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HYSTERESIS IN UNEMPLOYMENT:
EVIDENCE FROM OECD ESTIMATES OF THE NATURAL RATE

Laurence M. Ball
Joern Onken

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ABSTRACT

This paper studies the dynamics of unemployment (u) and its natural rate (u*), with u* measured by real-time estimates for 29 countries from the OECD. We find strong evidence of hysteresis: an innovation in u causes u* to change in the same direction, and therefore has permanent effects. For our baseline specification, a one percentage point deviation of u from u* for one year has a long-run effect of 0.16 points on both variables. When we allow asymmetry, we find, perhaps surprisingly, that decreases in u have larger long-run effects than increases in u.

Laurence M. Ball
Department of Economics
Johns Hopkins University
Baltimore, MD 21218
and NBER
lball@jhu.edu

Joern Onken
Department of Economics
University College London
Drayton House
30 Gordon St
London WC1H 0AX
United Kingdom
joern.onken.20@ucl.ac.uk

1. INTRODUCTION

According to mainstream macroeconomics, the long-run level of unemployment, or natural rate, is determined by imperfections in the labor market. Monetary policy and other determinants of aggregate demand can influence the fluctuations of unemployment around the natural rate but not the natural rate itself. This theory underlies the policies of many central banks, which focus on the inflation rate because it is a variable that monetary policy controls in the long run.

Some economists, however, dissent from this mainstream view. They argue that unemployment exhibits hysteresis: its short-run fluctuations cause the natural rate to change, for reasons including the scarring effects of unemployment on workers and changes in the number of insiders in wage bargaining. The concept of hysteresis was introduced by Blanchard and Summers (1986) and has been promoted over the years by authors such as Ball (1999), Stockhammer and Sturn (2011), and Gali (2020). Hysteresis implies that shifts in aggregate demand have long-run effects on unemployment, which potentially has profound implications for monetary policy. Gali, for example, presents a model in which hysteresis makes it optimal for a central bank to target employment rather than inflation.[1]

This paper seeks new evidence on the existence and strength of hysteresis effects. To that end, we examine real-time estimates of the natural rate of unemployment in 29 countries published by the Organization for Economic Cooperation and Development (OECD). We look for hysteresis effects in the evolution of these natural-rate measures over the period from 2002 to 2019.

We use the OECD series to estimate a simple statistical model of unemployment (u) and its

[1] Evidence supporting hysteresis is also reported by Leon-Ledesma (2002), Ball (2009), Blanchard, Cerutti and Summers (2015), and Girardi, Meloni, and Stirati (2020), among others.

natural rate (u*). These variables respond to shocks to u*, which we interpret as arising from changes in labor markets, and shocks to u for a given u*, which we interpret as arising from shifts in aggregate demand. We assume that u and u* move together in the long run: they are cointegrated. They deviate from each other in the short run but are drawn back together by one or both of two effects: u is pulled toward u*, as in mainstream models, and/or u* is pulled toward u through hysteresis effects.

Our central finding is that shocks to u have permanent effects. After a positive shock, u rises above u* for about eight years, and during that period u* is pulled to a higher level. We use our estimates of these effects to calculate the "degree of hysteresis" as defined by Ball, DeLong, and Summers (2017): the ratio of the long-run change in u* to the cumulative deviations of u from u*. Our baseline estimate of this parameter is 0.16, which means that u* changes by 0.16 percentage points when u deviates from u* by one point for one year.

Our main specification assumes that positive and negative shocks to u have symmetric effects, but we also estimate a model that relaxes that assumption. With that specification, we estimate that the degree of hysteresis is larger after a decrease in unemployment than after an increase (0.29 compared to 0.13). This finding is somewhat surprising in light of the hysteresis literature, which has focused on increases in unemployment, but it is consistent with Okun's (1973) idea that a "high pressure" economy has permanent benefits.

An important caveat to our analysis is that the natural rate of unemployment is not directly observable, and there is surely considerable measurement error in the OECD's natural rate series. We show, however, that this problem does not necessarily cause bias in our estimates of hysteresis effects. In particular, our results are robust to measurement error if the OECD's natural-rate estimates are optimal given all information available when the estimates are made. We cannot rule out the possibility of bias arising from some kind of systematic errors in the OECD series.

2. THE DYNAMICS OF U AND U*

Friedman (1968) introduced the concept of the natural rate of unemployment. He defined it as the level of unemployment consistent with inflation equal to expected inflation, a level that the actual unemployment rate must approach in the long run. Friedman's idea is captured by the expectations-augmented Phillips curve, a simple version of which is:

$$(1) \qquad \pi_t = \pi^e_t + \alpha(u_t - u^*_t) + \upsilon_t , \quad \alpha < 0 ,$$

where π and π^e are actual and expected inflation, u and u* are unemployment and its natural rate, and υ captures transitory shocks to inflation arising from supply-side factors. In this theory, u* is a time-varying intercept term in the short-run relationship between unemployment and inflation.

We seek to model the dynamic behavior of u and u*. In many countries, these variables wander up and down in ways that suggest they are non-stationary. This impression is confirmed formally by univariate analyses of the unemployment rate, which find that this variable follows an I(I) process (e.g., Leon-Ledesma [2002] and Gali [2020]).

At the same time, there is a strong theoretical presumption that the unemployment gap u-u* is stationary: the unemployment rate does not wander arbitrarily far from its natural rate. Most procedures for estimating u* build in this assumption. We therefore model u and u* as cointegrated variables. The long-run cointegrating relation is simply u = u*.

The dynamics of cointegrated variables can be captured by an error-correction model in which changes in the variables are influenced by the lagged deviation from the cointegrating relation

(Engle and Granger, 1987). We assume specifically that Δu and Δu^*, the changes in u and u*, are determined by:

$$(2) \qquad \Delta u^*_t = \beta_1(\Delta u^*)_{t-1} + \beta_2(\Delta u)_{t-1} + \beta_3(u_{t-1} - u^*_{t-1}) + \eta_t ;$$

$$(3) \qquad \Delta u_t = \gamma_0(\Delta u^*)_t + \gamma_1(\Delta u^*)_{t-1} + \gamma_2(\Delta u)_{t-1} + \gamma_3(u_{t-1} - u^*_{t-1}) + \varepsilon_t .$$

To interpret these equations, notice first that each one includes $u_{t-1} - u^*_{t-1}$, the error-correction term. We presume that $\gamma_3 < 0$ and $\beta_3 \geq 0$. With $\gamma_3 < 0$, u is pulled toward u*, an effect that arises from the economy's natural equilibrating forces and/or monetary policy that seeks to stabilize unemployment. If $\beta_3 > 0$, then there are also forces that pull u* toward u: there is hysteresis.

Our model allows Δu^*_t to affect Δu_t: a change in the natural rate can affect actual unemployment contemporaneously. To obtain a recursive structure that identifies our equations, we do *not* allow Δu_t to affect Δu^*_t: each period's natural rate is determined before the current innovation in actual unemployment. An Appendix to this paper shows that our main results are robust to relaxing this assumption. In that analysis, we partially identify our model through sign restrictions and find that the degree of hysteresis estimated for our baseline specification is a lower bound on the range of estimates consistent with the restrictions.

