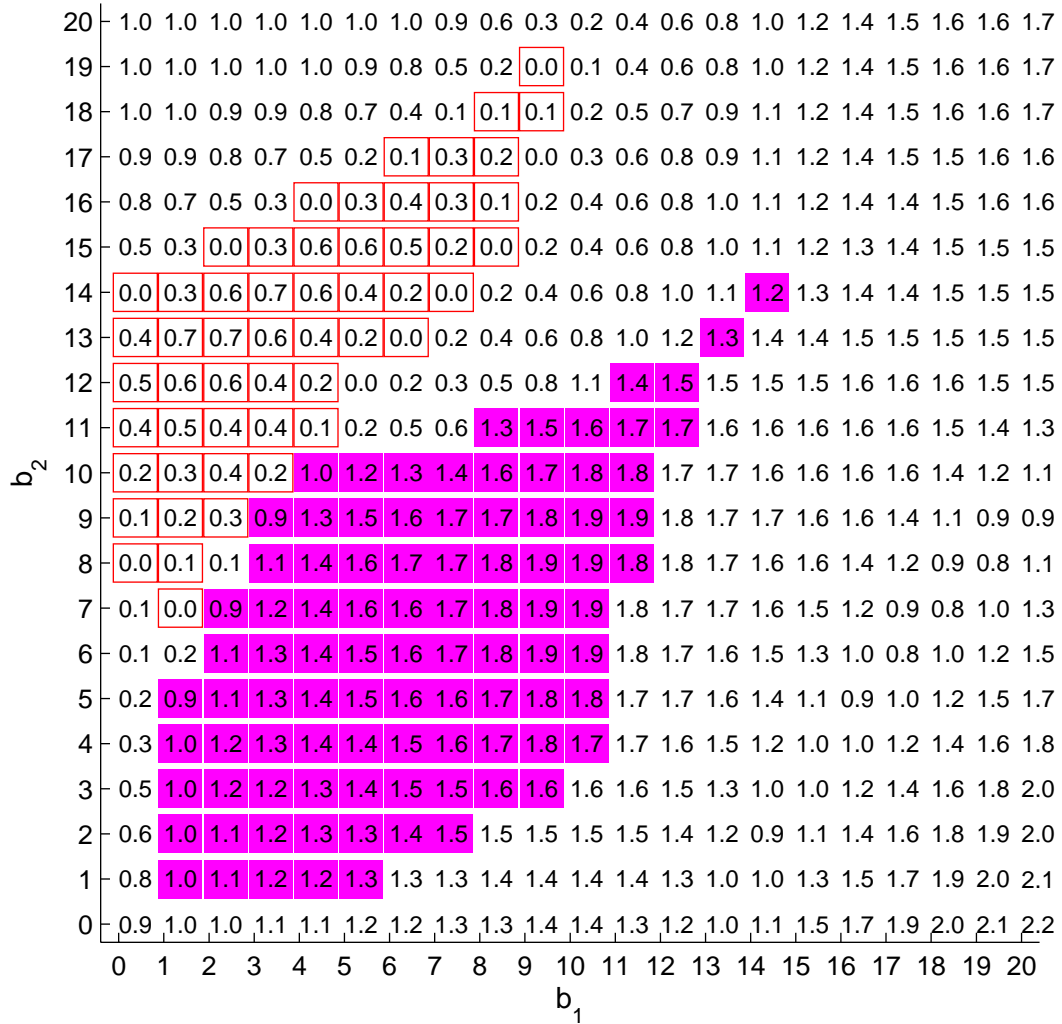


Figure 7. Price function  $p_1(b_1, b_2)$ .  $v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.08$ ,  $\theta = 3$ ,  $\lambda = 1$ .

Compatibility is indicated by shaded background, negative prices are boxed.



