The Coase Conjecture in Continuous Time:
Imperfect Durability, Endogenous Durability, and Aftermarkets

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INTRODUCTION

With the exceptions of Bond and Samuelson (B&S) (1984) and Bulow (1986), the extensive literature on the Coase (1972) conjecture deals only with goods of infinite durability. As Coase argued, in a continuous time setting where the monopolist can change price without notice, price instantly falls to marginal cost if the good is infinitely durable. But we know little about how this intuition carries over to limited durability, which is the case with virtually all real-world goods. Stokey (1981) introduced two analytical tools to the Coase discussion: (i) perfect rational-expectations equilibrium (PREE), and (ii) discrete time in the form of a "trading period," a repeated exogenous time interval during which the monopolist is unable to cut the price.

A PREE is a time-consistent profit-maximizing time path of production, subject to the restriction that consumers' expectations regarding future output be always realized, and furthermore that following any perturbation from the profit-maximizing path, expectations will continue to be realized along the new profit-maximizing path.

Stokey shows that with perfect durability and continuous time, the only PREE is instant market saturation, as Coase had originally argued.

This leads Stokey to introduce the trading period. Sure enough, by partially restoring the commitment power which Coase had taken away from his monopolist, monopoly profit is partially restored. The longer the trading period, the greater is the restoration. Stokey finds that price approaches marginal cost, after a sufficient number of trading periods has elapsed,regardless of the length of the individual trading period (denoted as **z**).

The literature offers no economic rationale for the trading period, nor any plausible mechanism whereby a monopolist can credibly precommit to a trading period.[1] Neither Stokey nor any subsequent author offers a serious discussion of where the trading period comes from. Thus the literature leaves us with the twin artificial constructs of infinite durability and a finite (and quite long) trading period.

Note in particular that menu or price-adjustment costs would not give rise to a finite trading period. Menu costs, once incurred, are sunk, and *ex post* the firm will want to cut price immediately after sales have dried up from the previous price cut (i.e., in the twinkling of an eye).

We believe it is more fruitful to retain Coase's reasonable assumption of continuous time[2] and relax the unreasonable assumption of infinite durability, rather than the other way around. Our goods are of limited durability, and time is continuous; the firm is given no free commitment power with which to save itself from time inconsistency. In Part I, the durability of the good is exogenous. We find that the firm retains substantial monopoly power except for very durable goods, even without the free commitment power given by discrete time.

In Part II, the level of durability is set by the monopolist. Given the negative relationship between durability and ability to retain monopoly power, this monopolist cuts durability below its efficient level to save itself from the Coase temptation. As a result it produces the good in a

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[1]As we will show, trading periods must be quite long (in the range of a year or more) in order to have much of a damping effect on the Coase discipline.

[2]By "continuous time," we mean that the firm has the power to cut price at any instant, regardless of his history of prior price cuts.

technically inefficient manner and recovers much of the monopoly profit that it would have lost had durability been exogenous. As Bulow (1986) noted in a two-period setting, the Swan (1972) independence result breaks down in the presence of the Coase temptation.

In Part III we consider the important case in which the lifetime of the good is jointly determined by built-in durability (as in Part II, above) and consumers' maintenance decisions (as in Schmalensee (1974) and Rust (1986)). In this setting we deal specifically with the case in which the monopolist has power not only over the original equipment but also over some replacement/repair parts. We find that the monopolist charges a much higher markup on parts than on original equipment. To a good approximation, he escapes the Coase temptation by pricing the durable good at marginal cost (giving in to the temptation) and taking a high markup on the flow demand for parts. Power over parts further restores the sales monopolist's credibility and profit, at the cost of a still more inefficient production plan.

Within real-world ranges of durability, even without a trading period, we find no plausible circumstance in which the durable-goods sales monopolist behaves approximately competitively. Durability gives no reason to believe that monopoly is benign; "competition from installed base" should not be accepted as a part of an antitrust defense.

I. EXOGENOUS DURABILITY

Consider a product which lasts for a timespan **d** and then dies like the one-hoss shay. Unit production cost is **C**. Inverse demand for the service flow is

$$\mathbb{R}_t = \alpha - \gamma X_t \qquad \textbf{(1)}$$

Without physical depreciation (until death at **d**), X_t is the aggregate of production over the timespan from **t-d** to **t**:

$$X_t = \int_{\tau = t-d}^{t} x_\tau \, d\tau \qquad \textbf{(2)}$$

where X_t is the stock and x_t is the flow. The firm chooses a path of x_t to

$$\max \Pi_t = \int_{\tau = t}^{\infty} x_\tau (p_\tau - C) e^{-\rho \tau} \, d\tau \qquad \textbf{(3)}$$

subject to the time-consistency requirement that at each future date the remaining path be optimal. p_τ is the price of the good at time τ, which is in turn the present value of consumers' expected future R(X).

Definitions

x_t : the instantaneous rate of production at time t.

q_t : an instantaneous production "spike" at time t

X_t: the stock of the good at time t.

$$X_t = \sum_{t-d}^{t} q_t + \int_{t-d}^{d} x_t dt$$

H_t: the history of production from t-d to t; i.e., the age distribution of the standing stock.

Expectations

Under infinite durability, or exponential-decay depreciation, X_t is a sufficient statistic for H_t; the age distribution of the stock is irrelevant. But under one-hoss-shay depreciation, the age of the stock matters; a given stock age d-ϵ is not the same as the same stock age ϵ, though both stocks yield the same service at time t. Consumers' expectations for future production must be conditioned on the age distribution of the stock, H, not just the scalar X. They form expectations rationally; that is, they expect the firm to maximize future profits, subject to a time consistency constraint. Expectations are represented as

$$G(s,t,H_t) = x_s, q_s | H_t ; \quad \forall s > t$$

As in Stokey, we require expectations to be consistent; if expectations up to time s are realized, then expected production at time s is not revised.

Proposition 1: There is a unique **Rational-Expectations Steady State (RESS)**, given the cost and demand parameters of the problem. A RESS is a production plan of the following form:

$$\frac{H_0 = \begin{matrix} x = \hat{x} & \forall t \leq 0 \\ q = 0 & \forall t \leq 0 \end{matrix};}{\begin{matrix} x = \hat{x} & \forall t > 0 \\ q = 0 & \forall t > 0 \end{matrix}}$$

A steady state is a smooth age distribution over all relevant History, and for all future time. The proposition states that there exists a steady state, characterized by the constant flow rate x*, such that if the rate x* has persisted from t = -d to t = 0, the firm will not deviate from this production rate in the future.

Following Stokey, we specify (point) expectations as follows:

$$G(s,t,X) \; = \; E(X_s \mid X_t) \tag{4}$$

where G(·) defines consumers' expected stock at any future date **s**, as of time **t**, given the time-t stock, $\mathbf{X_t}$. The trading period **z** disappears from the function since it is equal to zero.