As is standard in error-correction models, we include lags of Δu and Δu^* in our equations to capture short-run dynamics. For our annual data, the first lags of Δu and Δu^* appear sufficient: longer lags are never significant.

We interpret the shock η_t in the Δu^* equation as arising from changes in supply-side variables such as demographics, productivity, and labor market frictions. The shock ε_t in the Δu equation arises from shifts in monetary policy and other determinants of aggregate demand.[2]

3. OECD ESTIMATES OF THE NATURAL RATE

Various academics and official agencies have developed techniques for estimating the natural rate of unemployment. In most of this work, the basic approach is to define the natural rate as an unobserved term in the Phillips curve and estimate its path along with the Phillips curve parameters. The Phillips curve is typically a more complex version of equation (1) that includes measures of expected inflation, lags of unemployment and inflation, and supply shocks such as changes in oil prices. The Kalman filter is used to disentangle movements in u* from transitory shocks to inflation.[3]

This study uses natural rate estimates from the OECD because they are published for a large number of countries. Most of our analysis is based on real-time data. Specifically, for each year we use estimates of current natural rates (also called "structural unemployment rates") that appear in the year's second issue of the OECD *Economic Outlook*, which is usually published in November. All of the estimates are produced with the same basic Kalman filter approach, but some details of the methodology change over our sample period (such as the modeling of inflation expectations). We interpret each year's estimates as the OECD's best effort to measure natural rates given the available data and current views about the Phillips curve.[4]

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[2] The Δu and Δu^* equations do not include constant terms, which means we do not allow a deterministic trend in the unemployment rate. If constants are added, they are insignificant.

[3] Early papers using this approach include Staiger, Stock and Watson (1997) and Gordon (1997). Official sources of natural rate estimates include the European Community and the U.S. Congressional Budget Office.

[4] The OECD's evolving methodology is described in Turner et al. (2001), Gianella et al. (2008), Guichard and Rusticelli (2011), and Rusticelli et al. (2015). The last of these papers describes the change in the modeling of inflation expectations, from backward-looking to

We also measure the actual unemployment rate u with real-time data from the second issue of the *Economic Outlook*. When we analyze the effects of measurement error, it will be important that the measures of u and u* in a given year are produced at the same time.

Our data cover 29 countries over the period 2001-2019 (with later start dates for recent joiners of the OECD). The total number of country-year observations is 494. When we estimate our error correction model, the sample starts in 2003 because we need two lags of u and u* to construct the lags of Δu and Δu*.[5]

Figure 1 shows the series for u and u* in four countries. In three of them–Spain, Ireland, and the US–unemployment rose sharply after the 2008 financial crisis and fell later in the sample. The natural rate also rose and then fell, although the size and timing of the u* movements varied. In the last country, Germany, both u and u* fell fairly steadily. Overall the graphs give the impression that u and u* movements are related in some way.

4. BASELINE ESTIMATES

Here we present baseline estimates of our model, equations (2) and (3). We make two strong assumptions:

• We assume that the OECD's real-time estimates of u and u* are the true values of these variables. That is, we ignore the measurement error that surely exists in our data.

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anchored expectations.

[5] The countries in our sample are Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, United Kingdom, and United States.

• We assume that the coefficients in the equations are the same for all countries, ignoring possible heterogeneity in hysteresis effects. This allows us to estimate the equations with a single pooled sample.

We relax these assumptions in later parts of the paper.

While we assume common coefficients, we allow the variances of the errors ε and η to differ across countries (which is strongly suggested by the data). We also allow the errors in a given year to be correlated across countries. We estimate (2) and (3) by two-step generalized least squares (that is, the parameters of the error covariance matrices are estimated from OLS residuals).[6]

Table 1 presents the coefficient estimates. There is strong evidence that movements in actual unemployment affect the natural rate: in the equation for Δu^*, the coefficients on lagged $(u\text{-}u^*)$ and on lagged Δu are both positive and highly significant. In the Δu equation, there is a significantly negative coefficient on lagged $(u\text{-}u^*)$, confirming that u is pulled toward u^* as well as vice versa. Δu also depends positively on its own lag, meaning that unemployment movements exhibit momentum.

Figure 2 presents impulse response functions. We use our estimated equations to derive the responses of Δu and Δu^* to shocks, and then cumulate to derive the responses of the levels of u and

[6] For each equation, we estimate the variance of a country's errors with the OLS residuals for the country. We assume that the correlation of errors across countries is the same for every country pair and estimate it by averaging the correlations of OLS residuals for the (29)(28) pairs in our sample. The correlation is 0.174 for equation (2) and 0.307 for equation (3). These correlations and the country-specific variances determine the variance-covariance matrices of the errors in (2) and (3), which we use for GLS.

We have also tried a specification that allows the correlation of errors across two countries to vary depending on which of three groups each country belongs to: the euro area, Europe outside the euro area, and outside Europe. This change has little effect on our results. Our qualitative results are also similar if we simply estimate (2) and (3) by OLS.

u*. The Figure also shows 95% confidence intervals computed with Monte Carlo methods.[7]

To show the nature of hysteresis, Figure 3 compares the responses of u and u* to an innovation in u, which again we interpret as a demand shock. After a shock that raises u by one percentage point, u continues to rise for two more years (reflecting the momentum in Δu) and then starts to decline slowly. During this process, the high levels of u pull u* up. The paths of u and u* converge after about eight years and both variables end up three quarters of a point above their initial level.

How strong are the effects of u on u*? Following Ball, DeLong, and Summers (2017), we define the "degree of hysteresis" h as the ratio

h = [long-run effect of u shock on u*] / [cumulative deviations of u from u*]

The denominator of h is the total amount of cyclical unemployment caused by a shock, and the numerator is the long-run effect of this cyclical unemployment. The ratio h is the long-run effect of a one percentage point deviation of u from u* for one year.

For the impulse response functions in Figure 3, the numerator of h is 0.73 and the denominator is 4.65, which imply h = 0.16. This value of h means, for example, that if u exceeds u* by two percentage points for three years, u* rises by about one percentage point (0.16 x 2 x 3 = 0.96). A 95% confidence interval for h based on Monte Carlo simulations is [0.12, 0.19].[8]

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[7] Our Monte Carlo method follows Romer and Romer (2004). For each of our equations, we take 10,000 draws of the coefficients from a multivariate normal distribution with mean and covariance matrix given by our GLS point estimates and their covariance matrix. We compute impulse response functions for each of the 10,000 draws. For each point in an impulse response function, our 95% confidence interval extends from the 251st smallest value to the 9750th smallest value.

[8] We compute h for each of the 10,000 simulations described in footnote 7. The 95% confidence interval for h extends from the 251st smallest value to the 9750th smallest.

5. CROSS-COUNTRY DIFFERENCES IN HYSTERESIS

So far we have assumed that the dynamics of u and u* are the same in all countries. In reality, hysteresis effects may vary because of differences in national labor markets. Many economists have suggested, for example, that hysteresis is stronger in European countries than in the United States because of European institutions such as strong labor unions and generous unemployment insurance.