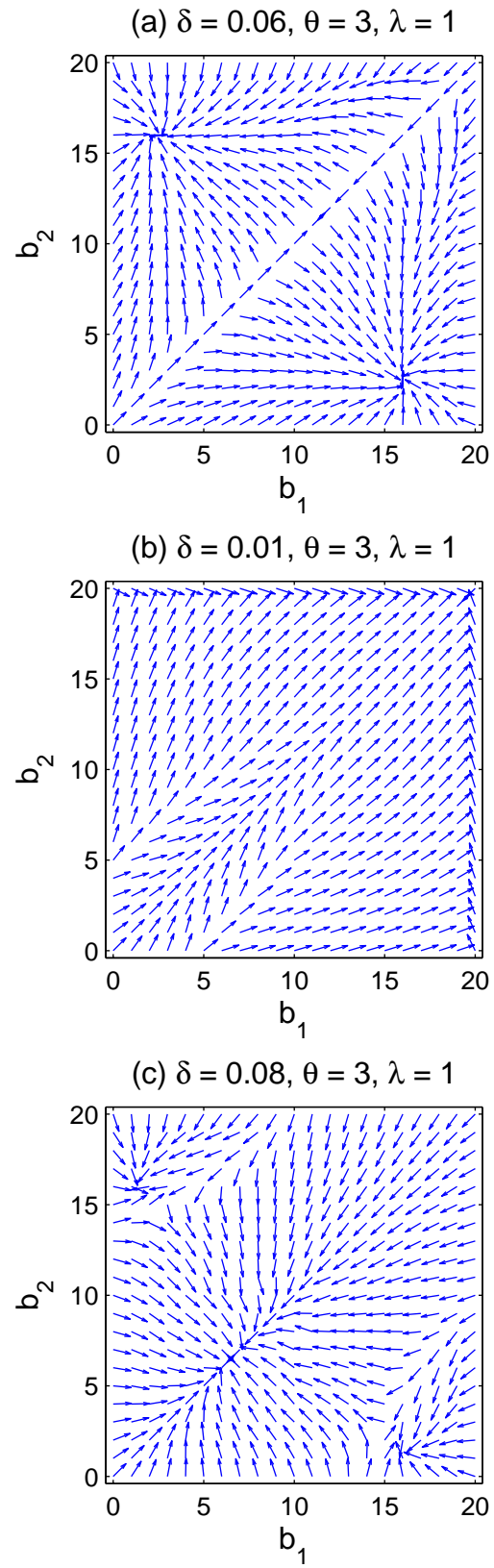


Figure 8. Resultant forces.  $v_0 = 0, \sigma = 1$ .