The typical research strategy at this stage of the analysis is to pick a functional form for G(·) and solve for its parameters. Generally (following Stokey) the form is postulated to be

$$G(\cdot) \; = \; X + \mu^{s-t}(X - \bar{X}) \; ; \tag{5}$$

that is, a constant fraction of the gap between current and limiting stock is closed in each trading period. $\bar{X}$ is the limiting stock which the monopolist approaches asymptotically. The model can then be solved for μ and $\bar{X}$, the rate of adjustment and the limiting stock.

Note that there is nothing "rational" about consumers' choice of the functional form (5); therefore, the solutions to these models are not necessarily unique nor even very plausible. The choice of the G(·) functional form is a more serious matter than, for example, choosing a functional form for a demand function. As we argue below, at least in the case of continuous time rationality itself dictates the functional form for G.

In continuous time, faced with rational consumers, the firm cannot extract any profit by holding X below the limiting stock, even for the twinkling of an eye. This of course is the original Coase insight. Thus, in continuous time, the only functional form for G(·) consistent with rationality is the first line of (6):

$$G(s,t,X) = \bar{X} \quad \forall \quad s \geq t, \quad X < \bar{X}$$
$$G(s,t,X) = X \quad \forall \quad s \geq t, \quad X \geq \bar{X} \qquad \textbf{(6)}$$

To see this, suppose $X < \bar{X}$. If the firm were to close only part of the gap in the first instant (hoping to keep the price up), it would be tempted to close more of the gap in the next instant. Consumers, knowing both $\bar{X}$ and the firm's inability to commit, wait a few instants. Thus $p(X) = p(\bar{X}) \ \forall \ X$ so the monopolist maximizes profit by immediately setting $X = \bar{X}$. So (6) (top line) is the only rational expectations function in continuous time. Clearly, then, the continuous-time rational consumer's only problem is to solve for $\bar{X}$.[3]

Expectations when $\mathbf{X \geq \bar{X}}$

Now note the second line of (6), which defines expectations if current stock exceeds $\bar{X}$. In Stokey, and in Bond and Samuelson, expectations are simply not defined if $X > \bar{X}$. Algebraically, (5) implies that stock gradually shrinks if X should be perturbed to a point beyond $\bar{X}$, but in the case of perfect durability this is impossible; the stock never dies. And in the case of imperfect durability, satisfying (5) may also be impossible, since the implied rate of decline in the stock might exceed the rate at which the stock depreciates. In the Stokey-inspired literature, equilibria are only perfect "from below;" the equilibria are simply not defined if $X > \bar{X}$. Of course, profit-maximizing behavior never takes us into this range; nevertheless, perfection of the

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[3]Note that this is true in both Coase and continuous-time Stokey. Without a trading period, the only thing to solve for is $\bar{X}$. Indeed, Coase's original insight, though not expressed in quite these terms, is that (6) is the only functional form consistent with rationality in continuous time.

equilibrium requires that we tidy this detail.

In the range where $X > \overline{X}$, we assume static expectations: consumers believe that if the firm somehow does push X beyond $\overline{X}$, it will not further expand the stock. For their part, of course, firms would not find it profitable to further expand the stock given consumers' expectations. The firm might, however, wish to shrink the stock. We rule this out by assuming the firm never conducts "sales." Price cuts, once announced, are irreversible. Given this assumption, static expectations are rational in the range $X > \overline{X}$; once the firm has inadvertently overshot $\overline{X}$, the best it can do is to refrain from expanding further. In the appendix we discuss the rationale for the no-sales assumption; here we simply note that this assumption is present, though unstated, in all of the Coase literature, and that we adopt it explicitly.

In fact we make virtually no use of the expectations assumption in the region $X > \overline{X}$. Equilibrium does not occur in this region, nor does any equilibrium path of output enter it (starting from any $X \leq \overline{X}$). And no starting point, $X \leq \overline{X}$, generates consumer expectations that X will ever exceed $\overline{X}$.

Solving for $\overline{\mathbf{X}}$

If (6) is *ex post* correct for all time, then consumers must have solved for it at t = 0. Consumers approach this problem as follows: Consider a candidate stock, X*; is it $\overline{X}$? Let us suppose that X* has been maintained by a constant flow of production,

$$x^*_\tau = \frac{X^*}{d} \quad ; \quad t-d \leq \tau \leq t \qquad \textbf{(7)}$$

The age distribution is uniform; if future production is held at x_τ* the stock will remain fixed at X*. Suppose the firm contemplates a small deviation from x_τ*, increasing output above X*. In particular, the firm contemplates a production spike, x**, at t = 0, t = d, t = 2d, etc. This instantly raises the stock to X*+x** and holds it there in perpetuity. X* = $\overline{X}$ if a small increase in stock, x**, is not profitable. Thus we (and consumers) solve for $\overline{X}$ by finding that value of X from which a small permanent increase is not profitable, given expectations. The firm's value at t is

$$\Pi_t = \left(\frac{x^*_\tau}{\rho} + \frac{x^{**}}{1 - e^{-\rho d}} \right) \cdot ([\alpha - \gamma(X^* + x^{**})] \cdot b(\rho,d) - C) \tag{8}$$

where (·) is the discounted flow of future production, [·] is the consumers' rental price given the stock (will permanently be) X*+x**, and

$$b(\rho,d) = \int_0^d e^{-\rho\delta} d\delta = \frac{1 - e^{-\rho d}}{\rho} \tag{9}$$

is the present value operator for the timespan **d**. Thus [·]· b(ρ,d) = price.

$\overline{X}$ is that X at which

$$\frac{\partial \Pi}{\partial x^{**}} = 0 \quad at \quad x^{**} = 0 \tag{10}$$

The solution is

$$\bar{X}^S = \left(\alpha - \frac{C}{b} \right) \left(\frac{d}{(b+d) \cdot \gamma} \right) \tag{11}$$

where the arguments ρ and **d** are henceforth suppressed in b(ρ,d) for compactness.

The competitive and rental-monopoly outputs are characterized by analogs to (11):

$$RENTAL\ \bar{X}^R = \left(\alpha - \frac{C}{b}\right)\left(\frac{1}{2\gamma}\right)$$

$$COMPETITION\ \bar{X}^C = \left(\alpha - \frac{C}{b}\right)\left(\frac{1}{\gamma}\right)$$

$$\therefore \frac{\bar{X}^S}{\bar{X}^R} = 2\left(\frac{d}{b+d}\right) \qquad \textbf{(12)}$$

$$\frac{\bar{X}^C}{\bar{X}^R} = 2$$

Proposition 1:

- As d → ∞, $\bar{X}^S \to \bar{X}^C$.
- As d → 0, $\bar{X}^S \to \bar{X}^R$

Proof:

$$\frac{d}{b+d} \to 1 \ \ as\ d \to \infty$$

$$\frac{d}{b+d} \to 1/2 \ \ as\ d \to 0$$

$$since$$

$$b \mapsto d \ \ as\ \ d \mapsto 0\ ;$$

$$b \mapsto 1/\rho \ \ as\ \ d \mapsto \infty$$

$\bar{X}^S/\bar{X}^R$ depends only upon **d** and ρ; Figure 1 shows $\bar{X}^S/\bar{X}^R$(d) respectively for ρ = .05 and 0.10. Note in particular that monopoly output remains quite low even for product lifetimes as high as 15 to 20 years. For any real-world durable good, even in continuous time, monopoly power remains significant for a sales monopolist.