Here, we allow country-specific coefficients in equations (2) and (3) by interacting each right-side variable with a full set of country dummies. We estimate the resulting equations by two-step GLS, assuming the same covariance structure of the error terms as in our baseline analysis. (This approach is more efficient than country-by-country OLS because of the correlation of errors across countries.)

To study heterogeneity in hysteresis, we first test the hypothesis that the three coefficients in the Δu^* equation are the same for all countries (which implies 84 restrictions on 87 country-specific coefficients). This hypothesis is strongly rejected (the Wald statistic is 152, which has a p-value below 0.001). We also test and reject the hypothesis of equal coefficients for all countries in the Δu equation (112 restrictions on 116 coefficients; Wald statistic = 149 and $p = 0.02$). We conclude, therefore, that our baseline model with equal coefficients is misspecified.

The natural next step is to ask which countries have the strongest hysteresis effects and why. Unfortunately, most of our estimates of country-specific coefficients are quite imprecise, reflecting the fact that we have 17 or fewer observations per country. The imprecise coefficient estimates lead to imprecise estimates of impulse response functions and wide confidence intervals for the degree of hysteresis. 95% confidence intervals for h include zero for 14 of the 29 countries in the sample. In sum, we do not have enough data to reliably measure the cross-country differences in hysteresis.

We can, however, derive precise estimates of the *averages* of coefficients across the 29 countries. These average coefficients, which we interpret as the coefficients for a typical country in our sample, are reported in Table 2. The average coefficients are similar to the coefficients in our baseline specification with homogeneity across countries. The average effect of lagged (u-u*) on Δu* is highly significant, which provides strong evidence of hysteresis without the dubious restriction of equal coefficients.

The impulse response functions implied by the average coefficients (not shown here) are similar to those in our baseline specification. The degree of hysteresis is 0.21, a bit higher than our baseline estimate of 0.16, with a 95% confidence interval of [0.16, 0.25].[9]

As a final exercise, we estimate average coefficients separately for the 15 countries that belonged to the European Union in 2003 and the 14 countries that did not. The two sets of estimates are quite similar, and we cannot reject the hypothesis that all the average coefficients are the same for the two groups (p=0.18 for the Δu* equation and p=0.30 for the Δu equation). The degree of hysteresis is 0.20 for the EU countries (95% confidence interval of [0.15, 0.25]) and 0.22 for the non-EU countries ([0.14, 0.29]). Thus our data do *not* suggest that hysteresis effects are stronger in Europe than elsewhere.

6. ASYMMETRY IN HYSTERESIS?

Our linear model imposes the restriction that increases and decreases in unemployment have symmetric effects on the natural rate. It is not obvious whether this restriction is reasonable. Most

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[9] This confidence interval is derived using the covariance matrix of the average coefficients, which is determined by the covariance matrix of the country-specific coefficients, and the simulation method described in notes 7 and 8.

research on hysteresis has focused on the adverse effects of increases in u. Some work reports that decreases in u reduce u* (e.g., Girardi et al., 2020), but the strength of these "reverse hysteresis" effects is unclear.

To explore this issue, we allow the behavior of u* in our statistical model to be asymmetric. We do so by interacting each right-side variable in the Δu* equation, (2), with dummies for positive and negative values of the variable. For simplicity, we continue to assume that the Δu equation, (3), is symmetric (a restriction that we cannot reject). We also return to our baseline assumption that the coefficients in (1) and (2) are the same for all countries.

Table 3 reports estimates of the expanded model, and Figure 4 shows the impulse response functions for shocks to Δu. We derive separate responses for positive and negative shocks, which reflect the different effects of positive and negative terms in the Δu* equation.

Qualitatively, the effects of both positive and negative shocks are similar to those in the baseline specification with symmetry. A positive shock to Δu causes a persistent increase in u, and u* is pulled up over time. The response to a negative shock is the mirror image: u falls persistently and u* is pulled down. In both cases the long-run effect on u* is highly significant, indicating hysteresis in both directions.

Perhaps surprisingly, the magnitude of these effects is larger for negative shocks. The long-run effect on u* of a one-percentage-point shock to Δu is 0.59 points for a positive shock and -1.35 points for a negative shock. The degree of hysteresis is 0.13 for a positive shock (95% confidence interval of [0.08, 0.17]) and 0.29 for a negative shock (confidence interval of [0.20, 0.39]). These results reflect the fact that the error correction term in the Δu* equation, which captures the pull of u on u*, has a larger coefficient when the term is negative.

It is plausible that hysteresis effects work in both directions. For example, just as high unemployment can erode workers' skills and attachment to the labor force, low unemployment can pull marginally attached workers into the labor force and help them build up their skills (Okun, 1973). That said, it is not obvious why hysteresis effects should be larger for unemployment decreases than for increases. Future research should explore the robustness of this result and seek explanations.

7. THE EFFECTS OF MEASUREMENT ERROR

The true natural rate u* is unobservable, and it is well known that estimates of u* contain considerable measurement error (e.g., Staiger, Stock, and Watson, 1997). It is natural to wonder whether measurement error in the OECD's u* estimates could cause some kind of bias in our estimates of hysteresis effects.

Here, we analyze the effects of measurement error under the assumption that the OECD's estimates of u* are optimal in the sense of Hyslop and Imbens (2001): the estimates minimize the expected squared deviations from the true u*s, given available information. In this case, our baseline analysis may produce biased estimates of hysteresis, but bias is eliminated by a modest modification of our approach. Our modified procedure uses OECD estimates of the natural rate one year in the past and one year in the future as well as the current natural rate. The results confirm our central finding that hysteresis effects are substantial.

The assumption of optimal u* estimates is arguably a reasonable one in light of the resources and expertise that the OECD devotes to estimating the natural rate. But we cannot rule out the possibility that some systematic deviation from optimal estimation produces a spurious finding of hysteresis. Future research could explore possible shortcomings of the OECD's methods for estimating u*.

Analysis of Possible Biases

The assumption of optimal u* estimates has a simple but powerful implication: the measurement error in an estimate is uncorrelated with any variable observed when the estimate is made, including the estimate itself. If that were not so, the variable could be used to adjust the estimate and reduce the sizes of errors. This property of optimal estimates is stressed by Mankiw and Shapiro (1986) and Hyslop and Imbens (2001).

To see the implications for our analysis, we introduce some notation. Let u_{t-i} and u^*_{t-i} be the true values of u and u* in year t-i, and let ${}_{t-j}u_{t-i}$ and ${}_{t-j}u^*_{t-i}$ be OECD estimates of these variables made in year t-j. The measurement errors in the estimates are ${}_{t-j}m_{t-i} = {}_{t-j}u_{t-i} - u_{t-i}$ and ${}_{t-j}m^*_{t-i} = {}_{t-j}u^*_{t-i} - u^*_{t-i}$.