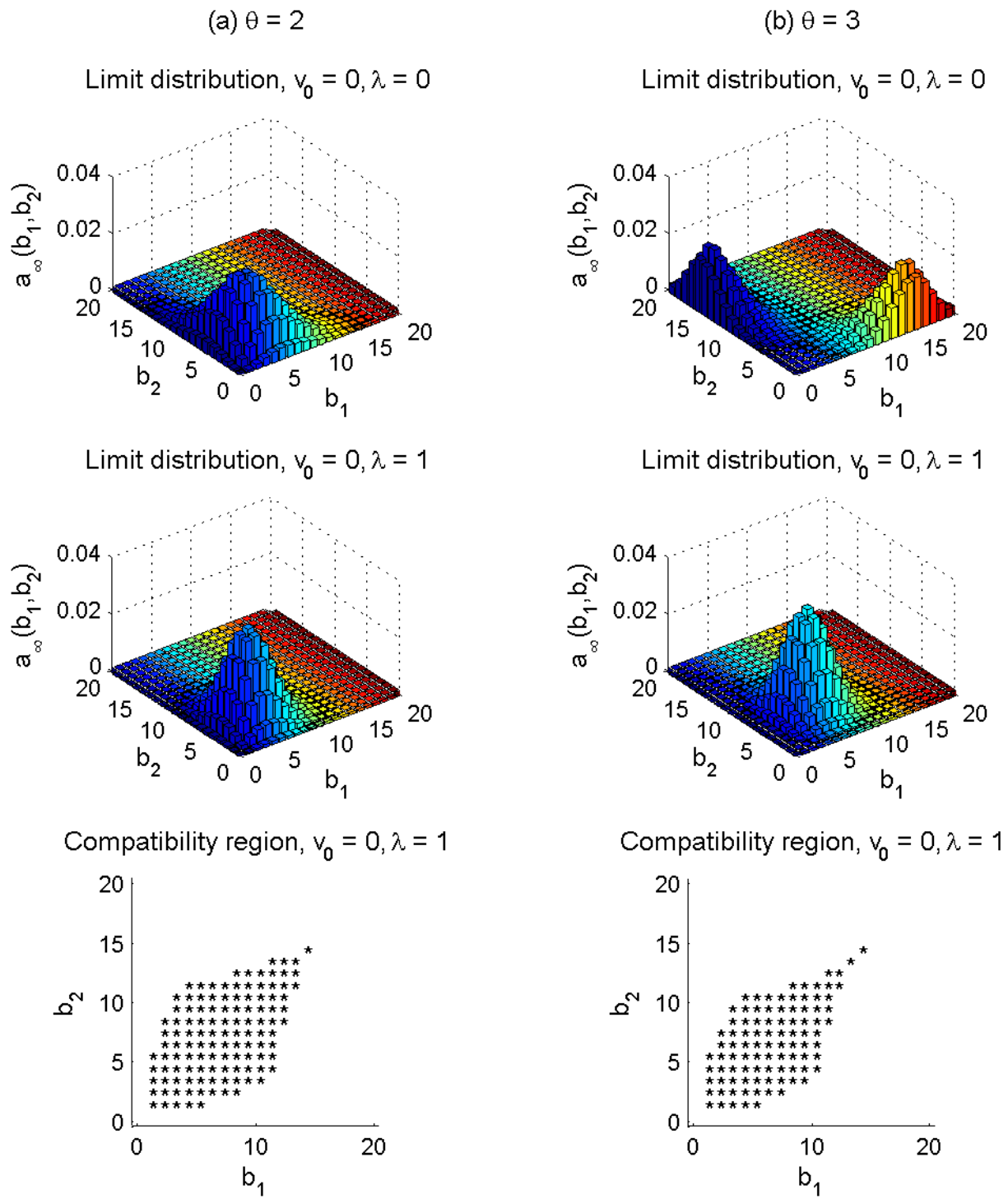


Figure 9. Limit distribution and compatibility region.

\* indicates compatibility.  $\sigma = 1, \delta = 0.08$ .

Figure 10. Long-run Herfindahl Index ( $H^\infty$ ). Solid lines:  $\lambda = 0$ , dashed lines:  $\lambda = 1$ .