Figure 1 depends only minimally on functional forms; the only arbitrary functional form is the assumed linearity of the demand function. The message is stark and clear: The Coase temptation offers only modest relief to consumers, for goods with less than about a 15-year lifetime.[4]

Recall that the PREE is found by solving (10). It is worth noting that (10), in addition to being our solution method, is a natural characterization of the rational and informed consumer's decision-making problem: When faced with a stock of X*, the consumer's natural question is this: "Has the Coase force spent itself, or will the firm raise X above X* in the next instant?" When (10) is satisfied, the consumer knows that the Coase force can offer him no more help.

II ENDOGENOUS DURABILITY

So long as durability is exogenous, welfare and consumers' utility unambiguously rise when rental-only is prohibited (as in current antitrust enforcement), though the welfare improvement is much more modest than the literature suggests. As shown above, and illustrated in Figure 1, a sales monopolist enjoys considerable monopoly profit even if its product lasts 15 to 20 years. Nevertheless, such a firm, while still enjoying profit, loses substantial profit to Coase. If $\rho = .05$ and $\mathbf{d} = 20$ years, the sales monopolist loses approximately 40% of potential profit to Coase. In this section we explore the possibility that a sales monopolist reduces durability to recapture his power over Coase.

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[4]The question we pose is essentially identical to that of Bond and Samuelson, but our results are strikingly different. In the Appendix we show how and why our results differ from theirs.

Suppose production cost is given by

$$C = c_0 + c_1 d \tag{14}$$

where C is unit cost, and c_1 is the cost of added durability. Socially optimal durability (which is produced by both the rental monopolist and the competitive industry) requires

$$\frac{(c_0 + c_1 d)e^{-rd}}{b(d,\rho)} - c = 0 \tag{15}$$

Assume now that the monopolist can commit to a given level of **d**; his problem is to decide on the level of **d**, knowing that **d** influences not only the cost of producing the service flow (optimized in equ. (15)) but also his ability to withstand the Coase temptation. For any given level of durability, sales-monopoly flow of output is

$$x = \frac{1}{\gamma\cdot(b+d)}\left(\alpha - \frac{c_0 + c_1 d}{b}\right) \tag{16}$$

(derived above in equation (11)) and the flow of profit is given by

$$\pi = x(\cdot)[(\alpha - \gamma x d)b - (c_0 + c_1 d)] \tag{17}$$

Differentiating equation (17) with respect to **d** and setting = 0,

$$\left[\frac{(c_0 + c_1 d)e^{-\rho d}}{b} - c\right] = \frac{x'}{x}\left((\alpha - 2\gamma x d)b - (c_0 + c_1 d)\right) + \gamma b x\left(1 - e^{-\rho d}\right)$$

$$= \frac{x'}{x}\left(MR - MC\right) + \gamma b x\left(1 - e^{-\rho d}\right) \qquad \textbf{(18)}$$

where x' = ∂x/∂d (the derivative of (16) with respect to d):

$$\frac{\partial x}{\partial d} = -x\frac{(1 + e^{-\rho d})}{d + b} + \left[\frac{(c_0 + c_1 d)e^{-\rho d}}{b} - c\right]\left(\frac{1}{(d+b)\gamma b}\right) \qquad \textbf{(19)}$$

The expression inside [] in equations (18) and (19) is the net reduction in cost of producing service flow as **d** is increased. For compactness, henceforth [] = H (where H stands for efficiency gain from added durability). ∂H/∂d < 0 and H = 0 for optimal durability. Thus if H > 0, durability is less than optimal. It is now straightforward (but messy) to show that H > 0 for the sales monopolist. Substituting (19) into (18) and rearranging:

$$H\left(1 - \frac{MR-MC}{(d+b)\gamma b}\right) = -\left(\frac{1+e^{-\rho d}}{d+b}\right)(MR-MC) + \gamma b x\left(1 - e^{-\rho d}\right) \qquad \textbf{(20)}$$

$$\qquad + \qquad\qquad + \qquad\qquad - \qquad\qquad +$$

Hence H > 0, confirming that the sales monopolist produces less than optimal durability.

In principle, we can solve (20) for **d**, and from that solve for the monopolist's steady-state output and profit. However, this expression is very unwieldy and gives little insight into behavior.

On the other hand, it is very easy to obtain numerical results.[5]

Numerical Solutions

Figure 2 shows the profit-durability relationship for a sales and rental monopolist respectively, for the parameter values given in Column 1 of Table 2. For every **d**, sales profit is lower than rental, because of the Coase temptation. And as discussed above, the (percentage) gap rises with **d**. The sales monopolist's profit reaches its maximum at a lower **d** than does the rental monopolist's, representing the sales monopolist's (partial) escape from Coase by cutting **d**.

Numerical results, for a variety of parameter values, are given in Table 2 below:

Table 2
Sales and Rental Monopoly
with Endogenous Durability

column	1	2	3	4	5	6	7	8
α	100	200	100	100	100	200	100	100
γ	1	1	1	1	1	1	1	1
c_0	50	50	50	50	10	100	50	50
c_1	20	20	20	5	65	3	0	-10
ρ	.05	.05	.10	.05	.05	.05	.05	.05
d^S	5.9	4.4	4.3	6.4	2.2	6.5	6.5	6.9
d^R	9.2	9.2	6.3	17.2	2.4	28.1	∞	∞
V^S/V^R	.833	.861	.771	.790	.94	.766	.470	.410
C^{sS}/C^R	1.036	1.101	1.034	1.270	1.00	1.76	3.60	*
X^S/X^R	1.054	1.034	1.081	1.039	1.025	1.025	1.008	.76

*: $C^S = -3.26$; $C^R = -47.82$
Note: In Col.9, V^R and X^R grow without bound as d rises (since cost goes to $-\infty$ as d rises). We have set d = 100 for the rental monopolist in Col. 9.

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[5]Unlike many numerical results, this work can readily be reproduced on a simple spreadsheet program. For **d** from 1 to 100, calculate x(d) in equ. (16) on a spreadsheet, for various parameter values. Substitute into (17), giving maximum profit as a function of **d**. Since the firm can commit to any **d**, the value of **d** at which profit is maximized is the equilibrium. Choice of **d** takes account of lost Coase immunity as **d** rises, as well as the effect of **d** on production cost.

First consider Column 1, the parameter values for Figure 2. The sales monopolist reduces **d** from its optimal value (9.2 years) to 5.9, raising production cost by 3.6%. Profit is 83.3% that of the rental monopolist.

Column 2 reveals that raising monopoly power ($\alpha = 200$) causes the sales monopolist to deviate further from optimal durability. With greater (potential) profit, the monopolist has a greater incentive to cut durability. In Column 3 we raise the discount rate to .10; this makes the sales monopolist more subject to the Coase temptation; it responds by cutting durability more than when $\rho = .05$, and it earns less profit .