We assume that our model, equations (2)-(3), holds for the true u_{t-i} and u^*_{t-i} with i = 0,1,2. In our empirical work above, our proxies for these variables are estimates with j=i: we measure each year's u and u* with estimates made in that year. Changes in u and u* are measured with changes in these current-year estimates (for example, Δu^*_t is measured with ${}_tu^*_t - {}_{t-1}u^*_{t-1}$). Using (2)-(3) and the definitions of the errors w and w*, we can derive the relationships among the variables in our regressions:

$$(4) \quad {}_tu^*_t - {}_{t-1}u^*_{t-1} = \beta_1({}_{t-1}u^*_{t-1} - {}_{t-2}u^*_{t-2}) + \beta_2({}_{t-1}u_{t-1} - {}_{t-2}u_{t-2}) + \beta_3({}_{t-1}u_{t-1} - {}_{t-1}u^*_{t-1})$$
$$+ [{}_tw^*_t - (1+\beta_1-\beta_3)({}_{t-1}w^*_{t-1}) + \beta_1({}_{t-2}w^*_{t-2}) - (\beta_2+\beta_3)({}_{t-1}w_{t-1}) + \beta_2({}_{t-2}w_{t-2}) + \eta_t] ;$$

$$(5) \quad {}_tu_t - {}_{t-1}u_{t-1} = \gamma_0({}_tu^*_t - {}_{t-1}u^*_{t-1}) + \gamma_1({}_{t-1}u^*_{t-1} - {}_{t-2}u^*_{t-2}) + \gamma_2({}_{t-1}u_{t-1} - {}_{t-2}u_{t-2}) + \gamma_3({}_{t-1}u_{t-1} - {}_{t-1}u^*_{t-1})$$
$$+ [-\gamma_0({}_tw^*_t) + (\gamma_0-\gamma_1+\gamma_3)({}_{t-1}w^*_{t-1}) + \gamma_1({}_{t-2}w^*_{t-2}) + {}_tw_t - (1+\gamma_2+\gamma_3)({}_{t-1}w_{t-1}) + \gamma_2({}_{t-2}w_{t-2}) + \varepsilon_t] .$$

In each of these equations, the first line shows the variables in our regression and the second line, in brackets, is the error term in the regression. As is standard, the error terms include components reflecting measurement error in the dependent and independent variables.

For our baseline regressions to yield unbiased estimates of the β and γ coefficients, the regressors in (4)-(5) must be uncorrelated with all the terms in the errors. This may not be the case, even with optimal estimation of u and u*, because the regressors include variables that are *not* known when some measurement errors are made. Specifically, both equations include estimates that are made at t-1, which may be correlated with errors at t-2 (${}_{t-2}w_{t-2}$ and ${}_{t-2}w^*_{t-2}$). In addition, equation (5) includes ${}_{t}u^*_{t}$, which may correlated with errors at both t-2 and t-1.

There is, however, a simple way to avoid such correlations: by measuring u and u* in years t, t-1, and t-2 with estimates that are all made in the same year, specifically t-1. This is possible because each issue of the *Economic Outlook* reports estimates of u and u* in the previous and following years as well as the current year. If all of these estimates are optimal given available information, then we can use them to obtain unbiased estimates of the β's and γ's.

To see this point, we derive a version of our two equations with all right-hand-side variables measured at t-1:

$$(6)\qquad {}_{t}u^*_{t} - {}_{t-1}u^*_{t-1} = \beta_1({}_{t-1}u^*_{t-1} - {}_{t-1}u^*_{t-2}) + \beta_2({}_{t-1}u_{t-1} - {}_{t-1}u_{t-2}) + \beta_3({}_{t-1}u_{t-1} - {}_{t-1}u^*_{t-1})$$
$$+ [{}_{t}w^*_{t} - (1+\beta_1-\beta_3)({}_{t-1}w^*_{t-1}) + \beta_1({}_{t-1}w^*_{t-2}) - (\beta_2+\beta_3)({}_{t-1}w_{t-1}) + \beta_2({}_{t-1}w_{t-2}) + \eta_t]\ ;$$

$$(7)\qquad {}_{t}u_{t} - {}_{t-1}u_{t-1} = \gamma_0({}_{t-1}u^*_{t} - {}_{t-1}u^*_{t-1}) + \gamma_1({}_{t-1}u^*_{t-1} - {}_{t-1}u^*_{t-2}) + \gamma_2({}_{t-1}u_{t-1} - {}_{t-1}u_{t-2}) + \gamma_3({}_{t-1}u_{t-1} - {}_{t-1}u^*_{t-1})$$
$$+ [-\gamma_0({}_{t-1}w^*_{t}) + (\gamma_0-\gamma_1+\gamma_3)({}_{t-1}w^*_{t-1}) + \gamma_1({}_{t-1}w^*_{t-2}) + {}_{t}w_{t} - (1+\gamma_2+\gamma_3)({}_{t-1}w_{t-1}) + \gamma_2({}_{t-1}w_{t-2}) + \varepsilon_t]\ .$$

In these equations, we replace estimates of u_{t-2} and u^*_{t-2} made at t-2, which appear in (4)-(5), with estimates of these variables made at t-1. In addition, on the right side of the Δu equation, we replace the estimate of u^*_t made at t with an estimate at t-1, that is, a forecast from the previous year. These changes in regressors imply corresponding changes in the measurement errors that appear in the equations.

With these changes, all the regressors in our equations are estimates made at t-1, and all measurement errors are made at either t-1 or t, when the regressors are known. Assuming optimal estimation of u and u*, the regressors are uncorrelated with the errors, and therefore GLS regressions based on (6)-(7) yield unbiased estimates of the β's and γ's.

Estimates of the Modified Equations

Table 4 reports the results of estimating equations (6)-(7). The β coefficients in the Δu* equation are similar to the estimates for our baseline specification in Table1. Once again, both lagged (u-u*) and lagged Δu have highly significant effects on Δu*, indicating hysteresis.

In the Δu equation, by contrast, the coefficients on Δu* and lagged Δu* have large standard errors, so they are statistically insignificant despite sizable point estimates. The estimate of the Δu* coefficient is negative. These results reflect the fact that the version of Δu* in the Δu equation is a forecast of a future change in u* (an estimate at t-1 of the change at t), which varies much less than the change in real-time u* in our baseline specification. In addition, the Δu* and lagged Δu* terms are highly correlated.

Figure 5 shows the responses of u and u* to a shock to u, along with 95% confidence intervals. The estimated responses are similar to those for our baseline specification, but the confidence intervals are larger, as one would expect from the imprecision of some coefficient estimates. The long-run effect of a u shock on u* is still highly significant, reflecting the strong evidence of hysteresis in the Δu* equation. The estimated degree of hysteresis is 0.18 with a 95% confidence interval of [0.13, 0.25].

8. CONCLUSION

This paper studies the relationship between unemployment and its natural rate in 29 advanced economies. We find strong evidence of hysteresis: an innovation in actual unemployment changes the natural rate in the same direction, and therefore has permanent effects. In our baseline specification, a one percentage point deviation of actual unemployment from the natural rate for one year changes the natural rate by 0.16 points.