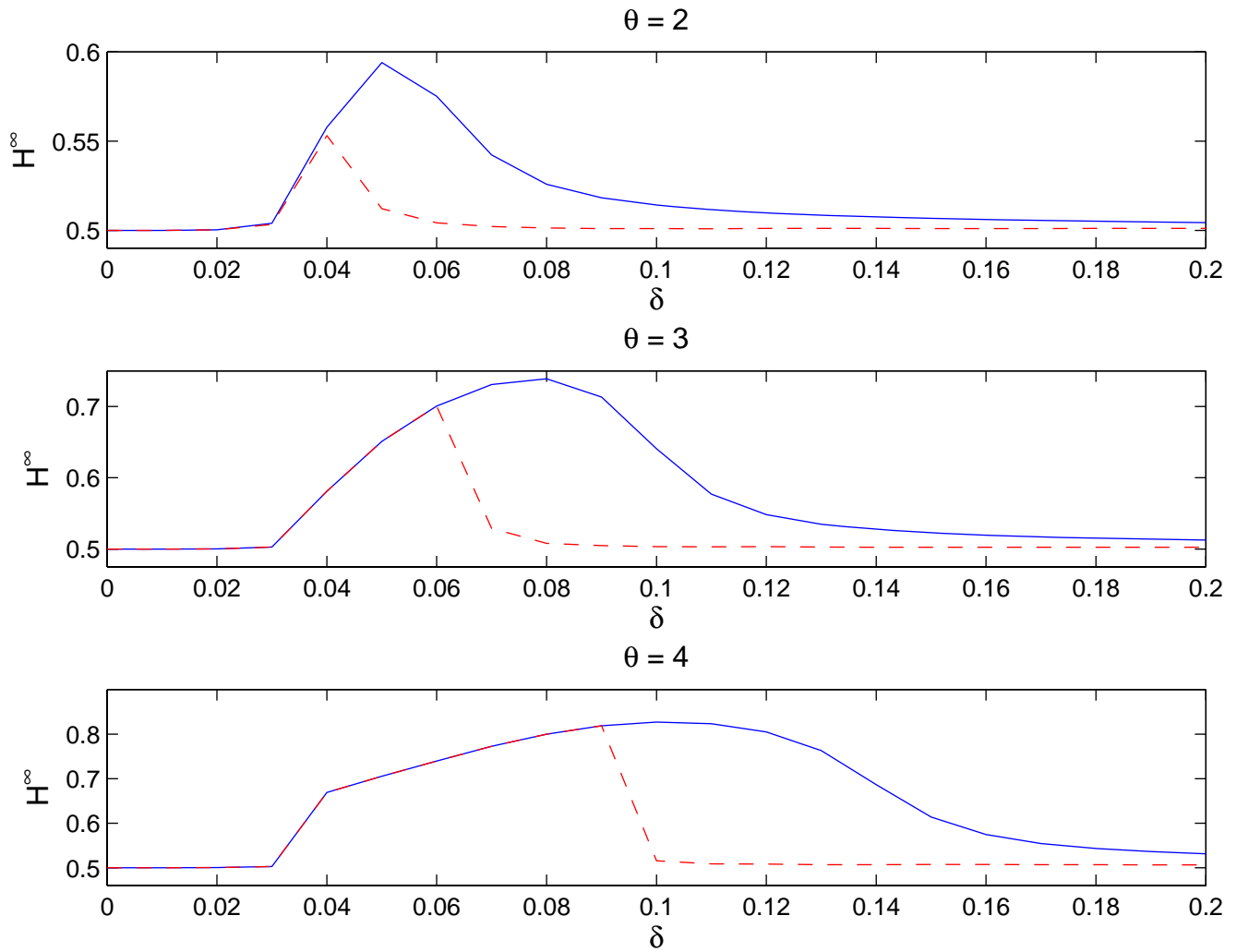