In Column 4 we create a very durable good (optimal lifetime = 17.2 years, possibly an automobile). The sales monopolist sets **d** = 6.4. Production cost of the service flow is 27% above optimal. Profit is 79% that of the rental monopolist.

Columns 5 and 6 present extremes of plausible durability. In Column 5 optimal durability is 2.4 years; the sales monopolist's behavior is very little different from the rental monopolist's. In Column 6, optimal durability is 28.1 years. Were the sales monopolist to produce this durability, its profit would only be 48% of rental profit. Despite a 76% increase in production (of service flow) cost, it chooses to cut durability back to 6.5 years, to find a safe harbor from Coase competition, earning 76% of the rental monopolist's profit.

In Column 7, production cost is independent of durability ($c_1 = 0$). The rental monopolist obviously produces infinite durability; the sales monopolist still produces a good of 6.5 years' durability, at a cost 3.6 times that of the rental monopolist.

In Column 8, it is costly to **reduce** durability; $c_1 = -10$. Even in this extreme version, where at $d = \infty$ $C = -\infty$, the sales monopolist produces a good of 6.9 years durability, and earns

41% as much profit as the rental monopolist.

Finally note the bottom row, which shows the ratio of sales to rental stock (if the sales monopolist fully fell victim to Coase, this ratio would be 2). In no case does X^S/X^R rise above 1.088. This crucial ratio remains small for the following reasons:

- If optimal durability is small (say in the range of 5 years) then sales output is not much higher than rental in any event (an extension of the Part-I finding);
- If optimal durability is high, the sales monopolist tends to cut durability, which depresses output for two reasons:
    - i. At lower durability the sales monopolist faces less Coase temptation, and
    - ii. As durability is reduced, cost is increased, further depressing output.

Thus regardless of whether optimal durability is high or low, sales-monopoly output is only very slightly above rental output. The results (confirmed by other numerical analyses over a wide range of parameter values) can be summarized quickly:

- Sales-monopoly durability falls relative to rental when
    - i. optimal durability rises,
    - ii. potential monopoly power (the intercept of the demand curve) rises,
    - iii. the interest rate rises.

In particular, whenever optimal durability is great enough to seriously erode sales-monopoly profit, durability is cut back dramatically, even if such a cutback is quite costly.

- Sales-monopoly stock, over a very wide range of parameter values, is less than 10% higher than rental-monopoly stock.

The results of Sections I and II strongly suggest that Coase discipline offers little relief to consumers. If durability is exogenous, sales monopolists capture a substantial fraction of rental monopolist's profit despite the Coase discipline, so long as durability is not "too great." Certainly for products with lifetimes of 10 to 15 years, sales monopolists enjoy a substantial fraction of the profit of rental monopolists. If the monopolist can set durability, it continues to enjoy substantial monopoly profit even for goods which should last for many decades. It simply reduces the durability of the good and escapes the Coase temptation.

With endogenous durability, prohibition of rental-only is not generally welfare enhancing. Though the sales monopolist usually slides a (tiny) bit farther down the demand curve than does the rental monopolist, he produces the product in an inefficient manner.

III MAINTENANCE

In Sections I and II, the lifetime of the product is determined on the factory floor. But for an important class of goods, lifetime is jointly determined by built-in durability and maintenance, which is performed by consumers if the good has been sold, during the lifetime of the good. And for many such goods, the original equipment manufacturer (OEM) is the sole supplier of some of the parts and expertise required to maintain the good.[6] In this section we consider the behavior of a durable-goods monopolist (respectively sales and rental) which possesses such aftermarket power. We find that such a monopolist can use its parts-pricing power to recover much of the power it would otherwise lose to Coase.

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[6]Some of the most contentious recent antitrust litigation has centered on OEMs' efforts to protect their aftermarkets. See AMI *v* IBM 93-1586 3rd (1994) and Eastman Kodak Co. *v* ITS, *et al* 112 S. Ct. 2072 (1992). IBM and Kodak had each attempted to exclude independent service organizations from performing maintenance and upgrade work on their original equipment (respectively large scale mainframe computers and copiers).

A. Technology

Unit cost of producing the durable good is

$$C = c_0 + \frac{c_1}{a} \tag{21}$$

where **a** is an index of maintenance required to keep the product alive. The level of maintenance required at age **t** is given by

$$m_t = a e^{gt} \tag{22}$$

Maintenance, in turn, is produced according to a CES technology:

$$a = \left(\delta \pi_0^{-\psi} + (1-\delta) s_0^{-\psi}\right)^{-1/\psi} \tag{23}$$

The inputs are respectively "parts" π, and "service" **s**. Parts are produced and priced exclusively by the OEM, and service is marketed competitively.

B. Pricing

The price of original equipment (the product) is

$$P = (1+\theta)\left(c_0 + \frac{c_1}{a}\right) \tag{24}$$

where θ is the monopolist's markup over production cost. The price of parts is

$$\mathbb{P}_{(parts)} = 1 + \phi \qquad \textbf{(25)}$$

where the cost of manufacturing parts is normalized to unity and ϕ is the markup over cost. The cost (to the consumer) of maintenance is

$$\mu_t = \mu_0 \, e^{g t} \qquad \textbf{(26)}$$

where

$$\mu_0 = s_0 + (1 + \phi)\, \pi_0 \qquad \textbf{(27)}$$

C. Consumer Behavior

In the PREE, θ, ϕ, a are constant. Given θ, ϕ, a, consumers choose the following:

(1) the mix of parts π and service s in the production of maintenance;

(2) the scrapping age, **d**, of the product, and

(3) the quantity of the product to purchase.

The consumer's cost-minimizing mix of maintenance inputs is given by

$$s_0 = \left(\frac{\delta}{1 - \delta} \frac{1}{1 + \phi} \right)^{-\sigma} \qquad \textbf{(28)}$$

$$\pi_0 = a \left(\delta + (1 - \delta) \left(\frac{\delta}{1 - \delta} \frac{1}{1 + \phi} \right)^{\psi \sigma} \right)^{1/\psi} \qquad \textbf{(29)}$$

Age-0 maintenance expenditure, μ_0, can thus be expressed as

$$\mu_0 \quad = \quad a\left(\delta + (1-\delta)\left(\frac{\delta}{1-\delta}\frac{1}{1+\phi}\right)^{\psi\sigma}\right)^{1/\psi}\left((1+\phi) + \left(\frac{\delta}{1-\delta}\frac{1}{1+\phi}\right)^{-\sigma}\right) \qquad \textbf{(30)}$$

$$\begin{aligned} \mu_0 \quad &= \quad a \cdot f_1(\phi;\delta,\sigma) \cdot f_2(\phi;\delta,\sigma) \\ &= \quad a \cdot f(\phi;\delta,\sigma) \end{aligned} \qquad \textbf{(31)}$$

where $\sigma = 1/(1+\psi)$ is the elasticity of substitution between parts and service in the production of maintenance, $f_1(\cdot)$ and $f_2(\cdot)$ are the two bracketed expressions to the right of **a** in equation (30), and $f(\cdot) = f_1 \cdot f_2$.