We find that the degree of hysteresis varies across countries, although our estimates for individual countries are imprecise. We also find evidence of an unexpected asymmetry: the natural rate is affected more strongly by decreases in unemployment than by increases. Future research could further explore cross-country differences and asymmetries in hysteresis.

Our analysis uses OECD estimates of the natural rate that are likely to contain considerable measurement error. We show that our results are robust to this problem if the estimates are optimal given the information available when they are made. We cannot, however, rule out the possibility that some deviation from optimality causes bias in our estimates of hysteresis. Future work could examine this possibility and also consider alternative approaches to measuring the natural rate.

APPENDIX: ALTERNATIVE IDENTIFYING ASSUMPTIONS

Our main analysis identifies the responses of u and u* to shocks by excluding contemporaneous Du from the equation for Du*. Here we consider the robustness of our results with different identifying restrictions.

We start with an error-correction model that allows both Du and Du* to affect each other contemporaneously:

$$\text{(A1)} \qquad \Delta u^*_t = \beta_0(\Delta u)_t + \beta_1(\Delta u^*)_{t-1} + \beta_2(\Delta u)_{t-1} + \beta_3(u_{t-1} - u^*_{t-1}) + \eta_t \, ;$$

$$\text{(A2)} \qquad \Delta u_t = \gamma_0(\Delta u^*)_t + \gamma_1(\Delta u^*)_{t-1} + \gamma_2(\Delta u)_{t-1} + \gamma_3(u_{t-1} - u^*_{t-1}) + \varepsilon_t \, .$$

Our main analysis assumes that β_0, the coefficient on Δu in the Δu^* equation, is zero. A natural alternative is to leave β_0 unconstrained and achieve identification by setting γ_0, the coefficient on Δu^* in the Δu equation, to zero. Table A1 reports estimates of our model with γ_0=0, and Figure A1 shows the implied responses to a Δu shock. The results are qualitatively similar to those for our baseline specification, but the degree of hysteresis, 0.22, is somewhat larger than our baseline estimate of 0.16.

It makes sense that the estimated degree of hysteresis is larger for this specification. The assumption that γ_0=0 means that co-movements of Δu and Δu^* are ascribed to an effect of u on u*–a hysteresis effect–whereas the baseline assumption of β_0=0 implies the opposite.

To find evidence of hysteresis, we do not need to impose either β_0=0 or γ_0=0. Instead, following Faust (1998) and Uhlig (2017), we can partially identify our model with weak sign restrictions. Specifically, we assume that both β_0 and γ_0 are non-negative: an increase in actual unemployment does not decrease the natural rate, and an increase in the natural rate does not decrease actual unemployment. To see what these restrictions imply, we estimate our model for a range of assumed values of $\beta_0 \geq 0$ and for a range of assumed $\gamma_0 \geq 0$. Choosing either one of these parameters identifies the model and allows us to estimate the responses to shocks.[10]

When we estimate the model for different values of $\beta_0 \geq 0$, we find that the degree of hysteresis h is smallest for β_0=0, in which case the model reduces to our baseline specification (2)-(3). Therefore, the estimate of h for that specification, 0.16, is a lower bound on h implied by $\beta_0 \geq 0$. When we impose different values of $\gamma_0 \geq 0$, we find that h is largest for γ_0=0, which is the specification in Table A1. The estimate of h for that specification, 0.22, is an upper bound on h implied by $\gamma_0 \geq 0$. Combining the lower and upper bounds, we conclude that our sign restrictions imply a degree of hysteresis in the range [0.16, 0.22].

[10] If we set β_0 at a value β_0', we can define $X_t = \Delta u^*_t - \beta_0'(\Delta u)_t$ and rearrange equations (A1) and (A2) to obtain a recursive system in X and Δu. We estimate this system by GLS and use the coefficients to derive the βs and γs in (A1) and (A2). We use a similar procedure when we set a value for γ_0.

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Table 1
Error-correction model for u and u*, baseline specification

	(2) Δu^*	(3) Δu
Δu^*		0.536
		(0.062)
$\Delta u^*(-1)$	-0.063	0.038
	(0.045)	(0.059)
$\Delta u(-1)$	0.112	0.428
	(0.028)	(0.048)
$u(-1) - u^*(-1)$	0.149	-0.215
	(0.018)	(0.031)
Observations	436	436
RMSE	0.587	0.893

Equations estimated by GLS. Standard errors in parentheses.

Table 2
Averages of country-specific coefficients

	Δu^*	Δu
Δu^*		0.634
		(0.085)
$\Delta u^*(-1)$	-0.089	0.027
	(0.051)	(0.079)
$\Delta u(-1)$	0.048	0.421
	(0.041)	(0.053)
$u(-1) - u^*(-1)$	0.216	-0.308
	(0.027)	(0.041)

Country-specific coefficients estimated by GLS. Standard errors of average coefficients in parentheses.

Table 3
Δu^* equation with asymmetries

	Δu^*
$\Delta u^*(-1)$ x pos. dummy	-0.071
	(0.069)
$\Delta u^*(-1)$ x neg. dummy	-0.049
	(0.071)
$\Delta u(-1)$ x pos. dummy	0.122
	(0.041)
$\Delta u(-1)$ x neg. dummy	0.091
	(0.049)
$(u(-1) - u^*(-1))$ x pos. dummy	0.117
	(0.023)
$(u(-1) - u^*(-1))$ x neg. dummy	0.279
	(0.046)
Observations	436
RMSE	0.579

Equation estimated by GLS. Standard errors in parentheses.

Table 4
Error-correction model with all regressors measured at t-1

	(6) Δu^*	(7) Δu
Δu^*		-0.638
		(0.445)
$\Delta u^*(-1)$	0.106	0.599
	(0.137)	(0.339)
$\Delta u(-1)$	0.120	0.606
	(0.031)	(0.057)
$u(-1) - u^*(-1)$	0.136	-0.145
	(0.018)	(0.031)
Observations	436	436
RMSE	0.586	0.953

Equations estimated by GLS. Standard errors in parentheses.

Table A1
Error-correction model for u and u*, alternative identification

	Δu^*	Δu
Δu	0.201	
	(0.026)	
$\Delta u^*(-1)$	-0.058	-0.013
	(0.043)	(0.063)
$\Delta u(-1)$	0.006	0.499
	(0.030)	(0.051)
$u(-1) - u^*(-1)$	0.188	-0.119
	(0.018)	(0.030)
Observations	436	436
RMSE	0.543	0.967

Equations estimated by GLS. Standard errors in parentheses.