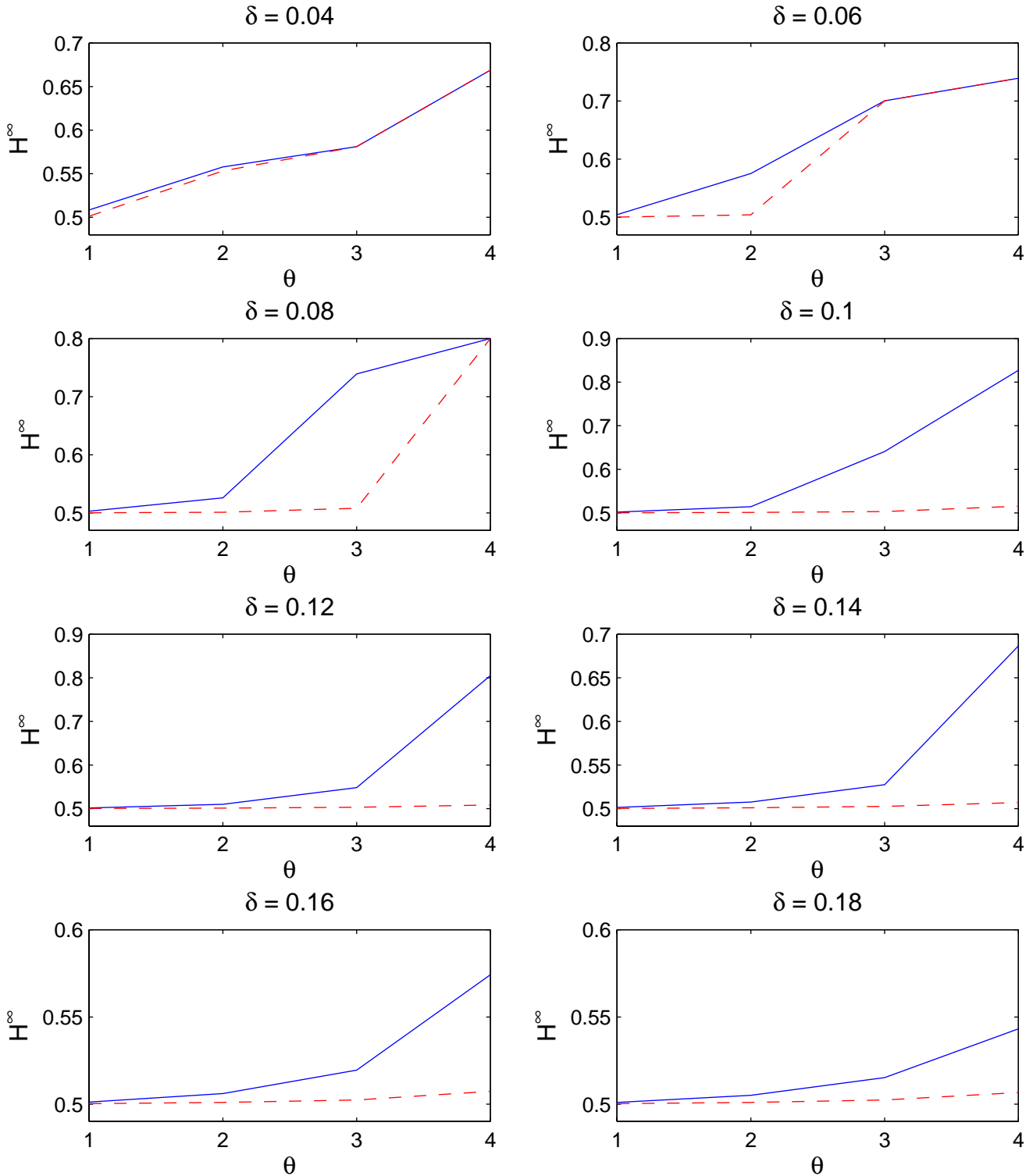