Next the consumer determines the optimal scrapping age, **d**.

$$d \quad = \quad \underset{\lambda}{argmin} \frac{P}{1-e^{-\rho\lambda}} \; + \; \frac{a \cdot f(\cdot)}{g-\rho} \cdot \frac{e^{(g-\rho)\lambda}-1}{1-e^{-\rho\lambda}} \qquad \textbf{(32)}$$

where the right hand side of (32) is the perpetual rental cost of the product as a function of λ.[7] Having solved for **d**, rental cost is found by substituting **d** for λ in the right hand side of (32):

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[7](12) does not yield a closed-form solution for **d**; the implicit solution is given in equation (23) in the Appendix.

$$\mathbb{R} = \frac{\mathbb{P}}{1-e^{-\rho d}} + \frac{a \cdot f(\cdot)}{g-\rho}\,\frac{e^{(g-\rho)d}-1}{1-e^{-\rho d}} \qquad \textbf{(33)}$$

As in Parts I and II, demand for the services of the product is linear in R:

$$X = \alpha - \gamma\mathbb{R} \qquad \textbf{(34)}$$

The Perfect Rational Expectations Equilibrium

As above, we analyze a PREE (Perfect Rational Expectations Equilibrium). Formally, we assume

1. Consumers know all of the parameters of the model;

2. Consumers do not anticipate any "sales;" i.e., price cuts followed by price increases;

3. Time is continuous; the firm is free to change θ,ϕ,a at any time without notice.

As in Parts I and II, there is only one functional form for expectations, consistent with rationality in continuous time:

$$\begin{aligned} \theta_s &= G_\theta(s,t,\theta_t,\phi_t,a_t) = \theta^* \quad \forall \quad s,\theta,\phi,a \\ \phi_s &= G_\phi(s,t,\theta_t,\phi_t,a_t) = \phi^* \quad \forall \quad s,\theta,\phi,a \\ a_s &= G_a(s,t,\theta_t,\phi_t,a_t) = a^* \quad \forall \quad s,\theta,\phi,a \end{aligned} \qquad \textbf{(35)}$$

Consumers solve for {*} using the same method we employed in Parts I and II: they test a vector {θ",ϕ",a"}[8] for steady-state-ness by solving the firm's problem:

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[8]In Sections I and II, consumers' expectations were defined over X; here we find it more convenient to define expectations over prices (and built-in durability, **a**).

$$\max_{\theta,\phi,a} V(\theta,\phi,a;\theta'',\phi'',a'') \tag{36}$$

The maximization problem is interpreted as follows: {θ",ϕ",a"} has persisted for the relevant past (at least for a timespan **d**, the lifetime of the good). Calculate the value of the firm, given this history, assuming that the new vector {θ,ϕ,a} will be permanent once announced. If

$$\begin{aligned} \theta &= \theta'' \\ \phi &= \phi'' \\ a &= a'' \end{aligned} \tag{37}$$

then {θ",ϕ",a"} is {θ*,ϕ*,a*}; satisfaction of (37) ensures that no permanent price change from {θ",ϕ",a"} is profitable. Thus {*} is a PREE by the same reasoning employed in parts I and II.

Solving the Model

As in Section II, the model does not yield tractable analytical solutions. Therefore, in this section we report and discuss numerical values of {θ*,ϕ*,a*}.

The problem is solved piecemeal: First hold ϕ and **a** constant and solve for θ*. Repeat this operation for a wide range of values of ϕ and **a** to obtain a relationship θ*(ϕ,a). That is, for any given ϕ and **a**, θ*(·) gives the profit-maximizing permanent value for θ. By the same method, obtain ϕ*(θ,a) and a*(θ,ϕ). The solution to these three equations is our PREE vector, {*}.[9]

In Tables 3 and 4 we present the results from the parameter vector shown. The good is designed to be similar to that of Column 4 of Table 2. The first row shows the competitive

[9]Derivation of θ*(ϕ,a), ϕ*(θ,a), and a*(θ,ϕ), and the solution to these expressions, is described in the Appendix.

outcome; the second shows the rental monopolist's outcome. Row 3 presents the results for the sales monopolist with parts power. The original-equipment markup is $\theta = .54$, and the parts markup ϕ is 43.84.[10] Cost to the consumer, R, is 3.62% **above** that for a rental monopolist, and **d** = 8.36 (efficient **d** = 18.64). Social cost of the service flow is 37% above optimum. The value of the firm is 92.4% that of the rental monopolist.

Table 3
Equilibrium With Aftermarket Power
$g=.1$; $\rho = .05$; $c_0 = 50$; $c_1 = 5$; $\sigma = 0.1$; $\alpha = 800$; $\gamma = 1$

	V	R	θ	ϕ	a	d	cost
Competition	0	73.75	0	0	.285	18.64	73.75
Rental Monopoly	132132	436.8			.285	18.64	73.75
Sales Monopoly	121128	452.6	0.54	43.48	.277	8.36	101.1

Table 4
Rental/Sales Comparisons

X^S/X^R	.957
C^S/C^R	1.37
V^S/V^R	.917

Note that the sales monopolist takes virtually all of his monopoly profit via the markup on parts rather than original equipment, because a high parts price is intertemporally credible, whereas a high original-equipment price is not.[11] The result is a socially excessive markup on parts and a too-early scrapping age.

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[10]The parts markup is huge for the standard Marshallian reasons: the elasticity of substitution between parts and service, and also between maintenance and original equipment, is small; and parts' share in the cost of the product's service is small as well. With different parameter values the parts markup is smaller.

[11]Crandall, *et al* (1986) estimate that the price markup on automobile parts is generally 3 to 8 times the markup on automobiles themselves.

Finally, note that our sales monopolist produces a **smaller** stock than does the rental monopolist. Although cost of the flow of services is 37% above optimal, profit is only 8.3% below that of the rental monopolist.

In virtually all of the Coase literature, the playing of the Coase game reduces deadweight loss, helps consumers, and harms the firm. Coase is the friend of both efficiency and consumers. Unfortunately, the aftermarket is a privately effective but socially wasteful anti-Coase weapon. Consumers may be even worse off than under rental monopoly (as they are in Table 3), and there is a large waste of resources via early scrappage. The Coase benefit to consumers is overwhelmed by the sales monopolist's technically wasteful response to that temptation.

The role of the installed base is intriguing and somewhat surprising. Most antitrust observers believe that a substantial installed base, not owned by the OEM, serves as a check on monopoly power. New equipment must compete with this used equipment.[12] But in the presence of aftermarket power, a substantial base, even if owned by consumers, is a powerful **antidote** to the Coase temptation; it enables the firm to credibly charge a monopoly markup on parts.

Other Parameter Values

We believe that the only tractable approach to this problem is simulation[13]. This of course leaves us without analytical solutions, and without the insight into general patterns which analytical solutions can give. In Part II we presented results for a wide range of parameter values;

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[12]The issue was first discussed (and rejected) by Judge Learned Hand in <u>US vs. Alcoa</u> (1945) and is the basis for many court rulings requiring OEM's to sell rather than only lease their products.