Figure 1. u and u* for selected countries (real-time OECD data)

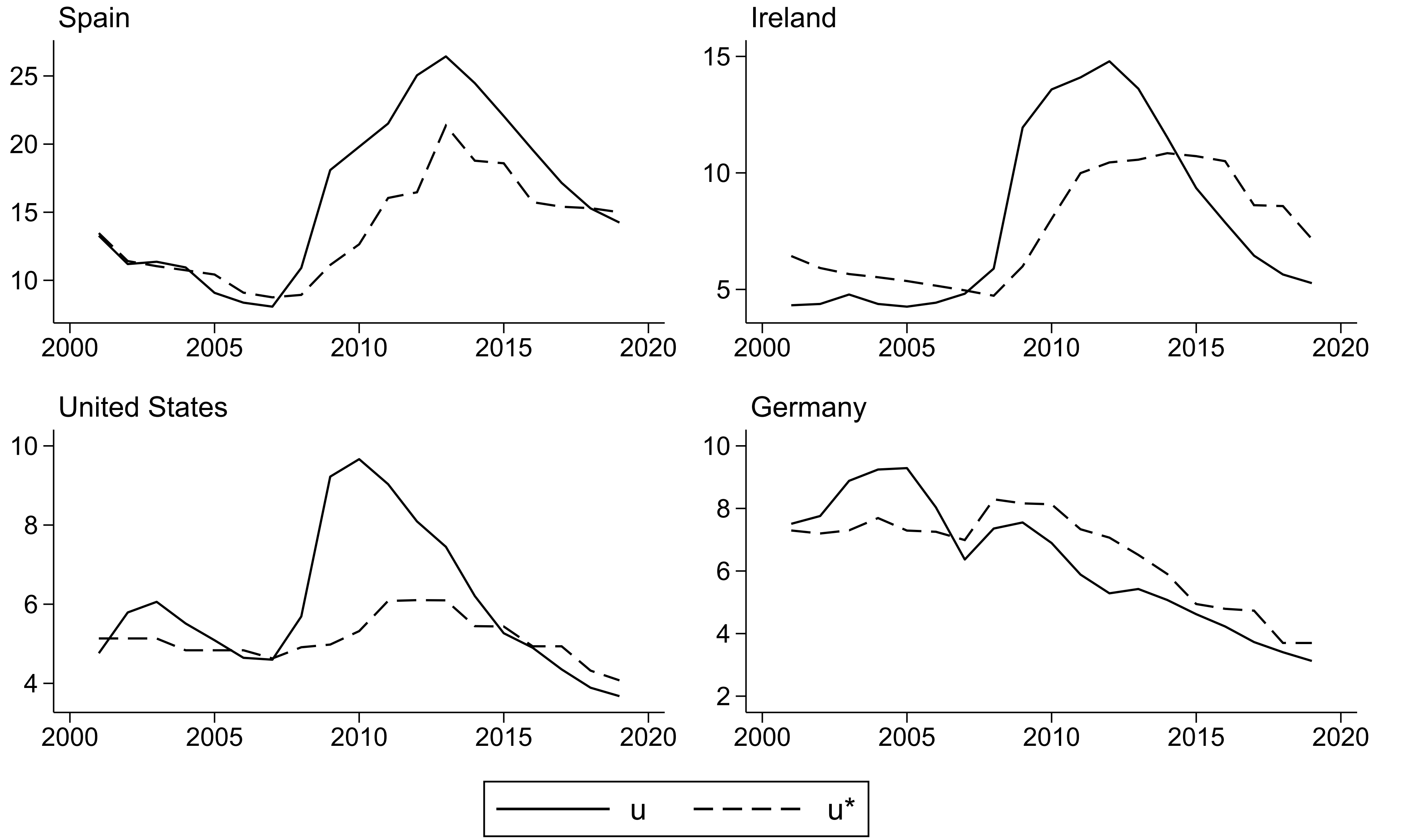
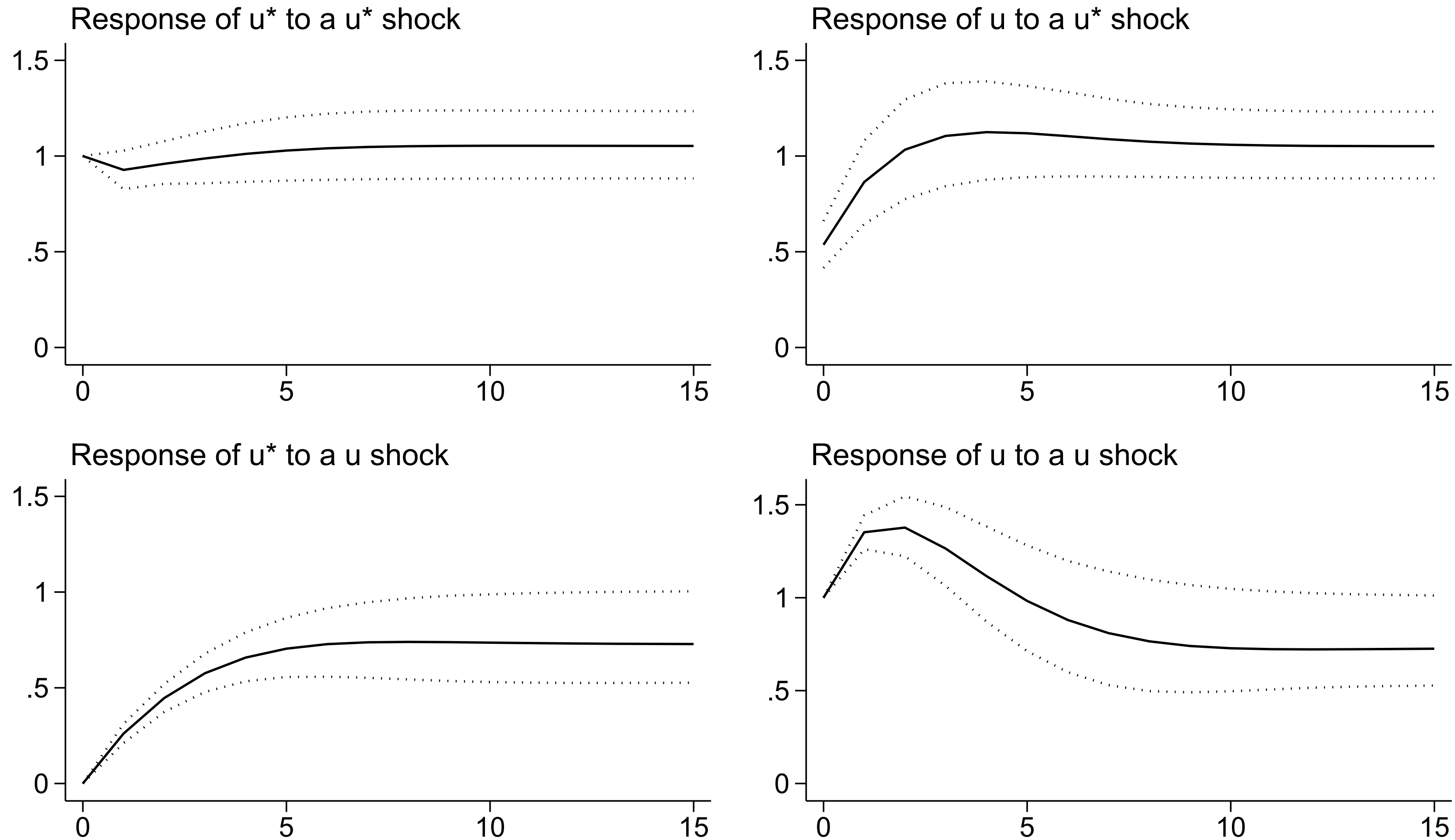
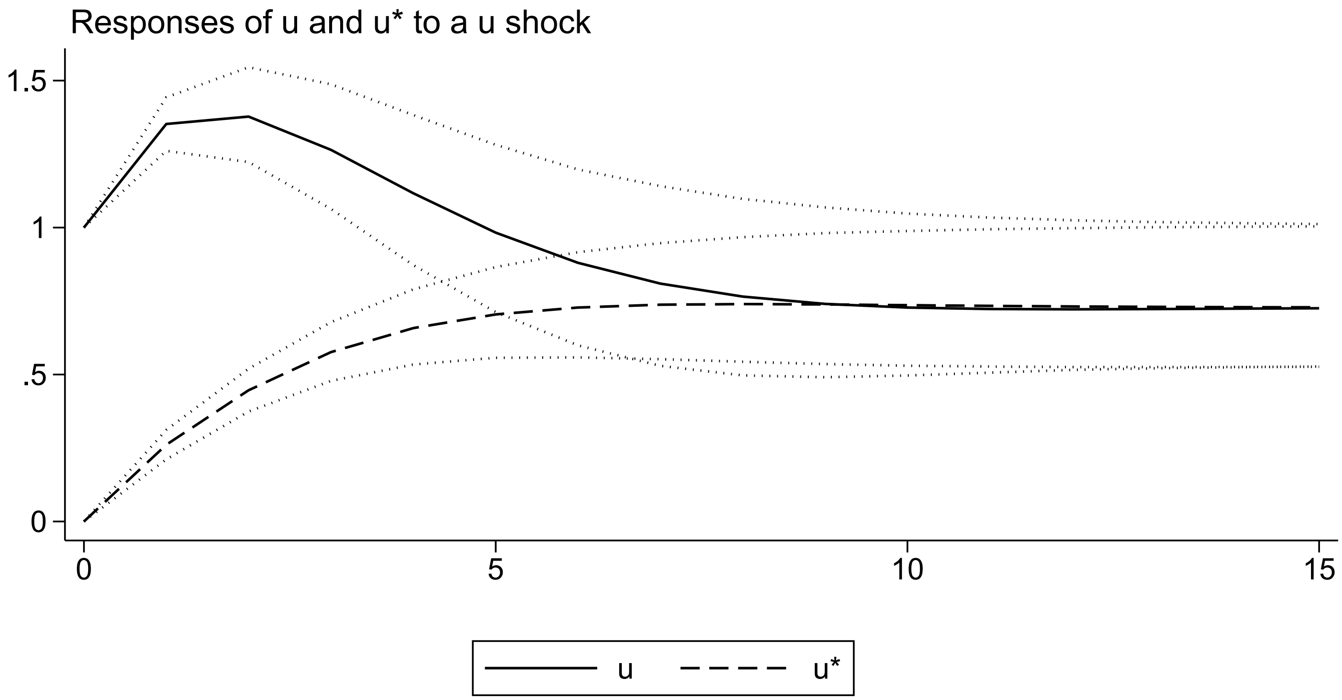