Figure 11. Long-run Herfindahl Index ( $H^\infty$ ). Solid lines:  $\lambda = 0$ , dashed lines:  $\lambda = 1$ .



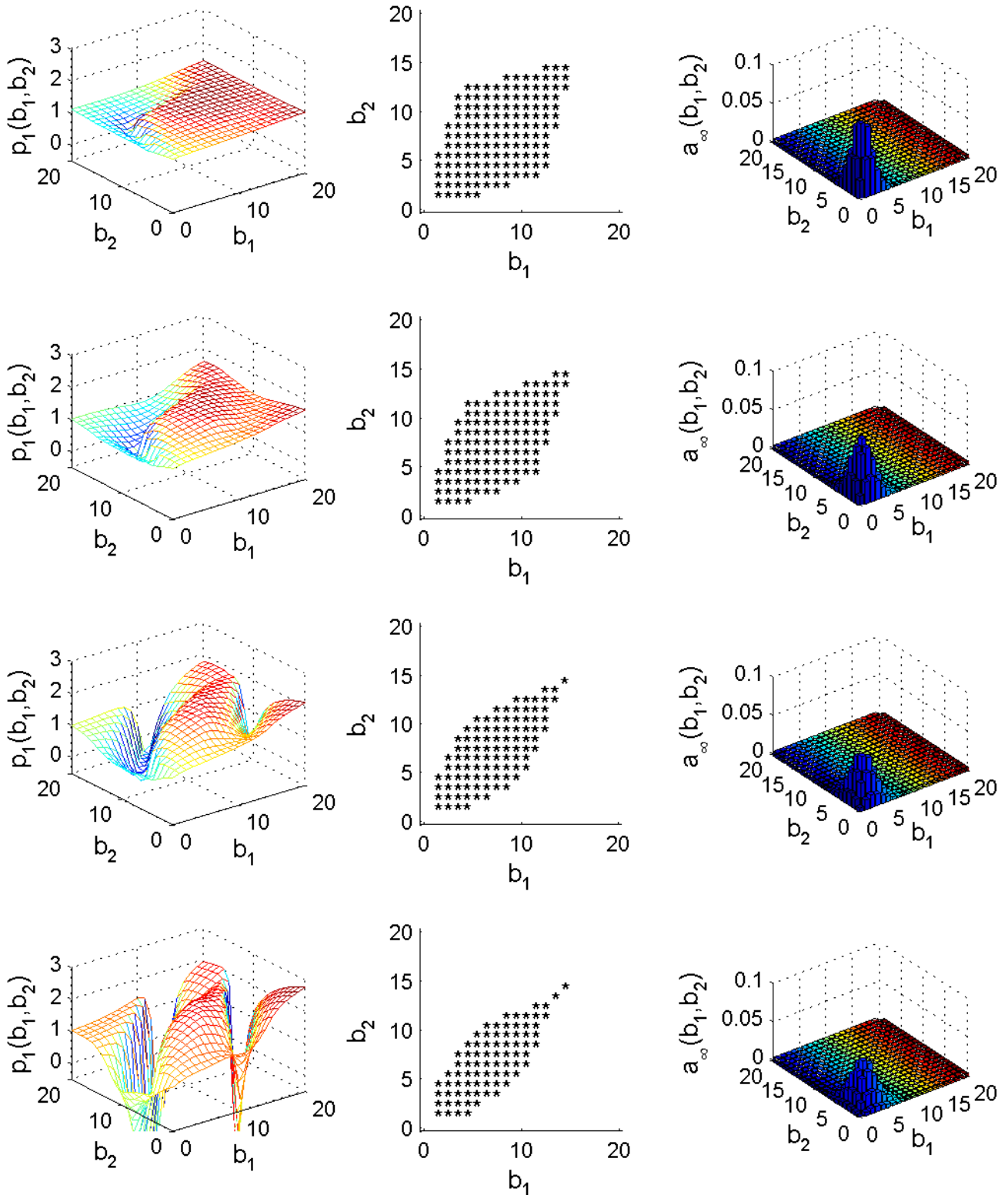


Figure 12. Policy function, compatibility region, and limit distribution.

\* indicates compatibility.

$v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.14$ ,  $\theta = 1, 2, 3, 4$  (from top to bottom),  $\lambda = 1$ .