[13]Unlike the numerical analysis of Part II, this simulation is extremely tedious; see Appendix.

in Part III we present only the parameter values depicted in the table. The product underlying Tables 3 and 4 is similar to that of Column 4 in Table 2; it "should" last between 15 and 20 years, and the sales monopolist cuts it back to somewhere between 7 and 9 years. In other simulations we have determined that the model of Part III responds to different parameter values in essentially the same way as that of Part II. Whenever the monopolist of Part II would cut durability, the monopolist of Part III raises the parts price, which in turn induces the consumer to shorten the life of the good. Thus for example, when optimal lifetime is long (in excess of about 10 years), the monopolist faces a substantial Coase threat and reduces durability in the Part-II model or raises parts prices in the Part-III model. On the other hand, when optimal durability is less than a decade, the Coase temptation is small in any event, and the monopolist gains little by reducing durability or setting a disproprotionately high parts price. Monopolistic parts pricing, like durability reduction, is a weapon to be employed only if Coase losses would otherwise be substantial.

IV CONCLUSION

This paper explores the characteristics of perfect rational-expectations equilibria for monopolists producing finitely durable goods in a continuous-time setting. We model this problem because we believe that both limited durability and continuous ability to change prices are more realistic than the standard paradigm of infinite durability and discrete (and quite long) trading periods. Even if durability is exogenous, we find that sales monopolists retain a great deal of monopoly power, except for goods with lifetimes of several decades or more.

If the monopolist has the power to determine the level of built-in durability, he can further recover monopoly profit by reducing durability below its optimal level.

Finally, if the monopolist has control over the market for repair/replacement parts which are required to maintain the good, the sales monopolist recovers still more profit.

Since altering built-in durability and excessively raising parts prices result in technical inefficiency which a rental monopolist would not find necessary, it is possible that consumer welfare is higher under rental than sales monopoly.

In no plausible case do we find that the Coase discipline comes close to replicating the welfare benefits of competition, and in many instances it does more harm than good.

Appendix

I. Why No Sales?

All of our equilibria are PREE, subject to the restriction that there be no sales. In parts I and II no-sales gives rational consumers static expectations for $X \geq \overline{X}$; in Part III it takes the form of a restriction that a price cut below θ^* be irreversible. The same assumption underlies the work of Stokey, and of Bond and Samuelson, though it is unstated in their work.

Of course, Stokey's monopolist would never intentionally let $X > \overline{X}$. In the case of imperfect durability, however, a monopolist might want to engage in sales; i.e., to capture residual demand by a temporary price cut. We choose not to analyze sales for the following reasons:

- Our assumption is in keeping with the (unstated) assumption in the rest of the literature;
- We wish to analyze the importance of the Coase force in our setting (imperfect durability and parts power), and we interpret the Coase force to be the **inexorable** downward pressure on price due to the time consistency problem. Regardless of whether there are occasional sales, the Coase forces will not permanently push stock above $\overline{X}$ in Parts I and II or price below θ^* in Part III.
- We are not able to formally characterize a PREE in which sales are permitted. It is not clear how to model consumers' expectations in the presence of sales, as we will discuss. Most important, it is difficult to imagine a plausible predictable pattern of sales in which anybody buys at the regular price.
- Despite our inability to formally model sales, we offer the following strong informal reason to believe that sales are unimportant in this context.

  First, consider a regular pattern of sales in the world of exogenous durability. In a prior regime without sales, the flow of output has been constant at x*. At time t, the firm begins to hold sales, hoping to capture residual demand. It is natural to think of these sales as occurring on the **d** interval; i.e., to repeat the sale when the residual demanders' goods die. But nonresidual demanders will anticipate this pattern of sales and retime their purchases. If **d** = 10 years and ρ = .05, it is easy to show that just over 20% of the in-service stock will be prematurely retired to take advantage of the sale price. A consumer whose good will require replacement at time t faces a rental stream of

$$R_f = \frac{P\, e^{-\rho(d-t)}}{1 - e^{-\rho d}}$$

if he waits and buys at full price. If instead he retires his good early to take advantage of the sale (and the series of repeat sales which he anticipates on the **d** interval), his stream is

$$R_s = \frac{P_s}{1 - e^{-\rho d}}$$

Setting $R_f = R_s$ and solving for t,

$$t = \frac{\ln \frac{P_s}{P}}{\rho} + d$$

If **d** = 10 years, ρ = .05, and P_s = .9 P, just over 21% of the stock will be prematurely retired to take advantage of the new cycle of sales. And before the second (anticipated) sale many more consumers will postpone replacement to get onto the sale cycle. Thus it is extremely difficult, with a pattern of anticipated sales, to target residual demanders and preserve the high price for replacement demanders. But if replacement demanders buy at the sale price, then the sale price is the regular price.

Now consider the effect of a once-only unanticipated sale, in the model of a monopolist with aftermarket power.[14] Assume that consumers recognize that the sale is temporary (as they should, if they are rational), and that they expect it never to be repeated. Consequences are:

(1) Sale to residual demanders (the Coase target), and

(2) Early retirement of older installed base to take advantage of the temporary sale price.

The firm must weigh the profit of selling to residual demanders against two costs:

(i) selling some replacement product at the sale price rather than the regular price, and

(ii) the loss of parts revenue brought about by the early retirement induced by the sale. If the sale is (correctly) anticipated,

(3) Some holders of installed base will postpone retirement so as to cash in on the sale.

If the net effect of (1) and (2) (and of (3) if the sale is anticipated) is positive, a sale, which will never be repeated (and which somehow consumers know will never be repeated) is profitable. But the firm must also weigh the effect of the sale on consumers' expectations. Suppose for example that the first sale causes consumers to expect a sale every **d** periods (if the firm had a sale to capture residual demanders at $t = 0$, it will want to recapture those same residual demander when their products die in **d** years).

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[14]A "sale" takes the form of a once-only reduction in the markup, θ, on the product.

If the sale is to be repeated every **d** years, this increases the number of nonresidual demanders who will retime their replacement so as to get onto the sale cycle. In other words, the firm, by offering a sale, is opening a can of expectational worms. The spot profitability of a sale must be weighed against the effect of such a sale on expectations.

In the model which we solve in the next section, we check for the profitability of a once-only sale, assuming the sale itself has no effect on subsequent expectations. We find that a sale of up to 20% increases spot profit by approximately 1%; any deeper sale reduces profit. We find that no sale, predictably repeated every **d** periods, is profitable.