Figure 2. Impulse response functions, baseline specification



Dotted lines show 95% confidence intervals.

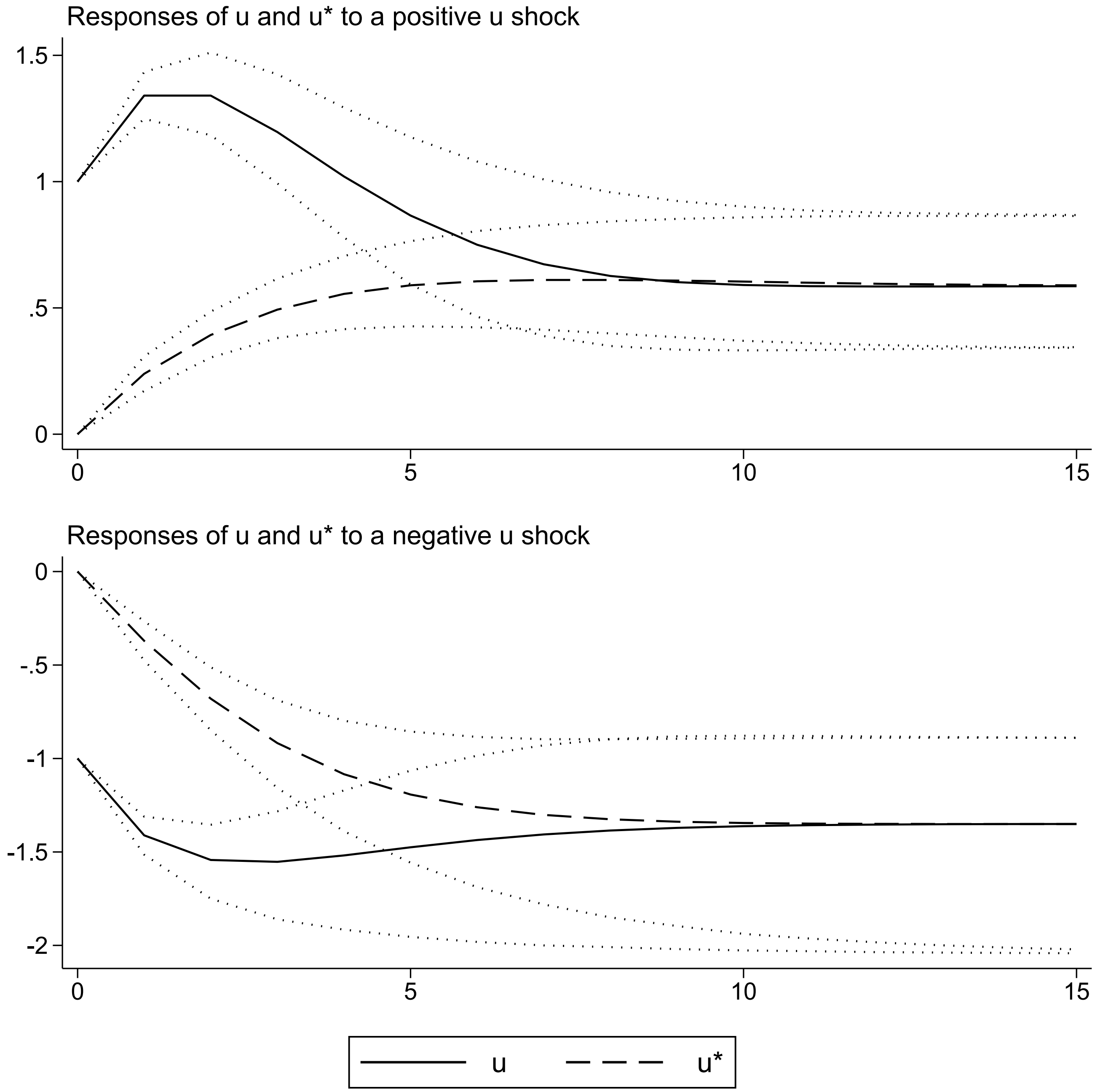
Figure 3. Measuring hysteresis, baseline specification



Dotted lines show 95% confidence intervals.