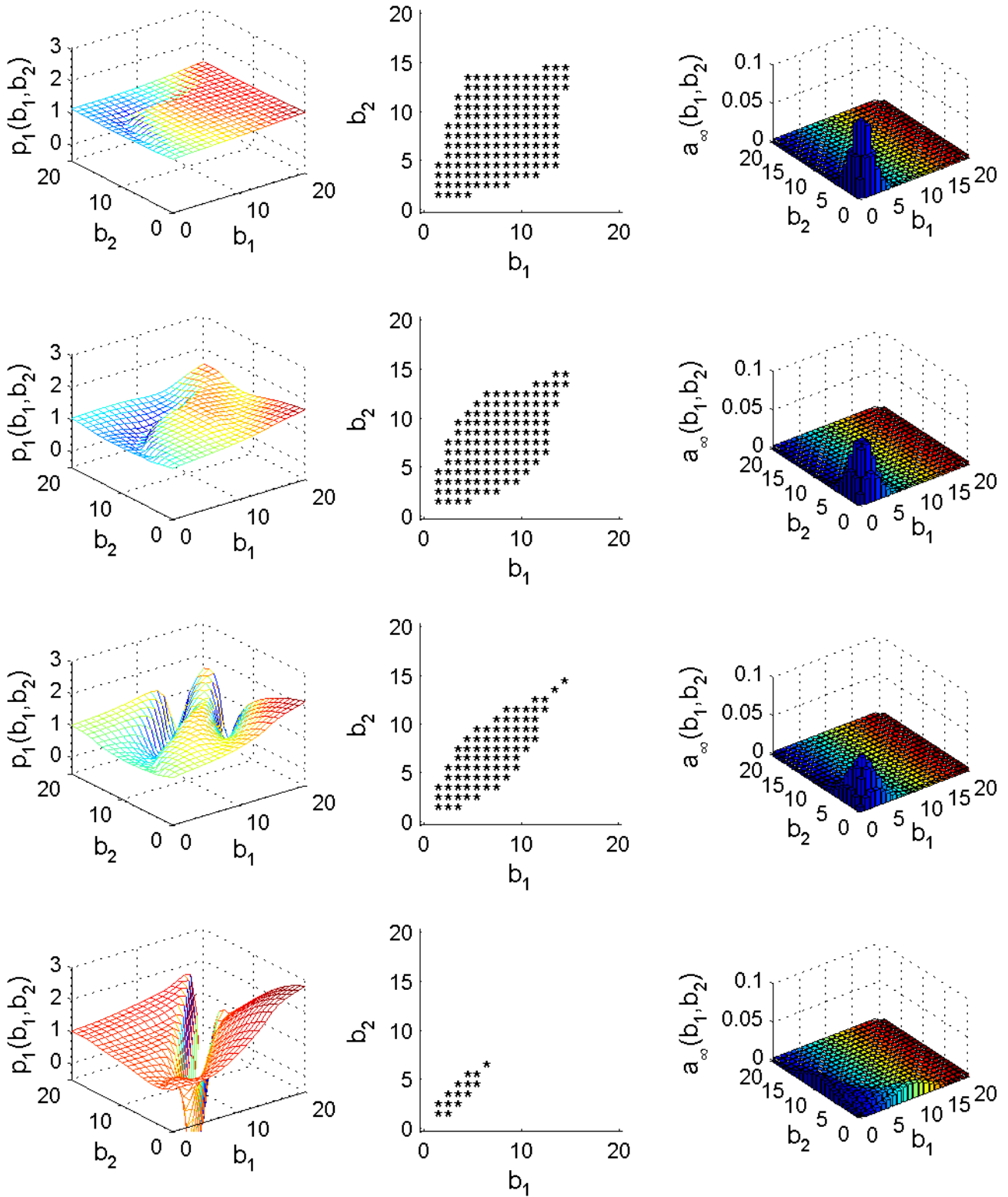


Figure 13. Policy function, compatibility region, and limit distribution.

\* indicates compatibility.

$v_0 = 0$ ,  $\sigma = 1$ ,  $\delta = 0.14$ ,  $\theta = 1, 2, 3, 4$  (from top to bottom),  $\lambda = 0.5$ .

Figure 14. Probability of compatibility.  $\lambda = 1$ .