II. Solving the Maintenance Model

We seek an inherited vector, {θ,ϕ,a}, from which no permanent deviation in any parameter is profitable. First, for a given {ϕ,a} vector we seek a time-consistent θ as follows: Given a vector {θ,ϕ,a}, assume {ϕ,a} remain fixed but θ is cut to θ'. Such a cut reduces both R and **d**, resulting in the output flow depicted in Figure 3. θ is reduced at time 0; stock which otherwise would have been retired at age **d** is now retired at age Δ. The size of this early-retirement (and therefore replacement at time 0) stock is a_1. It generates a production spike at t = 0, Δ, 2Δ, etc., of magnitude a_1. There is another production spike, on the same time interval, resulting from the reduction in R. Call this spike a_2. The present value of future output, pvq, is given by

$$pvq = \frac{q}{\rho d} + (a_1 + a_2)\frac{1}{1 - e^{-\rho\Delta}} \qquad \textbf{(41)}$$

where

$$a_1 = \frac{q}{d}(d - \Delta);$$

$$a_2 = \gamma\,(R' - R)$$

where R’ = R(θ’,ϕ,a). Now define

$$V_1 = pvq_1\left(c_0 + \frac{c_1}{a}\right)\theta'$$

is the profit, from time 0 forward, from selling the original equipment. Profit from aftermarket sales on this original equipment, V_2, is

$$V_2 = pvq\phi\cdot f_1\, a\cdot\left(\frac{e^{(g-\rho)\Delta} - 1}{g-\rho}\right)$$

The remaining element of profit, V_3, is the aftermarket profit from the base already installed at time 0, but not yet retired:

$$V_3 = \frac{q}{d}\cdot\phi\cdot f_1\cdot a\cdot\int_0^{\Delta} e^{g\delta}\int_{\delta}^{\Delta} e^{(t-\delta)(g-\rho)}\,dt\,d\delta$$

$$= \frac{q\cdot\phi\cdot a\cdot f_1}{d\,(g-\rho)}\left(\frac{1-e^{gd}}{g} + e^{gd} - \frac{e^{\Delta g-\rho)}}{\rho}\right)$$

where δ is the age (ranging from 0 to Δ) of the installed base at time 0. The firm’s problem is to

$$\max_{\theta'} V(\theta';\theta,\phi,a)$$

where $V = V_1+V_2+V_3$.

Given ϕ and **a**, we search over θ to find a value of θ at which $\theta = \theta'$. This gives us $\theta^*(\phi,a)$. By varying ϕ and **a**, and repeating this process, we trace out a relationship

$$\theta^*(\phi,a) \qquad \textbf{(47)}$$

By the same method (described below) we find the relationships

$$\phi^*(\theta,a); \qquad a^*(\theta,\phi) \qquad \textbf{(48)}$$

We then solve expressions (44) and (45) to obtain the vector {θ^*,ϕ^*,a^*).

$\phi^*(\theta,a)$ is found as follows. A cut in ϕ reduces R, creating a spike in demand, $\mathbf{a_2}$, at t = 0, Δ, 2Δ, etc., and increases **d**, creating a timespan from 0 to (Δ-d) during which no output is sold. The present value of output, after a cut in ϕ, is

$$pvq = \left(a_2 + \frac{q}{\rho d}\left(e^{-\rho(\Delta - d)} - e^{-\rho\Delta}\right)\right)$$

$$V_1 = \theta\, pvq\left(c_0 + \frac{c_1}{a}\right)$$

$$V_2 = \frac{\phi' \cdot a \cdot f_1 \cdot pvq}{g-\rho}\left(e^{(g-\rho)\Delta} - 1\right)$$

and

$$V_3 = \frac{\phi' \cdot a \cdot f_1 \cdot q}{d} \int_0^d e^{g\delta} \int_0^\Delta e^{(g-\rho)(t-\delta)} d\delta \, dt$$

$$= \frac{\phi' \cdot a \cdot f_1 \cdot q}{(g-\rho)d} \left(\frac{1 - e^{gd}}{g} + \frac{e^{gd + (\Delta - d)(g-\rho)} - e^{(g-\rho)\Delta}}{\rho} \right)$$

$$V = V_1 + V_2 + V_3$$

Holding θ and **a** fixed we find the value of ϕ such that ϕ = ϕ' where ϕ' maximizes V. Repeating over many values of θ and **a** gives us the relationship ϕ(θ,**a**). By the same procedure we find a'(θ,ϕ). First, define the pvq which flows after a rise in **a** (which reduces **d** and increases R):

$$pvq = \frac{q - \gamma \cdot (R' - R)}{d\rho} + \frac{e^{-\rho\Delta}(q - \gamma \cdot (R' - R))}{(1 - e^{-\rho\Delta})d} + \frac{1 - e^{-\rho(d-\Delta)}}{\rho}$$

V_1 and V_2 take the same form as above. V_3, the present value of aftermarket profit from installed base, is given by

$$V_3 = \frac{a \cdot f_1 \cdot \phi \cdot q}{d} \left(\int_0^d e^{g\delta} \int_\delta^d e^{(g-\rho)(t-\delta)} d\delta \, dt \right)$$

$$= \frac{a \cdot f_1 \cdot \phi \cdot q}{d(g-\rho)} \left(\frac{1 - e^{dg}}{g} - \frac{e^{d(g-\rho)} + e^{dg}}{\rho} \right)$$

Using the same technique as above, we find a'(θ,ϕ). We now have a system of three equations

$$\phi' = \phi'(\theta, a)$$
$$\theta' = \theta'(\phi, a)$$
$$a' = a'(\theta, \phi)$$

Solving these equations through manual search, we are able to solve the system for the vector {θ',ϕ',a'}, which is equal to the time-consistent PREE vector {θ*,ϕ*,a*}.

III. Why Didn't Bond and Samuelson Find This?

Bond and Samuelson model the behavior of a Coase monopolist, whose good depreciates at the exogenous rate δ. They adopt the Stokey discrete-time formulation and expectations function, equ. (5), and derive

B&S Proposition 1:

a. For any value of z (the trading period), the limiting stock falls with δ (the less durability, the more monopoly power);

b. For any value of δ, the limiting stock falls with **z** (the longer the trading period, the greater the ability to retain monopoly power);

c. For any value of δ, the limiting stock approaches the competitive level as $z \to 0$ (as the trading period approaches 0, then **regardless of durability**, monopoly power approaches 0).

In fact, even for goods with very modest durability (e.g., δ = .8), B&S require a very long trading period to sustain much monopoly power. Table A1 gives the ratio of B&S's $\overline{X}^S/\overline{X}^R$, for δ = .2, δ = .5, and δ = .8 for trading periods of various lengths. (The competitive value of this ratio is 2; the perfectly Coase-immune ratio is 1).

Table A1
B&S X^S/X^R
z and δ

z	δ=.2	δ = .5	δ = .8
0.1	1.82	1.74	1.68
0.3	1.72	1.58	1.50
0.5	1.65	1.48	1.38
0.7	1.60	1.42	1.32
1	1.53	1.34	1.24
2	1.40	1.20	1.10
5	1.20	1.02	1.00

Even with 80% depreciation, at a one-year trading period $\overline{X}^S$ is 24% greater than $\overline{X}^R$; with a trading period of 4 months, $\overline{X}^S$ is 50% higher than $\overline{X}^R$. In other words, B&S tell us that the Coase temptation is quite destructive of monopoly power, even for goods with virtually no durability at all, so long as the trading period is not greater than about 6 months to a year. The real message from Bond and Samuelson is not that the Coase temptation dissipates with imperfect durability, but that even for goods with little durability, consumers receive much protection from monopoly exploitation by the forces of Coase. Antitrust enforcement, it would seem, could be limited to ensuring that trading periods remain shorter than 2 to 3 months.[15]

The question we address above is identical to that of Bond and Samuelson, and our assumptions are identical except for the following:

(1) Our good is one-hoss-shay whereas theirs physically depreciates exponentially;

(2) Our time is continuous; their monopolist has commitment power for an exogenous period z

[15]With linear demand and constant marginal cost, deadweight loss is proportional to the square of the monopoly output restriction, so a 50% increase in output reduces deadweight loss by 75%.