Degree of hysteresis = 0.157
(95% confidence interval = [0.124,0.189])

Figure 4. Asymmetry in hysteresis

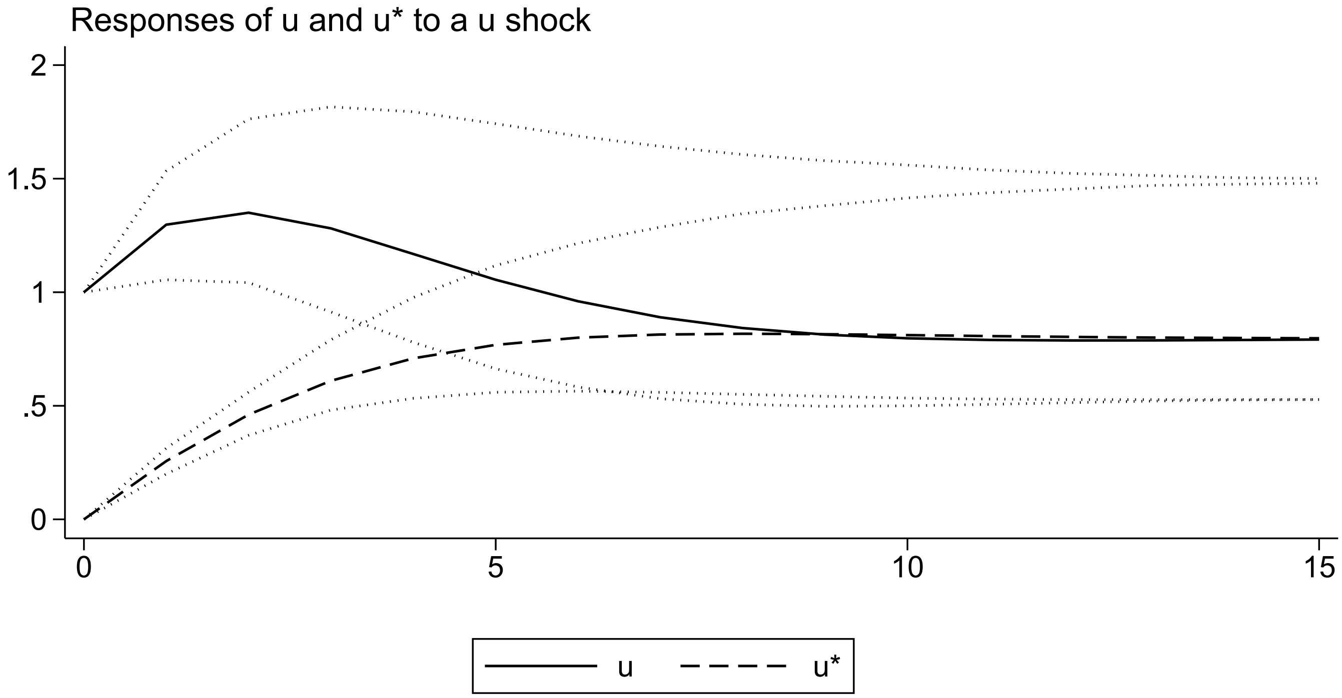


Dotted lines show 95% confidence intervals.

Positive shock: Degree of hysteresis = 0.126
(95% confidence interval = [0.081,0.167])

Negative shock: Degree of hysteresis = 0.290
(95% confidence interval = [0.203,0.385])

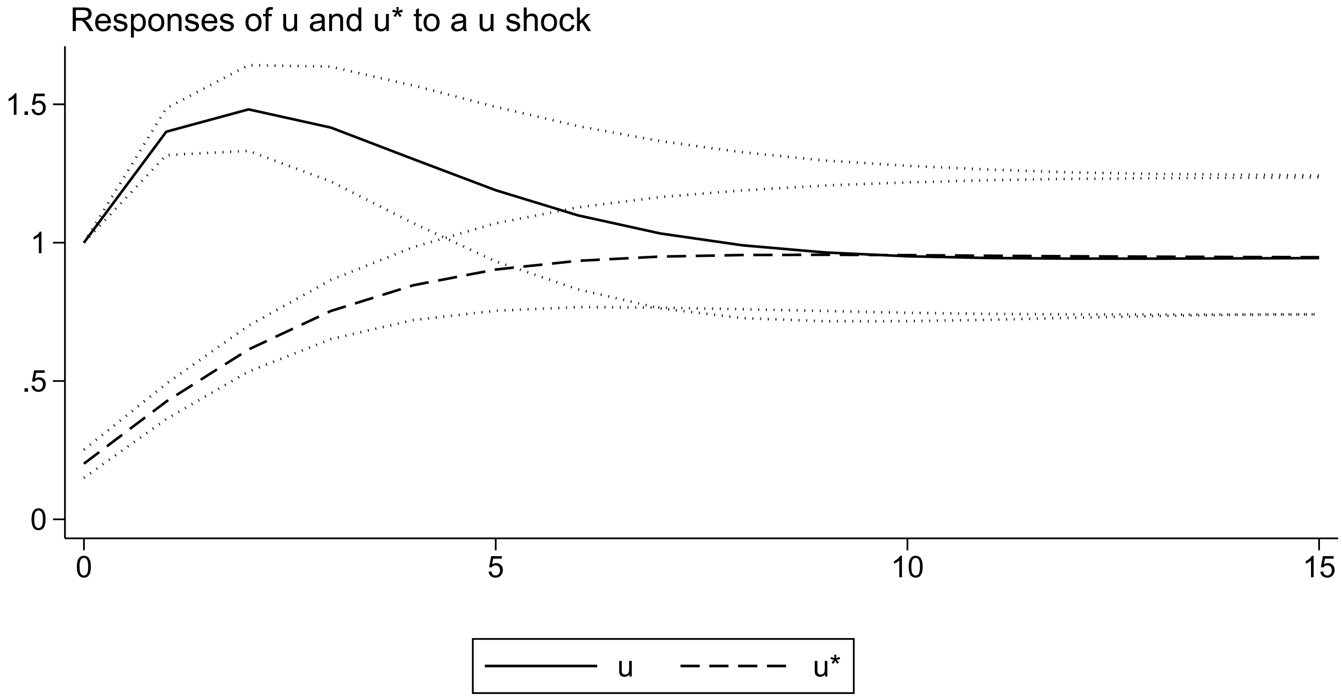
Figure 5. Hysteresis with all regressors measured at t-1



Dotted lines show 95% confidence intervals.

Degree of hysteresis = 0.176
(95% confidence interval = [0.128,0.254])

Figure A1. Measuring hysteresis, alternative identification



Dotted lines show 95% confidence intervals.

Degree of hysteresis = 0.220
(95% confidence interval = [0.181,0.261])