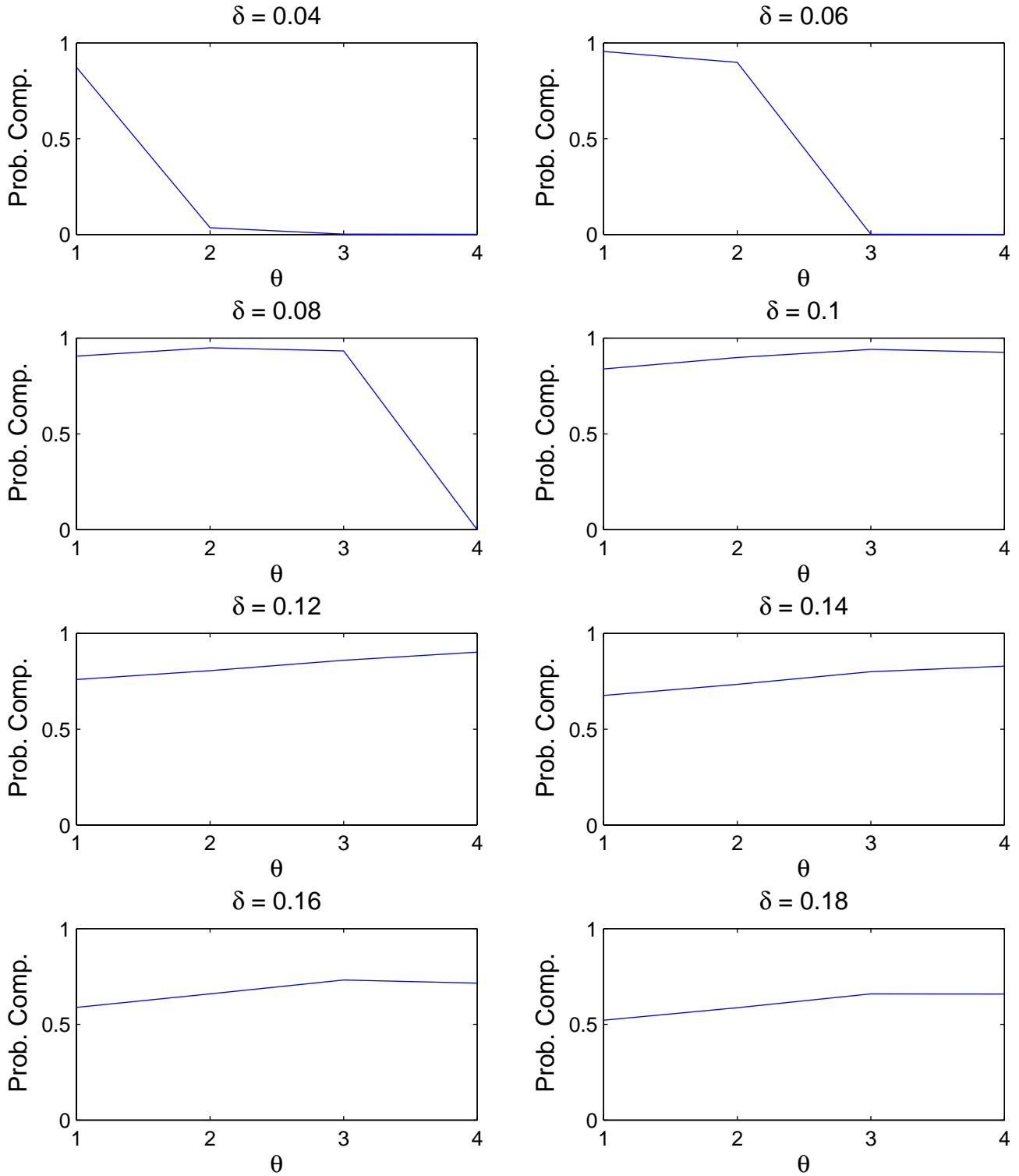


Table 1

Parameter	Range	Description
$\theta$	0, 1, 2, 3, 4	Network effect
$M$	20	Maximum base
$m$	15	Base where network effect maxes out
$\lambda$	0, .5, 1	Degree of compatibility or spillover
$\delta$	0, .01, ..., .20, .25, ..., 1	Customer turnover or depreciation
$\sigma$	.5, 1	Horizontal differentiation
$\alpha$	1	Price sensitivity
$v_0$	$-\infty, -3, -1, 0$	Value of outside option
$v$	0	Intrinsic product quality
$c$	0	Marginal cost
$\beta$	1/1.05	Discount factor
$N$	2	Number of firms

Table 2. Type of Equilibrium,  $v_0 = 0$ ,  $\sigma = 1$

$\lambda = 0$		$\delta$																				
		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
$\theta$	1	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
	2	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
	3	R	R	R	RT	RT	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
	4	R	R	R	RT	RT	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T

$\lambda = 0.5$		$\delta$																				
		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
$\theta$	1	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
	2	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
	3	D	D	R	RT	RT	T	T	T	T	T	TD	TD	D	D	D	D	D	D	D	D	D
	4	R	R	R	RT	RT	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T

$\lambda = 1$		$\delta$																				
		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
$\theta$	1	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
	2	D	D	D	D	D	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
	3	D	D	D	RT	RT	T	T	D	D	D	D	D	D	D	D	D	D	D	D	D	D
	4	R	R	R	RT	RT	T	T	T	T	T	D	D	D	D	D	D	D	D	D	D	D

F: Flat  
R: Rising  
T: Trenchy  
D: Dual Trenchy

RT: Morphing from Rising to Trenchy  
TD: Morphing from Trenchy to Dual Trenchy  
M: Mild Dual Trenchy

**Table 3. Mode of the limit distribution,  $v_0 = 0$ ,  $\sigma = 1$ .**

$\delta$	$\theta = 2$		$\theta = 3$		$\theta = 4$	
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$
0	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)
0.01	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)
0.02	(19,20)	(19,20)	(20,20)	(20,20)	(20,20)	(20,20)
0.03	(16,16)	(16,16)	(18,18)	(18,18)	(18,20)	(18,20)
0.04	(6,16)	(6,16)	(5,20)	(5,20)	(4,20)	(4,20)
0.05	(3,15)	(9,9)	(3,18)	(3,18)	(2,20)	(2,20)
0.06	(3,11)	(8,8)	(2,16)	(2,16)	(1,20)	(1,20)
0.07	(4,5)	(6,6)	(1,15)	(8,8)	(1,17)	(1,17)
0.08	(3,4)	(5,5)	(1,14)	(7,7)	(1,16)	(1,16)
0.09	(3,3)	(4,4)	(1,12)	(5,5)	(0,15)	(0,15)
0.1	(2,3)	(3,3)	(1,9)	(4,4)	(0,15)	(6,6)
0.11	(2,2)	(3,3)	(3,3)	(4,4)	(0,14)	(5,5)
0.12	(2,2)	(2,2)	(2,3)	(3,3)	(0,13)	(4,4)
0.13	(2,2)	(2,2)	(2,2)	(3,3)	(0,11)	(4,4)
0.14	(1,2)	(2,2)	(2,2)	(2,2)	(0,9)	(3,3)
0.15	(1,1)	(2,2)	(1,2)	(2,2)	(2,2)	(3,3)

1. For an asymmetric mode, there is a second mode in which the installed bases of the two firms are reversed.
2. The cases in which there is a bimodal distribution for  $\lambda = 0$  and a unimodal distribution for  $\lambda = 1$  are highlighted.