(for B&S continuous time is the special case where $z = 0$; as $z \to 0$ in B&S, the sales monopolist's behavior approaches competitive behavior, regardless of durability);

(3) They assume (without deriving it) a specific functional form for consumers' expectations; we infer our functional form as a consequence of consumers' rationality.

Despite these apparently modest differences in modelling, our findings are almost completely opposite to theirs.

Since B&S Propositions 1(a) and (b) above refer to finite trading periods, they have no analog in our model. Their Proposition 1(c), however, suggests that as $z \to 0$ (as time approaches continuous), monopoly output collapses to the competitive output, regardless of durability[16]. Our conclusion, by contrast, is that with no trading period the durable-good monopolist's price is much greater than marginal cost, except for goods which last several decades.

The B&S results differ from ours only because of the difference in choice of $G(\cdot)$ function and their discrete time assumption. As we have emphasized, this functional form for expectations rests on no optimization decision. By contrast, we assume that consumers expect instant saturation: $G(\cdot) = \overline{X}\ \forall\ X < \overline{X}$,s,t, as dictated by consumers' rationality.

$G(X) = \overline{X}$ in Bond & Samuelson

Here we solve the B&S model, but replace discrete with continuous time and function (5) with (6). Their modified model yields results essentially identical to our Part-I results.

The B&S first order condition for profit maximization (their equation (9)), is

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[16]When time is literally continuous in B&S ($z = 0$), $\overline{X}$ is not equal to the competitive level; it is indeterminate (see their equ. 11).

$$\alpha(z,\delta) \;-\; \beta(z,\delta)G(\cdot) \;-\; c(1 - e^{-r+\delta)z} \;-$$

$$\beta(z,\delta)[G(\tau,\cdot) \;-\; e^{-\delta z}G(\tau-1,\cdot)]\cdot[\sum_{n=0}^{\infty} e^{-(\rho+\delta)zn}\,G_3(\cdot)] \qquad \textbf{(57)}$$

where

$$\alpha(z,\delta) \;=\; \alpha\cdot\int_0^z e^{-(\rho+\delta)t}dt \;=\; \alpha\,\frac{1 - e^{-(\rho+\delta)z}}{\rho+\delta}$$

$$\beta(z,\delta) \;=\; \beta\cdot\int_0^z e^{-(\rho+2\delta)t}dt \;=\; \beta\,\frac{1 - e^{-(\rho+2\delta)z}}{\rho+2\,\delta}$$

are the values of the demand function parameters, integrated over the trading period z

and

$$G_3 \;=\; \frac{\partial G}{\partial X}$$

Equilibrium requires that $G(\cdot) = \overline{X}\ \forall$ s,t,$X < \overline{X}$, and at $\overline{X}$ any increase in X is regarded as permanent, so $G_3 = 1$. Substituting $G_3 = 1$ into (57),

$$\alpha(z,\delta) - \beta(z,\delta)\,\bar{X}(1-e^{-\delta z})\left(\sum_{n=0}^{\infty} e^{-(\rho+\delta)zn}\right)\cdot 1 - c(1-e^{-(r+\delta)z}) = 0:$$

$$\alpha(z,\delta) - \beta(z,\delta)\,\bar{X}\left(\frac{1-e^{-\delta z}}{1-e^{-(\rho+\delta)z}}\right) - c(1-e^{-(r+\delta)z}) = 0 \qquad \textbf{(60)}$$

The assumed expectations function is rational only if z = 0 (continuous time). As z → 0, (60) →

$$\hat{\alpha} - X\beta\left(1 + \frac{\delta}{\delta+\rho}\right) - \hat{c} = 0 \qquad \textbf{(61)}$$

where $\hat{\alpha}$ and β are the demand function parameters for the instantaneous flow of service, and $\hat{c}$ is the cost of producing the flow of service from the good. Using B&S's expressions for $\bar{X}^R$ and $\bar{X}^C$, (equs (5) and (7) in B&S) we can easily see that

$$\frac{\bar{X}^S}{\bar{X}^R} = 2\left(\frac{\delta+\rho}{2\delta+\rho}\right)$$

Qualitatively and quantitatively, this is virtually identical to the analogous condition we derived in (12), and an analogous proposition holds:

Modified B&S Proposition A1:

As δ → 0, $\bar{X}^S \to \bar{X}^C$

As δ → ∞, $\bar{X}^S \to \bar{X}^R$.

Thus we see that the B&S structure, properly cast in continuous time with the only expectations function consistent with continuous time, yields the same result as we found in Part I: **With no trading period**, as durability approaches 0, the loss of monopoly power to Coase approaches 0.

Quantitatively, this modified B&S formulation is also very similar to ours. Figure 3 plots $\overline{X}^S/\overline{X}^R$ as a function of 1/δ, respectively for ρ = .05 and .10 (thus Figure 3 is the modified-B&S analog to Figure 1). If we think of 1/δ as approximately equal to **d** in the Part-I formulation, then one can directly compare Figures 1 and 3. For 1/δ = 10, $\overline{X}^S/\overline{X}^R$ = 1.2 (again, for complete loss of monopoly power this ratio equals 2).

B&S's finding that regardless of durability, sales-monopoly power goes to zero in continuous time is an artifact of their expectations function; this function, in turn, is not consistent with rationality in continuous time.

References

AMI v IBM 93-1586 3rd (1994).

Bond, Eric, and Larry Samuelson, "Durable Goods Monopolies with Rational Expectations and Replacement Sales," RAND Journal of Economics, Vol. 15 (1982), pp. 336-345.

Bulow, Jeremy, ""The Economic Theory of Planned Obsolescence," Quarterly Journal of Economics, Vol. 101 (1986), pp. 729-750.

Coase, Ronald, "Durability and Monopoly," Journal of Law and Economics Vol. 15 (1972), pp. 143-149.

Crandall, Robert, Howard Gruenspecht and Lester Lave, Regulating the Automobile, Brookings, Washington, D. C. (1982).

Eastman Kodak Co v ITS, *et al* 112 S. Ct. 2072 (1992).

Rust, John, "When is it Optimal to Kill off the Market for Used Durable Goods," Econometrica Vol. 54 (1986), pp. 65-86.

Schmalensee, Richard, "Market Structure, Durability, and Maintenance Effort," Review of Economic Studies, Vol. 41 (1974), pp. 277-287.

Stokey, Nancy, "Rational Expectations and Durable Goods Pricing," Bell Journal of Economics, Vol. 12 (1981), pp. 112-128.

Swan, Peter, "Optimum Durability, Second Hand Markets, and Planned Obsolescence," Journal of Political Economy, Vol. 80 (1972), pp. 575-585.

Unites States v Aluminum Company of America, *et al* 148 F.2nd 416 (1945).